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QUANTUM ELECTRONICS

Continuous Photodetection Processes

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Abstract. The theory of continuous photodetection processes [Srinivas and Davies (1981)] is modified in order to avoid inconsistencies. The new approach results in a bounded counting rate, in contrast with the previous theory.

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1. Introduction

Since the pioneering work of Mandel [1] the aim of photodetection theories is to develop a framework for photocount probability distributions consistent with actual photodetection processes. Only in the 80's, after the Srinivas and Davies (SD) theory [2], photodetection was devised as a continuous measurement process. Despite prevailing over former theories [1,3–5], the SD theory [2] has its own inconsistencies. It gives unbounded counting rates, leading to ill-defined probability distributions, such as the coincidence probability density [2].

In this paper we present a first step in modifying the SD photocount theory in order to obtain a bounded counting rate. Our motivation finds ground on the recent discussion about the role of the 'annihilation' operator a in quantum optics [6]. Instead of a and $a^\dagger$ the *exponential phase operators* E_- and E_+ should be considered as real 'annihilation' and 'creation' operators in the photocounting theory. The introduction of these operators in the continuous photocounting theory, besides eliminating inconsistencies in the SD proposal, may lead to new interesting results related to the counting statistics.

2. Photodetection Processes and Exponential Phase Operators

To obtain a bounded counting rate we define the infinitesimal time-duration one-count operator as

$$J\rho = E_-\rho E_+ \,, \tag{1}$$

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where $E_- \equiv (a^\dagger a+1)^{-1/2}a$ and $E_+ \equiv a^\dagger(a^\dagger a+1)^{-1/2}$ are the *exponential phase operators*, introduced by London [7] and first used by Susskind and Glogower [8]. J is a bounded operator and the system state immediately after the 1-count process in the time interval $[0,t)$ is transformed into

$$\tilde{\rho}(t^+) = \frac{J\rho(t)}{\mathrm{Tr}[J\rho(t)]} = \frac{J\rho(t)}{1-p_0}\,, \tag{2}$$

where $p_0 \equiv \langle 0|\rho(t)|0\rangle$ is the probability for the vacuum state.

The mean number of photons in the state $\tilde{\rho}(t)$ (2) is

$$\tilde{n}(t^+) = \frac{\overline{n}(t)}{1-p_0} - 1\,, \tag{3}$$

so, whenever a state ρ has none, or very small, contribution from the vacuum state, the counting operation extracts exactly one photon from the system, independently of the field statistics. For example, for the *Fock state* $\rho = |m\rangle\langle m|$ $(m \neq 0)$, $\tilde{n} = m-1$, and for the *coherent state* $\rho = |\alpha\rangle\langle\alpha|$ $(\alpha \neq 0)$, $\tilde{n} = \overline{n}/\left(1-e^{-\overline{n}}\right) - 1$, with $\overline{n} = |\alpha|^2$. On the other hand, for the *thermal state*

$$\rho = \frac{1}{1+\overline{n}} \sum_{n=0}^{\infty} \left(\frac{\overline{n}}{1+\overline{n}}\right)^n |n\rangle\langle n| \tag{4}$$

we obtain $\tilde{n} = \overline{n}$, i.e. the mean number of photons is not changed, as expected. This is a correct description of a thermal system since taking out a single photon from a reservoir should not change its average number. Note that using $J\rho = a\rho a^\dagger$ one obtains the weird result $\tilde{n} = 2\overline{n}$. So one perceives that using a and $a^\dagger$ for constructing a continuous photocount measurement in the original SD theory leads to some inconsistent results.

The time evolution between sequential counts is represented by $S_t \equiv N_t(0)$, a superoperator defined in terms of ordinary Hilbert space operators

$$S_t\rho = e^{Yt}\rho e^{Y^\dagger t}\,. \tag{5}$$

The deduction of S_t is conditioned to the relation $\mathrm{Tr}[J\rho] = \mathrm{Tr}[\rho R]$, where R is the rate operator, related to Y by $\mathrm{Tr}(\rho R) = \mathrm{Tr}(Y\rho + \rho Y^\dagger)$. The theory requires that in the absence of counts the system has a unitary evolution, whose dynamics is governed by the free-field Hamiltonian $H = \hbar\omega a^\dagger a$. Thus, the convenient choice is

$$Y = -iH - R/2 = -iH - \frac{\gamma}{2}E_+E_-\,. \tag{6}$$

The continuous counting of k photons from a field in a time interval $[0,t)$ is represented by a linear operator $N_t(k)$:

$$\tilde{\rho}^{(k)}(t) = \frac{N_t(k)\rho(0)}{\mathrm{Tr}\left[N_t(k)\rho(0)\right]}\,, \tag{7}$$

where $\rho(0)$, or simply ρ, is the field state prior to the counting process and $P(k,t) = \mathrm{Tr}(N_t(k)\rho)$ is the probability of counting k photons in t. The linear operator $N_t(k)$ can be written in terms of the operators S_t and J as

$$N_t(k) = \int_0^t dt_k \int_0^{t_k} dt_{k-1} \cdots \int_0^{t_2} dt_1 S_{t-t_k} J S_{t_k - t_{k-1}} \cdots J S_{t_1} \,. \tag{8}$$

After performing the first $k-1$ integrations, Eq. (8) can be written as

$$N_t(k)\rho = \mathcal{U}_t \int_0^t dt' e^{-\gamma t'} \frac{(t')^{k-1}}{(k-1)!} e^{-\frac{1}{2}\gamma(t-t')\Lambda} \left(J^k \rho\right) e^{-\frac{1}{2}\gamma(t-t')\Lambda} \,, \tag{9}$$

where $\mathcal{U}_t\rho \equiv e^{-iHt}\rho e^{iHt}$, $t' \equiv t_k$ and the probability of occurrence of k counts in a time interval t is given by

$$P(k,t) = \mathrm{Tr}\left(N_t(k)\rho\right) = p_k \left[1 - e^{-\gamma t} \sum_{j=0}^{k} \frac{(\gamma t)^j}{j!}\right] + \frac{e^{-\gamma t}(\gamma t)^k}{k!} \sum_{n=k}^{\infty} p_n \,, \tag{10}$$

which verifies the normalization condition $\sum_{k=0}^{\infty} P(k,t) = 1$. The limiting value of Eq. (10)

$$\lim_{t\to\infty} P(k,t) = \langle k|\rho|k\rangle = p_k \tag{11}$$

means that, asymptotically, the counting statistics coincides with the photon statistics (as it should), since we do not consider here the possibility of photons lost to the surroundings or the failure to count any photon exiting the cavity.

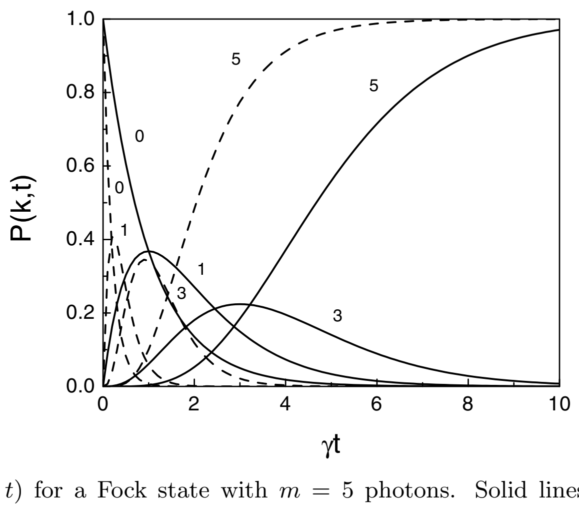

Fig. 1. $P(k,t)$ for a Fock state with $m = 5$ photons. Solid lines are for the present model while dashed ones are for the original SD theory. Numbers above the curves correspond to the k-event

In Fig. 1 we compare $P(k,t)$, from (10), with the corresponding expression in the SD theory for a Fock state. In SD theory and Fock states, $P(k,t)$ is a binomial distribution, while (10) contains elements of Poisson statistics. Thus in the new photocounting model every photon leaving the cavity is counted, in contrast to the SD theory.

3. Conclusion

We proposed modifications in the SD photocount theory in order to obtain a bounded counting rate. Our central assumption was in the choice of the exponential phase operators E_- and E_+ as real 'annihilation' and 'creation' operators in the photocounting process. The introduction of these operators in the continuous photocount theory, besides eliminating inconsistencies in the SD proposal, leads to new interesting results related to the counting statistics. A remarkable result, which is responsible for all the physical consistency of the model, is that in this new form the photocount operation J really takes out one photon from the field, unless it is in a thermal state.

We leave for an extended work a more detailed study of photocounting processes [9]. Many problems in the literature, such as [10–15] have adopted the SD theory of photodetection. The applications of our consistent model of photodetection to these problems may, indeed, bring new and important results.

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