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QUANTUM ELECTRONICS

Direct Measurement of Quasiprobabilities in Lossy Cavities

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Abstract. We show that the state of an electromagnetic field in a lossy cavity can be directly reconstructed by means of a simple scheme, allowing complete knowledge of the state of the field despite dissipation.

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1. Introduction

In recent years there have been proposals to measure electromagnetic fields inside cavities [1, 2] and vibrational states in ion traps [1, 3]. In fact the full reconstruction of nonclassical states of the electromagnetic field have been experimentally accomplished [4].

The presence of noise and dissipation has normally destructive effects. Schemes that treat a lossy cavity have been proposed [2] that involve a physical process that allows the storage of information about the quantum coherences of the initial state in the diagonal elements of the density matrix of a transformed state. Here we show, that although it is not possible to reconstruct the Wigner function, it is still possible to recover whole information about the initial state via, for instance, the Q-function.

2. Hamiltonian of the System

Let us first consider the ideal case of no dissipation. We consider then the Hamiltonian for the interaction between a quantized field and a two-level atom reads (we have set $\hbar = 1$)

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$$\hat{H} = \omega\hat{a}^\dagger\hat{a} + \frac{\omega_{eg}}{2}\hat{\sigma}_z + \lambda(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+)\,, \tag{1}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators for the field mode, respectively, obeying $[\hat{a}, \hat{a}^\dagger] = 1$. $\hat{\sigma}_+ = |e\rangle\langle g|$ and $\hat{\sigma}_- = |g\rangle\langle e|$ are the raising and lowering atomic operators, respectively, $|e\rangle$ being the excited state and $|g\rangle$ the ground state of the two-level atom. The atomic operators obey the commutation relation $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$. ω is the field frequency, ω_{eg} the atomic frequency and λ is the interaction constant. When we have the condition on the detuning, $\delta = \omega_{eg} - \omega$, $|\delta|/\lambda \gg \sqrt{n+1}$ for any "relevant" photon number, we can obtain an effective interaction Hamiltonian in the dispersive limit (see for instance [5])

$$\hat{H}_I^{\text{eff}} = \chi\hat{a}^\dagger\hat{a}\hat{\sigma}_z\,, \tag{2}$$

with $\chi = \lambda^2/\delta$.

We will consider an initial state given by

$$|\psi(\alpha;0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)|\psi_F(\alpha;0)\rangle\,, \tag{3}$$

where $|\psi_F(\alpha;0)\rangle = \hat{D}(\alpha)|\psi_F(0)\rangle$ with $\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ is the displacement operator with amplitude α and $|\psi_F(0)\rangle$ the state of the field to be arbitrary.

3. Dissipative Interaction

In the interaction picture, and in the dispersive approximation, the master equation that governs the dynamics of a two-level atom coupled with an electromagnetic field in a high-Q cavity is

$$\frac{d}{dt}\hat{\rho} = -i[\hat{H}_I^{\text{eff}}, \hat{\rho}] + \hat{\mathcal{L}}\hat{\rho}\,, \tag{4}$$

where

$$\hat{\mathcal{L}}\hat{\rho} = 2\gamma\hat{a}\hat{\rho}\hat{a}^\dagger - \gamma\hat{a}^\dagger\hat{a}\hat{\rho} - \gamma\hat{\rho}\hat{a}^\dagger\hat{a} \tag{5}$$

and $\hat{\rho}$ the density matrix of the system.

We define the superoperators

$$\hat{L}\hat{\rho} = -\hat{\Gamma}\hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{\Gamma}^\dagger\hat{a}^\dagger\hat{a}, \qquad \hat{J}\hat{\rho} = 2\gamma\hat{a}\hat{\rho}\hat{a}^\dagger\,, \tag{6}$$

where we have defined

$$\hat{\Gamma} = \gamma\hat{1}_A + i\chi\hat{\sigma}_z\,, \tag{7}$$

with $\hat{1}_A = |e\rangle\langle e| + |g\rangle\langle g|$. It is not difficult to show that

$$[\hat{J}, \hat{L}]\hat{\rho} = -\hat{S}_\Gamma\hat{J}\hat{\rho}\,, \tag{8}$$

where the superoperator $\hat{S}_\Gamma$ is defined as

$$\hat{S}_\Gamma\hat{\rho} = \hat{\Gamma}\hat{\rho} + \hat{\rho}\hat{\Gamma}^\dagger\,. \tag{9}$$

The solution to Eq. (4) subject to the initial state (3) is then given by

$$\hat{\rho}(t) = e^{(\hat{L}+\hat{J})t}\hat{\rho}(\alpha;0) = e^{\hat{L}t}e^{\hat{f}(t)\hat{J}}\hat{\rho}(\alpha;0)\,, \tag{10}$$

where

$$\hat{f}(t)\hat{\rho} = \frac{1-e^{-\hat{S}_\Gamma t}}{\hat{S}_\Gamma}\hat{\rho} \tag{11}$$

and $\hat{\rho}(\alpha;0) = |\psi(\alpha;0)\rangle\langle\psi(\alpha;0)|$.

From (10) we calculate $\langle\hat{\sigma}_x\rangle$ [6]

$$\langle\hat{\sigma}_x\rangle = \frac{1}{2}\sum_{m=0}^{\infty}\frac{\left(\gamma(1-e^{-2\xi t})/\xi\right)^m}{m!}\sum_{k=0}^{\infty}e^{-2k\xi t}|\langle k|\hat{a}^m\hat{D}(\alpha)|\psi_F(0)\rangle|^2 + \text{c.c.} \tag{12}$$

with $\xi = \gamma + i\chi$. After some algebra we obtain

$$\langle\hat{\sigma}_x\rangle = \sum_{M=0}^{\infty}\mu^M\cos(M\theta)\langle M|\hat{\rho}(0;\alpha)|M\rangle \tag{13}$$

with the definitions

$$\theta = \tan^{-1}\left(-\frac{\eta + e^{-2\eta\tau}[\sin(2\tau) - \eta\cos(2\tau)]}{\eta^2 + e^{-2\eta\tau}[\cos(2\tau) + \eta\sin(2\tau)]}\right) \tag{14}$$

and

$$\mu = \left(\frac{\eta^2 + e^{-4\eta\tau} + 2\eta e^{-2\eta\tau}\sin(2\tau)}{1+\eta^2}\right)^{1/2} \tag{15}$$

and with $\tau = \chi t$ and $\eta = \gamma/\chi$. Note that for μ equal to zero we obtain the Q-function and therefore complete information about the initial state. The value at which μ is equal to zero may be obtained numerically and is $\eta_{\mu=0} \approx 0.274457$. The parameter γ is the only one parameter that fixes all the other parameters: once known the rate at which the cavity decays, the cavity has to be tuned to obtain an effective interaction constant χ, given by $\chi = \gamma/0.274457$ and the atom has to traverse the cavity in a time $t = 3\pi/(4\chi)$.

Equation (13) may be also written as

$$\langle\hat{\sigma}_x\rangle = \sum_{M=0}^{\infty}\tilde{\mu}^M\langle M|\hat{\rho}(0;\alpha)|M\rangle + \sum_{M=0}^{\infty}(\tilde{\mu}^*)^M\langle M|\hat{\rho}(0;\alpha)|M\rangle \tag{16}$$

with $\tilde{\mu} = \mu e^{i\theta}$ and therefore, the terms that form the equation above are proportional to s-parametrized quasiprobability distributions with s a complex number.

4. Conclusions

In conclusion, we have solved the dispersive interaction between a quantized electromagnetic field and a two-level atom in the case of a real cavity. We have shown that even in the dissipative case we can still obtain information about the initial cavity field by means of the Q-function. Therefore we have been able to extend the range of parameters in which complete information may be obtained in CQED, from $\gamma = 0$ (ideal case) to $\gamma \approxeq 0.274457\chi$, i.e. by doing a right tuning complete information of the cavity field may be obtained despite of cavity losses.

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6. The average value of $\hat{\sigma}_x$ can be written as
$$\langle\hat{\sigma}_x\rangle = Tr[\hat{\sigma}_x\hat{\rho}] = Tr[\hat{R}\hat{\sigma}_z\hat{R}^\dagger\hat{\rho}] = Tr[\hat{\sigma}_z\hat{R}^\dagger\hat{\rho}\hat{R}] = Tr[\hat{\sigma}_z\hat{\rho}_R]\,,$$
i.e. the expectation value of $\hat{\sigma}_z$ (the atomic inversion or the probability of finding the atom in the excited state minus the probability of finding it in the ground state) for a *rotated* (in the atomic basis) density matrix (see for instance [1]). $\hat{R} = \exp[(\hat{\sigma}_- - \hat{\sigma}_+)\pi/4]$.
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