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RESEARCH PAPER

PERFECT NONLINEAR OBSERVERS OF FRACTIONAL DESCRIPTOR CONTINUOUS-TIME NONLINEAR SYSTEMS

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Abstract

Perfect nonlinear fractional descriptor observers for fractional descriptor continuous-time nonlinear systems are proposed. Necessary and sufficient conditions for the existence of the observers are established. The design procedure of the nonlinear fractional observers is given. It is based on the elementary row (column) operations and reducing the singular matrix of the system to upper (lower) triangular form. The effectiveness of the procedure is demonstrated on a numerical example.

MSC 2010: Primary 34K37; Secondary: 93C10, 93C35, 93C41

Key Words and Phrases: fractional descriptor nonlinear systems, design, perfect, descriptor, fractional, observer

1. **Introduction**

The fractional linear systems have been considered in many papers and books [8], [10], [15], [18], [25]. Positive linear systems consisting of n subsystems with different fractional orders have been proposed in [16], [18]. Descriptor (singular) linear systems have been investigated in [1], [2], [4], [3], [5], [6], [7], [12], [13], [20], [21], [22], [23], [26], [27]. The eigenvalues and invariants assignment by state and input feedbacks have been addressed in [5], [12], [20]. The computation of Kronecker's canonical form of a singular pencil has been analyzed in [26].

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A new concept of perfect observers for linear continuous-time systems has been proposed in [11]. Observers for fractional linear systems have been addressed in [19], [24]. Fractional descriptor full-order observers for fractional descriptor continuous-time linear systems have been proposed in [9].

Perfect full-order observers for fractional descriptor continuous-time linear systems have been proposed in [14] and perfect reduced-order observers for fractional descriptor linear systems in [17].

In this paper the concept of perfect observers will be extended for nonlinear fractional descriptor continuous-time systems.

The paper is organized as follows. In Section 2 the basic definitions and theorems of fractional descriptor nonlinear continuous-time systems are recalled and their perfect observers are introduced. The necessary and sufficient conditions for the existence of the perfect fractional nonlinear observers are established and a design procedure of the perfect nonlinear fractional observer is proposed in Section 3. An illustrating example is given in Section 4. Concluding remarks are given in Section 5.

2. **Fractional descriptor nonlinear systems and their perfect observers**

Consider the fractional descriptor continuous-time linear system

$$E\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = Ax + f(x,u), \quad x_0 = x(0), \tag{2.1a}$$

$$y = Cx, \tag{2.1b}$$

where $\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha}$ is the fractional α order derivative defined by Caputo [18], [25]:

$${}_0D_t^\alpha x(t) = \frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{\frac{\mathrm{d}^n x(\tau)}{\mathrm{d}\tau^n}}{(t-\tau)^{\alpha-n+1}}\,\mathrm{d}\tau,$$

$n-1 < \alpha < n \in N = 1,2,\ldots$, $\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}\mathrm{d}t$ is the Gamma function ($\mathrm{Re}(x) > 0$), $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}^m$, $y = y(t) \in \mathbb{R}^p$ are the state, input and output vectors, respectively, $E, A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{p\times n}$, $f(x,u) \in \mathbb{R}^n$ is a continuous vector function of x and u.

It is assumed that $\det E = 0$ and

$$\det[E\lambda - A] \neq 0 \quad \text{for some} \quad \lambda \in \mathbb{C} \text{ (the field of complex numbers)}.$$

Let U be the set of admissible inputs $u(t) \in U \subset \mathbb{R}^m$ and $X_0 \subset \mathbb{R}^n$ be the set of consistent initial conditions $x_0 \in X_0$, for which the equation (2.1) has a solution x for $u \in U$.

Definition 2.1. The fractional descriptor continuous-time linear system

$$E\frac{\mathrm{d}^\alpha \hat{x}}{\mathrm{d}t^\alpha} = F\hat{x} + f(x,u) + Hy, \tag{2.2}$$

where $\hat{x} = \hat{x}(t) \in \mathbb{R}^n$ is the estimate of $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $f(x,u) \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ are the same vectors as in (2.1), $E, F \in \mathbb{R}^{n\times n}$, $\det E = 0$, $H \in \mathbb{R}^{n\times p}$ is called a (full-order) perfect nonlinear state observer for the system (2.1), if

$$x(t) = \hat{x}(t) \quad \text{for} \quad t > 0.$$

3. **Design of perfect fractional descriptor nonlinear observers**

The following elementary row (column) operations will be used, see [13], [18]:

(1) Multiplication of the ith row (column) by a real number c. This operation will be denoted by $L[i \times c]$ ($R[i \times c]$).
(2) Addition to the ith row (column) of the jth row (column) multiplied by a real number c. This operation will be denoted by $L[i + j \times c]$ ($R[i + j \times c]$).
(3) Interchange of the ith and jth rows (columns). This operation will be denoted by $L[i, j]$ ($R[i, j]$).

LEMMA 3.1. *If*

$$\operatorname{rank} E = r < n, \tag{3.1}$$

then by the use of the elementary row and column operations the matrix E can be reduced to the following upper triangular form

$$P_1 E Q_1 = \begin{bmatrix} 0 & E_{12} \\ 0 & 0 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1r} \\ 0 & e_{22} & \cdots & e_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e_{rr} \end{bmatrix}, \tag{3.2a}$$

or lower triangular form

$$P_2 E Q_2 = \begin{bmatrix} 0 & 0 \\ E_{21} & 0 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} e_{11} & 0 & \cdots & 0 \\ e_{21} & e_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ e_{r1} & e_{r2} & \cdots & e_{rr} \end{bmatrix}, \tag{3.2b}$$

where P_k and Q_k, $k = 1, 2$ are the matrices of elementary row and column operations.

P r o o f. If the condition (3.1) is satisfied, then by elementary row and column operations the matrix E can be reduced to the form

$$\begin{bmatrix} 0 & E'_{12} \\ 0 & 0 \end{bmatrix}, \quad E'_{12} \in \mathbb{R}^{r\times r}.$$

Next, applying the elementary column operations we can reduce the matrix E'_{12} to the upper triangular form E_{12}. The proof for (3.2b) is similar. □

DEFINITION 3.1. The smallest nonnegative integer q is called the nilpotent index of the nilpotent matrix N if $N^q = 0$ and $N^{q-1} \neq 0$.

LEMMA 3.2. *If*

$$\det E = r < \frac{n}{2},$$

then the nilpotent index q of the matrix E is

$$q = 2 \quad \textit{for} \quad r = 1, 2, \ldots, \frac{n}{2} - 1.$$

P r o o f. If $r < \frac{n}{2}$, then by Definition 3.1 and (3.2a) we have

$$(P_1 E Q_1)^2 = \begin{bmatrix} 0 & E_{12} \\ 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{for} \quad r = 1, 2, \ldots, \frac{n}{2} - 1.$$

The proof for (3.2b) is similar. □

LEMMA 3.3. *If the nilpotent matrix $N \in \mathbb{R}^{n\times n}$ has the index q ($N^q = 0$) and*

$$D = \det[d_1, \ldots, d_n], \quad d_k \neq 0, \quad k = 1, 2, \ldots, n, \tag{3.3}$$

then the solution $x(t)$ of the fractional differential equation

$$N \frac{\mathrm{d}^\alpha \hat{x}}{\mathrm{d}t^\alpha} = Dx, \quad 0 < \alpha < 1 \tag{3.4}$$

satisfies the condition

$$x(t) = \begin{cases} -\sum_{k=0}^{q-2} \overline{N}^{(k+1)} \delta^{(k+1)\alpha - 1}(t) x_0 & \textit{for} \quad t = 0 \\ 0 & \textit{for} \quad t > 0 \end{cases}, \tag{3.5}$$

where $\delta^{(k)}(t)$ is the k-order derivative of the Dirac function $\delta(t)$.

P r o o f. Applying the Laplace transform to (3.4) and taking into account that

$$\mathcal{L}\left[\frac{\mathrm{d}^\alpha x(t)}{\mathrm{d}t^\alpha}\right] = \int_0^\infty \frac{\mathrm{d}^\alpha x(t)}{\mathrm{d}t^\alpha} e^{-st} \mathrm{d}t = s^\alpha X(s) - s^{\alpha-1} x_0,$$

we obtain

$$N s^\alpha X(s) - N s^{s-1} x_0 = DX(s), \tag{3.6}$$

where $X(s) = \mathcal{L}[x(t)] = \int_0^\infty x(t) e^{-st} \mathrm{d}t$ and $x_0 = x(0)$.

Pre-multiplying (3.6) by the inverse matrix D^{-1}, we obtain

$$X(s) = -\left[\mathbb{I}_n - \overline{N}s^{\alpha}\right]^{-1}\overline{N}s^{\alpha-1}x_0, \tag{3.7}$$

where $\overline{N} = D^{-1}N$ and $\overline{N}^q = D^{-q}N^q = 0$.

Taking into account that

$$\left[\mathbb{I}_n - \overline{N}s^{\alpha}\right]^{-1} = \sum_{k=0}^{q-1}\overline{N}^k s^{k\alpha},$$

from (3.7) we obtain

$$X(s) = -\sum_{k=0}^{q-2}\overline{N}^{k+1}s^{(k+1)\alpha-1}x_0. \tag{3.8}$$

Applying the inverse Laplace transform to (3.8), we obtain (3.5), since $\mathcal{L}^{-1}\left[s^{k\alpha}\right] = \delta^{(k\alpha)}(t)$. □

Let

$$e(t) = x(t) - \hat{x}(t).$$

Then using (2.1) and (2.2), we obtain

$$\begin{aligned} E\frac{\mathrm{d}^{\alpha}e}{\mathrm{d}t^{\alpha}} = E\frac{\mathrm{d}^{\alpha}x}{\mathrm{d}t^{\alpha}} - E\frac{\mathrm{d}^{\alpha}\hat{x}}{\mathrm{d}t^{\alpha}} &= Ax + f(x,u) - (F\hat{x} + f(x,u) + HCx) \\ &= (A - HC)x - F\hat{x} \end{aligned}$$

and

$$E\frac{\mathrm{d}^{\alpha}e}{\mathrm{d}t^{\alpha}} = Fe, \tag{3.9}$$

if

$$F = A - HC. \tag{3.10}$$

By Lemma 3.1 using the elementary row and column operations the singular matrix E can be reduced to a suitable nilpotent matrix N and from (3.9) we obtain

$$E\frac{\mathrm{d}^{\alpha}\overline{e}}{\mathrm{d}t^{\alpha}} = \overline{F}\overline{e},$$

where

$$N = PEQ, \quad \overline{F} = PFQ, \quad \overline{e}(t) = Q^{-1}e$$

and P, Q are matrices of elementary row and column operations.

If we choose the matrix H so that

$$\overline{F} = D, \tag{3.11}$$

where D is given by (3.3) then by Lemma 3.3 $\overline{e} = 0$ for $t > 0$ and the fractional descriptor observer (2.2) will be a perfect observer for the nonlinear system (2.1).

Theorem 3.1. *There exists the perfect fractional descriptor nonlinear observer* (2.2) *of the fractional descriptor nonlinear system* (2.1) *if and only if*

$$\operatorname{rank}\begin{bmatrix}\overline{A} - D\\ \overline{C}\end{bmatrix} = \operatorname{rank}[\overline{C}], \tag{3.12}$$

where

$$\overline{A} = PAQ, \quad \overline{C} = CQ \tag{3.13}$$

and the matrices P, Q satisfy (3.11).

P r o o f. To design the perfect observer (2.2) for the system (2.1) with given matrices A, C we have to choose the matrices F, H of the observer so that the conditions (3.10) and (3.11) are met. The conditions (3.10) and (3.11) are met if and only if

$$\overline{A} - \overline{HC} = D, \tag{3.14}$$

where $\overline{H} = PH$.

The equation (3.14) has a solution $\overline{H}$ (and $H = P^{-1}\overline{H}$) for given $\overline{C}$ and D if and only if the condition (3.12) is satisfied. Therefore, there exists the perfect observer (2.2) for the system (2.1) if and only if the condition (3.12) is satisfied. □

From the above considerations we have the following procedure for designing of the perfect nonlinear observer (2.2) for the nonlinear system (2.1).

Procedure 3.1.

Step 1. Find the matrices P and Q of the elementary row and column operations reducing the matrix E to its nilpotent form $N = PEQ$.

Step 2. Knowing the matrices P, Q compute $\overline{A}$, $\overline{C}$ defined by (3.13).

Step 3. Choose a diagonal matrix (3.3) and check the condition (3.12). If the condition is satisfied then there exists the perfect observer (2.2) for the system (2.1).

Step 4. Knowing the matrices $\overline{A}$ and $\overline{C}$ find the solution $\overline{H}$ of the equation (3.14).

Step 5. Compute the matrices of the perfect observer (2.2)

$$F = A - HC, \quad H = P^{-1}\overline{H}.$$

4. **Example**

Consider the fractional descriptor nonlinear system (2.1) with the matrices

$$\begin{gathered} E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}, \\ C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad f(x,u) = \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \\ f_3(x,u) \end{bmatrix}, \end{gathered} \tag{4.1}$$

where $f(x,u) \in \mathbb{R}^n$ is arbitrary continuous vector function of x and u.

The descriptor system is regular, since

$$\det[Es - A] = \begin{vmatrix} s-2 & 0 & -1 \\ -3 & s & -2 \\ 0 & -2 & 0 \end{vmatrix} = 2(1-2s) \neq 0.$$

To design the perfect fractional descriptor nonlinear observer for the system we use Procedure 3.1 and we obtain the following:

Step 1. In this case we have

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$N = PEQ = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

Step 2. Using (3.14) and (4.1) we obtain

$$\overline{A} = PAQ = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad \overline{C} = CQ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Step 3. In this case we choose

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

and the condition (3.12) is satisfied, since

$$\operatorname{rank} \begin{bmatrix} \overline{A} - D \\ \overline{C} \end{bmatrix} = \operatorname{rank} \begin{bmatrix} -2 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = 2 = \operatorname{rank}[\overline{C}] = \operatorname{rank} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Therefore, there exists the perfect nonlinear observer (2.2) for the system (2.1) with (4.1).

Step 4. The equation

$$\overline{HC} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \overline{A} - D = \begin{bmatrix} -2 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

has the solution

$$\overline{H} = \begin{bmatrix} -2 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} = H, \tag{4.2}$$

since $P = \mathbb{I}_3$.

Step 5. Using (2.2), (4.1) and (4.2) we obtain

$$F = A - HC = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}.$$

The perfect observer is described by the equation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{d}^\alpha \hat{x}}{\mathrm{d}t^\alpha} = \begin{bmatrix} 0 & 0 & 3 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \\ f_3(x,u) \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} y.$$

5. Concluding remarks

Perfect fractional descriptor nonlinear observers for fractional descriptor continuous-time nonlinear systems have been proposed. Necessary and sufficient conditions for the existence of perfect observers for the fractional descriptor nonlinear systems have been established. Designing procedure of the fractional descriptor observers has been proposed and illustrated on a numerical example. The considerations can be easily extended to fractional descriptor discrete-time nonlinear systems. An open problem is an extension for fractional descriptor 2D continuous-discrete linear and nonlinear systems.

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