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RESEARCH

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Multi-objective design optimization of polymer spur gears using a hybrid approach

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Abstract

Polymer gears are used in applications requiring small to moderate loads to effectively transmit power and use the limited place available as possible. Various commercial standards have been provided designers with the rating criteria and acquaintance of different polymer material properties for the process of design. However, the result was unsatisfactory in terms of economy, time, and the optimality of the product. Thus, classic and stochastic algorithms have been embraced to reach the best design of polymer gears. Taking advantage of the former and latter algorithm' methods, optimal design of gears could be attained with an increased gear life span and decreased failure modes. In this study, polyoxymethylene (POM) spur gear set has been optimized combining the mathematical model from plastic standards and hybrid optimization approach of multi-objective genetic algorithm (MOGA) and sequential quadratic programming (SQP). Weight and power loss were the objective functions. Five design variables were optimized with the satisfaction of bending and contact stresses, temperature, wear, and deformation as constraints. Solutions of the problem were formulated as Pareto optimal set. The results of multi-objective were compared with previously published single-objective optimization. The results favored multi-objective optimization (82.67%, 31.39% reduction in weight and power loss respectively) as it gave the best applicable solution fitting in real life situations. The results also went hand in hand with literature confirming the efficiency of the proposed algorithm. With the variation of operating parameters, various optimal designs could be obtained where the designers can choose the design that is suitable for a particular application.

Keywords: Polyoxymethylene, Hybrid optimization approach, Multi-objective genetic algorithm, Sequential quadratic programming, Spur gears, Weight, Power loss

Introduction

There has been an upswing in the utilization of polymer gears in the field of industry since 1950s [1]. The reasonable causes for this rise are easier production with lower manufacturing costs, the ability to work without lubrication, good damping properties, and corrosion resistivity. All these potentials have made polymer gears competitors to their equivalents, i.e., metal gears [2, 3]. By substituting metal gears with polymeric ones, one can reduce up to 70% of mass, 80% of inertia, and a relatively 9% of consumed fuel [4]. On the contrary, there have been discernible disadvantages which make them readily available for limited applications such as food and office appliances. The main cons are lower load capacities and the deterioration of mechanical properties such as fatigue life and material modulus of elasticity due to the elevated temperatures [1, 2, 5]. Polymer gears are extremely sensitive to temperature because of their smaller thermal conductivity and, as a result, significant heat generation during the mesh with low dissipation of this heat amount. Temperature produced during gear pair mesh comprises of flash and bulk temperature. Flash temperature according to [5, 6] is the localization of instant temperature rise around heat source, while bulk temperature is the temperature rise through the gear structure [7]. Although estimation of gear pair temperature using numerical methods escalates the computational costs and is full of intricacies, it is ubiquitous in the literature. Mao [8] estimated the flash temperature using numerical method. The model was assumed to be in an unsteady state where the heat source is not constant and depending on sliding velocity at each mesh position and load per unit width. This assumption was unsimilar to the paradigm by Blok [6]. A combination of analytic and numerical methods was used in [9] to figure out a model for flash temperature. The limitations for this model could be the constant coefficient of friction which in reality depends on sliding speed and contact pressure, and its validation is for moderate load conditions only. On the other hand, it is applicable to various geometries of spur gear types. In a study conducted by [7], the flash and bulk temperatures were predicted using semi-analytical model and numerical finite element method (FEM) analysis, respectively, measured using infrared camera. The authors determined both temperatures as a function of cycle time and movement of the heat source. The comparison established between the theoretical and the experimental models revealed a difference of 25% in the case of bulk temperature estimation. In another study of Roda-Casanova et al. [10], a developed numerical model for heat transfer based on convection method was compared with a model used by [7]. The authors estimated the heat transfer coefficients for each surface or part of a gear geometry in connection with surrounding air. The difference between the two models is the exclusion of face width of gear for [7]. This model is used for accurately estimating bulk temperature of the gear pair under consideration. Another paradigm was developed using 2D finite element analysis (FEA) in [11]. It was developed to overcome the limitations that appeared from analytical models in the literature. The model is based on solutions of two problems, mechanical and thermal ones. The mechanical problem was to determine the amount of frictional heat generated during teeth mesh, while using this heat as a thermal load to solve the thermal model by FEM and obtain the average temperature of the gear pair. Although this study showed good agreement with experimental work in the literature, this model is inapplicable in case of small width gears as well as increments in the computational costs. Various studies [5, 12, 13] reported that any temperature rise accompanied by increase in load and speed would significantly increase wear rate and thus thermal damage to the gear teeth. It was recorded that wear is the most predominant type of failure in polymer gears followed by tooth melting thermal bending at elevated torques, and tooth root/pitch fracture [14–25]. Addition of fiber reinforcements to the polymer materials for gears has been also studied [13–15, 26, 27]. These reinforcements appeared to improve the mechanical properties of materials such as increasing modulus of elasticity, improving fatigue life of the gears, and lowering coefficient of friction between mating gears. Material to be selected for driver and driven gear was investigated by number of researchers [18–20] where it was revealed to be important factor affecting level of polymer gear wear rates. *Polyoxymethylene (POM)* gears have been extensively studied in the literature due to good mechanical and chemical properties such as high stiffness, stability in dimension as it is a lower moisture-absorbent, corrosion resistance, and high fatigue strength [28]. However, when they were thoughtfully tested by authors [17, 20, 21], wear rate increased drastically at critical torques. This in turn led to tooth melting especially when the surface temperature reached melting point of 175 °C of the material. Based on the aforementioned studies, predicting the transition torque at which wear is shifted from low to high is extremely essential for better performance of POM gears. Mao et al. [21] developed a model based on the estimation of bulk, flash, and ambient temperature through which critical torque could be determined. This model correlated well with experimentally measured surface temperature. By assessing the critical load, excessive wear could be avoided. Moreover, Kalin and Kupec [1] claimed that carefully controlled tooth root temperature even at high loads could increase the fatigue life of the polymer gear. In order to expand the limiting use of polymer gears, designers and engineers tried to devise polymer gears based on rating criteria such as bending and contact strength from standards and handbooks as prevalent practice of metal gears; however, the process did not give the optimum solution [8]. This was due to other circumstances i.e., temperature sensitivity led to wear, and degradation of mechanical properties. It was pinpointed that frictional losses resulted from relative motion of pinion and gear has a significant impact on temperature rise [2]. As such, researchers adopted optimization methods to decline any power losses with reduced weight gear pair as possible to fit specific applications especially, when it successfully worked with metallic gears. By applying optimization techniques, designers will have the ability to consider all the rating criteria affecting gear pair such as bending and contact strength, wear, temperature, and deformation. Few studies have taken a pace in the optimization of polymer gears. As a consequence, Singh et al. [29] optimized wear rate, surface temperature, and efficiency of three different polymeric materials meshing with pinion steel gears. The authors varied operating parameters such as speed and torque where they obtained 27 alternatives. The optimized gear pair was attained through a hybrid multi-criteria decision-making approach. Optimization of e-bike drive was applied by Tavcar et al. [30]. This is done through multicriteria function containing the previously mentioned rating criteria with an aim of decreasing the center distance to fit the place specified. Zorko et al. [31] attempted to optimize a polymer gear meshing with steel pinion through testing new material, i.e., polyether ether ketone (PEEK). Although it excels other plastic materials in terms of life span and fatigue strength, testing these materials was time-consuming, and numerical solution did not consider the temperature when the stress was calculated. Newly optimization techniques have been adopted by researchers to enhance the performance of gear sets with optimized parameters. By delving into the literature, it was found that stochastic techniques such as genetic algorithm and particle swarm were used extensively in optimizing metallic gears (more information on metal gear optimization on [32]). Nevertheless, to the best of the authors' acquaintance, few studies using population-based algorithms were mentioned in the literature such as the one by Miler et al. [33]. In their research, a pair of POM gears has been optimized using genetic algorithm. Frictional power losses and volume were used as design objectives. The problem was formulated as a multi-objective optimization. In this research, polymer spur gear is optimized using a more accurate approach to reach the global optima quickly and precisely, which is multi-objective genetic algorithm (MOGA) with sequential quadratic programming (SQP): "hybrid optimization approach (HOA)." Frictional power losses and weight of the gear pair are formulated as design objectives. The optimum design will be chosen from the Pareto optimal set. Also, a comparison between multi-objective and single-objective optimization will be conducted for the best geometric parameters' solution as well as a comparison to the multi-objective optimization in the literature. The rest of this article is organized as follows. The "Mathematical model" section elucidates the mathematical model of POM spur gear pair based on previously mentioned rating criteria, the "Optimum design problem formulation" section presents multi-objective problem formulation, the "Results and discussion" section discusses results with holistic analysis of the multi-objective problem, and lastly the "Conclusions" section concludes the main findings.

Methods

Mathematical model

The mathematical model adopted in this research is based on rating criteria from VDI [34] and ISO[35] standards. A pair of spur gear made of POM were optimized using hybrid optimization approach (HOA). The gear pair was in the form of solid discs operating at speed of 500 rpm and a torque of 8 Nm in dry conditions as recommended by other researchers [8] testing POM against POM in dry conditions where above critical torque, wear would increase as the worn particles were shown in larger size at the beginning of the operation resulting in melting the gear teeth especially when the surface temperature reached melting point of POM. The model is summarized in this section. A schematic of the spur gear pair with basic geometric parameters such as pitch circle radii $R_{\mathrm{p1,2}}$, addendum circle radii $R_{\mathrm{a1,2}}$, dedendum circle radii $R_{\mathrm{d1,2}}$, and pressure angle α is shown in Fig. 1.

Polymer spur gear is firstly rated through root bending stress not exceeding the permissible strength of the material according to the following equation:

$$\sigma_F = K_F Y_F Y_S Y_\varepsilon \frac{F_t}{bm} \tag{1}$$

where F_t is tangential force, b is the face width of the gear pair, and Y_ε is contact ratio factor.

It is worth mentioning that the form factor Y_F which depends on the teeth number for both pinion and gear respectively and stress correction factor Y_S which takes stress concentration into account can be calculated according to method B available in [35, 36].

According to VDI [34], strength of the material can be calculated as follows:

$$\sigma_{FlimN} = 26 - 0.0025\vartheta_{root}^2 + 400N_L^{-0.2} \tag{2}$$

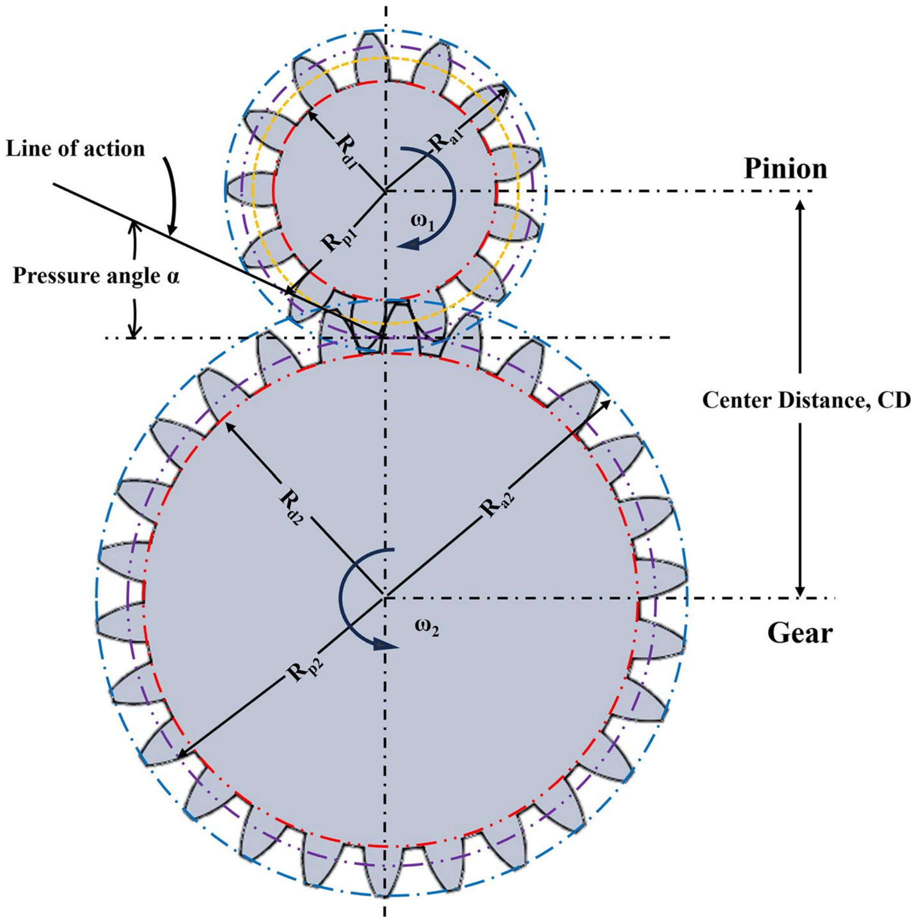

Fig. 1 A schematic illustrating spur gear pair geometry

where ϑ_{root} is root temperature determined analytically or numerically based on previously mentioned literature.

Application factor K_A is roughly equal to tooth root load K_F as suggested by [33, 34]. Because of good damping behavior of POM gears, factors K_v, $K_{F\beta}$, and $K_{F\alpha}$ were diminished:

$$K_F = K_A\, K_v\, K_{F\beta}\, K_{F\alpha} \approx K_A \tag{3}$$

where K_v is dynamic factor, and $K_{F\beta}$and$K_{F\alpha}$ are width and face factors, respectively.

Secondly, temperature is the second rating criterion, installing in the model due to its detrimental effects on polymer gears, so it should be less than the permissible temperature $\vartheta_{perm.}$ of 80 °C as suggested by Miler[33]:

$$\vartheta_i = \vartheta_0 + P_A\, \mu\, H_V \left(\frac{K_{\nu\prime i}}{b\, z(\nu\, m)^{0.75}} + \frac{R_{\lambda\prime G}}{A_G} \right) ED^{0.64} \tag{4}$$

where ϑ_0 is the ambient temperature, $R_{\lambda\prime G}$ is a factor for heat transfer resistivity = 0 for open housing, A_G is a housing factor, and $K_{\nu\prime i}$ represent root and flank heat transfer coefficients. P_A is the input power.

It is worth noting that coefficient of friction μ is assumed constant through the whole mesh cycle as recommended in various literature and VDI standard [2, 8, 9, 11, 34] although it depends on sliding velocity, temperature, and contact stress.

The degree of heat loss H_V can be calculated according to the following equation [34, 37, 38]:

$$H_V = \frac{(u+1)}{z_1 u \cos(\beta_b)}(1 - \varepsilon_\alpha + \varepsilon_1^2 + \varepsilon_2^2) \tag{5}$$

where β_b is base circle helix angle, ε_α is theoretical contact ratio, and $\varepsilon_1, \varepsilon_2$ are partial contact ratio of pinion and gear, respectively.

Wear should also be considered as third rating criterion because it is the most frequent type of failure of POM gears running in dry condition. Correspondingly, it should not outpace permissible wear $W_{Perm.} = 0.1\text{m}$ [33, 34]:

$$W_m = \frac{2\pi T_1 N_L H_V k_w}{bzl_{fl}} \tag{6}$$

where k_w is wear coefficient, and, l_{fl} is length of active tooth flank.

Finally, deformation should be rated also, due to lower values of elasticity modulus of polymers than metals, causing deviation of pitch and noise. Therefore, it should not exceed permissible deformation $\lambda_{perm.} = 0.07\text{m}$ [33, 34]:

$$\lambda = \frac{7.5\, F_t}{b\cos(\beta)}\left(\frac{1}{E_1} + \frac{1}{E_2}\right) \tag{7}$$

where $E_{1/2}$ are moduli of elasticity for pinion and gear, and β is helix angle = 0 for spur gears.

Optimum design problem formulation

Conventionally, classical methods such as goal programming have been adopted extensively for metallic gears in the literature; however, they started with a single initial point and trapped in the local optima [39]. Consequently, there has been a leap in attaining the best solutions of various design problems using meta-heuristic algorithms. Meta-heuristic algorithms have proven themselves to outperform conventional methods. They are capable of handling discrete and continuous design variables as well as converging rapidly towards global optima [40]. One of the most common evolutionary meta-heuristic algorithms is GA. GA has been adopted in many research papers for solving various design problems such as in [41]. Since its invention by Holland in 1975, GA still seamlessly excels in obtaining the global optimum solutions [42]. GA is based on biological evolution and natural selection with the inclusion of inspired operators such as mutation, crossover, and inversion. Optimization problem can be either a series of single-objective or multi-objective problem; however, single-objective optimization is not efficient in real-life applications [43]. This is due to optimizing competing objectives, so a solution for one objective will not be acceptable relative to other objectives. The optimum design of POM spur gear pair can be formulated as a general multi-objective problem according to Deb [44] as follows:

$$
\begin{array}{ll}
\text{minimize } F_\kappa(\chi) & \kappa = 1, 2, \ldots, \mathrm{K} \\
\text{subject to } g_\omega(\chi) \le 0, & \omega = 1, 2, \ldots, \Omega \\
h_\sigma(\chi) = 0, & \sigma = 1, 2, \ldots, \Sigma \\
\chi_i^L \le \chi_i \le \chi_i^U, & i = 1, 2, \ldots, n
\end{array}
\tag{8}
$$

where $F(\chi) = [F_1(\chi), F_2(\chi), \ldots, F_\mathrm{K}(\chi)]^T$ represents K objectives to be optimized and χ $=[\chi_1, \chi_2, \chi_3, \ldots, \chi_n]^T$ serves as a vector of n design variables. The problem also is subjected to nonlinear inequality and equality constraints designated as $g_\omega(\chi)$ and $h_\sigma(\chi)$ respectively. Unlike single-objective optimization which retains only a single solution point, multi-objective GA (MOGA) yields series of all non-inferior solutions called Pareto points at a single run [45]. Hybrid optimization approach (HOA) is a wise method to efficiently reach the global optima [40, 44]. Various hybrid approaches were presented in the literature such as simulated annealing with direct search in [46] and Nelder-Mead with GA in [47]. As MOGA is used to search for the global region through an iterative process, a more local search is applied using SQP resulting in faster convergence rate [40, 44]. Therefore, a more accurate gear pair design can be achieved.

In this paper, weight W and power loss P_{VZ} are formulated in the form of multi-objective problem. Five decision variables, namely, m, φ_m, z_1, x_1, and x_2, are bounded by upper and lower values, while the objective functions are constrained by material strength, wear, temperature, and deformation.

Objective functions can be represented mathematically as follows:

$$
\left.\begin{array}{l}
F_1(\chi) = W = \frac{\pi}{4} b\rho(d_1^2 + d_2^2) \\
F_2(\chi) = P_{VZ} = P_A \mu H_V
\end{array}\right]
\tag{9}
$$

where d_1, and d_2 are tip diameters of pinion and gear, respectively.

Decision variables can be shown as a vector:

$$
\chi = [m, \varphi_m, z_1, x_1, x_2]^T \tag{10}
$$

Representation of design variables bounded by lower and upper values is demonstrated in the following equation:

$$
\left.\begin{array}{c}
m_L \le m \le m_U \\
\varphi_{mL} \le \varphi_m \le \varphi_{mU} \\
z_{1L} \le z_1 \le z_{1U} \\
x_{1L} \le x_1 \le x_{1U} \\
x_{2L} \le x_2 \le x_{2U}
\end{array}\right]
\tag{11}
$$

The constraints need satisfaction in this design problem are presented as follows:

$$
\left.\begin{array}{c}
\sigma_F \le \frac{\sigma_{FlimN}}{S_f} \\
\vartheta_i \le \vartheta_{perm.} \\
W_m \le W_{Perm.} \\
\lambda \le \lambda_{perm.}
\end{array}\right]
\tag{12}
$$

The type of variables and the upper and lower boundaries utilized in this study can be represented in the following Table 1, while the input parameters to the optimization algorithm are demonstrated in Table 2.

Table 1 Lower and upper design variable boundaries

Design variables	Unit	Lower boundary	Upper boundary	Type of variable
Module (m)	mm	1	4	Discrete
Number of teeth (z_1)	-	14	24	Integer
Width ratio (φ_m) [33]	-	6	30	Continuous
Pinion shift coefficient	-	0	0.7	Continuous
Gear shift coefficient	-	-0.7	0.7	Continuous

Table 2 Input parameters to HOA

Parameter	Symbol	Value	Unit
Input torque	T_1	8	Nm
Input speed	n	500	RPM
Transmission ratio	u	2	-
Initial module	m_i	2	mm
Initial teeth number	z_{1i}	18	-
Initial width ratio	φ_{mi}	13	-
Initial pinion coefficient	x_{1i}	0.5	-
Initial gear coefficient	x_{2i}	0.5	-
Application factor	K_A	1.2	-
Heat transfer coefficient (root)	K_{vFUB}	2100	$K(\frac{m}{s})^{0.75}.mm^{1.75}.W$
Heat transfer coefficient (flank)	K_{vFLA}	9000	$K(\frac{m}{s})^{0.75}.mm^{1.75}.W$
Ambient temperature	ϑ_0	20	$°C$
Relative duty cycle	ED	100%	-
Wear coefficient (POM/POM)	k_w [33]	$60 * 10^{-8}$	mm^3/Nm
Root safety factor	S_f	1.3	-
Cycle number	N_L	10^6	Cycles
POM density	ρ	$1410 * 10^{-9}$	(Kg/mm^3)
Friction coefficient	μ[34]	0.28	-

Results and discussion

Multi-objective optimization of POM spur gear pair has been adopted in this study at speed of 500 rpm and torque of 8 Nm in dry conditions. Hybrid optimization approach (HOA) has been performed in this research to precisely and quickly converge toward the optimal solution. In the HOA, a combination of GA and SQP was applied in MATLAB Toolbox. The parameters used in MOGA were as follows:

- Population size: 500
- Generation number: 1000
- Crossover fraction: 0.8
- Mutation rate: 0.01
- Constraint tolerance: $10.^{-3}$

Solutions of the problem have been streamed as Pareto optimal set and the optimal objective functions have been represented as a convex Pareto front. This Pareto frontier represents more than one optimal solution of two conflicting objectives which were weight and power loss, unlike single-objective optimization which is giving only one optimal solution. However, a designer needs to select one solution among variety of solutions belonging to Pareto set. One way to choose a compromise solution from the attained Pareto frontier was a solution relative to a theoretical point called "Utopia or ideal point [48]." This point demonstrates the minimal value of each objective function. Several normalized vectors pointing from the ideal point to each solution on the Pareto front were calculated. The smallest vector magnitude means a neighboring solution to the utopia point and thus represented as compromise optimal solution. This is shown in Fig. 2. Due to the stochastic nature of the MOGA, a different design solution is attained at each run. Consequently, the hybrid algorithm has been run ten times before deciding on the final solution. The variation of the objective functions and design variables is shown in Fig. 3a, b. The optimization algorithm chose the upper limit of pinion teeth (z_1) which is 24 at each run, while the pinion and gear coefficients x_1 and x_2 were selected from lower boundary with values near zero for x_1 and the range between − 0.4 and − 0.5 for x_2. Module (m) was ranging between 2 and 3 mm where face width (b) had values between 30 and 50 mm.

Due to the variation attainable at each run, designer's preference plays a pivotal role in selecting the criterion that best fits a specific application. Therefore, two designs were performed based on minimum power loss designated as Des. (1) and Des. (2) based on minimum weight (see Fig. 2 left and right). Run number 1 demonstrates Des. (1) where run number 4 represents Des. (2). A comparison between single-objective (SO) and multi-objective (MO) optimization can be seen in Fig. 4a, b and Table 3. Results of multi-objective optimization are better than single-objective optimization in terms of design variables and the two conflicting objectives. The design variable solution in one objective gave unacceptable solution for another objective. As such, weight and power loss of the gear set were reduced by 82.67% and 31.39% respectively when comparing multi-objective to single-objective optimizations. Module and face width were minimized by 42.86% and 44.35% respectively (in case of optimizing power loss as a single objective), while single- or multi-objective optimization gave slight difference in case of gear shift or absolutely no difference in case of pinion teeth and pinion shift (single-objective optimization of either weight or power loss). From Table 3, it can be seen that minimized values of module with higher values of face width gave lower volume with high power loss; such conclusion was made by Miler et al. [33]. Moreover, increasing pinion shift and decreasing gear shift will increase the power loss factor H_v, and this will in turn increase the frictional power loss. Also, lower modules and higher pinion teeth at constant torque gave higher mesh efficiency with lower power loss. This result is in agreement with Walton et al. [49]. One of the parameters which significantly affects power loss is friction coefficient (μ); however, with a value of 0.28 adopted from VDI [34], the optimization algorithm tended to give an efficiency of 97% (a power loss of 13.6 W) in dry conditions. The small difference in optimal solutions in Fig. 2 was due to constant friction coefficient along the line of action, while in the study of Walton et al. [50], the efficiency value was 95% (experimentally) due to higher coefficient of friction used than the value mentioned in VDI. It is worth mentioning that this result was extracted from the figure of POM-POM material (efficiency value was not available in tables). Accordingly, a formula was developed by Miler et al. [51] for prediction of friction coefficient; however, it needs a

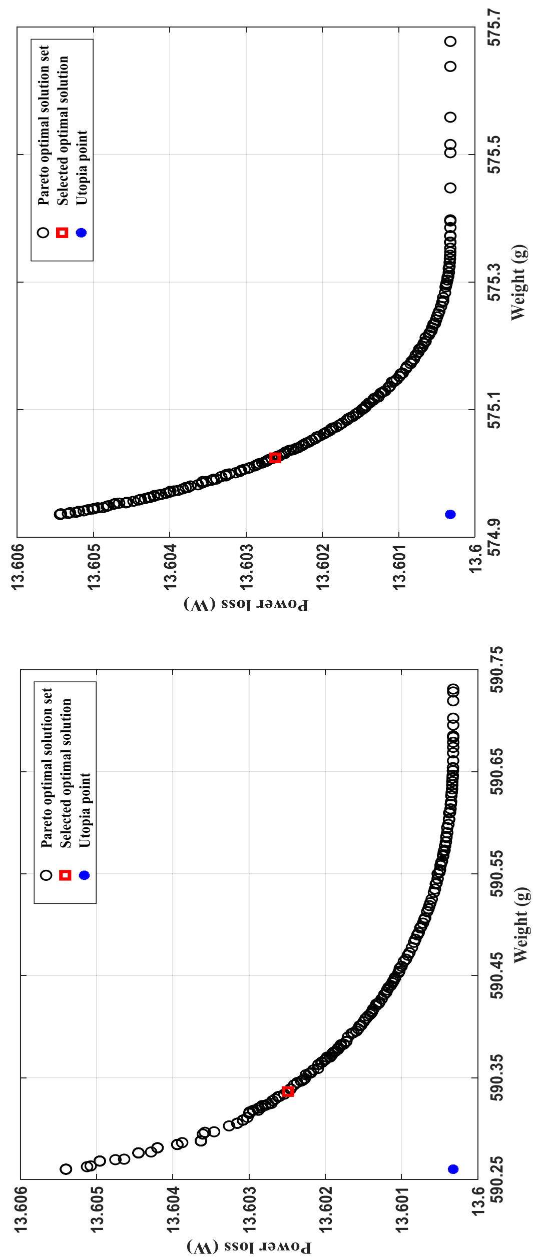

Fig. 2 Pareto front according to minimum power loss (left) and weight (right)

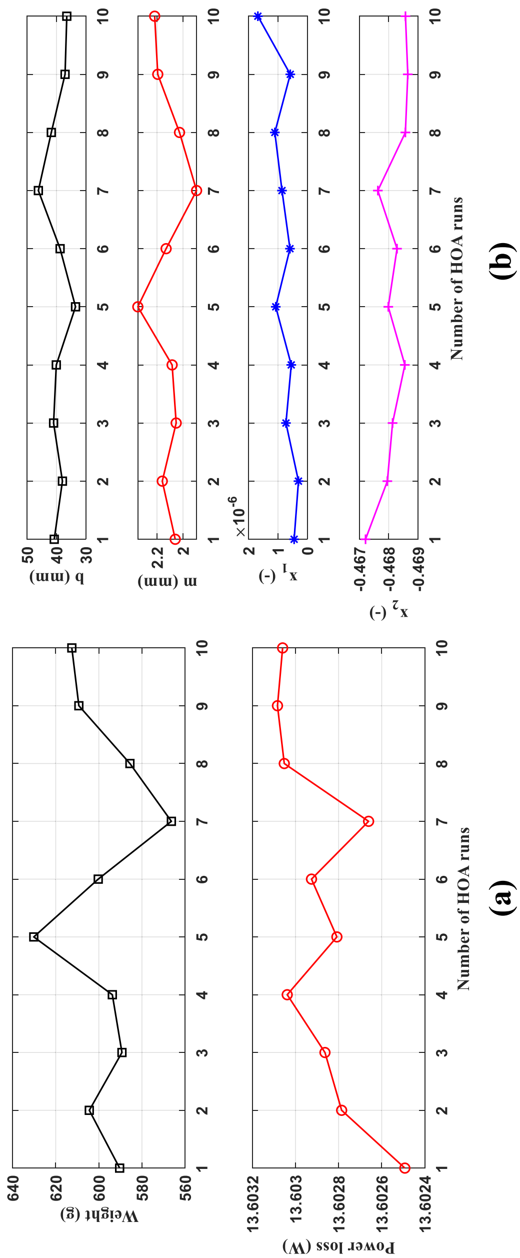

Fig. 3 Variation of objective functions (**a**) and design variables (**b**)

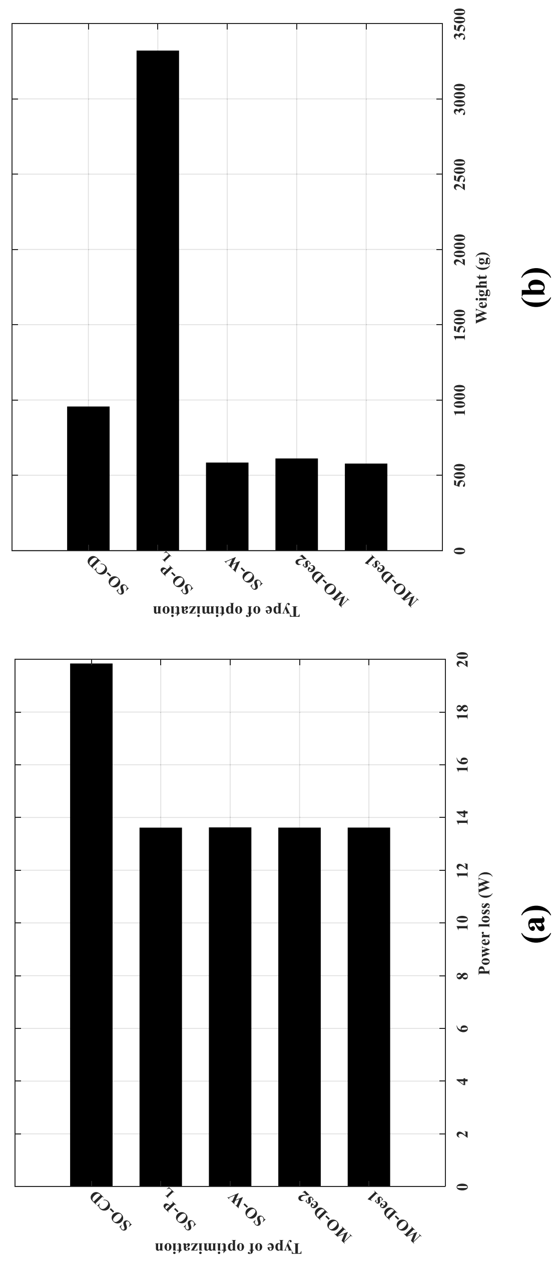

Fig. 4 Comparison between single-objective (SO) and multi-objective (MO) in case of **a** power loss and **b** weight

Table 3 Comparison between single-objective and multi-objective optimizations

Type of optimization	Multi-Objective MO-Des (1)	Multi-Objective MO-Des (2)	Single objective SO-W	Single objective SO-P_L	Single objective SO-CD
Weight [W] (g)	590.3	575	581.7	3317.9	954
Power loss [P_L] (W)	13.6025	13.6026	13.6091	13.6003	19.827
Face width (mm)	40.75	44.1	42.64	79.24	80.23
Module (mm)	2	2	2	3.5	2.75
Pinion teeth (-)	24	24	24	24	16
Pinion shift (-)	0	0	0	0	0.043
Gear shift (-)	-0.4672	-0.4675	-0.4857	-0.4559	-0.4730

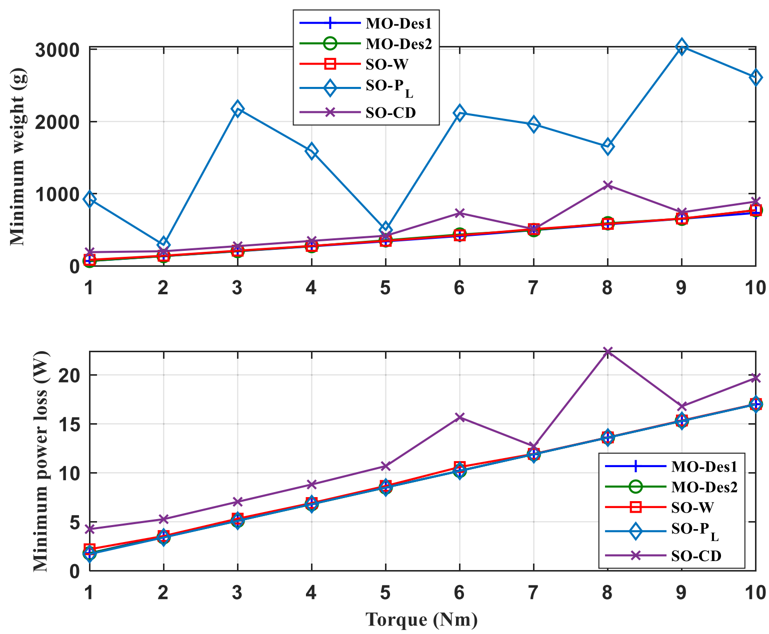

Fig. 5 Comparison between single-objective (SO) and multi-objective (MO) at $n = 500$ rpm and $u = 2$

validation on gear pairs, and it did not take temperature into account. The value of efficiency in the study of Baglioni et al. [37] was higher due to oil lubrication in the friction coefficient formula.

A study has been conducted when varying operational parameters such as torque (T_1), speed (n), and gear ratio (u) for the purpose of determining the effects of these parameters on objective functions in two cases: (I) single-objective optimization and (II) multi-objective optimization. Values of optimal weight and power loss at variation of torque up to 10 Nm can be seen in Fig. 5. It is obvious from Fig. 5 that values less than 1000 g for weight while values less than 20 W for power loss could be attainable, and these values were in compromise in comparison with single-objective optimization. Increasing torque will increase power loss according to Eq. (9) and according to weight of the gear pair due to the increase in number of pinion teeth at constant

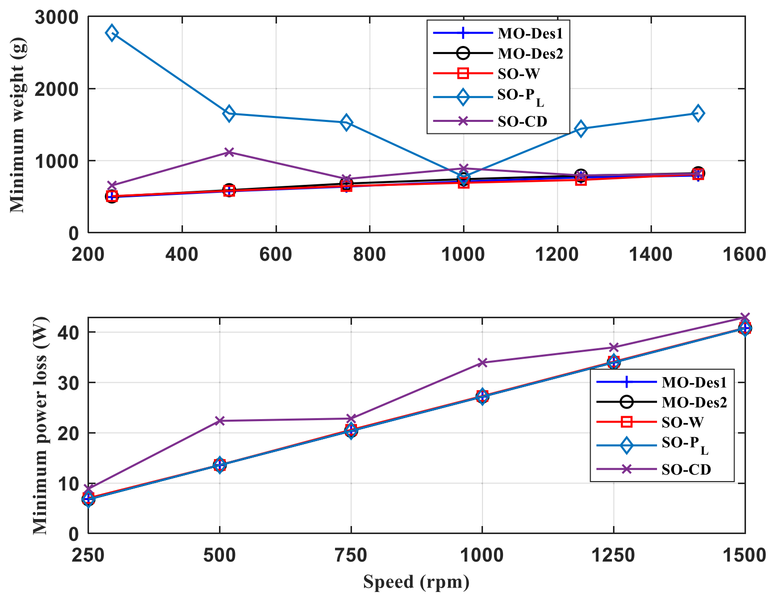

Fig. 6 Comparison between single-objective (SO) and multi-objective (MO) at $T_1 = 8$ Nm and $u = 2$

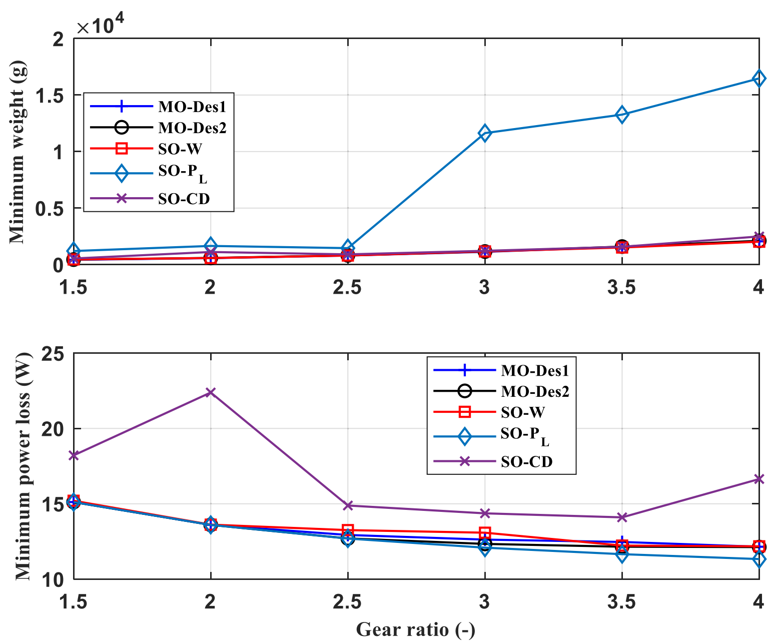

Fig. 7 Comparison between single-objective (SO) and multi-objective (MO) at $T_1 = 8$ Nm and $n = 500$ rpm

speed and gear ratio. This is in agreement with Walton et al. [50]. When varying rotational speed, values of power loss increased significantly in both single-objective or multi-objective optimizations. These values were up to 40 W (see Fig. 6). The last operational parameter was gear ratio as shown in Fig. 7. Increasing gear ratio will slightly increase weight and significantly decrease power loss, due to the decreased loss factor H_V with higher pinon teeth. So, objective functions are affected by speed variations followed by torque and gear ratio.

Conclusions

The problem of polymer spur gear pair was formulated in this work with consideration of bending and contact stress, wear, temperature, and deformation as design constraints. This problem was in the form of multi-objective optimization of two conflicting functions, i.e., weight and frictional power loss. The optimization was performed using HOA combing GA and SQP. The adopted algorithm converged quickly and efficiently toward the optimal solution. The resultant optimal solutions were in the form of Pareto optimal set. Several successive runs have been applied to ensure global optima; then, SQP attempted to search locally to precisely obtain the best gear pair design. Two different designs were selected based on the minimal of each objective in all runs to best fit the requirements of a specific applications. When comparing multi-objective with single-objective optimization, the former gave compromise solutions regarding all objectives unlike the latter giving unrealistic solutions. Lowering module with slightly high face width gave lower volume and higher power loss. All runs gave higher pinion teeth which leads to lower power loss where the vice versa occurred in optimizing center distance as a single objective. The variation of torque, speed, and gear ratio was also studied in this work to give a holistic picture of the optimality of polymer gear pair design. As a result, designers can obtain a variety of optimal designs at different operational conditions, therefore gaining some flexibility to choose appropriate design that best suits a specific application. Future scope will be attempting to optimize spur gear pair with different polymer materials such as nylon (PA) or polyether ether ketone (PEEK) with the variable friction coefficient. The effect of hysteresis power loss could be added to frictional power loss in the future investigation for a more inclusive guideline.

Abbreviations

POM	Polyoxymethylene
HOA	Hybrid optimization approach
MOGA	Multi-objective genetic algorithm
SQP	Sequential quadratic programming
VDI	Verein Deutscher Ingenieure
ISO	International Organization for Standardization

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Authors' contributions
Elsiedy, MA, is a master's student who started the research by collecting data from literature, formulating the mathematical model of the gear set, adapting the mathematical model with the proposed algorithm, after that applying the algorithm on MATLAB and demonstrating results by figures and tables finally drafting the whole paper. Hegazi, HA, is a professor supervisor who helped in formulating the mathematical model and in proposing the hybrid approach technique used. He also participated in revising each section scientifically, with language proofreading. El-Kassas, AM, is a professor supervisor who helped during the data collection stage by explaining how to start the research and revising the whole paper as a final stage. Zayed, AA, is a supervisor who helped Elsiedy, MA, by proposing the meta-heuristic technique used in this article, through revising each section in the paper critically during the writing stage, and by editing the whole paper with language proofreading.

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Availability of data and materials
The datasets used or analyzed during the current study are available from the corresponding author upon reasonable request.

Declarations

Competing interests
The authors declare that they have no competing interests.

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