Shwan
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Microstructural interpretation of effective stress equations for unsaturated sands

B. J. Shwan*

*Correspondence: bestun.shwan@koyauniversity.org

Department of Geotechnical Engineering, Faculty of Engineering, Koya University, KOY45, Kurdistan Region, Koya, Iraq

Abstract

It was fairly accepted to interpret soil behaviour at the microstructural level by utilising the effective degree of saturation instead of the total degree of saturation in the effective stress equation for unsaturated fine-grained soils. However, this interpretation for pure sands was under-represented in the literature. On the basis of shear strength evolution with suction, this paper reports a series of experimental tests to evaluate the use of the soil microstructural interpretation for fine sands. The tests comprised direct shear and unconfined-compression tests at different suction profiles. The analysis of the experimental test results suggested that the contribution of the effective degree of saturation was trivial and was almost equal to the total degree of saturation, therefore, shear strength evolution, owing to the suction effect, can be related to the single stress variable, Bishop's equation. In addition, it was hypothesised that the shear strength evolution was attributed to the soil fabric changes.

Keywords: Shear resistance, Microstructural analysis, Effective stress, Effective degree of saturation

Introduction

The contribution of the degree of saturation (S_r) to the effective stress is considerable due to the microstructural effect. Evidence from literature confirmed the important relationship between the equivalent fluid pressure and microstructural level, [1]. This contribution is more significant for fine-grained soils than coarse-grained soils because of the proposition of the free water in the macrospores, [1].

Bishop [2] suggested a single stress state variable equation, Eq. 1, where the weighting parameter χ varies from 1 (for fully saturated soils) to 0 (for fully dry soils). In addition, it depended on the degree of saturation, soil structure and cycle of wetting and drying. It considered the fluid pressure in the effective stress, [3]. Therefore, the effective stress tensor for unsaturated soil can be expressed as:

$$\sigma'_{ij} = \left(\sigma_{ij} - u_a\delta_{ij}\right) + \chi(u_a - u_w)\delta_{ij} \tag{1}$$

The validity of Eq. 1, however, is limited for the isotropic stress tensor, discussed later in more detail. Based on Eq. 1, Bishop et al. [4] suggested shear strength equation for unsaturated soils as:

$$\tau = c\prime + (\sigma\prime + \chi s) \tan \phi\prime \tag{2}$$

where σ'_{ij} is the effective stress tensor, σ_{ij} is the total stress tensor, u_a is the air pore pressure, u_w is the water pore pressure, δ_{ij} is Kronecker's delta, τ is the shear strength (kPa), c' is the effective cohesion (kPa), σ' is the effective normal stress (kPa), s is the suction (kPa), and φ' is the effective internal friction angle (°). Khalili and Khabbaz [5] linked χ to the air-entry value of the soil on the drying path as follows:

$$\begin{aligned} &\chi = \left(\frac{s}{s_e}\right)^{-0.55} \quad for\, s \geq s_e \\ &\chi = 1 \quad for\, s \leq s_e \end{aligned} \tag{3}$$

where s_e is air entry suction on the drying path (kPa). Equation 3 was used to plot a relationship between χ and s for a soil having s_e = 200 kPa as shown in Fig. 1a. There was a significant reduction in χ once suction excessed the air entry suction.

The increase in shear strength due to unsaturated conditions is generally related to the increase in apparent cohesion $C\prime$ which is due to the second term of Eq. 2 and may expressed as:

$$C\prime = \chi s \tag{4}$$

The apparent cohesion can, therefore, be determined for a range suction profiles using Eq. 4. Figure 1b shows the relationship between apparent cohesion versus suction. χ was calculated based on Eq. 3. Although the inverse relationship between suction and χ, the increase in C' is monotonic. By analogy, there should be a decrease in apparent cohesion beyond the saturation suction due to desaturation.

It was commonly accepted to replace χ by the total degree of saturation, S_r, [6]. The term sS_r has been, therefore, studied extensively and it was linked to soil water retention curves (SWRC). For clayey soils, the term provided high values at high suctions which was unrealistic, consequently, a combination of S_r and χ was considered for microstructural interpretation, [1]. Shwan and Smith [7], therefore, considered the microstructural effect by proposing the following:

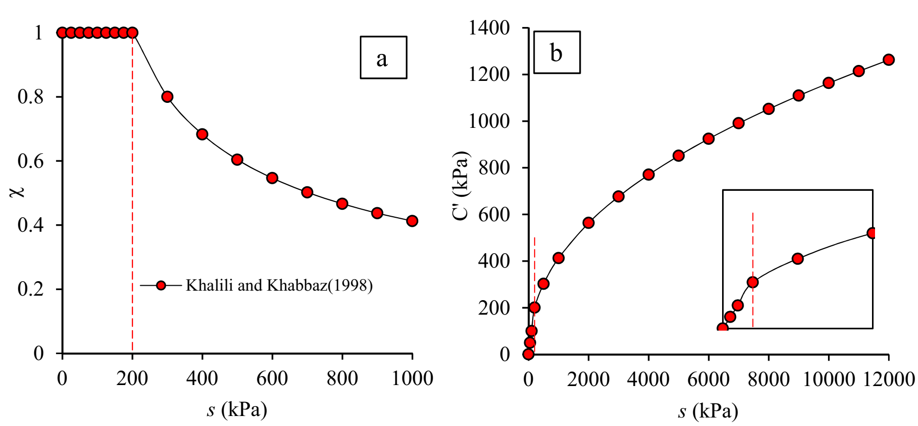

Fig. 1 **a** Weighting parameter (χ) versus suction and **b** apparent cohesion versus suction

$$\tau = c\prime + \left(\sigma\prime + sS_r^{\Gamma}\right)\tan\phi\prime \quad (5)$$

where Γ is a parameter accounts for the water filling microspores and S_r^{Γ} is the effective degree of saturation. The increase in shear strength due to unsaturated conditions in Eq. 5 can be, therefore, expressed as $C\prime\prime = c' + sS_r^{\Gamma}$. The term sS_r^{Γ} is similar to Eq. 4, but considered the microstructural effect. Further details on the parameter Γ and its microstructural effect is described below.

Figure 2a and b shows a range of S_r^{Γ}-S_r curves, based on $S_r = S_r^{\Gamma}$ equation for different values of Γ. Two sets of Γ values are used as high (from 0 to 1) and low (from 0 to 0.1). Figure 2 shows different trends where S_r it was not necessarily equal to S_r^{Γ} apart from the case where Γ = 1. That is, insignificant effect of Γ and therefore the effective degree of saturation was not related to the soil microstructure. Cases where $S_r \neq S_r^{\Gamma}$ showed higher values of S_r^{Γ} at a particular S_r and hence further shear enhancement using Eq. 5. The smaller the Γ value, the higher the S_r^{Γ}. The significant influence of Γ suggested that the effective stress equation should be linked to the soil microstructure.

To the best knowledge of the author, the only previous study investigating the effect of microstructure on the effective stress was by [1]. The study was mainly devoted to the fine grained soils, although four granular soils were also examined. However, these granular soils contained a significant amount of fines (silt and clay): 40%, 72%, 80%, the former soil (40% fines) was classified as a silty sand. Therefore, there was a lack of studies on the pure granular soils, e.g. pure sand.

The aim of this paper is, therefore, to evaluate the use of the effective degree of saturation in the effective stress equations, Eq. 5, on the basis of the soil microstructure for two pure sands, due to a lack of studies on pure granular soils, e.g. sands. A series of direct shear and unconfined compression tests for unsaturated sands within a small range of suction profiles are, therefore, considered and performed for the evaluation.

Experimental setup and validations

In order to evaluate the effect of effective degree of saturation for unsaturated sands (i.e. soil microstructure) the experimental programme, consisting of two sands 1 and 2, comprised a series of direct shear and drained unconfined compression tests for sand 1. In addition, for sand 2, direct shear data from the work of [8] were also utilised, conducted

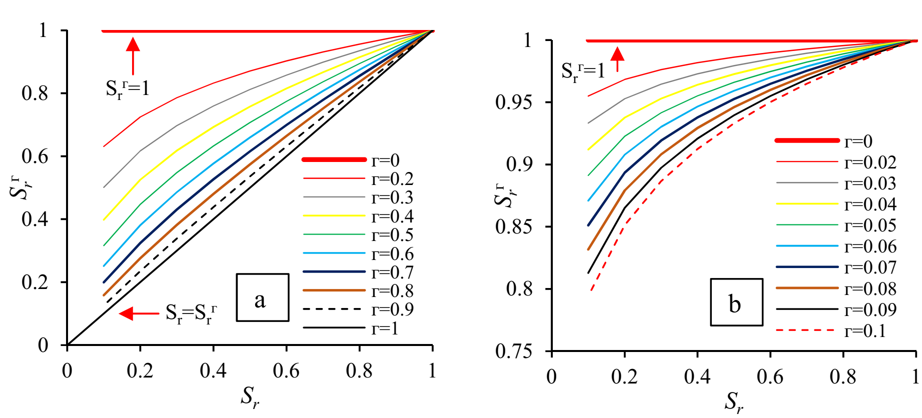

Fig. 2 S_r^{Γ}-S_r relationships at different Γ values from (**a**) (0–1) and (**b**) (0–0.1)

at a range of suction profiles: 0–6 kPa. The sieve analysis results for both sands are shown in Fig. 3. The sands are classified as poorly graded sands (SP) and fines according to the Unified Soil Classification System (USCS). It is clear that the particle size distribution curves of the used sands showed almost close ranges. This is to ensure studying the individual effect of the effective degree of saturation at the microstructural level where the effect of particle size can be negligible. In addition, to ensure results repeatability via utilising the second sand, i.e. sand 2.

For sand 1, the direct shear test was conducted, using a circular box, at constant degrees of saturation. The samples were prepared using the water pluviation technique, where the box was filled previously with the required amount of water that ensured the full saturation conditions. The gaps between the two halves of the box were sealed by a layer of silicon grease to prevent leakage. An amount of dry soil was, then, poured carefully into the box using a funnel. After the successful preparation of the sample, it was left in a controlled temperature room (25 °C) where its water was allowed to evaporate. By knowing the initial and current water contents (monitored via its total mass after evaporation), degree of saturation was calculated. After reaching the desired degree of saturation, the sample was left overnight for the equilibrium purposes and tested afterwards.

The unconfined test was conducted for sand 1 using a modified split spoon sampler of 38 mm diameter and 84 mm height. The modification was by inclusion of a layer of filter paper (8 mm thickness) at the bottom. The filter paper layer consisted of a piece of porous plastic and a thin layer of a silt layer, prepared following the procedure adopted by [9]. The box was connected to a burette, for the suction application, via a plastic tube attached to the bottom of the cylinder. Suction was applied, following the procedure described by [10], by lowering the burette to a required depth until the desired suction head was obtained.

After the successful application of the desired suction, then, the two halves of the cylinder were carefully opened and the sample was brought to failure by applying loads at the top of the sample in successive batches.

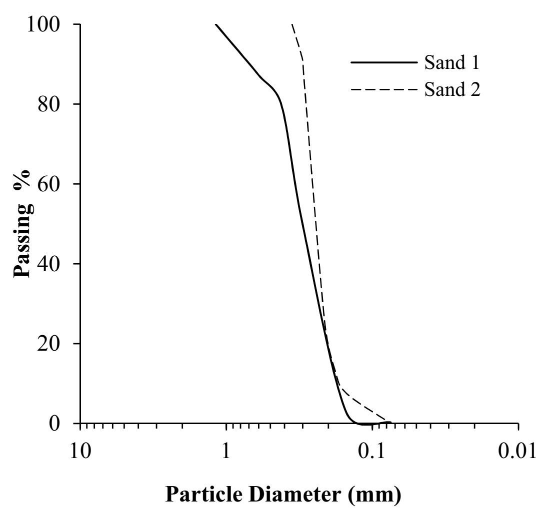

Fig. 3 Sieve analysis for the used sands

For sand 2, the test was conducted by [8] using a bespoke circular box made of Perspex, for the sample preparation. The box was modified by inclusion of a high air-entry disk of 100 kPa capacity at the bottom, which was connected with a burette via a plastic tube. The sample was prepared, using the water pluviation technique. After targeting the required void ratio of 0.7, the box was covered by a piece of latex membrane and left overnight for the equilibrium purposes. Suction was applied, following the same procedure described above. The tests were, then, conducted at three normal stresses: 50, 100 and 200 kPa. Further details about the apparatus modification and sample preparation is given by [8].

Direct shear test results-sand 1

In the following analysis, a series direct shear test at 50, 100 and 200 kPa normal stresses were conducted at constant degrees of saturations of: 0.95, 0.9, 0.85, 0.65, 0.55 and 0.4. The experimental SWRC for the sand was obtained by means of the hanging column technique. The obtained SWRC was fitted using an SWRC equation, proposed by Shwan and Smith [7]:

$$S_r = e^{-\kappa(s-s_o)} \tag{6}$$

where κ is a fitting parameter (kPa^{-1}), s_o is the air entry suction (kPa). Based on the obtained experimental SWRC, the sand had $s_o = 1.1$ kPa, $\kappa = 0.4$ kPa^{-1} and residual suction of ~ 7 kPa. By using the fitted SWRC and knowing the saturation values, suction values were obtained. The stated above saturation values were equivalent to suctions of: 1.226, 1.358, 1.505, 2.175, 2.593 and 3.35 kPa, respectively.

Figure 4 shows the shear resistance versus normal stress at the critical state at various suctions where the experimental data were validated against Eq. 4 (at $\Gamma = 0.1$ and 0.9) and against Eq. 2. In the following analysis, χ in Eq. 2 was calculated based on Eq. 3. The calculated shear resistance values were based on the obtained shear strength parameters at each suction magnitude and the three normal stresses. The results revealed an insignificant influence of Γ suggesting that the single stress variable (i.e. Equation 2, χ s) is retrieved and the effective degree of saturation is almost equal to the total degree of saturation.

Unconfined compression test-sand 1

The unconfined-drained tests for 28 samples were conducted at a range of suction profiles of 0 to ~ 2 kPa. The samples were prepared at a loose state. Figure 5 depicts the obtained experimental results, compared to the relevant obtained values using Eq. 5 at two Γ values: 0.1 and 1. In addition, shear strength values obtained using Eq. 2 were also plotted.

It is clear that the difference was insignificant, trivial effect of Γ. Figure 5 shows as well the zone of collapse at low suctions. This zone corresponded to zero strength, recorded where the samples suddenly collapsed once the two halves of the sampler tube were opened before the load application. The collapse load was only observed for samples where the applied suctions were less than the air entry suction. The behaviour was attributed to the effect of high degrees of saturation. In other words, the poor agreement in the collapse zone was attributed to the wetting behaviour where the single stress

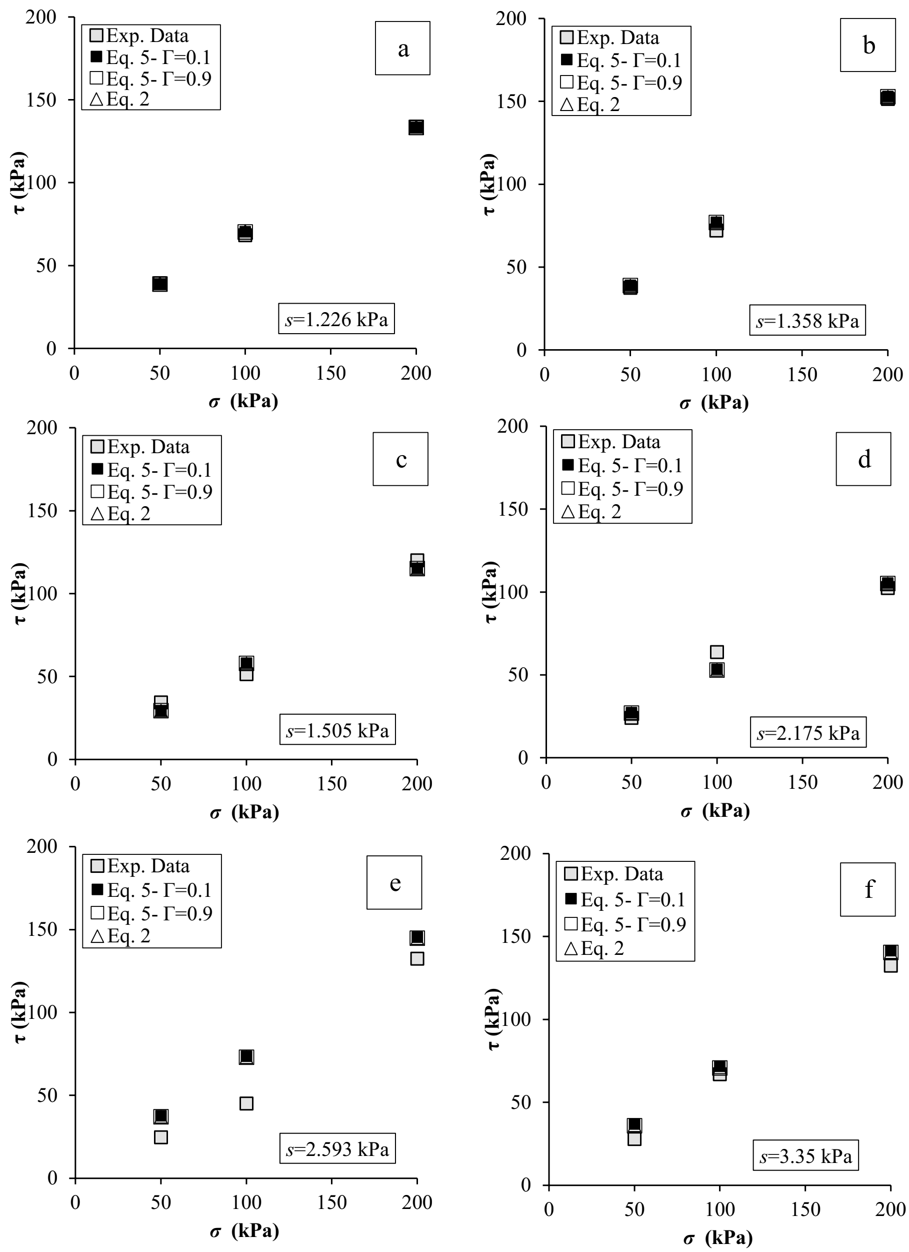

Fig. 4 Evaluation of the effective degree of saturation for sand 1at different suctions (**a**) $s=1.226$ kPa, (**b**) $s=1.358$ kPa, (**c**) $s=1.505$ kPa, (**d**) $s=2.175$ kPa, (**e**) $s=2.593$ kPa and (**f**) $s=3.35$ kPa

variable equation, Eq. 5, was unable to predict soil collapsibility, i.e., Eq. 5 was limited to represent the volume change in unsaturated collapsible soils.

Direct shear test results-sand 2

As stated previously, the data of this analysis were extracted from the work of [8]. The tests were performed at different constant suction values. By means of the hanging column technique, the suction head was measured as a distance from the water level in the hanging column (burette) to the sample's surface, [10]. The sand had an air entry suction

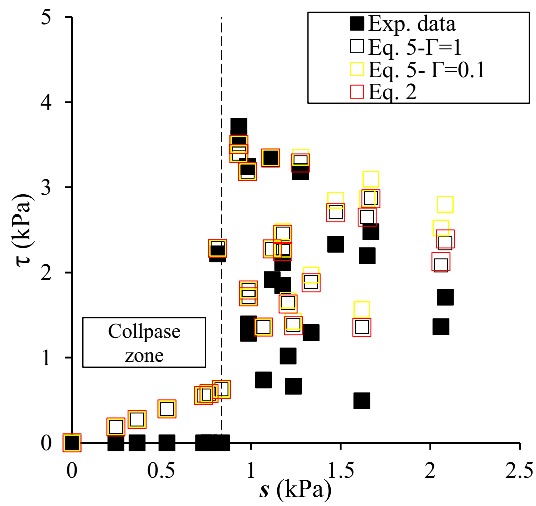

Fig. 5 Evaluation of the effective degree of saturation for the unconfined compression test results for sand 1

and residual suction of 2.3 kPa and 5.5 kPa, respectively. The experimental data represented the average of three repeated tests at the same effective normal stress at the critical state. The microstructural effect was investigated at five different Γ values: 0.1, 0.25, 0.5, 0.75 and 1.

The results are shown in Fig. 6. It is clear that the effective stress of the unsaturated sand did not relate to the soil microstructure, i.e. the influence of Γ was trivial and did not reasonably well coincide with the experimental data. This was a confirmation that the increase of shear resistance of the tested sand was not significantly influenced by Γ, but rather attributed to the influence of fabric changes as deduced by Shwan [8]. The finding here was also identical for sand 1. Other scholars also related the increase of shear resistance at the critical condition to the soil fabric changes, e.g. [12]. In addition, the effect of Γ was slightly more noticeable at the residual suction (i.e. $s = 5.5$ kPa) than near the air entry suction due to the higher applied suction.

The changes in soil fabric were attributed to formed liquid bridges between the solid grains at a specific degree of saturation, especially in the transition zone between the air entry suction and saturation suction (pendular regime). Therefore, assemblage and particle aggregation formed by the menisci suction. Consequently, it led to shear strength enhancement. The finding here is also reported by others, e.g. [13].

Equation 1, however, is only valid for cases where the stress tensor is isotropic as already stated previously. However, anisotropic capillary stress has been confirmed at the pendular regime, see for example [13] and [14]. The anisotropic effect mechanism is not fully understood and explained, [14]. This study, therefore, did not consider the anisotropic effect.

Conclusions

The contribution of degree of saturation on the effective stress for fine-grained soils has been interpreted differently, suggesting the use of effective degree of saturation instead of total degree of saturation. Evaluation on fine-grained soils confirmed the preceding suggestion. The effective stress at unsaturated conditions, therefore, has

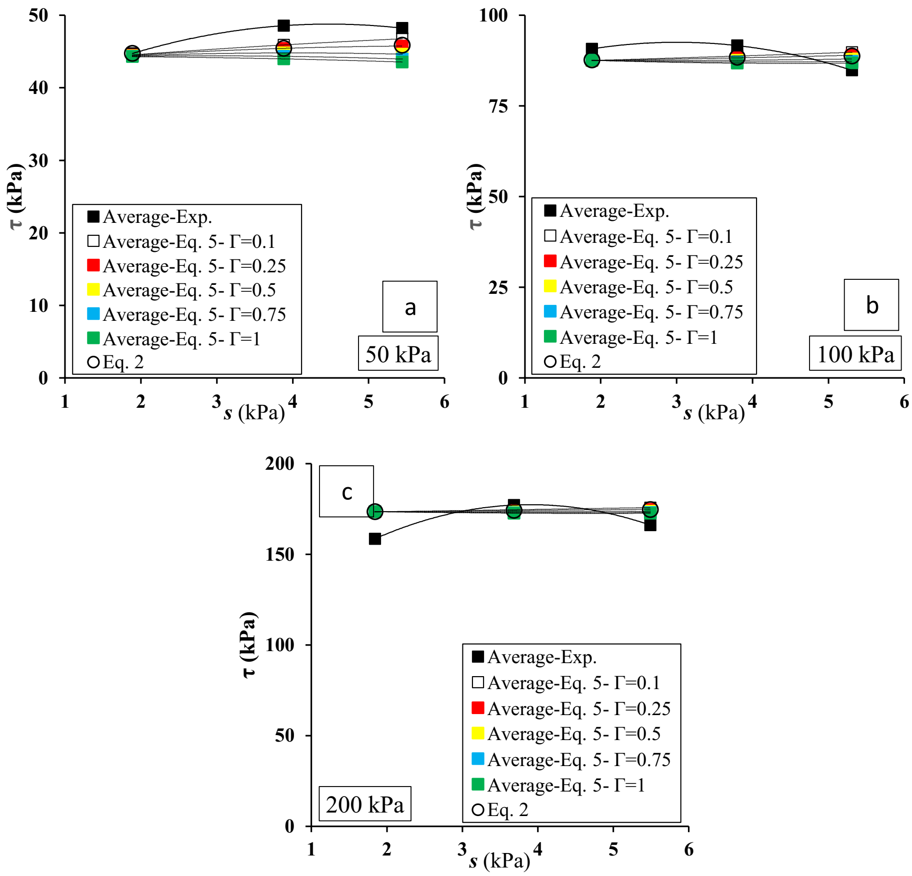

Fig. 6 Evaluation of the effective degree of saturation for the direct shear results for sand 2 at (**a**) σ = 50 kPa, (**b**) σ = 100 kPa and (**c**) σ = 200 kPa

been interpreted at the microstructural level. However, the use of effective degree of saturation for pure sands has been under-represented.

This paper, therefore, validated the use of the effective degree of saturation and its contribution to the effective stress at the microstructural level for two different sands. A series of direct shear and drained-unconfined compression tests were conducted.

The analyses showed a trivial difference between the use of effective degree of saturation and total degree of saturation. Therefore, the single stress variable, e.g. Bishop's equation, can be retrieved. The analysis hypothesised that shear strength evolution with suction should be associated with soil fabric changes.

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Declarations

Competing interests

I do confirm that there are no conflicts of interests to disclose in this paper.

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