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Thermal instability of a nanofluid layer under local thermal non-equilibrium

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Abstract

In this paper, we study the effect of local thermal non-equilibrium on the linear thermal instability in a horizontal layer of a Newtonian nanofluid. The nanofluid layer incorporates the effect of Brownian motion along with thermophoresis. A two-temperature model has been used for the effect of local thermal non-equilibrium among the particle and fluid phases. The linear stability is based on normal mode technique and for nonlinear analysis, a minimal representation of the truncated Fourier series analysis involving only two terms has been used. We observe that for linear instability, the value of Rayleigh number can be increased by a substantial amount on considering a bottom heavy suspension of nano particles. The effect of various parameters on Rayleigh number has been presented graphically. A weak nonlinear theory based on the truncated representation of Fourier series method has been used to find the concentration and the thermal Nusselt numbers. The behavior of the concentration and thermal Nusselt numbers is also investigated by solving the finite amplitude equations using a numerical method.

Keywords: Nanofluid; Instability; Natural convection

1 Background

Natural convection in fluids has been a topic of interest for researchers like Nield and Bejan [1], Pop and Ingham [2], Ingham and Pop [3], Vafai [4,5], Vadasz [6], due to its appearance in industry and machineries where heat transfer is encountered. In the present scenario where the focus has shifted from ordinary fluids to nanofluids to be used as heat transfer mediums, the phenomena needs to be critically studied for them [7]. Accordingly, natural convection has been studied in nanofluids by Buongiorno [8], Tzou [9,10], Kim et al. [11-13], and based on these results in the current decade by Nield and Kuznetsov [14], Kuznetsov and Nield [15] for the Horton-Rogers-Lapwood Problem of onset of thermal instability in a porous medium saturated by a nanofluid, using Darcy and Brinkman models, respectively, and incorporating the effects of Brownian motion and thermophoresis of Nanoparticles. They found that the critical thermal Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of

the nanoparticles. Based on these conservation equations, some recent studies have been performed by Agarwal et al. [16,17], Bhadauria and Agarwal [18,19], Agarwal and Bhadauria [20], Rana and Agarwal [21], Agarwal [22]. For the case of a simple nanofluid layer, studies have been performed by Yadav et al. [23] for fluid layer, Bhadauria and Agarwal [24], Agarwal and Bhadauria [25-27], for a rotating nanofluid layer.

In the above studies, investigations have been done assuming local thermal equilibrium(LTE) between the fluid and particle phases as well as fluid and solid-matrix phases, i.e., the temperature gradient at any location between the two phases is assumed to be negligible. But, as thermal lagging between the particle and fluid phases has been proposed by Vadasz as an explanation for the observed increase in the thermal conductivity of nanofluids, we need to study local thermal non equilibrium(LTNE) model. The LTNE model of convective heat transfer in porous medium has been dealt by Kuznetsov and Nield [15], Agarwal and Bhadauria [20], Bhadauria and Agarwal [18,19] to claim that the effect of LTNE can be significant for some circumstances but remains insignificant for typical dilute nanofluids.

Apart from the above few studies on thermal instability in nanofluids, no other study is available on this

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problem, therefore we intend to investigate this problem further. Assuming that the nanoparticles being suspended in the nanofluid using either surfactant or surface charge technology Nield and Kuznetsov [14], preventing the agglomeration and deposition of these on the porous matrix, in the present article, we study the linear and non-linear thermal instability in a nanofluid layer, using Rayleigh Bénard problem, considering LTNE between the fluid/particle interphase.

2 Methods

We consider a horizontal layer of nanofluid, confined between two horizontal boundaries at $z=0$ and $z=d$, heated from below and cooled from above. The boundaries are impermeable and perfectly thermally conducting. The fluid layer is extended infinitely in x and y -directions, and z -axis is taken vertically upward with the origin at the lower boundary. The local thermal non-equilibrium between the fluid and particle phase has been considered, thus heat flow has been described using two temperature model. T_h and T_c are the temperatures at the lower and upper walls respectively, the former being greater. The conservation equations for the total mass, momentum, thermal energy in the fluid phase, thermal energy in the particle phase, and nanoparticles, come out to be as below. A detailed derivation of the conservation equations has been dealt by Buongiorno, Tzou, and Nield and Kuznetsov:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\rho_f \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + [\phi \rho_p + (1-\phi) \{ \rho_f (1-\beta(T_f - T_c)) \}] g \tag{2}$$

$$(\rho c)_f \left[\frac{\partial T_f}{\partial t} + \mathbf{v} \cdot \nabla T_f \right] = k_f \nabla^2 T_f + (\rho c)_p \times \left[D_B \nabla \phi \cdot \nabla T_f + D_T \frac{\nabla T_f \cdot \nabla T_f}{T_f} \right] + \frac{h_{fp}}{1-\phi_0} (T_p - T_f) \tag{3}$$

$$\phi_0 (\rho c)_p \left[\frac{\partial T_p}{\partial t} + \mathbf{v}_D \cdot \nabla T_p \right] = \phi_0 k_p \nabla^2 T_p + h_{fp} (T_f - T_p) \tag{4}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = D_B \nabla^2 \phi + \frac{D_T}{T_c} \nabla^2 T_f \tag{5}$$

where $\mathbf{v} = (u, v, w)$ is the fluid velocity. In these equations, ρ is the fluid density, $(\rho c)_f$, $(\rho c)_p$, the effective heat capacities of the fluid and particle phases respectively, and k_f the effective thermal conductivity of fluid phase. D_B and D_T denote the Brownian diffusion coefficient and thermophoretic diffusion respectively. In the above equations, both Brownian transport and thermophoresis coefficients are taken to be time independent, in tune with the recent studies that neglect the effect of thermal transport attributed to the small size of the nanoparticles (as per recent arguments by Keblinski and Cahill [28]). Further, Thermophoresis and Brownian transport coefficients are assumed to be temperature - independent due to the fact that the temperature ranges under consideration are not far away from the critical value, and the volume averages over a representative elementary volume.

Assuming the temperature and volumetric fraction of the nanoparticles to be constant at the stress-free boundaries, we may assume the boundary conditions on T and ϕ to be:

$$\mathbf{v} = 0, \quad T = T_h, \quad \phi = \phi_1 \text{ at } z = 0, \tag{6}$$

$$\mathbf{v} = 0, \quad T = T_c, \quad \phi = \phi_0 \text{ at } z = d, \tag{7}$$

where ϕ_1 is greater than ϕ_0 . To non-dimensionalize the variables we take

$$(x^*, y^*, z^*) = (x, y, z)/d, \quad t^* = t \alpha_f / d^2, \quad \alpha_f = \frac{k_f}{(\rho c)_f}$$

$$(u^*, v^*, w^*) = (u, v, w) d / \alpha_f, \quad p^* = p d^2 / \mu \alpha_f,$$

$$\phi^* = \frac{\phi - \phi_0}{\phi_1 - \phi_0}, \quad T^* = \frac{T - T_c}{T_h - T_c},$$

Equations (1)–(7), then take the form (after dropping the asterisk):

$$\nabla \cdot \mathbf{v} = 0, \tag{8}$$

$$\frac{1}{Pr} \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla^2 \mathbf{v} - Rm \hat{e}_z + RaT \hat{e}_z - Rn \phi \hat{e}_z,$$

$$\frac{\partial T_f}{\partial t} + \mathbf{v} \cdot \nabla T_f = \nabla^2 T_f + \frac{N_B}{Le} \nabla \phi \cdot \nabla T_f + \frac{N_A N_B}{Le} \nabla T_f \cdot \nabla T_f + N_H (T_p - T_f), \tag{9}$$

$$\frac{\partial T_p}{\partial t} + \mathbf{v} \cdot \nabla T_p = \varepsilon \nabla^2 T_p + \gamma N_H (T_f - T_p) \tag{10}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = \frac{1}{Le} \nabla^2 \phi + \frac{N_A}{Le} \nabla^2 T_f, \tag{11}$$

$$\mathbf{v} = 0, \quad T = 1, \quad \phi = 1 \text{ at } z = 0 \tag{12}$$

$$\mathbf{v} = 0, \quad T = 1, \quad \phi = 1 \text{ at } z = 0 \tag{13}$$

$$\mathbf{v} = 0, \quad T = 0, \quad \phi = 0 \quad \text{at } z = 1 \quad (14)$$

Here

$$Pr = \frac{\mu}{\rho\alpha_f}, \text{ is the Prandtl number,}$$

$$Le = \frac{\alpha_f}{D_B}, \text{ is the Lewis number,}$$

$$Ra = \frac{\rho g \beta d^3 (T_h - T_c)}{\mu \alpha_f}, \text{ is the Rayleigh}$$

–Darcy number,

$$Rm = \frac{[\rho_p \phi_0 + \rho (1 - \phi_0)] g d^3}{\mu \alpha_f}, \text{ is the basic density}$$

Rayleigh number,

$$Rn = \frac{(\rho_p - \rho) (\phi_1 - \phi_0) g d^3}{\mu \alpha_f}, \text{ is the concentration}$$

Rayleigh number,

$$N_B = \frac{(\rho c)_p (\phi_1 - \phi_0)}{(\rho c)_f}, \text{ is the modified particle}$$

density increment,

and

$$N_A = \frac{D_T (T_h - T_c)}{D_B T_c (\phi_1 - \phi_0)}, \text{ is the modified diffusivity}$$

ratio which is similar to the Soret parameter that arises in cross diffusion in thermal instability,

$$N_H = \frac{h_{fp} d^2}{(1 - \phi_0) k_f}, \text{ is the interphase heat transfer}$$

parameter or the Nield number,

$$\gamma = \frac{(1 - \phi_0) (\rho c)_f}{\phi_0 (\rho c)_p}, \text{ is the modified thermal capacity}$$

ratio,

$$\varepsilon = \frac{k_p / (\rho c)_p}{k_f (\rho c)_f}, \text{ is the thermal diffusivity ratio.}$$

3 Basic solution

The basic state of the nanofluid layer is assumed to be at rest, therefore we have

$$\mathbf{v} = 0, \quad p = p_b(z), \quad T_f = T_{fb}(z), \quad T_p = T_{pb}(z), \quad \phi = \phi_b(z) \quad (15)$$

Substituting eq. (15) in Eqs. (10)–(12), we get

$$\frac{d^2 T_{fb}}{dz^2} + \frac{N_B}{Le} \frac{d\phi_b}{dz} \frac{dT_{fb}}{dz} + \frac{N_A N_B}{Le} \left(\frac{dT_{fb}}{dz} \right)^2 + N_H (T_p - T_f) = 0 \quad (16)$$

The second and third terms in equation (16) are small [Nield and Kuznetsov [29]], so we have:

$$\frac{d^2 T_{pb}}{dz^2} = 0, \quad \frac{d^2 T_{fb}}{dz^2} = 0, \quad \frac{d^2 \phi_b}{dz^2} = 0, \quad (17)$$

The boundary conditions for solving the above equation are found as

$$T_{fb} = T_{pb} = 1, \quad \phi_b = 1, \quad \text{at } z = 0, \quad (18)$$

$$T_{fb} = T_{pb} = 0, \quad \phi_b = 0, \quad \text{at } z = 1. \quad (19)$$

On solving eq. (17), subject to conditions (18) and (19), we obtain the basic temperature and concentration fields as

$$T_{fb} = T_{pb} = 1 - z \quad (20)$$

$$\phi_b = 1 - z. \quad (21)$$

4 Stability analysis

To superimpose the perturbations on the basic state we write

$$\mathbf{v} = \mathbf{v}', \quad p = p_b + p', \quad T_f = T_{fb} + T_f', \quad T_p = T_{pb} + T_p', \quad \phi = \phi_b + \phi'. \quad (22)$$

We substitute the expression (22) in Eqs. (9)–(12), and use the expressions (20) and (21). For simplicity, we consider the case of two dimensional rolls, assuming all physical quantities to be independent of y . The reduced dimensionless governing equations after eliminating the pressure term are

$$\frac{1}{Pr} \frac{\partial}{\partial t} (\nabla_1^2 \psi) = \nabla_1^4 \psi - Ra \frac{\partial T_f'}{\partial x} + Rn \frac{\partial \phi'}{\partial x} + \frac{1}{Pr} \frac{\partial (\psi, \nabla_1^2 \psi)}{\partial (x, z)} \quad (23)$$

$$\frac{\partial T_f'}{\partial t} + \frac{\partial \psi}{\partial x} = \nabla_1^2 T_f' + \frac{\partial (\psi, T_f')}{\partial (x, z)} + N_H (T_p - T_f) \quad (24)$$

$$\frac{\partial T_p'}{\partial t} + \frac{\partial \psi}{\partial x} = \varepsilon \nabla_1^2 T_p' + \frac{\partial (\psi, T_p')}{\partial (x, z)} + \gamma N_H (T_f - T_p) \quad (25)$$

$$\frac{\partial \phi'}{\partial t} + \frac{\partial \psi}{\partial x} = \frac{1}{Le} \nabla_1^2 \phi' + \frac{N_A}{Le} \nabla_1^2 T_f' + \frac{\partial (\psi, \phi')}{\partial (x, z)} \quad (26)$$

The equations (23)–(26) are solved subject to stress-free, isothermal, iso-nanoconcentration boundary conditions:

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T_f' = T_p' = \phi' = 0 \quad \text{at } z = 0, 1 \quad (27)$$

For linear stability analysis we use the normal mode technique and then critical Rayleigh numbers for

stationary and oscillatory onset of convection and the frequency of oscillations, ω , are given by

$$Ra^{st} = \frac{\delta^6}{\alpha_c^2} \left(1 + \frac{N_H(\varepsilon - 1)}{\varepsilon\delta^2 + (1 + \gamma)N_H} \right) - Rn \left\{ N_A - Le - \frac{N_H Le(\varepsilon - 1)}{\varepsilon\delta^2 + (1 + \gamma)N_H} \right\} \tag{28}$$

$$Ra^{osc} = \frac{\delta^2}{\alpha^2} \left[\delta^4 \left(1 + \frac{N_H(\varepsilon - 1)P_1}{P_1^2 + \omega^2} \right) - \frac{\omega^2}{Pr} \left(1 - \frac{N_H(\varepsilon - 1)\delta^2}{P_1^2 + \omega^2} \right) \right] - \frac{Rn}{(\delta^2/Le)^2 + \omega^2} \left[\frac{\delta^2}{Le} \left(P_2 - \frac{N_H(\varepsilon - 1)\delta^2 P_1}{P_1^2 + \omega^2} \right) - \omega^2 \left(1 - \frac{N_H(\varepsilon - 1)\delta^2}{P_1^2 + \omega^2} \right) \right] \tag{29}$$

$$\omega_c^2 = \frac{-X_2 + \sqrt{X_2^2 - 4X_1X_3}}{2X_1} \tag{30}$$

where

$$\delta^2 = \pi^2 + \alpha_c^2 \text{ and } \alpha_c = \frac{\pi}{\sqrt{2}}$$

Also

$$X_1 = P_3 + \frac{\delta^4}{\alpha^2}$$

$$X_2 = \frac{P_3\delta^4}{Le^2} + P_3P_1^2 + P_3N_HP_1(\varepsilon - 1) + \frac{\delta^8}{\alpha^2Le^2} + \frac{\delta^4P_1^2}{\alpha^2} - \frac{N_H(\varepsilon - 1)\delta^6}{\alpha^2} + RnP_2 + \frac{Rn\delta^2}{Le}$$

$$X_3 = P_3P_1^2\frac{\delta^4}{Le^2} + P_3P_1N_H(\varepsilon - 1)\frac{\delta^4}{Le^2} + \frac{\delta^8P_1^2}{\alpha^2Le^2} - \frac{\delta^{10}N_H(\varepsilon - 1)}{\alpha^2Le^2} + RnP_2P_1^2 - RnP_1N_H\delta^2(\varepsilon - 1) + \frac{RnP_1^2\delta^2}{Le} - \frac{RnN_H\delta^4(\varepsilon - 1)}{Le}$$

These expressions were obtained from Nield and Kuznetsov by dropping the terms pertaining to porous media.

For local nonlinear stability analysis we take the following Fourier expressions:

$$\psi = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{mn} \sin(m\alpha x) \sin(n\pi z) \tag{31}$$

$$T_f = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} B_{mn}(t) \cos(m\alpha x) \sin(n\pi z) \tag{32}$$

$$T_p = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C_{mn}(t) \cos(m\alpha x) \sin(n\pi z) \tag{33}$$

$$\phi = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} D_{mn}(t) \cos(m\alpha x) \sin(n\pi z) \tag{34}$$

In what follows we take the modes (1, 1) for stream function, and (0, 2) and (1, 1) for temperature and nanoparticle concentration

$$\psi = A_{11}(t) \sin(\alpha x) \sin(\pi z) \tag{35}$$

$$T_f = B_{11}(t) \cos(\alpha x) \sin(\pi z) + B_{02}(t) \sin(2\pi z) \tag{36}$$

$$T_p = C_{11}(t) \cos(\alpha x) \sin(\pi z) + C_{02}(t) \sin(2\pi z) \tag{37}$$

$$\phi = D_{11}(t) \cos(\alpha x) \sin(\pi z) + D_{02}(t) \sin(2\pi z) \tag{38}$$

where the amplitudes $A_{11}(t), B_{11}(t), B_{02}(t), C_{11}(t), C_{02}(t), D_{11}(t)$ and $D_{02}(t)$ are functions of time and are to be determined. Substituting equations (35)–(38) in equations (23)–(26) taking the orthogonality condition with the eigenfunctions associated with the considered minimal model, we get

$$\frac{dA_{11}(t)}{dt} = \frac{Pr}{\delta^2} \{ \alpha [RnD_{11}(t) - RaB_{11}(t)] - \delta^4 A_{11}(t) \} \tag{39}$$

$$\frac{dB_{11}(t)}{dt} = N_H [C_{11}(t) - B_{11}(t)] - [\alpha A_{11}(t) + \delta^2 B_{11}(t) + \pi\alpha A_{11}(t)B_{02}(t)] \tag{40}$$

$$\frac{dB_{02}(t)}{dt} = \frac{1}{2} \{ \pi\alpha A_{11}(t)B_{11}(t) - 8\pi^2 B_{02}(t) + 2N_H [C_{02} - B_{02}] \} \tag{41}$$

$$\frac{dC_{11}(t)}{dt} = - \{ \alpha A_{11}(t) + \varepsilon\delta^2 C_{11}(t) - \gamma N_H [B_{11}(t) - C_{11}(t)] + \pi\alpha A_{11}(t)C_{02}(t) \} \tag{42}$$

$$\frac{dC_{02}(t)}{dt} = \frac{1}{2} \{ \pi\alpha A_{11}(t)C_{11}(t) - 8\pi^2 \varepsilon C_{02}(t) + 2\gamma N_H [B_{02}(t) - C_{02}(t)] \} \tag{43}$$

$$\frac{dD_{11}(t)}{dt} = - \alpha A_{11}(t) - \frac{\delta^2}{Le} D_{11}(t) - \frac{N_A \delta^2}{Le} B_{11}(t) - \pi\alpha A_{11}(t)D_{02}(t) \tag{44}$$

$$\frac{dD_{02}(t)}{dt} = - \frac{4\pi^2}{Le} [D_{02}(t) + N_A B_{02}(t)] + \pi\alpha A_{11}(t)D_{11}(t) \tag{45}$$

The above system of simultaneous autonomous ordinary differential equations be subsequently solved numerically using Runge-Kutta-Gill method.

5 Heat and nanoparticle concentration transport
 The thermal Nusselt number for fluid phase, $Nu_f(t)$ is defined as

$$Nu_f(t) = \frac{\text{Heat transport by (conduction + convection)}}{\text{Heat transport by conduction}}$$

$$= 1 + \left[\frac{\int_0^{2\pi/\alpha_c} \left(\frac{\partial T_f}{\partial z} \right) dx}{\int_0^{2\pi/\alpha_c} \left(\frac{\partial T_b}{\partial z} \right) dx} \right]_{z=0} \tag{46}$$

substituting equations (20) and (36) in eq. (46), we get

$$Nu_f(t) = 1 - 2\pi B_{O2}(t) \tag{47}$$

The thermal Nusselt number for particle phase, $Nu_p(t)$, and the nanoparticle concentration Nusselt number, $Nu_\phi(t)$, are defined similar to the thermal Nusselt number. Following the procedure adopted for arriving at $Nu_f(t)$, one can obtain the expression for $Nu_p(t)$ and $Nu_\phi(t)$ in the form:

$$Nu_p(t) = 1 - 2\pi C_{O2}(t) \tag{48}$$

$$Nu_\phi(t) = (1 - 2\pi D_{O2}(t)) + N_A (1 - 2\pi B_{O2}(t)) \tag{49}$$

6 Results and discussion

The expressions for the Rayleigh number for both stationary and oscillatory convection have been presented analytically in equations (28) and (29) respectively.

Considering the expression for stationary convection in eq. (28), we have

$$Ra^{st} = \frac{\delta^6}{\alpha_c^2} \left(1 + \frac{N_H(\epsilon - 1)}{\epsilon\delta^2 + (1 + \gamma)N_H} \right) - Rn \left\{ N_A - Le - \frac{N_H(\epsilon - 1)\delta^2}{\epsilon\delta^2 + (1 + \gamma)N_H} \right\} \tag{50}$$

For thermal equilibrium condition $N_H = 0$, so we get

$$Ra^{st} = \frac{\delta^6}{\alpha_c^2} - Rn(N_A - Le) \tag{51}$$

This result agrees with the result obtained by Nield and Kuznetsov under LTNE.

In Figure 1, we present linear stability curves showing the stationary and oscillatory modes of convection. In the figure we see that the region of over stability under-lies the region of damped oscillations, i.e. at the start of instability, oscillatory or periodic pattern of motion prevails. Therefore we can say that the Principle of Exchange of stabilities is invalid [30,31]. This can be explained as since oscillatory motion is prevalent at the onset of convection, the restoring forces provoked are strong enough to prevent the system from tending away from equilibrium.

In Figure 2(a-f) we present neutral stability curves for Ra^{st} versus the wavenumber α for the fixed values of $Rn, Le, N_A, \epsilon, \gamma$ and N_H , respectively, with variation in one of these parameters. In all these plots, it is interesting to note that the value of Ra starts from a higher note, falls rapidly with increasing α , and then increases steadily. The Figure 2(a), (b), (d) and (e) correspond to the variation of Ra with respect to α at different values of concentration

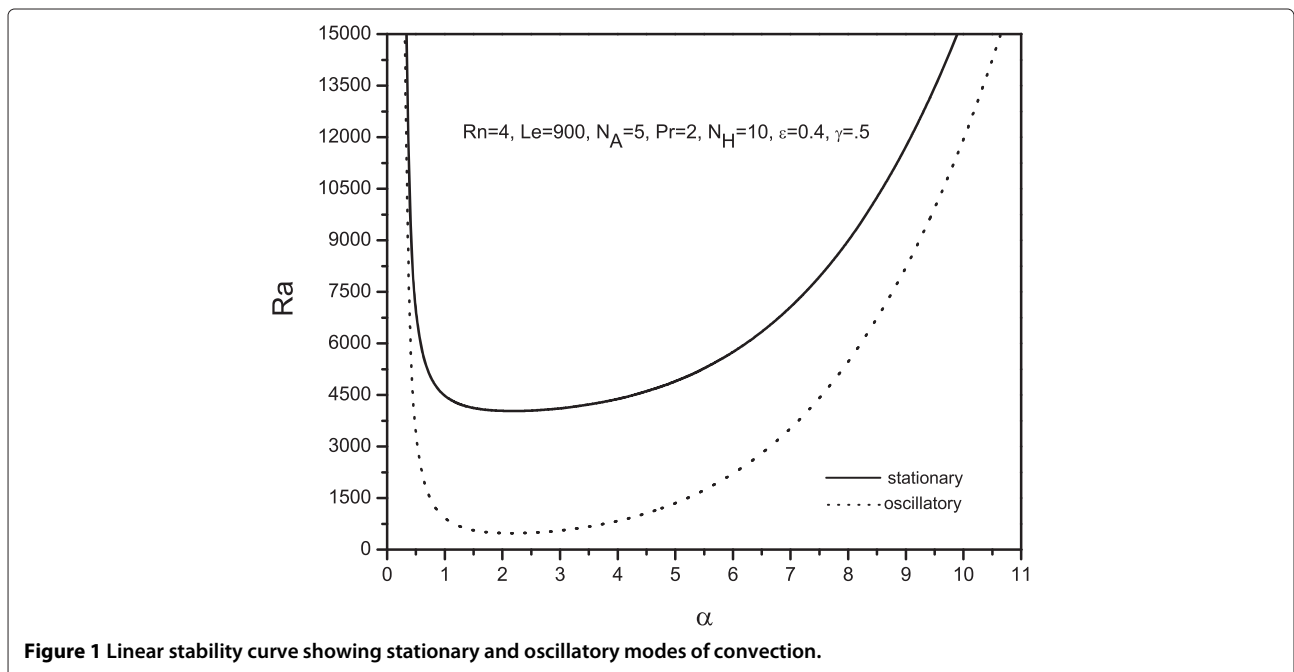
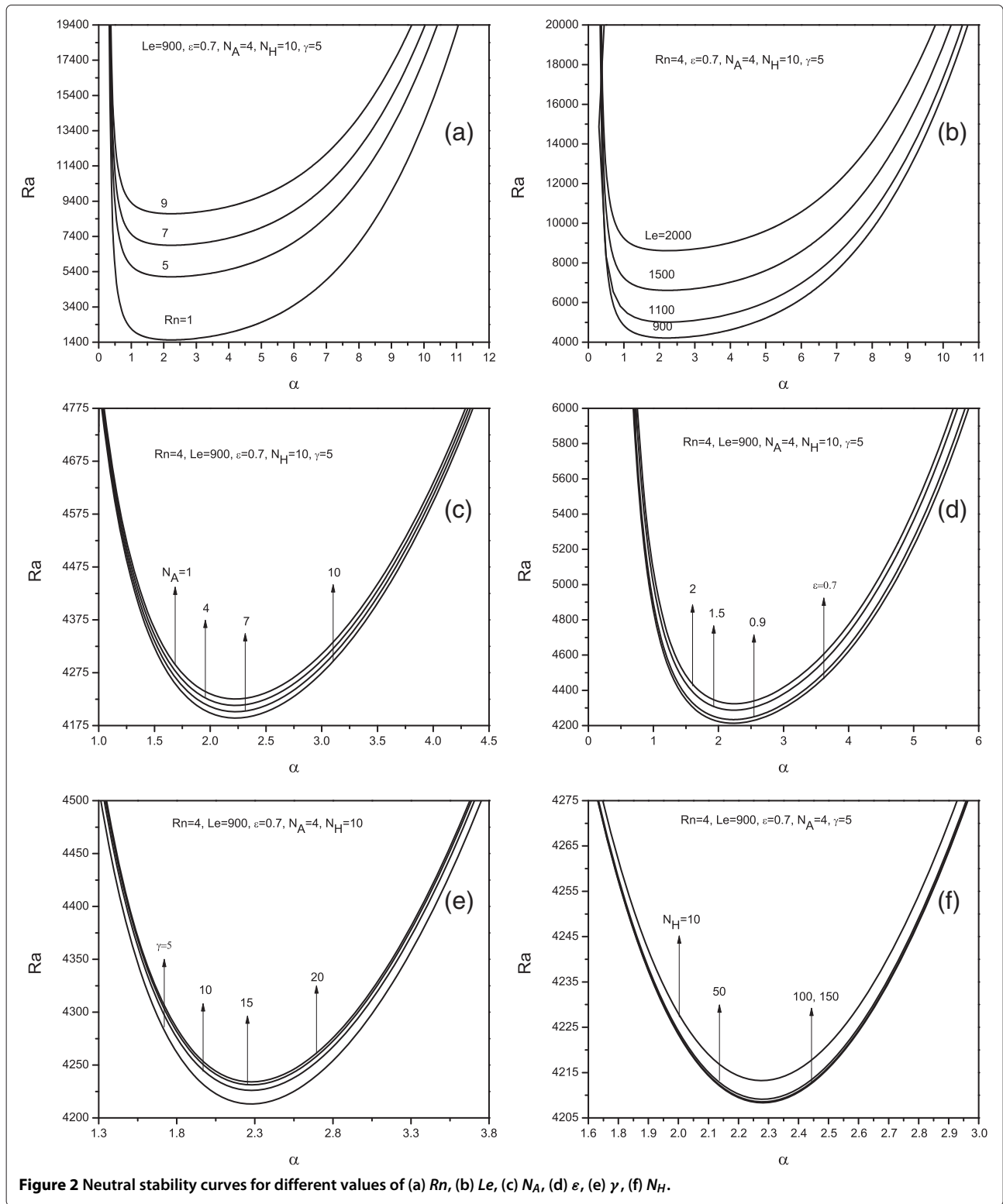


Figure 1 Linear stability curve showing stationary and oscillatory modes of convection.



Rayleigh number Rn , Lewis number Le , thermal diffusivity ratio ε , and modified thermal capacity ratio γ . These plots reveal that on increasing the value of these parameters, the value of Ra_{cr} increases, i.e. the system tends to stabilize.

However, in the Figure 2(c) and (f), it is to be noted that the effect of the parameters N_A and N_H is to destabilize the system. As we increase their value, the value of Ra_{cr} decreases, indicating that the convection starts earlier.

In the Figure 3, we draw the neutral stability curve for LTNE and LTE. We see that the value of Rayleigh number Ra is less in case of LTNE than LTE. This implies that convections starts earlier in the case of LTNE than LTE. The observed phenomenon may be attributed to the fact that because of temperature difference between the fluid and particle phases, there occurs transfer of energy between them. This leads to a chaotic state and enhances the onset of convection in case of LTNE.

In the Figure 4, we compare the Rayleigh number Ra for nanofluids with ordinary fluids under thermal nonequilibrium conditions. It is to be noted that the value of Rayleigh number is less in the case of ordinary fluid than nanofluid, or to say convection sets in earlier in ordinary fluids than nanofluids. This implies that the thermal conductivity of nanofluids is higher than ordinary fluids.

The nature of critical values of Rayleigh number Ra and the critical values of wave number α as functions of inter phase heat transfer parameter or Nield number for fluid/nanoparticle inter phase, N_H , for $Rn = 4, Le = 900, N_A = 1, \varepsilon = 0.04, \gamma = 5$, with a variation in the value of one of these parameters, are shown in Figures 5 and 6 respectively. For very small and large values of N_H , we observe that the stability criterion is independent of its value, and that the value of N_H play a significant role in the stability criterion only in the intermediate range. The reason behind this state being, that at $N_H \rightarrow 0$, there occurs almost zero heat transfer between fluid/nanoparticle inter phase, and the properties of nanoparticle do not interfere in the onset of convection. While, when $N_H \rightarrow \infty$, the two

have attained almost the equal temperatures and behave as a single phase. Between these two extremes, a LTNE effect is observed being attributed to N_H .

In the Figure 5, we present the variation of critical Rayleigh number Ra_{cr} with Nield number for the fluid/particle inter phase N_H for different parameters. The figure indicates that the value of Ra_{cr} decreases from high values for very small N_{HP} to small LTNE value for large N_{HP} . The system tends to destabilize for the intermediate values of N_{HP} . The effect of the parameters concentration Rayleigh number Rn , Lewis number Le , thermal diffusivity ratio ε , and modified thermal capacity ratio γ , on the system is to inhibit the decrease in the value of the critical Rayleigh number Ra_{cr} , thus preventing the system from destabilization. While for the other parameter, modified diffusivity ratio N_A , on increasing its values, the value of Ra_{cr} falls further trending the system towards destabilization.

In the Figure 6, we have exhibited the critical wavenumber α_c as a function of N_H for both stationary and oscillatory convection. We observe that value of critical wavenumber α_c decreases with increasing N_H from high values when N_H is small, to its minimum LTNE value for intermediate N_H , and finally bounces back to higher values for large N_H . This implies that the the value of critical wavenumber α_c approaches to its LTE value when $N_H \rightarrow 0$ and $N_H \rightarrow \infty$. This is quite obvious as the corresponding physical situation are anonymous. At $N_H \rightarrow 0$, the particle phase does not interfere with the thermal field of the fluid, which is free to act independently, while as $N_H \rightarrow \infty$, the particle/fluid phase have attained

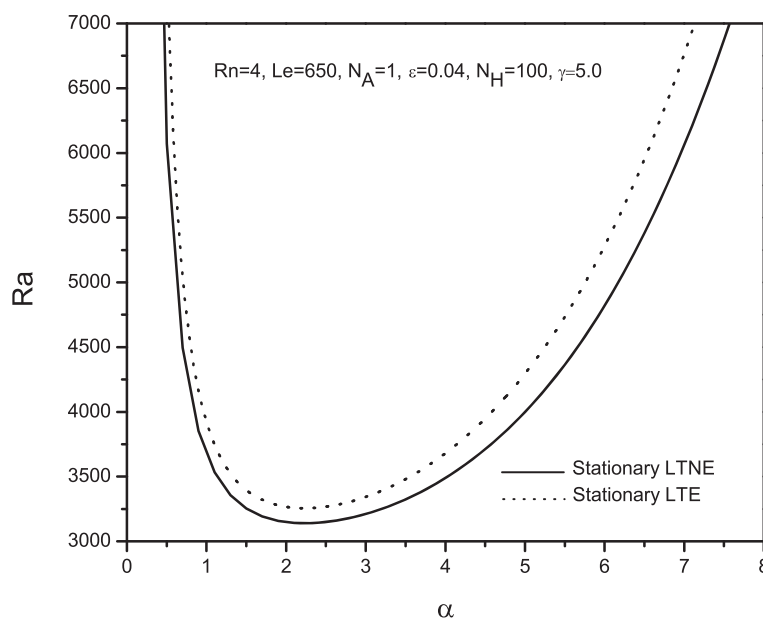


Figure 3 Comparison of the value of Rayleigh number Ra for LTNE and LTE.

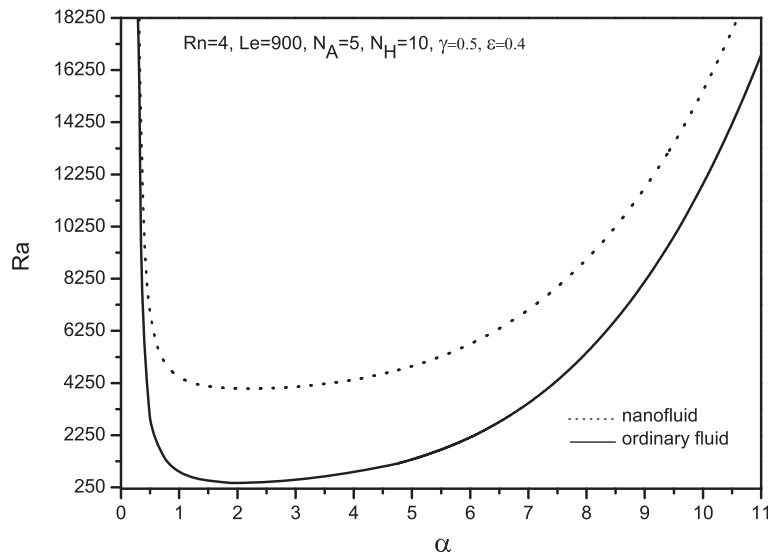


Figure 4 Comparison of the value of Rayleigh number Ra for Nanofluid with Ordinary fluid.

the identical temperatures, and behave as single phase only. We conclude that as time passes, and heat intensifies, the nanofluids behave more like a single phase fluid rather than like a conventional solid-liquid mixture. For the parameters, $Rn, Le, N_A,$ and $\epsilon,$ we see that an increase in their values has effect on the critical value of wave number α_c only when $N_H \rightarrow 0$ and $N_H \rightarrow \infty.$ For intermediate values of $N,$ i.e. when α_c falls to its minimum LTNE value, the value of α_c remains independent of these parameters. But for $\gamma,$ as we increase its value, minimum value of α_c decreases. Also, we notice that the minimum value of α_c is less for oscillatory convection as compared to stationary convection.

The variation of critical frequency ω_c^2 for the oscillatory mode of convection with Nield number N_H for different parameter values is shown in Figure 7(a)–(f). It is clear from these figures that the critical frequency decreases from a constant value, when N_H is very small to its minimum value, and then with further increase in $N_H,$ it bounces back to another constant value for large values of $N_H.$ Figure 7(a) displays the effect of concentration Rayleigh number Rn on the value of critical frequency $\omega_c^2.$ We can observe that an increase in the value of Rn inhibits the decrease in the value of critical frequency $\omega_c^2.$ A similar effect on critical frequency ω_c^2 has been observed in Figure 7(d), 7(e) and 7(f) with modified thermal capacity ratio $\gamma,$ thermal diffusivity ratio $\epsilon,$ and Prandtl number $Pr.$ The parameters, Lewis number $Le,$ and modified diffusivity ratio N_A do not have a very pronounced effect on the value of critical frequency $\omega_c^2.$ In general, the value of critical frequency ω_c^2 decreases from its LTE value to its LTNE value, and then increases back to attain another constant value in the intermediate range.

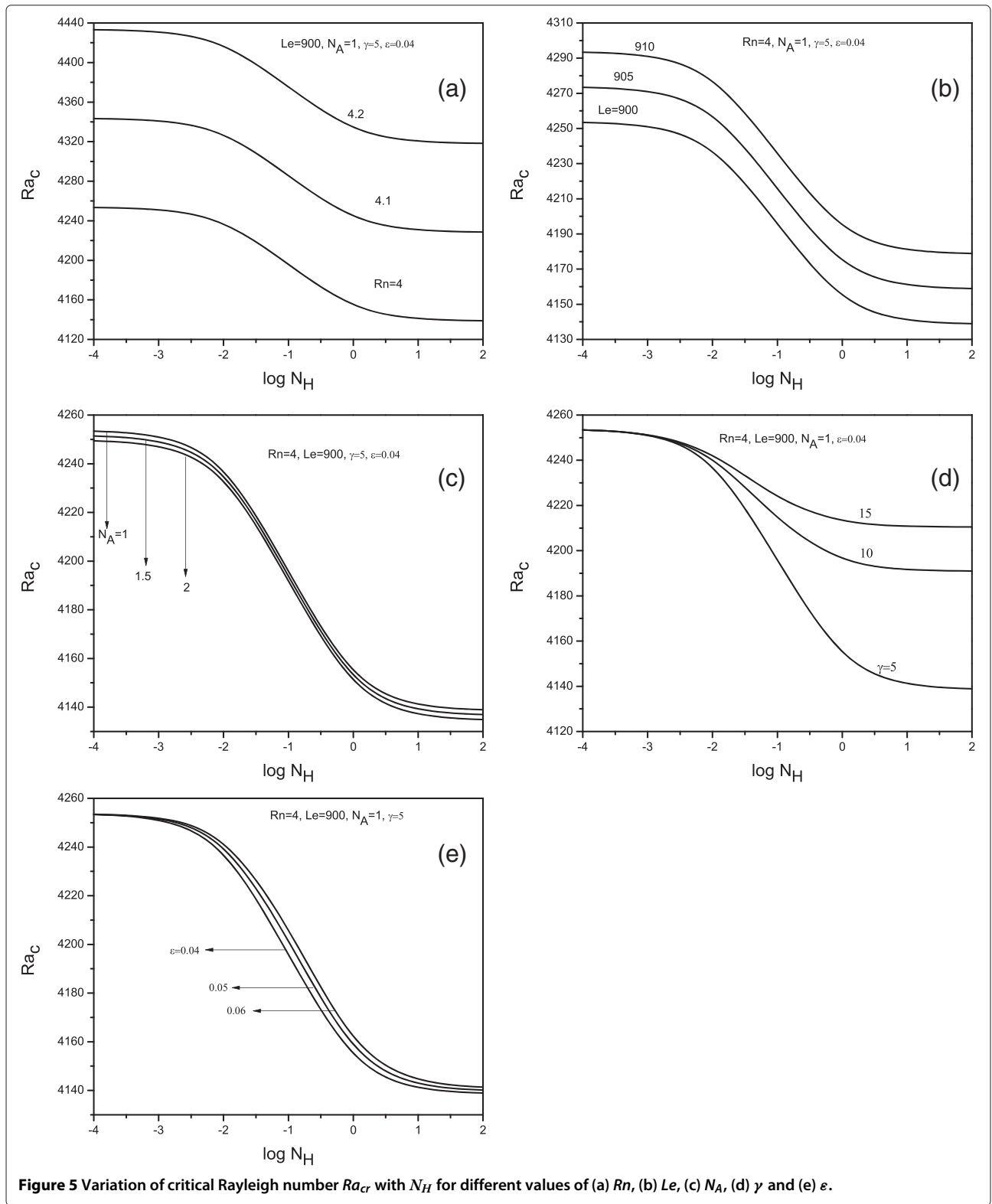
6.1 Non-linear unsteady stability analysis

The linear solutions exhibit a considerable variety of behavior of the system and the transition from linear to non-linear convection can be quite complicated, but interesting to deal with. We need to study the time dependent results to analyze the same. This transition can be well understood by the analysis of the seven coupled ordinary differential eqs. (39)–(45) whose solutions give a detailed description of the two-dimensional problem. We solve the eqs. (39)–(45) numerically, using Runge–Kutta–Gill Method, and calculate Nusselt numbers as function of time $t.$ Also the seven-mode Differential eqs. (39)–(45) have an interesting property in phase-space:

$$\begin{aligned} & \frac{\partial \dot{A}_{11}}{\partial A_{11}} + \frac{\partial \dot{B}_{11}}{\partial B_{11}} + \frac{\partial \dot{B}_{02}}{\partial B_{02}} + \frac{\partial \dot{C}_{11}}{\partial C_{11}} + \frac{\partial \dot{C}_{02}}{\partial C_{02}} + \frac{\partial \dot{D}_{11}}{\partial D_{11}} + \frac{\partial \dot{D}_{02}}{\partial D_{02}} \\ &= - \left[\delta^2 Pr + N_H + \delta^2 + 4\pi^2 + N_H + \gamma_p N_H \right. \\ & \quad \left. + \epsilon \delta^2 + 4\pi^2 \epsilon + \gamma N_H + \frac{\delta^2}{Le} + \frac{4\pi^2}{Le} \right] < 0 \end{aligned} \tag{52}$$

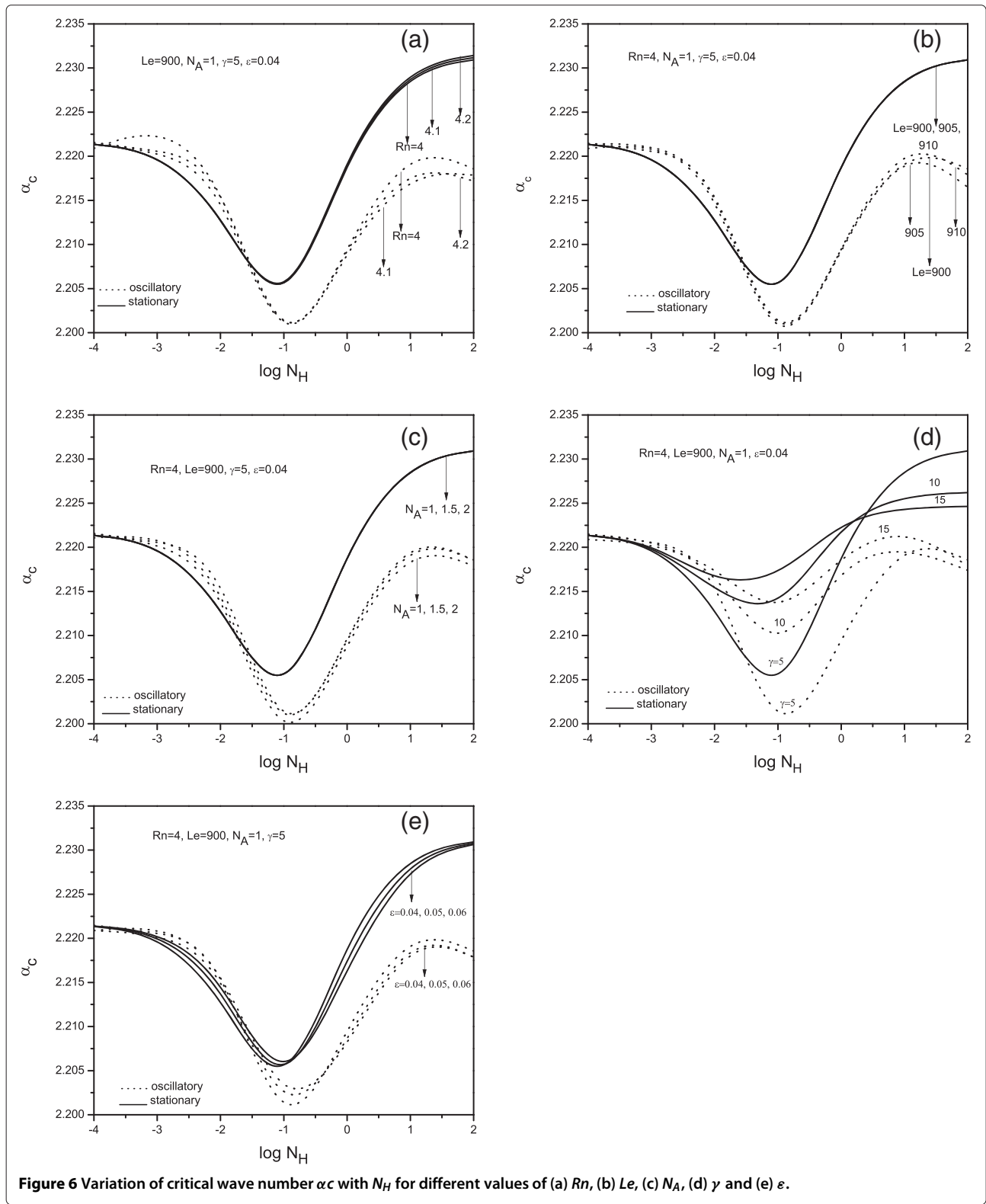
which indicates that the system is dissipative and bounded. This implies that the trajectories of the eqs. (39)–(45) will be attracted to a set of measure zero in the phase space, in other words they will approach to a fixed point, a limit cycle or to a unknown attractor.

The nature of Nusselt numbers, $Nu^\phi, Nu(\text{fluid}),$ and $Nu(\text{particle}),$ as a function of time $t,$ for $Rn = 4, Le = 10, N_A = 5, Ra = 5000, Pr = 0.75, \epsilon = 0.04, N_H = 20,$



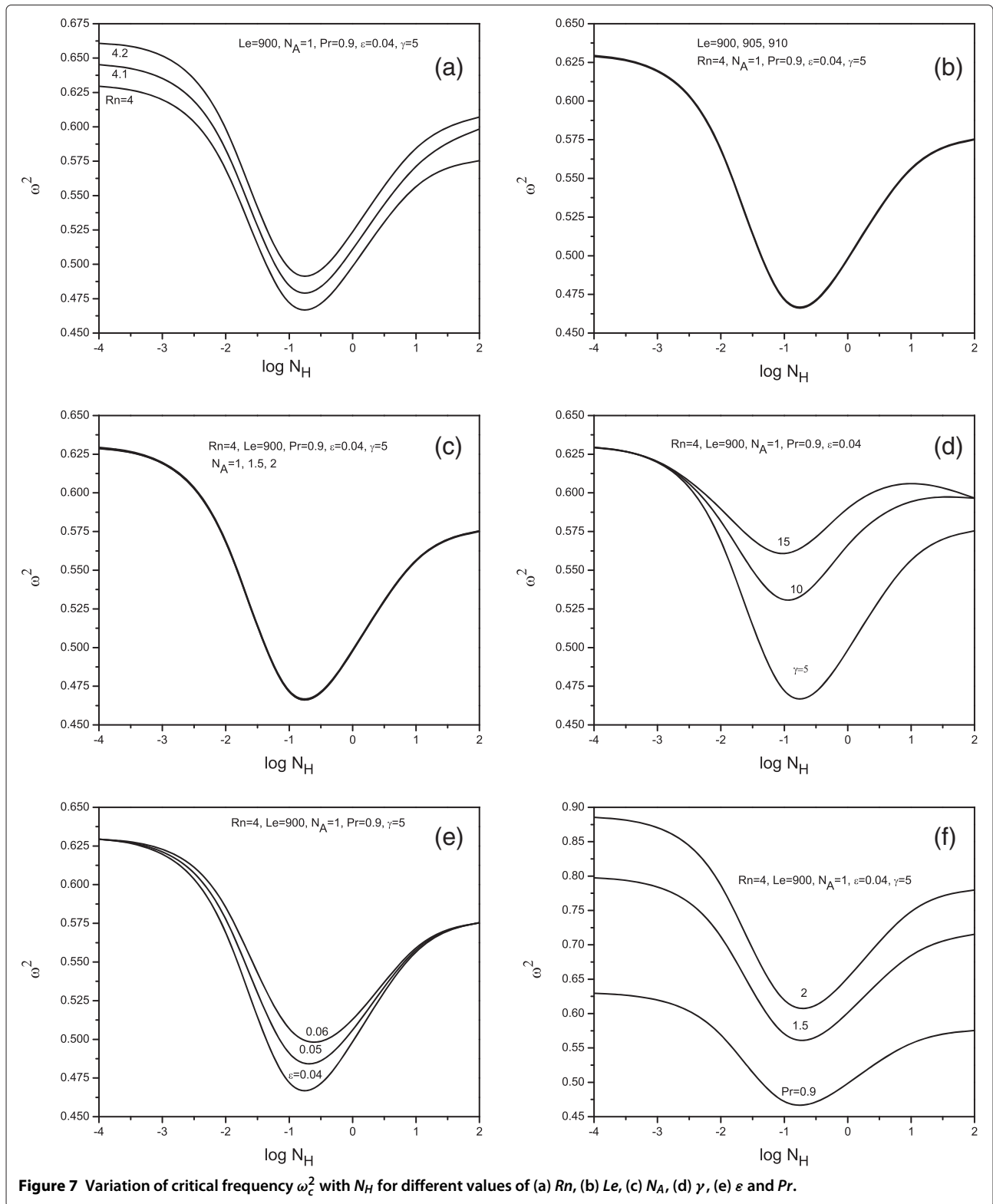
and $\gamma = 0.5$ with a variation in the value of one of these parameters, is shown in Figures 8 and 9 respectively. These figures indicate that initially, when time is

small, there occurs large scale oscillations in the values of Nu^ϕ , $Nu(\text{fluid})$, and $Nu(\text{particle})$, indicating an unsteady rate of mass and heat transfer in the fluid and particle



phases. As time passes by, these approach to steady values, corresponding to a near conduction instead of convection stage.

In the Figure 8, the transient nature of concentration Nusselt number or Sherwood number (as some researchers name it), is visible. We can observe that



the effect of increasing the value of modified diffusivity ratio or Soret parameter N_A , thermal Rayleigh number Ra , Prandtl number Pr , modified thermal capacity

ratio γ , and the Nield number N_H , on the amplitude of oscillations is to increase it, i.e., an increase in the value of these parameters brings about an increase

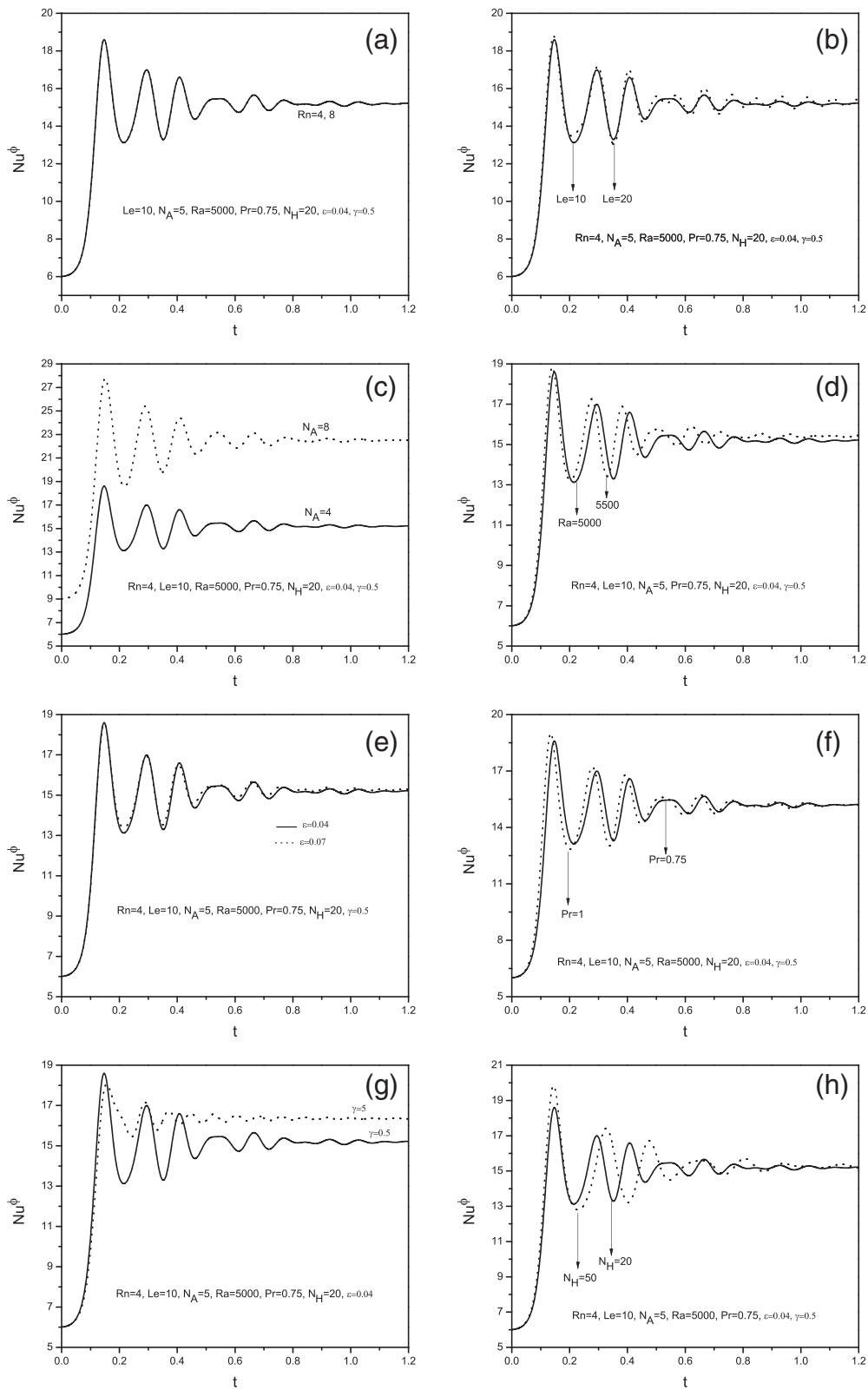


Figure 8 Variation of Concentration Nusselt Number Nu^ϕ with time t for different values of (a) Rn , (b) Le , (c) N_A , (d) Ra , (e) ϵ , (f) Pr , (g) γ , and (h) N_H .

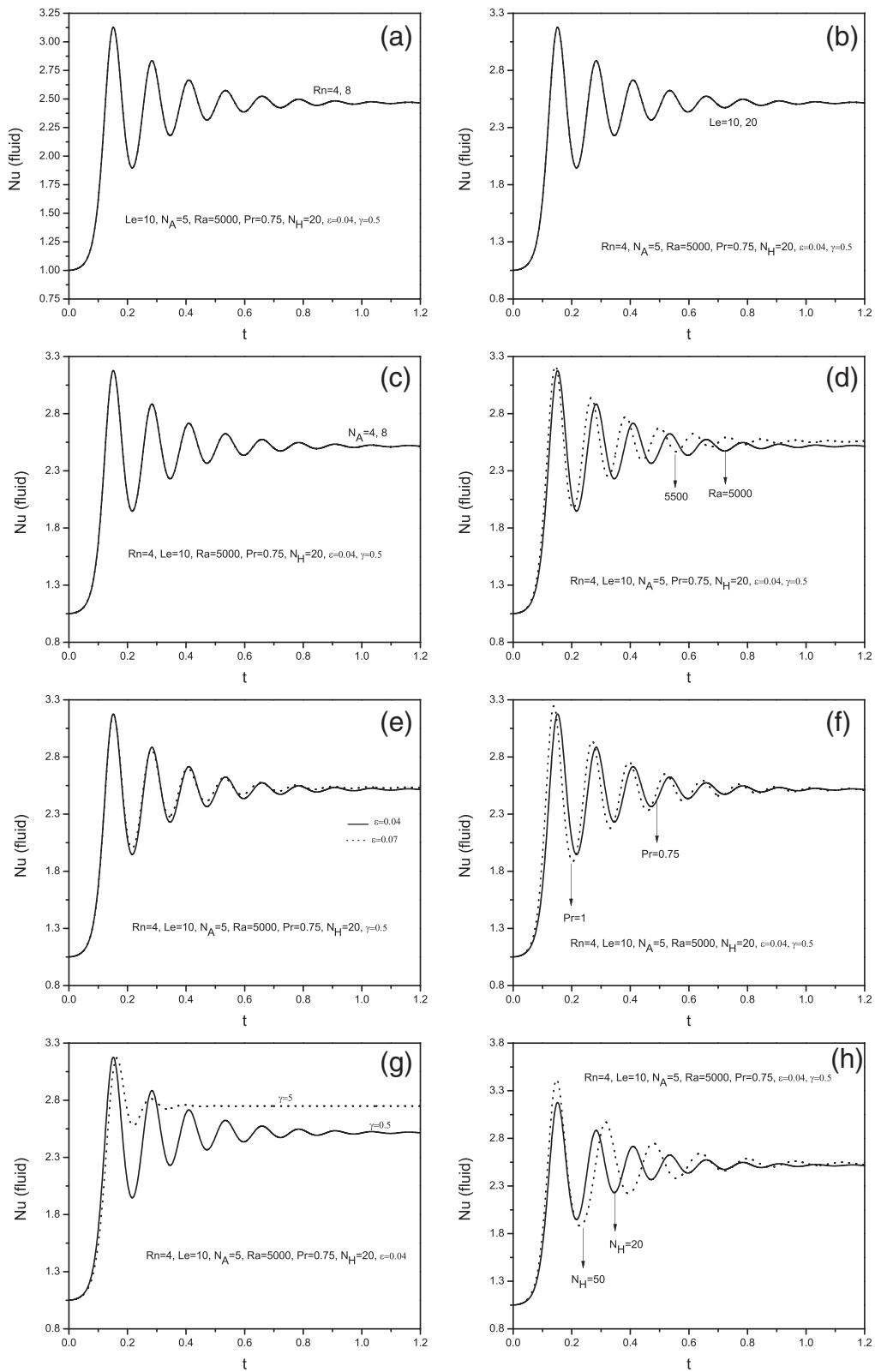


Figure 9 Variation of Nusselt Number $Nu(\text{fluid})$ with time t for different values of (a) Rn , (b) Le , (c) N_A , (d) Ra , (e) ε , (f) Pr , (g) γ , and (h) N_H .

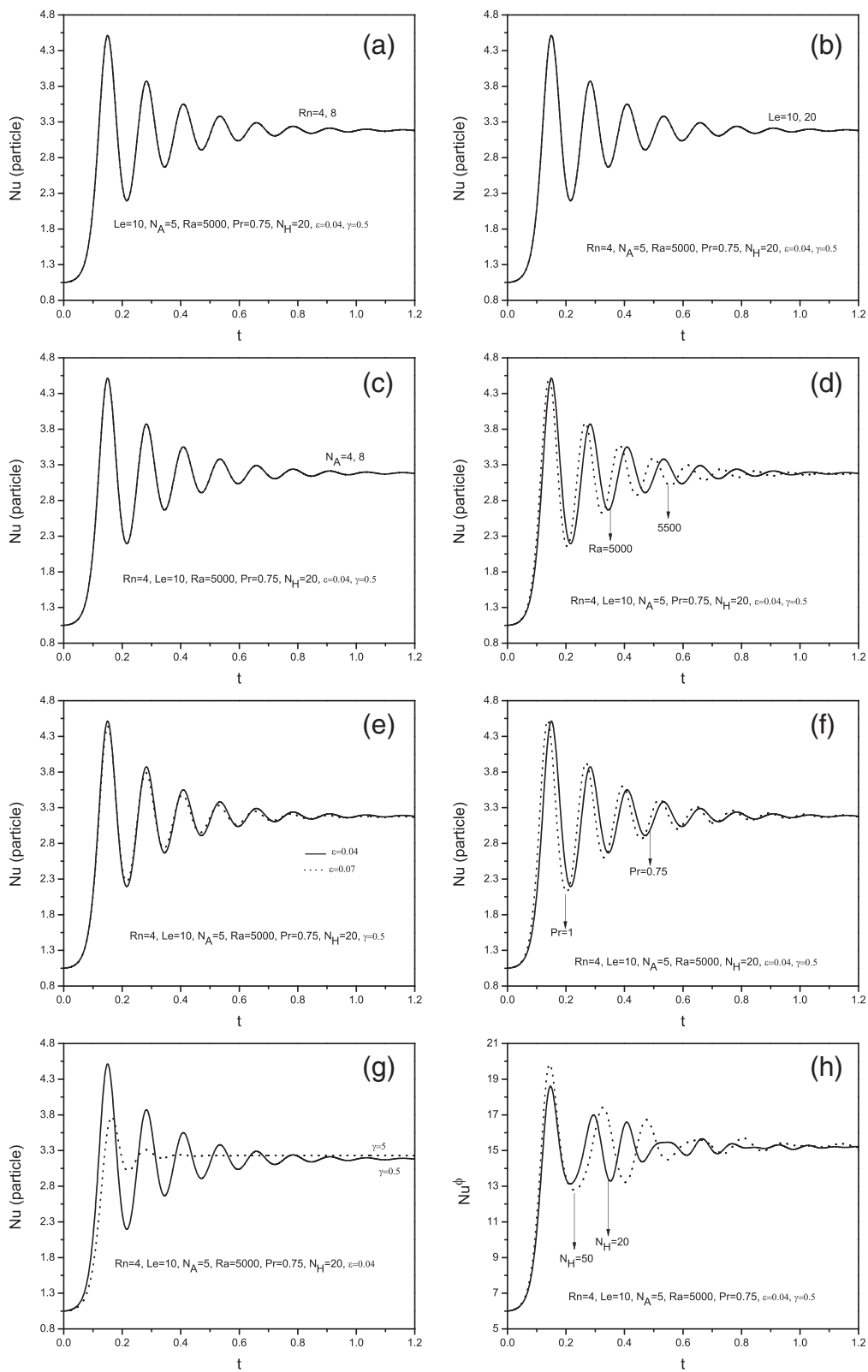


Figure 10 Variation of Nusselt Number Nu (particle) with time t for different values of (a) Rn , (b) Le , (c) N_A , (d) Ra , (e) ϵ , (f) Pr , (g) γ , and (h) N_H .

in the rate of mass transfer across the nanofluid layer.

Figure 9 depicts the transient nature of Nusselt number for the fluid phase. There occurs large amount of heat transfer in the fluid phase initially, and with large time the amount of heat transfer approaches a near constant value. We can observe that the effect of increasing the value of thermal Rayleigh number Ra , Prandtl number Pr , modified thermal capacity ratio γ , and the Nield number N_H , on the amplitude of oscillations is to increase it, i.e., an increase in the value of these parameters brings about an increase in the rate of heat transfer across the fluid phase.

The transient nature of Nusselt number for particle phase has been shown in Figure 10. We see that the amplitude of heat transfer in particle phase is small initially, but increases rapidly with time t to approach a steady value for large values of time t . The effect of the parameters Ra, Pr, γ , and N_H on the amplitude of oscillations is to increase them.

7 Conclusions

We considered linear stability analysis in a horizontal nanofluid layer, heated from below and cooled from above, incorporating the effect of Brownian motion along with thermophoresis, under non-equilibrium conditions. Further bottom heavy suspension of nanoparticles has been considered. Linear analysis has been made using normal mode technique, while for weakly nonlinear analysis, we have used a minimal representation of truncated Fourier Series involving only two terms. Then the effect of various parameters on the onset of thermal instability has been found. The results have been presented graphically. We draw the following conclusions:

1. The effect of the concentration Rayleigh number Rn , Lewis number Le , modified thermal capacity ratio γ , and thermal diffusivity ratio ε is to stabilize the system.
2. Convection sets in earlier for LTNE as compared to LTE.
3. The value of critical wave number α_c is lower for oscillatory convection than for stationary convection.
4. The effect of time on Nusselt numbers is found to be oscillatory, when t is small. However when time t becomes very large Nusselt number approaches the steady value.
5. On increasing the value of thermal Rayleigh number Ra , the rate of mass and heat transfer is increased.

Nomenclature

Latin Symbols

D_B Brownian Diffusion coefficient.
 D_T thermophoretic diffusion coefficient.

Pr Prandtl number.
 d dimensional layer depth.
 Le Lewis number.
 N_A modified diffusivity ratio.
 N_B modified particle-density increment.
 N_H Nield Number.
 p pressure.
 g Gravitational acceleration.
 Ra thermal Rayleigh-Darcy number,
 Rm basic density Rayleigh number,
 Rn concentration Rayleigh number,
 t time.
 T nanofluid temperature.
 T_c temperature at the upper wall.
 T_h temperature at the lower wall.
 \mathbf{v} nanofluid velocity.
 (x, y, z) Cartesian coordinates.

Greek symbols

α_f thermal diffusivity of the fluid.
 β proportionality factor.
 γ modified thermal capacity ratio.
 ε thermal diffusivity ratio.
 ψ stream function.
 μ viscosity of the fluid.
 ρ_f fluid density.
 ρ_p nanoparticle mass density
 $(\rho c)_f$ Heat capacity of the fluid.
 $(\rho c)_p$ Heat capacity of the nanoparticle material.
 ϕ nanoparticle volume fraction.
 α wave number.
 ω frequency of oscillations.

Subscripts

b basic solution.
 f Fluid phase.
 p Particle phase.
 c critical.

Superscripts

* dimensional variable
 ' perturbation variable

Operators

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

SA and BSB completed the analytic and numeric analysis of the problem and drafted the manuscript. All authors read and approved the final manuscript.

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Received: 9 July 2014 Accepted: 28 October 2014

Published online: 25 March 2015

References

1. DA Nield, A Bejan, *Convection in Porous Media (3rd edition)*. (Springer, New York, 2006)
2. I Pop, DB Ingham, *Convective Heat Transfer: Mathematical and Computational Modeling of Viscous Fluids and Porous Media*. (Oxford, Pergamon, 2001)
3. DB Ingham, I Pop, *Transport Phenomena in Porous Media III*. (Elsevier, Oxford, 2005)
4. K Vafai, *Handbook of Porous Media (2nd edition)*. (Taylor & Francis, New York, 2005)
5. K Vafai, *Porous Media: Applications in Biological Systems and Biotechnology*. (CRC Press, Hindawi Publications, New York, 2010)
6. P Vadasz, *Emerging Topics in Heat and Mass Transfer in Porous Media*. (Springer, New York, 2008)
7. L Godson, B Raja, DM Lal, S Wongwises, Enhancement of Heat Transfer Using Nanofluids - An Overview. *Renew. Sustain. Energy. Rev.* **14**, 629–641 (2010)
8. J Buongiorno, Convective transport in nanofluids. *ASME Jr. Heat Transfer.* **128**, 240–250 (2006)
9. DY Tzou, Instability of nanofluids in natural convection. *ASME, Jr. Heat Transfer.* **130**, 072401 (2008a)
10. DY Tzou, Thermal instability of nanofluids in natural convection. *Int. Jr. Heat Mass Transfer.* **51**, 2967–2979 (2008b)
11. J Kim, YT Kang, CK Choi, Analysis of convective instability and heat transfer characteristics of nanofluids. *Phys. Fluids.* **16**, 2395–2401 (2004)
12. J Kim, CK Choi, YT Kang, MG Kim, Effects of thermodiffusion and nanoparticles on convective instabilities in binary nanofluids. *Nanoscale Microscale Thermophys. Eng.* **10**, 29–39 (2006)
13. J Kim, YT Kang, CK Choi, Analysis of convective instability and heat transfer characteristics of nanofluids. *Int. J. Refrig.* **30**, 323–328 (2007)
14. DA Nield, AV Kuznetsov, Thermal instability in a porous medium layer saturated by nanofluid. *Int. Jr. Heat Mass Transfer.* **52**, 5796–5801 (2009)
15. AV Kuznetsov, DA Nield, Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman Model. *Trans. Porous Med.* **81**, 409–422 (2010)
16. S Agarwal, BS Bhadauria, PG Siddheshwar, Thermal instability of a nanofluid saturating a rotating anisotropic porous medium. *Spec. Top. Rev. Porous. Media Int. J.* **2**(1), 53–64 (2011). Begell House, USA
17. S Agarwal, NC Sacheti, P Chandran, BS Bhadauria, AK Singh, Non-linear convective transport in a binary nanofluid saturated porous layer. *Transport Porous Medium.* **93**(1), 29–49 (2012)
18. BS Bhadauria, S Agarwal, Natural convection in a nanofluid saturated rotating porous layer: a nonlinear study. *Transport Porous Media.* **87**(2), 585–602 (2011)
19. BS Bhadauria, S Agarwal, Convective transport in a nanofluid saturated porous layer with thermal non equilibrium model. *Transport Porous Media.* **88**(1), 107–131 (2011)
20. S Agarwal, BS Bhadauria, Natural convection in a nanofluid saturated rotating porous layer with thermal non equilibrium model. *Transport Porous Media.* **90**, 627–654 (2011). Springer
21. P Rana, S Agarwal, Convection in a binary nanofluid saturated rotating porous layer. *J. Nanofluids.* **4**(1), pp1-7 (2014 in press). American Scientific Publishers
22. S Agarwal, Thermal instability of a nanofluid saturating an anisotropic porous medium layer under local thermal non-equilibrium condition, *J. Nano Energy Power Res.* (2014, in press). American Scientific Publishers
23. Y Dhananjay, GS Agrawal, R Bhargava, Rayleigh Bénard convection in nanofluids. *Int. J. Appl. Math Mech.* **7**(2), 61–76 (2011)
24. BS Bhadauria, S Agarwal, Natural Convection in a Rotating Nanofluid Layer, *MATEC Web of Conferences, EDP Sciences.* **1**, 06001 (2012)
25. S Agarwal, BS Bhadauria, Convective heat transport by longitudinal rolls in dilute Nanofluids. *J. Nanofluids.* **3**(4), pp 1-11 (2014). American Scientific Publishers
26. S Agarwal, BS Bhadauria, Unsteady heat and mass transfer in a rotating nanofluid layer. *Continuum Mech. Therm.* **26**, 437–445 (2014)
27. S Agarwal, BS Bhadauria, Flow patterns in linear state of Rayleigh-Bénard convection in a rotating nanofluid layer. *Appl. Nanosci.* **4**(8), pp 935-941 (2013). doi:10.1007/s13204-013-0273-2
28. P Koblinski, DG Cahil, Comments on model for heat conduction in nanofluids. *Phy. Rev Lett.* **95**(209401) (2005)
29. DA Nield, AV Kuznetsov, The effect of local thermal nonequilibrium on the onset of convection in a nanofluid. *J. Heat. Tran.* **132**, 052405 (2010)
30. S Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*. (Oxford University Press, Oxford, 1961)
31. PG Drazin, DH Reid, *Hydrodynamic Stability*. (Cambridge University Press, Cambridge, 1981)

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