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Hybrid beluga whale optimization algorithm with multi-strategy for functions and engineering optimization problems

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Abstract

Beluga Whale Optimization (BWO) is a new metaheuristic algorithm that simulates the social behaviors of beluga whales swimming, foraging, and whale falling. Compared with other optimization algorithms, BWO shows certain advantages in solving unimodal and multimodal optimization problems. However, the convergence speed and optimization performance of BWO still have some performance deficiencies when solving complex multidimensional problems. Therefore, this paper proposes a hybrid BWO method called HBWO combining Quasi-oppositional based learning (QOBL), adaptive and spiral predation strategy, and Nelder-Mead simplex search method (NM). Firstly, in the initialization phase, the QOBL strategy is introduced. This strategy reconstructs the initial spatial position of the population by pairwise comparisons to obtain a more prosperous and higher quality initial population. Subsequently, an adaptive and spiral predation strategy is designed in the exploration and exploitation phases. The strategy first learns the optimal individual positions in some dimensions through adaptive learning to avoid the loss of local optimality. At the same time, a spiral movement method motivated by a cosine factor is introduced to maintain some balance between exploration and exploitation. Finally, the NM simplex search method is added. It corrects individual positions through multiple scaling methods to improve the optimal search speed more accurately and efficiently. The performance of HBWO is verified utilizing the CEC2017 and CEC2019 test functions. Meanwhile, the superiority of HBWO is verified by utilizing six engineering design examples. The experimental results show that HBWO has higher feasibility and effectiveness in solving practical problems than BWO and other optimization methods.

Keywords: Beluga whale optimization, Quasi-oppositional based learning, The adaptive and spiral predation strategies, Nelder-Mead simplex search, Engineering design

Introduction

Meta-heuristic algorithms (MAs, for short) have developed rapidly in recent years. It combines stochastic and local search algorithms and is widely used to solve global optimization problems in different fields. Compared with traditional algorithms, MAs can better deal with complex, multi-modal, non-continuous non-differentiable problems [1] in the real world due to their "random factors". For example, MAs can be used to solve problems in image processing [2], shape optimization [3], machine learning [4], deep learning [5], path planning [6], clustering [7], engineering problems [9, 10], and other fields. MAs simulate the relevant behavior of humans or animals and apply the rules and action principles in physics and chemistry to construct the mathematical model of the optimization algorithm. Therefore, MAs can be divided into four categories: evolution-based algorithms, human-based algorithms, physics-based algorithms, chemistry-based algorithms, and swarm-based intelligence algorithms [8].

Evolution-based algorithms are used to derive superior next-generation individuals through Darwinian evolution and the survival of the fittest in nature, thus enabling the population to progress collectively. Among the representative methods are Differential Evolution (DE) [12] and GA [13], which simulate the genetic laws in nature to find the optimal individual through natural derivation laws such as selection, crossover, and mutation. Some more specific methods are Gene Expression Programming (GEP) [14], Evolutionary Programming (EP) [15], etc.

Human-based algorithms are mainly inspired by human social behaviors, including teaching and learning, competitive, cooperative, etc. For example, Teaching–Learning-Based Optimization (TLBO) [16] Soccer League Competition (SLC) [18] simulates sports football matches and finds players with higher scores among fixed players and substitute players by ranking points, Socio Evolution & Learning Optimization (SELO) [19], Human Urbanization (HUS) [20] derived from human life, its purpose is to realize urbanization and have more convenient urban life, Growth Optimizer (GO) [21] its inspiration comes from learning and reflection in the process of growing up, Artificial Ecosystem-Based Optimization (AEO) [22], Harmony Search (HS) [17] simulate musicians to get wonderful music by constantly adjusting notes, etc.

Physics theorems and chemical experiments inspire algorithms based on physics and chemistry. The more classical ones include Simulated Annealing (SA) [23] and Gravitational Search Algorithm (GSA) [24] to simulate the motion between objects under the universal gravitation theorem. It also includes Lightning Attachment Process Optimization (LAPO) [25], Young's Double-Slit Experiment Optimizer (YDSE) [27], physical experiments from double-slit interference have shown the wave of light, Atomic Orbit Search (AOS) [28] Inspired by concepts such as quantum mechanics and quantum atom models in physics, Atom Search Optimization (ASO) [29], eta.

Swarm-based intelligence algorithms focus on finding the best solution by simulating the behavior of the group and learning the intelligence of the group. Classical swarm intelligence algorithms include Particle Swarm Optimization (PSO) [30], Ant Colony Optimization (ACO) [31], and Firefly Algorithm (FA) [11]. Due to their simple structure, they may need better convergence accuracy when dealing with complex optimization problems. With the deepening of scientific research, a large number of new swarm intelligence algorithms have emerged, including Harris Hawks Optimization (HHO) [32], mimicking the hunting process of a Harris hawk, Sparrow Search Algorithm (SSA) [35], Chameleon Swarm Algorithm (CSA) [33] simulates the way chameleons change color according to their environment when searching for food. These search methods can achieve excellent performance against benchmark functions but suffer from slow convergence when dealing with real engineering problems. Also included, Jellyfish Search (JS) [34] simulated jellyfish following ocean currents, the Coati Optimization Algorithm (COA) [36], the Gannet Optimization Algorithm (GOA) [37] a numerical model was constructed based on the unique behavior of gannets during foraging. Although these algorithms have good convergence speed and accuracy, they fall into local optimization in some high-dimensional problems and are more sensitive to control parameters. In addition, swarm-based intelligence includes Aphid-Ant Mutualism (AAM) [38], simulates the mutualism between aphids and ants in nature, Shrimp and Goby Association Search (SGA) [39] mathematical modeling based on the cooperative win–win relationship between shrimp and goby, Conscious Neighborhood-Based Crow Search (CCSA) [40], Manta Ray Foraging Optimization (MRFO) [41], Mountain Gazelle Optimizer (MGO) [42] it was inspired by the herd life of the mountain gazelle, Artificial Rabbits Optimization (ARO) [43], Artificial Hummingbird Algorithm (AHA) [44]. These algorithms have good global search capability and can handle continuous optimization problems, but solving discrete optimization problems is challenging.

The Beluga Whale Optimization (BWO) [45] was proposed by Changting Zhong et al. in 2022, and its inspiration comes from the three stages of beluga whale swimming, foraging, and whale fall. Beluga whales are social animals, and usually, many beluga whales migrate together every July. In summer, beluga whales gather in the estuary to hunt. Because beluga whales do not have sharp teeth, they generally eat salmon, cucurbit fish, cod, and smaller prey such as shrimp, squid and clams. Whale falling refers to the attack of natural enemies, polar bears, and killer whales during migration, as well as the harm of human beings to beluga whales and some irresistible factors that make beluga whales die and fall to the bottom of the sea. The above three stages correspond to BWO exploration, development, and whale fall, and a mathematical model is established accordingly.

The two algorithms, BWO and WOA, differ significantly in the process of constructing optimization models due to different inspirations [61]. Firstly, BWO was inspired by the swimming, foraging, and whale-fall processes of beluga whales and developed a mathematical model based on these three stages. In contrast, WOA modeled the foraging process of humpback whales in a bubble network. Further, BWO designed a Lévy flight process during the exploitation phase that simulates the foraging movement of beluga whales and considers whales falling under natural factors. Therefore, the significant differences between the two algorithms need to be squarely addressed.

Some experiments have strongly demonstrated the competitiveness of BWO in solving optimization problems. However, due to the increasing complexity of real problems, the original BWO is unable to handle these problems effectively and achieve a suitable result. An improved situation exists for effective enhancement. Therefore, this paper presents an improved BWO called HBWO, which introduces Quasi-oppositional based learning (QOBL) [46], adaptive and spiral predation strategy, and Nelder-Mead simplex search method (NM) [47 ~ 49]. Firstly, quasi-positional-based learning is introduced to obtain an optimal solution with higher probability. Secondly, the adaptive and spiral predation strategies improve the convergence speed while avoiding falling into local optima. Finally, the Nelder-Mead simplex search method is introduced so that the better individuals replace the worst ones.

In this paper, the CEC 2017 test function [50] and the CEC 2019 test function [51] are used to verify the superiority of HBWO. The main contributions of this paper are as follows:

1 In order to improve the performance of BWO, three strategies were added on the basis of BWO, and an improved Beluga optimization algorithm, HBWO, was proposed.
2 The performance of HBWO was evaluated in CEC2017 and CEC2019 test functions and compared with nine optimization algorithms and two improved optimization algorithms. The numerical results show that HBWO has a certain competitiveness.
3 Six practical engineering application problems were solved by HBWO. It further shows the superiority of HBWO and the high efficiency of solving practical problems.

This paper is composed in the following way: Sect. "Basic BWO" briefly describes the specific process of BWO. Sect. "Proposed HBWO" presents a detailed description of the proposed HBWO. Sect. "Experimental results and discussion" tests the performance of HBWO using CEC2017 and CEC2019 test functions, and the resulting experimental results are analyzed and evaluated. Sect. "HBWO for engineering optimization problems" solves six practical engineering examples with the HBWO. Sect. "Conclusion and Future" summarizes the paper.

Basic BWO

The Beluga Whale Optimization (BWO), proposed by Changting Zhong et al. in 2022. The algorithm is inspired by the three phases of beluga whale swimming, feeding and whale falling.

The exploration and exploitation stages of BWO are determined by B_f, which can be expressed as:

$$B_f = B_0(1 - T/2T_{\max}) \tag{1}$$

where B_0 is a random number between (0, 1). When $B_f > 0.5$ is the exploration phase of BWO, and when $B_f \leq 0.5$ is the exploitation phase of BWO. As T increases, B_f decreases from (0,1) to (0,0.5).

Exploration phase

The exploration phase mathematical model of BWO is inspired by beluga whale swimming. Based on the behavioral recordings of the belugas swimming, the two pairs of belugas swim closely together in a synchronized or mirrored fashion (Fig. 1a). Location update as follows:

Fig. 1 Behaviors of beluga whales **a** Swim **b** Foraging **c** Whale fall

$$\begin{cases} X_{i,j}^{T+1} = X_{i,pj}^{T} + (X_{r,p1}^{T} - X_{i,pj}^{T})(1 + r_1)\sin(2\pi r_2), j = even \\ X_{i,j}^{T+1} = X_{i,pj}^{T} + (X_{r,p1}^{T} - X_{i,pj}^{T})(1 + r_1)\cos(2\pi r_2), j = odd \end{cases} \tag{2}$$

where $X_{i,j}^{T+1}$ is the new position of the ith individual on the Jth dimension, $p_j (j = 1, 2, \cdots, d)$ is a number randomly selected from the D-dimension, $X_{i,pj}^{T}$ is the position of the ith individual on the p_j dimension, $X_{r,p1}^{T}$ is the current position of the rth individuals (r is randomly selected), r_1 and r_2 are the random numbers of (0,1).

Exploitation phase

The BWO exploitation phase was inspired by the beluga whale's predatory behavior (Fig. 1b). It is expressed as follows:

$$X_i^{T+1} = r_3 X_{best}^{T} - r_4 X_i^{T} + C_1 \cdot L_F \cdot (X_r^{T} - X_i^{T}) \tag{3}$$

where X_i^T and X_r^T are the current and the random beluga position, X_i^{T+1} is the updated beluga position, X_{best}^{T} is the beluga with the best position, and r_3 and r_4 are random numbers of (0,1), $C_1 = 2r_4(1 - T/T_{\max})$ is used to measure the random jump strength of Lévy flight. L_F is calculated as follows:

$$L_F = 0.05 \times \frac{u \times \sigma}{|v|^{1/\beta}} \tag{4}$$

$$\sigma = \left(\frac{\Gamma(1+\beta) \times \sin(\pi\beta/2)}{\Gamma(1+\beta)/2 \times \beta \times 2^{(\beta-1)/2}} \right)^{1/\beta} \tag{5}$$

where u and v are normal distributed random numbers and β is constant $\beta = 1.5$.

Whale fall

Whale fall refers to the attack of natural enemies such as polar bears and killer whales in the migration process, as well as the harm of humans and some irresistible factors that cause the death of beluga whales (Fig. 1c). A mathematical model of a whale's fall was established, expressed as follows:

$$X_i^{T+1} = r_5 X_i^{T} - r_6 X_i^{T} + r_6 X_{step} \tag{6}$$

$$X_{step} = (u_b - l_b)\exp(-C_2 T/T_{\max}) \tag{7}$$

where ($C_2 = 2W_f \times n$), W_f can be expressed as follows:

$$W_f = 0.1 - 0.05_2 T/T_{\max} \tag{8}$$

The whale falls when $B_f < W_{f.}$

Proposed HBWO

With the increasing complexity of optimization problems, BWO has some limitations in solving practical problems, given the shortcomings of the BWO, such as insufficient solution accuracy. In this section, an improved Beluga Whale Optimization HBWO is proposed by combining the following three strategies. (1) Improve the distribution quality of the initial population based on Quasi-opposition-based learning (QOBL); (2) Adaptive and spiral predation strategy. The adaptive part makes the beluga learn from the best individual and reduces the learning from random beluga. The spiral predation part makes the beluga expand the exploitation phase, which helps to prevent the algorithm from being premature and jumping out of the local best beluga individual; (3) Nelder-Mead simplex search method (NM), which makes better beluga whale individuals replace the worst beluga whale individuals through reflection, expansion, and contraction.

Quasi-opposition-based learning (QOBL)

Opposition-based learning (OBL) [52, 53] assumes that the initial population has a higher probability of getting the optimal solution than the random initial population. OBL has been extended to QOBL, Quasi-opposition-based learning (QOBL), and the initial beluga population obtained with QOBL has better optimization ability than the initial beluga population obtained with OBL and random initialization. QOBL is denoted by:

$$\begin{cases} x_{i,j}^{T+1} = H_{i,j} + (x_i^T - H_{i,j}) \times rand(0,1), x_i^T < H_{i,j} \\ x_{i,j}^{T+1} = x_i^T + (H_{i,j} - x_i^T) \times rand(0,1), x_i^T \geq H_{i,j} \end{cases} \tag{9}$$

where

$$x_{i,j}^T = (lb_j + lb_j) - X_i^T \tag{10}$$

$$H_{i,j} = (lb_j + lb_j)/2 \tag{11}$$

where $X_{i,j}^T$ is the position of the *i*th white whale on the *j*th dimension generated by random initialization.

Adaptive and spiral predation strategies

The adaptive part is an improvement made to the exploration part of the Beluga whale optimization. The original BWO exploration part ignores the learning from the optimal beluga individual, which leads to the weakness of the algorithm in finding the best, and there is some room for improvement. HBWO has made some improvements based on the framework of the original BWO exploration section. HBWO learns from random beluga individuals while also learning from optimal beluga individuals in some dimensions, increasing the algorithm's superiority-seeking ability in the exploration phase with the following mathematical formulation.

$$\begin{cases} x_{i,s1}^{T+1} = x_{g,s1}^{T} + (x_{i,s1}^{T+1} - x_{g,s1}^{T})(1 + r_1)\sin(2\pi r_2) \\ x_{i,s2}^{T+1} = x_{g,s2}^{T} + (x_{i,s2}^{T+1} - x_{g,s2}^{T})(1 + r_1)\cos(2\pi r_2) \end{cases} \tag{12}$$

where s_1 and s_2 are two random integers from 0 to D (number of dimensions), $x_{i,s1}^{T+1}$ the new position of the ith beluga in the s_1 th dimension, $x_{g,s1}^{T+1}$ is the position of the globally optimal beluga in the s_1 th dimension, and $x_{i,s1}^{T}$ is the current position of the ith beluga in the s_1 th dimension. s_2 same thing.

The spiral predation part is an improvement made to the exploitation part of the Beluga whale optimization (Fig. 2). Influenced by the Whale optimization algorithm, the spiral predation with cosine function is introduced in the exploitation stage of BWO to broaden the exploitation ability of beluga whales to enhance the diversity of later populations. It is calculated by Eq. (12).

$$x_i^{T+1} = r_3 x_{best}^{T} - r_4 x_i^{T} + C_1 \cdot L_F \cdot x_i^{T} \cdot \cos(w \cdot 2 \cdot \pi) \tag{13}$$

where x_i^T is the current beluga position, x_i^{T+1} is the position of the updated beluga, x_{best}^T the best solution, r_3 and r_4 is the random number of (0,1), $C_1 = 2r_4(1 - T/T_{\max})$, w is represented as follows:

$$w = (h - 1) \cdot r + 1 \tag{14}$$

$$h = T \cdot ((-1)/T_{\max}) - 1 \tag{15}$$

where r and h are random numbers between (0,1).

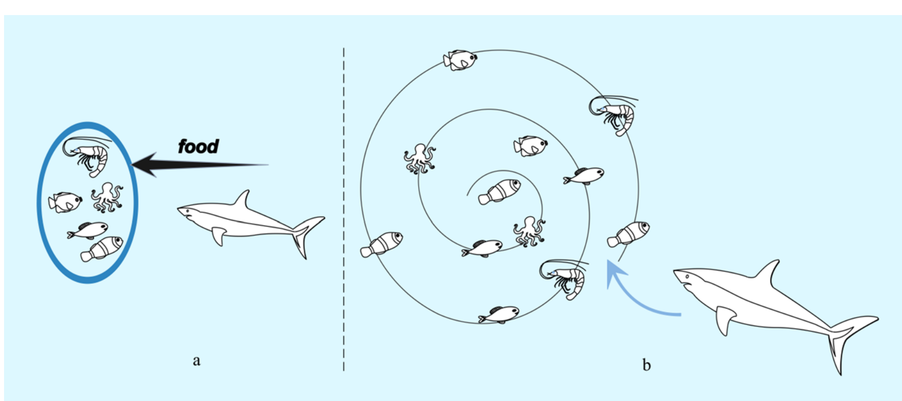

Fig.2 Beluga whale feeding (**a**) primitive (**b**) spiral feeding

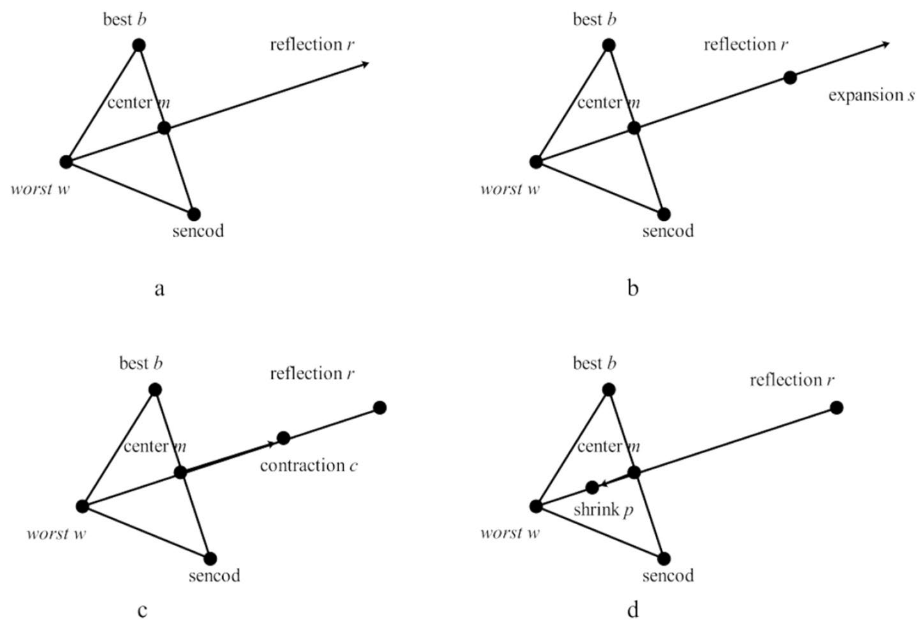

Fig. 3 NM **a** reflection. **b** expansion. **c** contraction**d** shrink

Nelder-Mead simplex search method (NM)

The method uses reflection, expansion, and compression to rescale the beluga individuals. By calculating the magnitude of these points and the corresponding point fitness values, a better beluga individual can be made to replace the worst beluga individual according to the relevant steps of NM, helping to find the optimal beluga individual (Fig. 3). The steps of NM are described below.

(1) Reflection. Calculate the reflection point $r = m + \alpha \cdot (m - w)$, α is the reflection coefficient, this paper $\alpha = 1$.
(2) Expansion. If $f_r < f_w$, calculate the expansion point $s = m + \beta \cdot (r - m)$, β is the expansion factor, this paper $\beta = 2$. If $f_s < f_r$, $w = s$, otherwise $w = r$.

where f_r、f_w、f_s denote the adaptation values of the corresponding points *rws.*

(1) Compression.
① If $f_b < f_r < f_m$, $w = r$.
② If $f_m < f_r < f_w$, contraction inward, $c = m + (r - m)/2$, if $f_c < f_w$, $w = c$.
③ Otherwise, shrink outward, $p = m - (r - m)/2$, if $f_p < f_w$, $w = p$, otherwise $w = r$.

where the magnitude of the fitness value of the point f_b、f_r、f_m、f_w、f_c、f_p to which b、r、m、w、c、p corresponds.

In order to reduce the error caused by using NM randomness, it can be run repeatedly. This paper runs 5 times. Algorithm 1 gives the pseudo-code.

Algorithm 1: Nelder-Mead simplex search method.

Input: Optimal value b, the worst value w

Output: A better value replaces the worst value

Take the average of the population, and call the result the centroid m

Calculating reflection points r; $r = m + \alpha \cdot (m - w)$

if $f_r < f_w$

  $s = m + \beta \cdot (r - m)$

  if $f_s < f_r$

    $w = s$

  else

    $w = r$

  end if

else if $f_b < f_r < f_m$

  $w = r$

else if $f_m < f_r < f_w$

  $c = m + (r - m)/2$

  if $f_c < f_w$

    $w = c$

  end if

else

  $p = m - (r - m)/2$

  if $f_p < f_w$

    $w = p$

  else

    $w = r$

  end if

end if

Detailed Steps for HBWO

Combining the three strategies based on quasi-opposition-based learning, adaptive and spiral predation strategy, and Nelder–Mead simplex search method with BWO, an enhanced belugas optimization, which is marked as HBWO, is proposed.

For HBWO, the three strategies introduced are important ways to ensure balance. First, Quasi-opposition-based learning strategies increase the likelihood that the population searches near the optimal solution by improving the initialization distribution of the population. This improves the search efficiency in the exploration phase. Adaptive and spiral predation strategies can increase search efficiency by introducing optimal position information in the exploration phase. The population constantly explores the optimal solution. In addition, the Nelder-Mead simplex search method relies on the relevant position information of the population in the exploration phase to determine the optimal position in the solution space.

The steps of HBWO are as follows:

Step1. Initialization, determining the parameters related to the enhanced beluga optimization algorithm;

Step2. Generating an initial population of belugas by Eq. (9);

Step3. Calculate the fitness value and get the current optimal individual;

Step4. Calculate B_f according to Eq. (1). If $B_f > 0.5$ calculates the position of individual beluga whale according to Eq. (12), else calculates the position of individual beluga whale according to Eq. (13);

Step5. The fitness value was calculated and sorted to find the current optimal belugas individual;

Step6. Calculate W_f according to Eq. (8), If $B_f < W_f$ according to Eq. (6) calculate the individual position of beluga whale;

Step7. Determine whether the upper and lower bounds of the position are exceeded; if the upper bound is exceeded, *ub* is used instead, and if the lower bound is exceeded, *lb* is used instead;

Step8. Replacement of the position of the worst beluga individual according to NM;

Step9. Find the current optimal individual position;

Step10. If $T \le T_{\max}$, then execute **Step 3**, otherwise execute **Step 11**;

Step11. Output the global optimal position and fitness values.

Algorithm 2 gives the pseudo-code of HBWO. Figure 4 shows the flow chart of HBWO.

Time complexity of HBWO

The computational complexity (O) of HBWO was determined by three processes: algorithm initialization, fitness evaluation value and beluga whale individual updating. Firstly, Quasi-opposition-based learning (QOBL) was added, degree complexity is $O(N_{pop} \times D)$. Secondly, the adaptive and spiral predation strategies are introduced, and the computational complexity is $O(N_{pop} \times T_{\max})$. The degree complexity of a whale fall is $O(N_{pop} \times T_{\max} \times 0.1)$. Finally, the Nelder–Mead simplex search method is added, and the computational complexity is

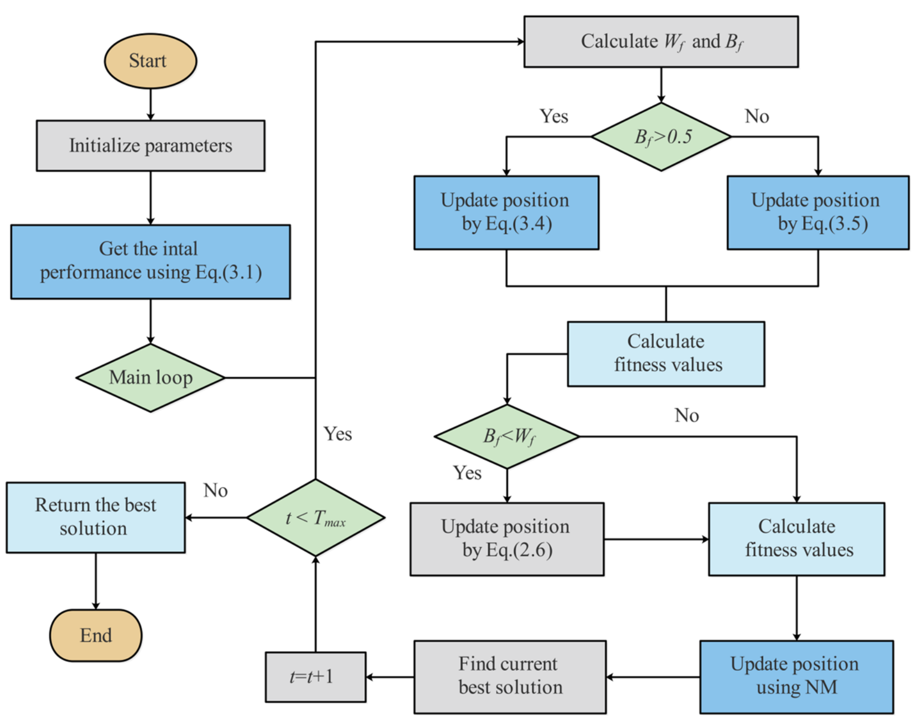

Fig. 4 Flow chart of HBWO algorithm

$O(5 \times N_{pop})$. Therefore, the complexity of the proposed HBWO is $O(\text{HBWO}) = O(N_{pop} \times (D + 1.1 \times T_{\max} + 5)$.

Algorithm 2: Proposed HBWO Algorithm.

Input: Algorithmic parameter (population,size,iteration)
Output: The best solution
Initialize the population $\begin{cases} x_{i,j}^{T+1} = H_{i,j} + (x_i^T - H_{i,j}) \times rand(0,1), x_i^T < H_{i,j} \\ x_{i,j}^{T+1} = x_i^T + (H_{i,j} - x_i^T) \times rand(0,1), x_i^T \ge H_{i,j} \end{cases}$
Evaluate fitness values, obtain the best solution
while $T \le T_{\max}$ **do**
Define the value of B_f according to Eq. (2.1)
Define the value of W_f according to Eq. (2.8)
  for each population (x_i) **do**
    if $B_f > 0.5$
      $\begin{cases} x_{i,s1}^{T+1} = x_{g,s1}^T + (x_{i,s1}^{T+1} - x_{g,s1}^T)(1 + r_1)\sin(2\pi r_2) \\ x_{i,s2}^{T+1} = x_{g,s2}^T + (x_{i,s2}^{T+1} - x_{g,s2}^T)(1 + r_1)\cos(2\pi r_2) \end{cases}$
    else if $B_f \le 0.5$
      Update the jump parameter $C_1 = 2r_4(1 - T / T_{\max})$
      Calculate the Levy flight function using Eq.(2.4)
      Calculate the parameter w according to Eq. (3.6)
      $x_i^{T+1} = r_3 x_{best}^T - r_4 x_i^T + C_1 \cdot L_F \cdot x_i^T \cdot \cos(w \cdot 2 \cdot \pi)$
    end if
    Check the boundaries of new position and evaluate the fitness values
  end for
  for each population (x_i) **do**
    if $B_f \le W_f$
      Calculate X_{step} ($X_{step} = (u_b - l_b)\exp(-C_2 T / T_{\max})$)
      Update step factor C_2 ($C_2 = 2W_f \times n$)
      $x_i^{T+1} = r_5 x_i^T - r_6 x_i^T + r_6 x_{step}$
      Check the boundaries of new position and calculate fitnsee value
    end if
  end for
  Using the Nelder − Mead simplex search method (NM) replace the worst beluga
  if $fitness\_x_i^{T+1} < fitness\_x_i^T$
    $fitness\_x_i = fitness\_x_i^{T+1}$
    $x_i = x_i^{T+1}$
  end if
  T=T+1
end while

Experimental results and discussion

In this section, by working with the more classical algorithms Particle Swarm Optimization (PSO), the more applied Whale Optimization Algorithm (WOA), Harris Hawk Optimizer (HHO), the more novel Sparrow Search Algorithm (SSA), Dandelion Optimizer(DO) [54], Sand Cat Swarm Optimization(SCSO) [55], Aquila Optimizer (AO) [56], Arithmetic Optimization Algorithm (AOA) [59], Improved Harris Hawk Optimizer Algorithm, Leader Harris Hawks Optimization (LHHO) [57], Improved Slime Mode Algorithm, Leader Slime Mode Algorithm (LSMA) [58] and the original BWO are compared as a way to verify the superiority of the proposed HBWO on the CEC2017 and CEC2019 test set. These two test sets contain single-peaked, multi-peaked, mixed, and composite functions, which are challenging and thus enable a more scientific measure of the algorithm's merit. The initial parameters of the optimization algorithm are shown in Table 1.

Sensitivity analysis of reflection and expansion coefficients

The NM strategy scales the positions of beluga individuals through reflection, expansion, and compression to prevent the algorithm from falling into a local candidate solution. The key to the NM strategy affecting the position update of beluga individuals lies in selecting the reflection parameter α and the expansion parameter β. The NM strategy is based on selecting the reflection parameter α and the expansion parameter β. In this section, we discuss and analyze the effects of the reflection parameter α and the expansion parameter β on the performance of the algorithm. We use the CEC2019 suite containing ten functions to investigate the effects of the reflection parameter α and the expansion parameter β. The values of reflection parameter α and expansion parameter β are categorized into [0.5, 1.5] and [1.5, 2.5] with 0.1 as

Table 1 parameter setting

Algorithm	Parameter	Value
PSO	Cognitive factor c_1 Social factor c_2 The inertia constant w decreases linearly	2 2 [0.8, 0.2]
SSA	Population size of producers and danger was the overall percentage Safety threshold ST	0.2 0.1 0.8
WOA	The control parameter a decreases linearly The value of constant b	[2, 0] 1
HHO	The initial energy E_0 range	[-1, 1]
DO	The adaptive parameter a decreases linearly The adaptive parameter k increases linearly The linear parameter δ increases linearly	[1, 0] [0, 1] [0, 2]
SCSO	Parameter S	2
AO	Exploitation adjustment factors a,δ	0.1,0.1
AOA	Parameter a, μ	5, 0.499
BWO	Parameters a, D Whale landing probability W_f linear decline	3/2, 0.05 [0.1,0.05]
LHHO	The initial energy E_0 changes the range	[-1, 1]
LMNS	Parameters N, z	20,0.03
HBWO	Parameters a, D, α, β, γ Whale landing probability W_f linear decline	3/2, 0.05,1,2,2 [0.1,0.05]

the step size. For the reflection parameter α and the expansion parameter β selected, the mean values and ordering of the test function solutions obtained by HBWO in 20 independent experiments are presented in Table 2 and 3, respectively. Bold indicates the scaling parameter with the smallest mean value.

From the results in Tables 2 and 3, it can be found that the best average is obtained for all six cases with $\alpha = 1$ and the lowest average is 1.7. It can be found that the closer α is to 1, the better the performance of the HBWO algorithm. In addition, all four cases with $\beta = 2$ obtained the best mean and the lowest mean value of 3. Hence, $\alpha = 1$ as well as $\beta = 2$ are the most appropriate results.

Performance benefits of improved strategies

In order to effectively analyze the impact of introducing each strategy on the optimization ability of the BWO algorithm and to demonstrate the synergistic effect of multiple strategies, an ablation analysis of multiple strategies is performed. In order to improve the drawbacks of BWO and enhance the performance of the algorithm, Quasi-poppositional-based learning, adaptive and spiral predation, and Nelder-Mead simplex search are introduced in the proposed HBWO and three other strategies. Table 4 gives the algorithms of the BWO variants using one or more fusion strategies. Where "1" indicates that the strategy is introduced and "0" indicates that the strategy is not introduced. The performance of multiple variants of the BWO algorithm

Table 2 Mean and ranking of the different reflection parameter α

Functions	The mean, rank of the different reflection parameter α										
	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$	$\alpha = 1.1$	$\alpha = 1.2$	$\alpha = 1.3$	$\alpha = 1.4$	$\alpha = 1.5$
cec01	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	1	**1**	**1**	**1**	**1**	**1**	**1**	**11**	**1**	**1**	**1**
cec02	4.79	4.57	4.50	4.57	4.53	**4.34**	4.45	4.71	4.59	4.67	4.53
	11	6	3	7	5	**1**	2	10	8	9	4
cec03	2.86	2.60	**2.13**	2.44	2.53	2.52	2.66	2.74	3.05	2.71	3.26
	9	5	**1**	2	4	3	6	8	10	7	11
cec04	25.88	19.85	19.39	16.34	14.51	**12.94**	13.53	14.41	13.55	15.63	15.00
	11	10	9	8	5	**1**	2	4	3	7	6
cec05	1.08	1.06	1.02	1.01	1.01	**1.00**	1.02	1.04	1.07	1.08	1.12
	9	7	5	2	3	**1**	4	6	8	10	11
cec06	3.12	2.88	1.93	1.51	1.46	1.45	**1.22**	1.53	2.08	1.93	2.41
	11	10	6	4	3	2	**1**	5	8	7	9
cec07	605.99	601.84	749.69	616.05	**597.50**	600.80	691.22	632.90	628.68	661.35	668.87
	4	3	11	5	**1**	2	10	7	6	8	9
cec08	4.02	4.17	3.88	3.91	3.93	**3.69**	3.98	3.96	3.94	4.05	3.84
	9	11	3	4	5	**1**	8	7	6	10	2
cec09	1.24	1.24	1.22	1.22	1.21	**1.16**	1.18	1.23	1.25	1.23	1.24
	10	9	4	5	3	**1**	2	7	11	6	8
cec10	18.53	17.08	13.38	13.13	**11.11**	15.03	20.44	20.49	20.25	19.55	21.26
	6	5	3	2	**1**	4	9	10	8	7	11
Mean rank	8.1	6.7	4.6	4	3.1	1.7	4.5	7.5	6.9	7.2	7.2

The bold represents the optimal values of the evaluation indicators

Table 3 Mean and ranking of the different reflection parameter β

Functions	The mean, rank of the different reflection parameter β										
	$\beta=0.5$	$\beta=0.6$	$\beta=0.7$	$\beta=0.8$	$\beta=0.9$	$\beta=1$	$\beta=1.1$	$\beta=1.2$	$\beta=1.3$	$\beta=1.4$	$\beta=1.5$
cec01	**1**	**1**	**1**	1	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	1	**1**	**1**	11	**1**	**1**	**1**	**1**	**1**	**1**	**1**
cec02	4.71	4.55	4.66	4.58	4.67	4.56	4.67	4.70	4.59	4.56	**4.39**
	11	2	7	5	9	4	8	10	6	3	**1**
cec03	2.67	**2.30**	2.60	2.72	2.64	2.41	2.65	2.66	2.77	2.52	2.47
	9	**1**	5	10	6	2	7	8	11	4	3
cec04	14.27	12.43	14.16	13.39	11.25	**10.37**	11.21	11.16	12.21	11.79	14.05
	11	7	10	8	4	**1**	3	2	6	5	9
cec05	1.01	1.01	1.01	1.01	1.00	**1.00**	1.01	1.02	1.01	1.02	1.01
	4	7	9	5	2	**1**	8	10	3	11	6
cec06	1.32	1.19	**1.16**	1.61	1.24	1.53	1.37	1.24	1.51	1.83	1.65
	5	2	**1**	9	3	8	6	4	7	11	10
cec07	603.55	578.12	595.27	614.05	627.40	**535.17**	706.69	703.65	620.42	615.88	549.06
	5	3	4	6	9	**1**	11	10	8	7	2
cec08	3.81	3.76	3.75	3.80	3.80	**3.75**	3.73	3.74	3.83	3.57	3.86
	9	6	5	7	8	4	2	3	10	1	11
cec09	1.19	1.22	1.20	**1.17**	1.19	1.18	1.19	1.18	1.19	1.20	1.18
	7	11	9	**1**	5	4	6	2	8	10	3
cec10	15.12	16.13	17.10	17.07	15.13	15.15	17.30	18.18	**14.01**	15.30	17.85
	2	6	8	7	3	4	9	11	**1**	5	10
Mean rank	6.4	4.6	5.9	6.9	5	3	6.1	6.1	6.1	5.8	5.6

The bold represents the optimal values of the evaluation indicators

Table 4 Various BWO variants with four strategies

Strategies	QHWO	AHWO	NMBWO	QABWO	QNMBWO	ANMBWO	HBWO
Quasi-oppositional based learning	1	0	0	1	1	0	1
Adaptive and spiral predation	0	1	0	1	0	1	1
Nelder-Mead simplex search	0	0	1	0	1	1	1

was analyzed in the CEC2019 test suite, and Table 5 lists the experimental results, including averages and rankings, for multiple variants of the BWO algorithm for the CEC2019 test suite.

As can be seen in Table 5, the performance of the BWO variant containing the NM strategy is relatively unstable. These experimental results show that these four strategies are useful in improving the performance of the original algorithm, especially the introduced Adaptive and spiral predation, but lack robustness. In addition, from the experimental results, it can be seen that ANMBWO, QABWO, and AHWO, which are ranked 2, 3, and 4, outperform the other BWO variants in terms of overall optimization performance, which also proves that the Adaptive and spiral predation strategy plays a greater and more consistent role among these four strategies. Compared to BWO, the proposed HBWO mainly improves the exploration and balancing ability and the ability to jump out of the local solution. Therefore, the combination of

Table 5 Results of various BWO on the CEC2019 test suit

Functions	QHWO	AHWO	NMBWO	QABWO	QNMBWO	ANMBWO	HBWO
cec01	1	1	1	1	1	1	1
	1	1	7	1	6	1	1
cec02	4.92	4.96	5.00	4.86	4.99	4.59	4.34
	4	5	7	3	6	2	1
cec03	1.88	2.09	5.44	2.14	5.54	2.70	2.52
	1	2	6	3	7	5	4
cec04	16.27	17.73	53.88	18.05	48.95	11.22	12.94
	3	4	7	5	6	1	2
cec05	1.37	1.30	6.25	1.33	11.72	1.01	1.00
	5	3	6	4	7	2	1
cec06	2.22	2.44	6.91	2.61	6.16	1.86	1.45
	3	4	7	5	6	2	1
cec07	635.25	582.66	1678.85	654.14	1695.30	551.07	600.80
	4	2	6	5	7	1	3
cec08	4.01	3.89	4.76	3.85	4.70	3.75	3.69
	5	4	7	3	6	2	1
cec09	1.26	1.25	1.91	1.27	1.99	1.18	1.16
	4	3	6	5	7	2	1
cec10	20.12	15.50	21.41	18.67	21.42	18.19	15.03
	5	2	6	4	7	3	1
Mean rank	3.5	3	6.5	3.8	6.5	2.1	1.6

Quasi-oppositional-based learning, adaptive and spiral predation, and Nelder-Mead simplex search moves the YDSE towards the optimal solution.

Experiments and analysis on the CEC2017 test set

Verify the strengths and weaknesses of the HBWO using the CEC2017 test set. First, the relevant parameters are set: the population size is 50, the number of dimensions is 30, and each algorithm is run 20 times for each test function. The results are shown, including the mean, standard deviation, best, worst, and rank.

The average rank of HBWO is 1.1724, as seen in Table 6, which ranks first. This indicates that the overall solution result of HBWO is better than the 30 test functions of CEC2017. As can be seen from Table 6, HBWO has significantly improved the merit-seeking ability for each test function at CEC2017 compared to BWO, indicating the effectiveness of introducing QOBL, adaptive and spiral predation strategy, and NM. On the single-peak test functions F1 and F3, the HBWO is superior to other algorithms. For the multi-peak test functions F4-F10, the HBWO algorithm ranked second after LSMA on F5 and F7. With composite test functions and mixed test functions, HBWO ranked second on F24, third on F27, and first on all other test functions. However, the proposed HBWO method has more runtime portion than the original BWO algorithm, which is mainly due to the added time complexity of the three added strategies.

From Table 6, it can be obtained that solving 30 dimensions on the CEC2017 test function, BWO ranks last, AOA ranks eleventh, LHHO ranks fifth, AO ranks fourth, SSA ranks third, LSMA ranks second, and HBWO ranks first, which fully indicates that the three strategies have significantly improved computational accuracy of the BWO. The

Table 6 Results of various HBWO on CEC2017

F	Index	Algorithm											
		PSO	SSA	WOA	HHO	DO	SCSO	AO	AOA	BWO	LHHO	LSMA	HBWO
F1	Ave	1.117E+10	2.150E+04	4.217E+08	1.892E+07	1.459E+09	3.718E+09	1.149E+08	4.966E+10	4.990E+10	1.530E+07	4.810E+03	**2.492E+03**
	Std	7.048E+09	2.202E+04	2.153E+08	3.635E+06	1.038E+09	2.621E+09	7.323E+07	8.532E+09	4.723E+09	3.849E+06	4.072E+03	3.666E+03
	Best	1.085E+09	3.843E+03	1.143E+08	1.265E+07	1.805E+08	4.250E+08	3.148E+07	3.408E+10	3.895E+10	1.007E+07	1.845E+02	1.013E+02
	Worst	2.602E+10	9.005E+04	9.417E+08	2.528E+07	3.769E+09	9.598E+09	3.683E+08	6.309E+10	5.555E+10	2.514E+07	1.697E+04	1.515E+04
	Rank	10	3	7	5	8	9	6	11	12	4	2	1
F3	Ave	5.735E+04	5.922E+04	2.493E+05	2.662E+04	8.119E+04	4.331E+04	4.374E+04	7.801E+04	7.532E+04	2.089E+04	3.975E+03	**7.572E+02**
	Std	1.949E+04	5.926E+03	6.312E+04	4.037E+03	6.074E+03	9.802E+03	6.677E+03	7.734E+03	6.075E+03	5.648E+03	2.249E+03	3.877E+02
	Best	2.843E+04	4.579E+04	1.563E+05	1.922E+04	6.592E+04	1.895E+04	3.315E+04	6.094E+04	6.014E+04	1.158E+04	1.205E+03	3.443E+02
	Worst	1.010E+05	6.905E+04	3.526E+05	3.609E+04	8.927E+04	6.497E+04	5.515E+04	8.847E+04	8.372E+04	3.136E+04	1.050E+04	2.421E+03
	Rank	7	8	12	4	11	5	6	10	9	3	2	1
F4	Ave	1193.4901	524.1194	692.0717	556.9365	682.9445	877.3982	588.8941	12804.9910	11706.3310	533.0430	500.9661	**484.5028**
	Std	612.2786	20.3094	81.9339	46.4416	118.9142	518.9321	50.2650	3226.8255	1331.0749	20.8540	19.4186	17.0210
	Best	642.5808	488.5944	586.7821	477.9126	558.9925	524.6050	523.6538	7611.4869	9557.0240	498.2239	478.1453	467.2975
	Worst	2861.2938	576.1173	881.1165	660.0705	1047.1917	2855.3264	736.1262	19043.9951	14381.7390	588.5292	563.4665	517.2056
	Rank	7	8	12	4	11	5	6	10	9	3	2	1
F5	Ave	714.8665	778.8699	820.4144	750.0392	788.3881	740.3144	675.9038	880.1379	913.3710	733.7390	**602.9851**	624.1913
	Std	45.0029	41.8028	53.5514	34.0208	28.7438	48.0607	29.7633	38.2791	18.4476	28.2254	28.9875	21.8096
	Best	618.9838	689.1764	730.8912	679.0856	739.0908	653.3828	618.2987	812.8303	877.0543	684.7845	561.7918	589.5656
	Worst	791.1411	823.3583	912.0708	809.5262	829.1142	816.8894	723.1441	982.3228	950.6040	793.6184	666.2277	646.5348
	Rank	4	8	10	7	9	6	3	11	12	5	1	2
F6	Ave	645.4623	659.8264	675.2904	664.8593	668.4228	661.8107	649.8268	674.9460	687.4726	661.4536	614.7668	**603.3094**
	Std	9.5723	9.0251	11.3517	5.4649	4.3619	8.4392	8.7552	7.4000	4.1215	4.8580	7.4938	1.9251
	Best	631.8421	633.8830	656.3361	654.2089	661.6524	647.1529	630.2593	664.0555	681.1609	653.4225	605.2460	600.3809
	Worst	669.8550	671.9930	693.2667	676.7000	678.9963	678.1189	666.8517	686.7597	696.8912	670.4959	634.2795	610.1477
	Rank	3	5	11	8	9	7	4	10	12	6	2	1

Table 6 (continued)

F	Index	Algorithm											
		PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**	**HBWO**
F7	Ave	1233.2470	1268.6096	1251.1970	1256.6749	1275.0880	1116.4781	1045.8198	1363.5532	1371.5558	1244.6440	**852.2571**	890.6840
	Std	193.8509	77.0029	105.6151	72.0228	85.2053	98.8470	45.3360	56.3962	23.1486	79.1381	29.7721	25.1362
	Best	958.3216	1032.3435	1099.2157	1060.8874	1092.0211	927.8141	958.5698	1245.0758	1318.3713	1109.2355	812.7436	839.7794
	Worst	1723.4123	1351.0352	1485.7560	1359.8450	1396.1940	1349.4336	1119.1388	1439.0308	1409.3233	1396.1934	913.3785	998.0526
	Rank	5	9	7	8	10	4	3	11	12	6	1	2
F8	Ave	978.8219	984.2392	1043.4621	966.3460	1002.9273	981.6329	942.1143	1089.8954	1135.3670	970.1768	909.8984	**904.0689**
	Std	51.5642	22.1450	68.9071	27.1292	22.3964	31.7130	23.0098	32.8884	13.9167	30.2885	27.1595	13.5051
	Best	892.0606	936.3103	918.6110	913.7056	955.3333	936.5612	902.9588	1040.7797	1104.2317	916.1991	860.2946	881.2173
	Worst	1081.7289	1020.8794	1194.4619	1012.2838	1031.4747	1050.5218	971.0618	1146.5256	1153.1203	1018.8170	953.8185	937.07436
	Rank	6	8	10	4	9	7	3	11	12	5	2	1
F9	Ave	4899.7002	5448.7506	9082.7267	7387.3795	6724.6856	5903.2591	5681.5669	6672.6114	10,324.1013	6635.5453	2970.4248	**2088.3688**
	Std	1615.1659	115.4332	3463.3655	1113.2126	434.4347	905.0906	1189.5706	930.8770	869.9235	938.8063	1363.7122	666.2224
	Best	2724.6157	5170.7584	5072.8615	5291.3602	5896.8021	3990.3181	3638.0528	5291.3287	8400.3419	4523.8199	1154.6737	914.1005
	Worst	7911.7378	5718.6937	17,371.1393	9234.3284	7455.9103	7665.7291	8350.6299	8413.4238	11861.7156	8563.4062	6409.3938	3737.2913
	Rank	3	4	11	10	9	6	5	8	12	7	2	1
F10	Ave	5383.0213	5846.2470	6721.8157	5864.6576	5700.7327	5668.0229	5325.3514	7298.3805	8482.6709	5737.5498	4895.1208	**4564.2864**
	Std	749.1992	653.8037	862.7071	801.1618	599.3269	555.2641	544.2342	546.1045	227.3939	761.8773	804.9351	531.1267
	Best	4060.3419	5005.8622	5278.0251	3935.8444	4574.1449	4627.9329	4155.2448	6394.8128	8011.0880	4193.2299	3361.7564	2909.6566
	Worst	6839.9826	7157.5190	8007.8280	7087.2998	6644.2146	6761.3703	6226.3990	8250.7158	8865.5418	7141.6112	6008.3384	5984.7154
	Rank	4	8	10	9	6	5	3	11	12	7	2	1
F11	Ave	1565.5178	1337.2066	4917.4890	1283.3919	2818.2733	1922.1087	1741.6028	7607.6140	6552.7070	1262.3180	1364.6989	**1199.0067**
	Std	176.4638	63.1468	2836.7528	35.5507	1117.7553	662.6147	242.6069	2204.5851	776.6216	46.7826	82.7906	35.2782
	Best	1297.1635	1229.8505	1690.1523	1219.8686	1434.0926	1356.0857	1373.4648	3880.7830	5204.6234	1181.6680	1215.6934	1142.2628
	Worst	1920.6600	1497.3442	10,511.4259	1374.4997	5078.6008	3955.0588	2222.4138	12432.6025	8325.1548	1359.9467	1507.9184	1267.478
	Rank	6	4	10	3	9	8	7	12	11	2	5	1

Table 6 (continued)

F	Index	Algorithm											
		PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**	**HBWO**
F12	Ave	8.983E+08	2.348E+06	1.867E+08	1.538E+07	6.677E+07	1.258E+08	4.830E+07	1.222E+10	9.381E+09	1.995E+07	5.326E+06	**2.168E+05**
	Std	9.977E+08	2.035E+06	1.513E+08	9.171E+06	4.129E+07	1.238E+08	3.192E+07	3.059E+09	1.640E+09	1.718E+07	3.031E+06	2.617E+05
	Best	3.401E+06	1.380E+05	3.030E+06	4.993E+06	8.524E+06	2.376E+06	1.066E+07	5.331E+09	6.517E+09	4.319E+06	1.229E+06	1.431E+04
	Worst	4.033E+09	7.872E+06	5.490E+08	4.454E+07	1.422E+08	3.874E+08	1.190E+08	1.708E+10	1.235E+10	5.378E+07	1.243E+07	1.443E+06
	Rank	10	2	9	4	7	8	6	12	11	5	3	1
F13	Ave	2.913E+08	2.318E+04	5.366E+05	7.083E+05	6.331E+05	2.116E+07	5.391E+05	7.725E+09	6.070E+09	4.337E+05	2.379E+05	**1.685E+04**
	Std	7.783E+08	1.858E+04	4.049E+05	8.556E+05	1.210E+06	6.660E+07	2.645E+05	4.486E+09	1.875E+09	2.063E+05	1.314E+05	1.322E+04
	Best	7.217E+03	1.931E+03	1.250E+05	2.735E+05	5.060E+04	5.202E+04	1.114E+05	1.789E+09	1.732E+09	1.476E+05	4.392E+04	2.769E+03
	Worst	3.306E+09	5.840E+04	1.679E+06	4.214E+06	5.603E+06	2.969E+08	1.102E+06	1.672E+10	9.214E+09	1.021E+06	5.431E+05	4.295E+04
	Rank	10	2	5	8	7	9	6	12	11	4	3	1
F14	Ave	5.437E+04	1.975E+05	1.259E+06	4.301E+05	1.014E+06	2.705E+05	4.447E+05	9.584E+05	2.248E+06	3.228E+05	9.378E+04	**2.100E+03**
	Std	4.775E+04	2.075E+05	1.055E+06	4.536E+05	5.118E+05	4.966E+05	4.235E+05	1.066E+06	8.975E+05	2.238E+05	3.765E+04	8.032E+02
	Best	7.605E+03	2.225E+04	5.225E+03	5.096E+03	1.182E+05	9.087E+03	2.919E+04	2.071E+04	9.449E+05	2.825E+04	1.166E+04	1.469E+03
	Worst	1.872E+05	7.443E+05	3.914E+06	1.884E+06	2.026E+06	1.860E+06	1.758E+06	4.423E+06	3.981E+06	7.381E+05	1.511E+05	4.269E+03
	Rank	2	4	11	7	10	5	8	9	12	6	3	1
F15	Ave	6.829E+04	8.021E+03	2.077E+05	7.787E+04	5.156E+04	1.301E+05	9.471E+04	2.146E+04	1.911E+08	6.405E+04	1.737E+05	**2.752E+03**
	Std	4.586E+04	9.615E+03	1.649E+05	3.567E+04	3.062E+04	1.949E+05	5.258E+04	6.614E+03	9.117E+07	4.002E+04	9.015E+04	9.848E+02
	Best	8.072E+03	1.659E+03	3.468E+04	2.780E+04	1.529E+04	1.238E+04	2.364E+04	1.519E+04	1.618E+07	2.475E+04	3.934E+04	1.696E+03
	Worst	1.676E+05	3.308E+04	5.872E+05	1.569E+05	1.136E+05	8.846E+05	2.444E+05	3.509E+04	3.363E+08	1.631E+05	3.998E+05	9.367E+03
	Rank	6	2	11	7	4	9	8	3	12	5	10	1
F16	Ave	3175.6419	2864.6308	3771.9360	3383.8883	3345.8875	3193.7475	3252.7742	4855.6554	5154.3382	3270.8437	2805.4281	**2584.6473**
	Std	429.9832	298.9653	592.7776	280.1114	365.5902	486.0102	323.0632	1032.5207	324.8035	351.6901	428.3570	233.0739
	Best	2425.2366	2210.8857	3106.6263	2722.2354	2761.7645	2505.4588	2617.0181	3356.7658	4541.8219	2564.1874	1975.0548	2049.4479
	Worst	3945.8229	3304.9890	5310.9360	3735.1019	3767.1828	4418.1806	3778.9187	6613.9165	5620.9109	4003.6757	3616.9408	3053.3518
	Rank	4	3	10	9	8	5	6	11	12	7	2	1

Table 6 (continued)

F	Index	Algorithm											
		PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**	**HBWO**
F17	Ave	2397.5599	2606.3295	2658.4199	2518.1441	2640.3216	2300.5276	2401.7174	3550.3685	3480.2497	2648.4641	2316.0067	**2096.2868**
	Std	251.4000	256.6690	288.8123	316.3164	228.6026	259.5862	301.6049	688.0100	350.0231	219.4118	217.3942	184.6278
	Best	1857.1365	2022.0996	2201.6420	1956.6759	2047.2521	1963.2543	1954.6091	2386.4196	2413.8635	2098.9281	2001.5227	1808.2824
	Worst	2795.1968	3012.2911	3151.8034	3138.5765	2982.3611	2870.3323	3219.7852	4423.2107	3879.8989	2987.7886	2909.5392	2329.5352
	Rank	4	7	10	6	8	2	5	12	11	9	3	1
F18	Ave	2.196E+06	2.444E+06	7.126E+06	2.242E+06	3.058E+06	1.144E+06	3.500E+06	1.101E+07	2.540E+07	1.469E+06	6.543E+05	**1.360E+04**
	Std	2.938E+06	3.156E+06	8.732E+06	3.375E+06	4.298E+06	1.114E+06	2.525E+06	1.141E+07	1.064E+07	8.213E+05	4.989E+05	8.622E+03
	Best	1.155E+05	5.369E+04	1.639E+05	1.296E+05	1.041E+05	7.926E+04	4.304E+05	3.732E+05	6.295E+06	2.914E+05	1.210E+05	5.406E+03
	Worst	1.091E+07	1.143E+07	3.255E+07	1.456E+07	1.720E+07	3.680E+06	8.029E+06	4.084E+07	4.217E+07	3.100E+06	1.824E+06	2.505E+04
	Rank	5	7	10	6	8	3	9	11	12	4	2	1
F19	Ave	8.572E+06	8.307E+03	6.957E+06	5.368E+05	9.055E+05	8.910E+06	1.492E+06	1.775E+06	2.893E+08	6.348E+05	8.557E+04	**3.762E+03**
	Std	2.411E+07	1.100E+04	5.658E+06	3.784E+05	7.014E+05	2.671E+07	1.156E+06	8.888E+04	1.691E+08	3.874E+05	4.212E+04	1.196E+03
	Best	3.318E+03	2.034E+03	1.047E+06	9.659E+04	8.499E+03	8.056E+04	2.922E+05	1.621E+06	5.081E+07	3.857E+04	1.210E+04	1.966E+03
	Worst	9.637E+07	5.362E+04	2.382E+07	1.437E+06	2.504E+06	1.172E+08	4.050E+06	1.935E+06	6.857E+08	1.533E+06	1.530E+05	1.069E+04
	Rank	10	2	9	4	6	11	7	8	12	5	3	1
F20	Ave	2684.6853	2817.5966	2955.9664	2852.8646	3004.1042	2646.5124	2626.6951	2734.1083	2924.1134	2693.3524	2523.3771	**2395.3218**
	Std	136.5414	173.4071	149.7941	195.5261	231.0104	191.1978	199.5179	205.9055	95.4253	230.2799	179.7403	135.7985
	Best	2400.6119	2541.4574	2712.1224	2355.2306	2534.6459	2306.1872	2342.4132	2308.7180	2724.1219	2193.5420	2158.2490	2211.1559
	Worst	2894.2039	3152.2455	3301.2341	3215.8494	3440.5879	2982.8525	2975.3871	3098.6288	3158.4874	3111.3410	2837.5962	2437.8654
	Rank	5	8	11	9	12	4	3	7	10	6	2	1
F21	Ave	2495.0140	2543.4689	2595.1513	2550.1752	2589.1329	2529.3685	2473.9158	2643.3748	2709.6139	2549.8522	2420.3104	**2388.6272**
	Std	39.4258	61.8986	51.2448	50.9354	40.6553	40.2883	27.1043	54.7449	33.1335	49.1951	31.0362	65.0481
	Best	2432.4175	2457.5473	2502.2677	2464.4466	2514.5785	2452.5180	2429.8940	2563.2657	2609.5053	2475.2025	2366.9397	2204.2430
	Worst	2581.8905	2657.3682	2715.1921	2622.3455	2668.6988	2630.8722	2517.3818	2795.7971	2756.7864	2658.6565	2504.3243	2658.6565
	Rank	4	6	10	8	9	5	3	11	12	7	2	1

Table 6 (continued)

F	Index	Algorithm											
		PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**	**HBWO**
F22	Ave	6109.8561	7049.7968	7497.8010	6183.1626	6075.6625	3542.5763	2399.3833	8626.2745	8450.1254	6049.6377	6244.6879	**2300.0013**
	Std	1464.5003	1221.4694	2022.1267	1934.7364	2471.3402	1305.1334	52.3730	939.7920	319.6964	2081.5806	735.1593	0.0001
	Best	2707.6686	2682.2575	2663.9489	2324.3727	2538.6859	2455.2465	2348.6327	6351.5468	7940.4044	2320.9101	5204.0229	2300.0011
	Worst	7636.3954	8338.8288	10002.1862	8029.5623	8522.8724	7029.1515	2594.0805	10265.9072	9003.5627	8690.1685	8048.4270	2300.0017
	Rank	6	9	10	7	5	3	2	12	11	4	8	1
F23	Ave	2965.4947	2996.4377	3111.7738	3134.4080	3037.1280	2931.2160	2926.2826	3490.0654	3299.2966	3138.3352	2755.9422	**2745.7592**
	Std	58.8406	101.9410	127.0545	92.5111	128.1638	71.3144	52.9785	131.0001	63.2793	117.4633	24.8429	83.4068
	Best	2827.0572	2853.7268	2879.4486	2956.7014	2867.2435	2813.7680	2812.9168	3282.8175	3154.1509	2885.6453	2710.7024	2400.0044
	Worst	3087.7605	3171.9445	3332.6742	3334.5761	3412.6120	3067.4194	3032.0425	3700.5069	3384.4321	3328.7348	2801.6992	2805.3067
	Rank	5	6	8	9	7	4	3	12	11	10	2	1
F24	Ave	3135.7638	3105.6899	3221.4059	3424.5504	3185.7998	3054.5138	3053.0035	3827.4739	3532.4220	3381.3365	**2929.1165**	2950.5972
	Std	99.0560	109.4686	100.9139	111.0490	59.8385	51.3528	61.8780	139.8378	69.1839	160.1375	20.4225	32.8631
	Best	3001.1931	2902.7451	3050.4266	3240.8308	3067.7619	2980.7664	2938.7462	3578.1437	3393.5560	3100.8671	2899.3903	2895.3582
	Worst	3392.7942	3345.9161	3395.2952	3614.6389	3281.4267	3169.4179	3170.3926	4082.3328	3615.0356	3740.2089	2957.2468	2999.8603
	Rank	6	5	8	10	7	4	3	12	11	9	1	2
F25	Ave	3211.2386	2912.6312	3038.9561	2927.6037	3046.9645	3088.7182	2960.2558	5347.8520	4317.3950	2922.2684	2898.1004	**2888.9003**
	Std	189.1999	19.7671	42.1076	25.1724	43.5660	74.4351	22.9186	750.4546	151.3576	19.5747	15.5549	3.8665
	Best	2977.2150	2885.5179	2976.4808	2892.7531	2973.0182	2993.5561	2918.0422	4009.7519	4021.1460	2886.6153	2884.0211	2883.4320
	Worst	3651.3781	2949.0394	3124.0680	2982.9068	3125.9017	3301.9208	2994.5043	6767.6739	4605.1009	2952.6203	2937.6835	2944.3881
	Rank	10	3	7	5	8	9	6	12	11	4	2	1
F26	Ave	7177.5898	6128.6915	8117.6243	7631.4761	6894.5618	6360.6456	4836.4678	10495.1393	10460.4689	6988.2442	4737.3528	**3824.6362**
	Std	824.4703	1546.8440	833.0835	864.7109	1528.3168	1144.9286	1392.4522	588.1445	492.3812	1361.7652	387.2618	1388.3413
	Best	5811.3798	2982.4986	6522.0429	4662.6442	3676.1662	4239.4786	3579.6881	9514.8141	9414.4121	3229.3168	4259.8877	2800.0085
	Worst	9376.1716	7954.9727	9774.8328	8790.8118	8616.2368	8460.1084	7979.2909	11364.0104	11141.6114	8756.2478	5779.2272	6532.4757
	Rank	8	4	10	9	6	5	3	12	11	7	2	1

Table 6 (continued)

F	Index	Algorithm											
		PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**	**HBWO**
F27	Ave	3373.5047	3277.6497	3437.7518	3432.7866	3433.4308	3356.8544	3343.1111	4457.8746	3890.2176	3389.2486	**3230.5663**	3308.6936
	Std	81.9472	32.7986	104.4579	116.8794	125.4298	67.5992	52.3217	287.4731	109.7212	106.7578	13.4763	67.6797
	Best	3262.2010	3217.3184	3265.0651	3249.3937	3303.7075	3255.1634	3252.0419	3906.0438	3631.9751	3235.2373	3208.9560	3216.9018
	Worst	3632.4337	3325.7841	3646.1658	3678.1669	3700.3381	3526.5854	3464.8142	4987.8711	4057.9684	3583.3277	3254.6518	3501.9857
	Rank	6	2	10	8	9	5	4	12	11	7	1	3
F28	Ave	4258.5695	3274.6367	3455.7334	3304.3000	3514.4206	3508.7626	3373.1211	6758.8366	6333.3964	3285.7390	3259.0707	**3203.8239**
	Std	684.3681	26.4744	73.6077	31.2571	113.5801	149.5453	44.4038	847.1102	297.8431	27.1039	42.5163	16.6326
	Best	3353.3882	3213.9182	3375.7620	3252.5198	3328.4440	3276.1625	3316.1926	5158.4442	5597.6340	3228.9611	3206.2804	3141.6422
	Worst	5961.2007	3325.2994	3608.0957	3368.3748	3830.9219	3831.3515	3464.6534	8110.9727	6805.3403	3352.9978	3339.7512	3257.4963
	Rank	10	3	7	5	9	8	6	12	11	4	2	1
F29	Ave	4672.2286	4300.6611	4892.7109	4608.7586	4933.8050	4552.3682	4527.9831	6373.5770	6596.6544	4464.6782	4016.1799	**3751.9158**
	Std	263.9605	260.0620	529.8430	408.6676	301.8095	358.3775	319.9550	955.9632	548.1144	320.0715	207.8239	135.2264
	Best	4210.6044	3781.3729	4115.1667	4169.8561	4272.6269	3835.2997	3978.6854	4936.8693	5533.2657	4080.9198	3658.9006	3461.4182
	Worst	5219.4107	4797.1159	6077.6722	5653.0543	5721.8974	5404.2310	5277.7161	8625.2794	7500.7505	5427.7954	4334.5970	3959.5377
	Rank	8	3	9	7	10	6	5	11	12	4	2	1
F30	Ave	6.884E+06	2.116E+04	2.930E+07	3.939E+06	1.572E+07	1.167E+07	1.011E+07	1.291E+09	7.671E+08	3.144E+06	1.017E+06	**1.442E+04**
	Std	6.403E+06	8.307E+03	2.309E+07	3.084E+06	7.532E+06	6.361E+06	8.489E+06	1.272E+09	2.961E+08	1.370E+06	7.075E+05	7.865E+03
	Best	1.424E+05	8.302E+03	7.179E+06	1.027E+06	1.806E+06	2.371E+06	9.072E+05	8.538E+07	3.483E+08	6.671E+05	1.071E+05	7.018E+03
	Worst	2.669E+07	3.773E+04	1.138E+08	1.512E+07	2.942E+07	2.411E+07	3.002E+07	4.277E+09	1.377E+09	5.186E+06	2.398E+06	5.081E+04
	Rank	6	2	10	5	9	8	7	12	11	4	3	1
Average rank		6.2069	5	9.4828	6.7241	8.2759	6.0345	5.0345	10.6207	11.3103	5.4828	2.6552	1.1724
Final rank		7	3	10	8	9	6	4	11	12	5	2	1

The bold represents the optimal values of the evaluation indicators

performance ranking HBWO > LSMA > SSA > AO > LHHO > SCSO > PSO > HHO > DO > WOA > AOA > BWO of the twelve algorithms has been fully verified.

The Wilcoxon rank sum test values are given in Table 7. Bold data is used to indicate $p > 0.05$. The results are obtained as 0/0/29, 0/2/27, 0/0/29, 0/0/29, 0/0/29, 0/0/29, 0/0/29, 0/0/29, 0/0/29, 0/0/29, 3/5/21. HBWO compared with SSA on test functions F13 and F27 $p > 0.05$, and outperformed SSA on all 27 test functions. Compared with LSMA, it is $p > 0.05$ on F8, F10, F21, F23, and F26 test functions and is better than LSMA on 21 test functions. Comparison with other algorithms outperformed the other algorithms. Therefore, the HBWO has a better ability to find the 30 dimensions of the 2017 test functions.

The convergence curves for each test function algorithm in CEC2017 are shown in Fig. 5. The convergence of HBWO is better on the single-peak test function. With multi-peak test and mixed test functions, HBWO suffers from a weaker convergence rate at the beginning of the iteration. However, due to the addition of spiral predation, the later iterations make HBWO jump out of the local most. It shows convergence still downward at the later stages, which makes HBWO have some competitiveness and superiority, especially on the F6, F9, F12, F14, F18, F19, and F22 test functions have better performance. For the composite test functions, HBWO has significantly better finding ability than other algorithms on the F26 and F30 test functions.

In addition, the box line plots are given in Fig. 6. It can be seen that the box corresponding to HBWO is smaller and lower in most cases, which indicates that the results solved by HBWO are better and more stable. HBWO only has slightly higher box positions than LSMA on the CEC2017 test functions F5, F7, F24, and F27, and the median box on the F8 test function is slightly lower than the LSMA and has a smaller box. The box of HBWO is smaller and lower than that of the other algorithms, especially on the test functions F14, F16, F17, F20, F22, and F29. In general, the box line plot shows that HBWO has a significant improvement over BWO.

The radar plot drawn by the ranking of the twelve optimization algorithms on the CEC2017 test function is given in Fig. 7. It can be seen that the HBWO has the smallest shaded area, and BWO has the largest area. It further illustrates that the performance of BWO combined with the three strategies post-computation has a great improvement, which shows the superiority of HBWO.

Tables 8, 9, and 10 give the results of HBWO, OBCWOA [87], and BWO runs on CEC2017 test functions on 10, 50, and 100 dimensions. From the table, we can see that HBWO has improved performance on each test function in 50 dimensions, especially F1, F4, F12, F13, F14, F15, F19, and F30, which have very obvious improvements. It shows some superiority in 100 dimensions on the F1, F3, F12, F13, F14, F15, F17, F18, F19, and F30. It further illustrates that BWO combines the three strategies to obtain richer populations and escape local optimal solutions in high-dimensional space.

Experiments and analysis on the CEC2019 test set

The performance of the HBWO was further tested on the CEC2019 test set. The algorithm-related parameters were first set with a population size of 40 and run 20 times, and the results obtained are shown in Table 11. From Table 11, the average rank of HBWO is 1.1, which is the first overall rank. HBWO ranked second only on the F2 test

Table 7 Wilcoxon rank sum test results of other optimization algorithms CEC2017 test set based on HBWO

F	Algorithm										
	PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**
F1	6.79560E-08	1.20090E-06	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	0.0071135
F3	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	1.65710E-07
F4	6.79560E-08	1.04730E-06	6.79560E-08	6.01480E-07	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	7.94790E-07	0.0036388
F5	9.12660E-07	6.79560E-08	6.79560E-08	7.89800E-08	6.79560E-08	1.06460E-07	0.000011045	6.79560E-08	6.79560E-08	6.79560E-08	0.0065572
F6	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	2.95980E-07
F7	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	9.17280E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	0.0003048
F8	5.87360E-06	6.79560E-08	1.06460E-07	2.95980E-07	6.79560E-08	6.79560E-08	5.87360E-06	6.79560E-08	6.79560E-08	1.65710E-07	**0.4093600**
F9	2.56290E-07	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	0.0315170
F10	0.0003048	1.91770E-07	6.79560E-08	7.57740E-06	9.12660E-07	2.68980E-06	0.0001159	6.79560E-08	6.79560E-08	2.35570E-06	**0.2732900**
F11	6.79560E-08	1.91770E-07	6.79560E-08	6.91660E-07	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	0.000082924	3.93880E-07
F12	6.79560E-08	3.06910E-06	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
F13	4.53900E-07	**0.3369200**	6.79560E-08	6.79560E-08	7.89800E-08	7.89800E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	7.89800E-08
F14	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
F15	6.79560E-08	0.0065572	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
F16	0.000066104	0.0036388	7.89800E-08	1.91770E-07	7.94790E-07	0.000023025	7.94790E-07	6.79560E-08	6.79560E-08	1.04730E-06	0.0315170
F17	0.00033819	2.35570E-06	6.91660E-07	0.00002596	6.91660E-07	0.012345	0.00068682	7.89800E-08	7.89800E-08	4.53900E-07	0.0023413
F18	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
F19	5.22690E-07	0.00604030	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
F20	5.16580E-06	3.41560E-07	6.79560E-08	4.53900E-07	2.21780E-07	1.79360E-04	5.09070E-04	0.000011045	6.79560E-08	0.00012941	0.0179390
F21	9.17280E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	2.21780E-07	6.79560E-08	6.79560E-08	6.79560E-08	**0.0764310**
F22	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
F23	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	**0.5608500**
F24	9.17280E-08	4.54010E-06	6.79560E-08	6.79560E-08	6.79560E-08	2.95980E-07	1.04730E-06	6.79560E-08	6.79560E-08	6.79560E-08	0.0467920
F25	6.79560E-08	2.47060E-04	6.79560E-08	1.43090E-07	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	2.06160E-06	0.015479
F26	1.23460E-07	7.57740E-06	6.79560E-08	1.91770E-07	4.54010E-06	6.67370E-06	0.0083548	6.79560E-08	6.79560E-08	6.01480E-07	**0.0764310**

Table 7 (continued)

F	Algorithm										
	PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**
F27	0.0097865	**0.285300**	0.000082924	0.0004155	0.0004155	0.033718	0.0438800	6.79560E-08	6.79560E-08	0.0097865	5.87360E-06
F28	6.79560E-08	1.23460E-07	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	2.06160E-06
F29	6.79560E-08	2.56290E-07	6.79560E-08	6.79560E-08	6.79560E-08	1.91770E-07	7.89800E-08	6.79560E-08	6.79560E-08	6.79560E-08	0.00014438
F30	6.79560E-08	0.0065572	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
+/=/−	0/0/29	0/2/27	0/0/29	0/0/29	0/0/29	0/0/29	0/0/29	0/0/29	0/0/29	0/0/29	3/5/21

The bold represents the optimal values of the evaluation indicators

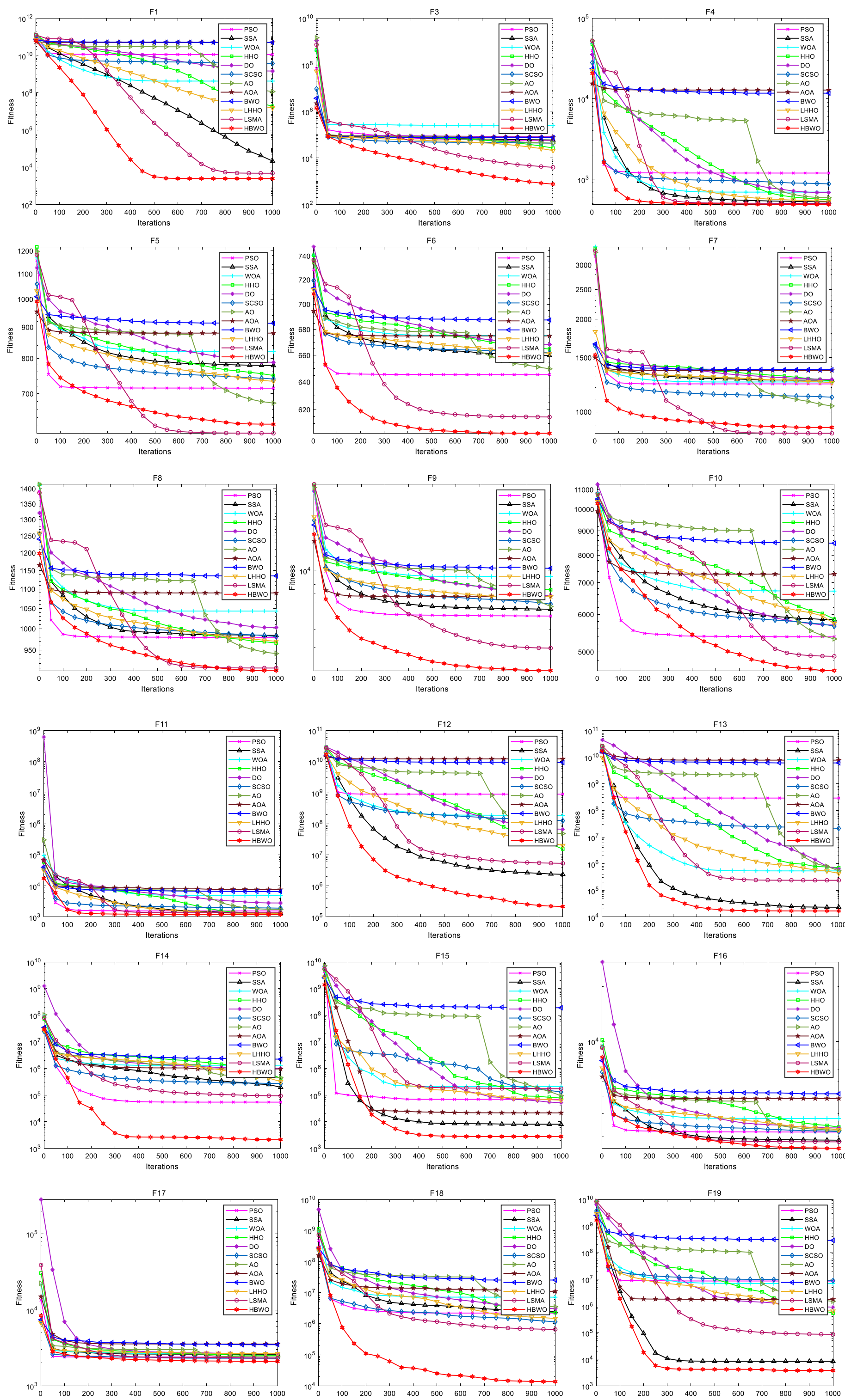

Fig. 5 Convergence curves of HBWO and other algorithms on CEC2017 test set

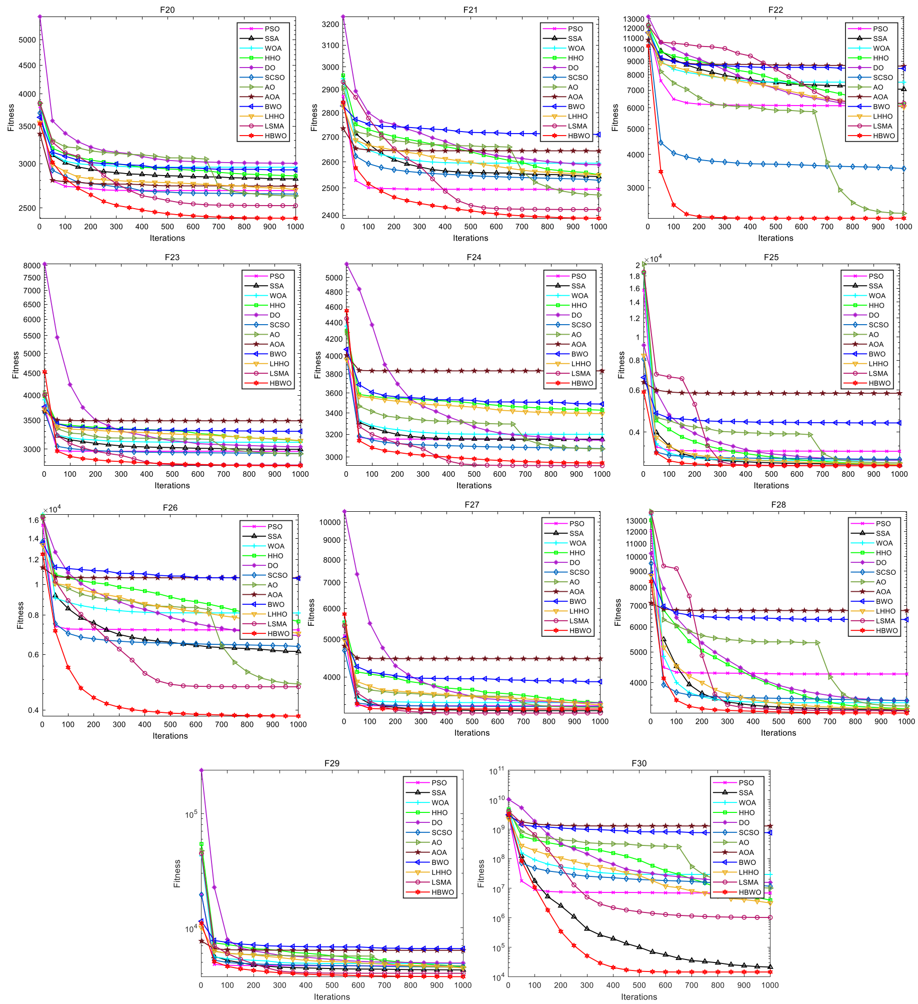

Fig. 5 continued

function, and SCSO ranked first. However, HBWO ranks first on other test functions, and all experimental results are better than the SCSO algorithm. In general, HBWO is a great improvement over BWO. However, HBWO still needs to improve its accuracy for some single-peak functions. The main reason for this is that the proposed algorithm effectively balances the exploration and development process, thus resulting in an exploration time that is too short.

From Table 11, it can be seen that the HBWO ranks first, the LSMA ranks second, the SCSO ranks third, the HHO ranks fourth, the BWO ranks eleventh, and the AOA ranks twelfth. Therefore, the performance ranking of the twelve algorithms for solving the CEC2019 can be obtained as HBWO > LSMA > SCSO > HHO > AO > LHHO > PSO > SSA > DO > WOA > BWO > AOA.

The Wilcoxon rank sum test values are given in Table 12. According to the last row of Table 12, the results are obtained as 0/1/9, 0/1/9, 0/1/9, 0/1/9, 0/1/9, 0/2/8, 0/2/8, 0/0/10, 0/1/9, 0/1/9, 0/3/7. It can be seen that HBWO is better than PSO, SSA, WOA, HHO, DO, BWO, and LHHO on 9 test functions; HBWO showed better results in the CEC2019 test set.

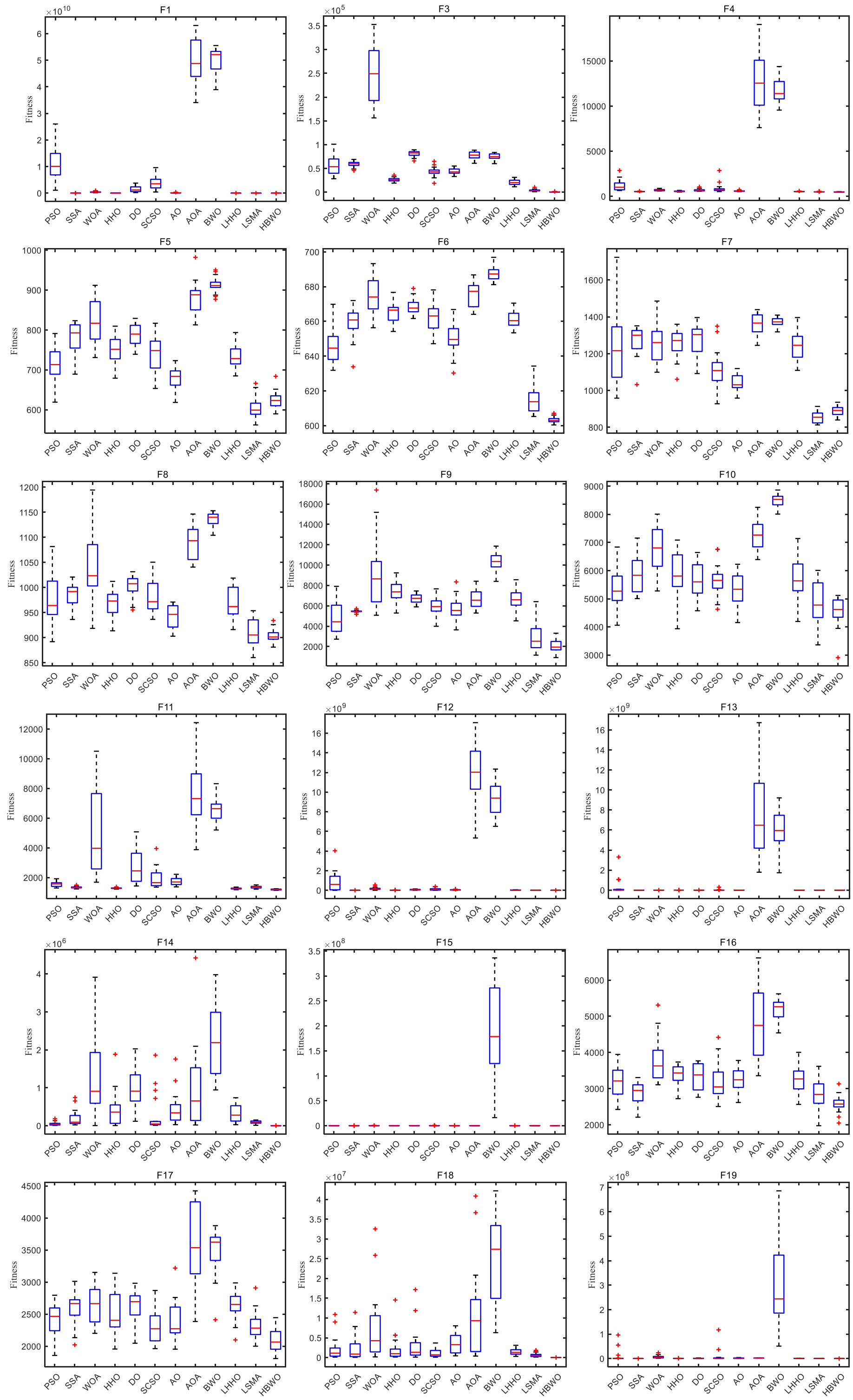

Fig. 6 Boxplot of HBWO and other algorithms on CEC2017 test set

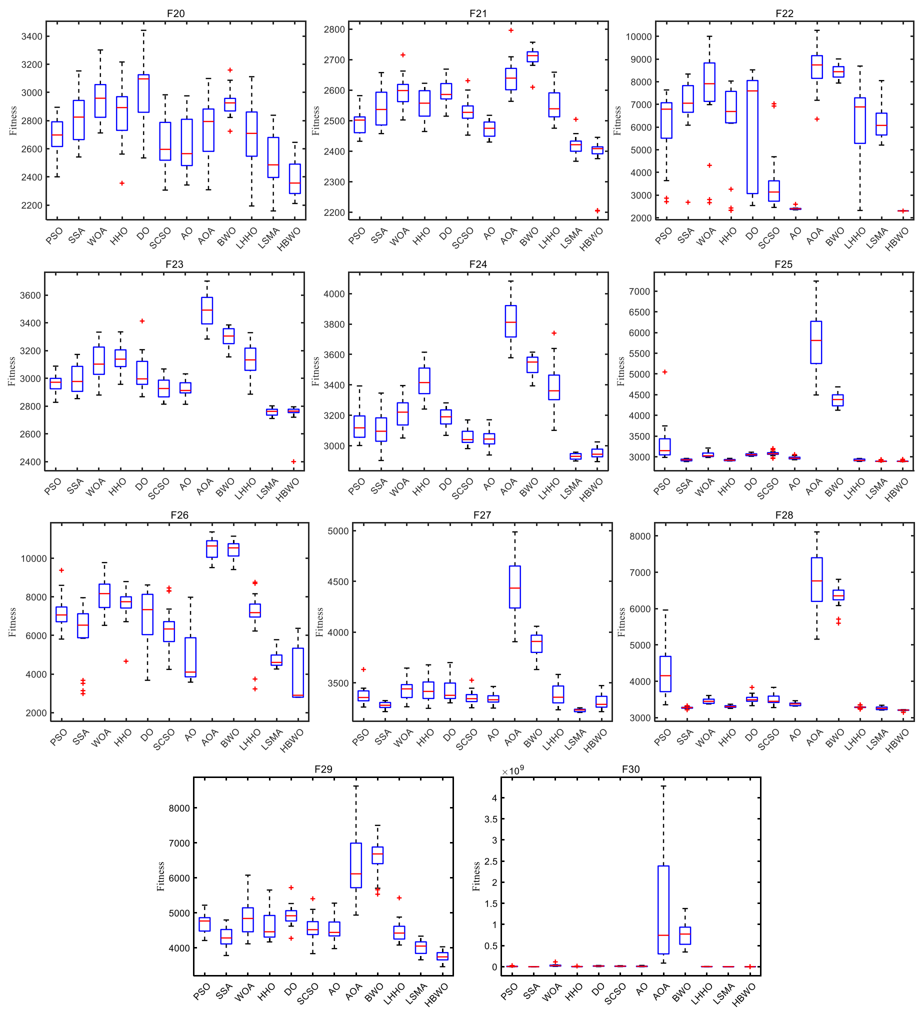

Fig. 6 continued

The convergence profile of HBWO on the CEC2019 test function is given in Fig. 8. It can be observed from Fig. 8. On the test functions F4, F6, F7, and F8, the HBWO algorithm not only converges fast but also converges with high accuracy, showing certain advantages. On the test functions F4, F6, F7, F8, and F10, HBWO's convergence speed is weaker than PSO in the early stage, but it still converges downward and jumps out of the local optimal value in the later iteration, and its optimization ability is stronger than PSO. HBWO has little difference in convergence speed and convergence accuracy with SSA, HHO, DO, SCSO, AO, BWO, LHHO, and LSMA on F1, F2, and F3 test functions, which is also the place that needs to be improved in the future. However, compared with the above algorithms on other test functions, HBWO is significantly superior to other algorithms.

Figure 9 presents the boxplots of each algorithm on the CEC2019 test functions. Figure 9 shows that HBWO has lower and smaller boxings on most of the test functions, which indicates the superiority of the algorithm. These test functions have obvious superiority, especially in the F3, F4, and F6. On the test functions F7 and F8, HBWO has a

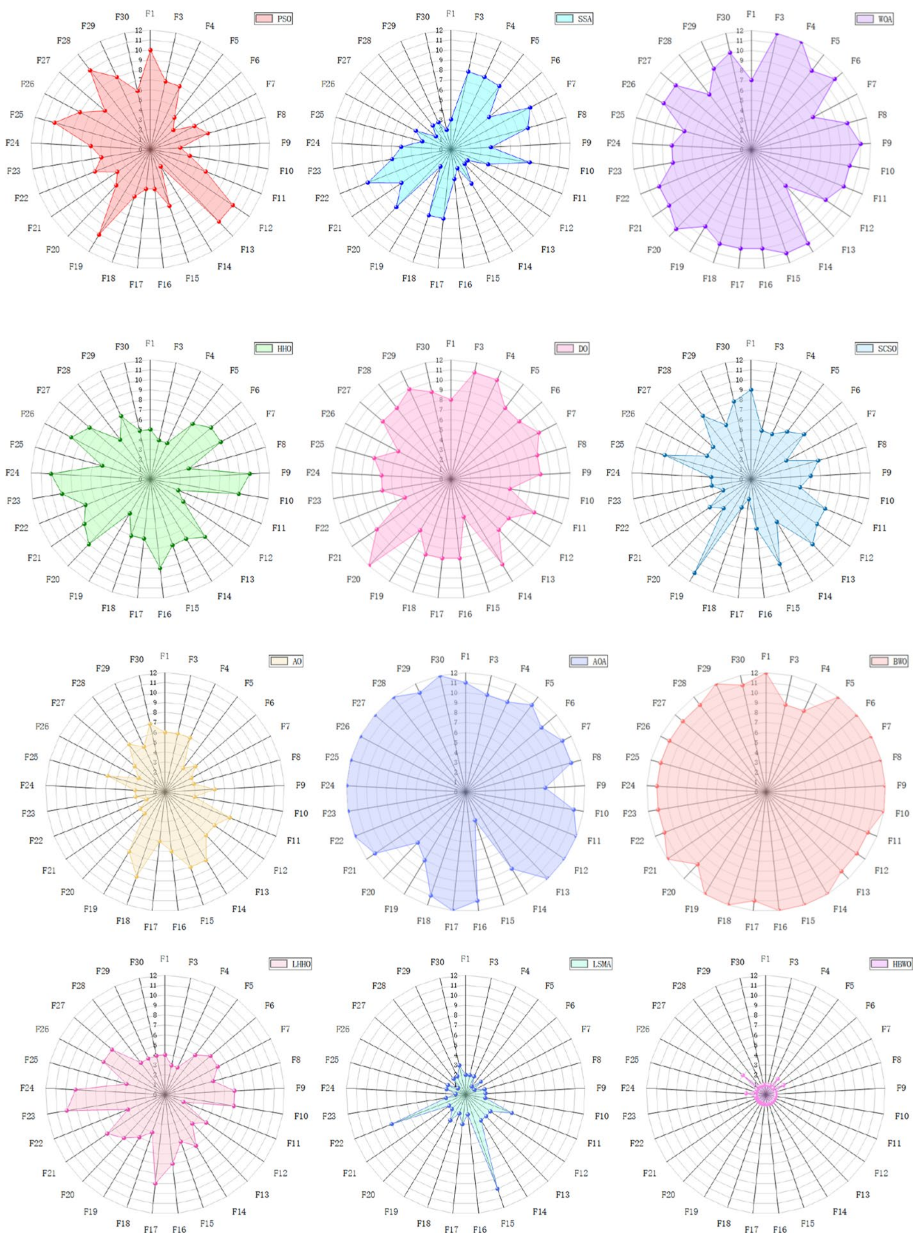

Fig. 7 Radar graph of HBWO and other algorithms on CEC2017 test set

larger box, but obviously, it can be observed that HBWO has a lower box with a smaller mean value.

The radar plot drawn by the ranking of the twelve optimization algorithms on the CEC2019 test function is given in Fig. 10. It can be seen that the HBWO has the smallest shaded area, BWO is ranked eleventh in terms of area, and AOA has the largest area. It further illustrates that the performance of BWO combined with the QOBL, adaptive and spiral predation strategies, and NM post-computation has shown a great improvement.

Table 8 Results of HBWO, OBCWOA and BWO for solving 10 dimensional CEC2017 test set

F	Index	Algorithm			F	Algorithm			F	Algorithm		
		BWO	OBCWOA	HBWO		BWO	OBCWOA	HBWO		BWO	OBCWOA	HBWO
F1	Mean	6.75E+07	4.68E+08	1.00E+02	F11	1.13E+03	5.32E+03	1.11E+03	F21	2.23E+03	2.58E+03	2.23E+03
	Std	4.92E+07	2.41E+08	2.22E-01		9.59E+00	2.36E+03	1.49E+00		4.95E+01	6.32E+01	4.71E+01
	Best	2.43E+07	1.70E+08	1.00E+02		1.11E+03	1.75E+03	1.10E+03		2.20E+03	2.46E+03	2.20E+03
	Worst	2.37E+08	9.75E+08	1.01E+02		1.15E+03	1.19E+04	1.11E+03		2.33E+03	2.71E+03	2.31E+03
F2	Mean	NA	NA	NA	F12	7.04E+05	1.39E+08	6.65E+03	F22	2.32E+03	7.10E+03	2.30E+03
	Std	NA	NA	NA		4.64E+05	1.06E+08	3.91E+03		1.64E+00	1.84E+03	1.05E-02
	Best	NA	NA	NA		5.68E+04	2.31E+07	1.48E+03		2.31E+03	2.62E+03	2.30E+03
	Worst	NA	NA	NA		1.70E+06	3.51E+08	1.29E+04		2.32E+03	8.69E+03	2.30E+03
F3	Mean	1.36E+03	2.38E+05	3.00E+02	F13	1.72E+04	6.88E+05	1.31E+03	F23	2.62E+03	3.11E+03	2.61E+03
	Std	4.76E+02	8.97E+04	4.64E-08		1.02E+04	6.38E+05	3.09E+00		5.01E+00	8.71E+01	2.98E+00
	Best	8.29E+02	1.12E+05	3.00E+02		2.76E+03	1.62E+05	1.31E+03		2.61E+03	2.92E+03	2.61E+03
	Worst	2.73E+03	4.73E+05	3.00E+02		3.33E+04	2.91E+06	1.32E+03		2.63E+03	3.33E+03	2.62E+03
F4	Mean	4.11E+02	6.71E+02	4.00E+02	F14	1.76E+03	1.30E+06	1.43E+03	F24	2.66E+03	3.24E+03	2.62E+03
	Std	1.12E+00	8.35E+01	5.95E-02		2.89E+02	1.23E+06	1.81E+01		1.15E+02	1.06E+02	1.22E+02
	Best	4.09E+02	5.70E+02	4.00E+02		1.44E+03	3.47E+04	1.40E+03		2.52E+03	3.07E+03	2.50E+03
	Worst	4.13E+02	8.70E+02	4.00E+02		2.67E+03	3.64E+06	1.48E+03		2.77E+03	3.43E+03	2.75E+03
F5	Mean	5.24E+02	7.99E+02	5.10E+02	F15	2.20E+03	2.24E+05	1.51E+03	F25	2.94E+03	3.05E+03	2.93E+03
	Std	4.38E+00	4.46E+01	4.24E+00		7.57E+02	2.27E+05	1.64E+01		1.62E+01	3.23E+01	2.03E+01
	Best	5.16E+02	7.03E+02	5.03E+02		1.55E+03	2.88E+04	1.50E+03		2.91E+03	3.00E+03	2.90E+03
	Worst	5.35E+02	8.80E+02	5.18E+02		4.95E+03	9.90E+05	1.58E+03		2.95E+03	3.13E+03	2.95E+03
F6	Mean	6.05E+02	6.75E+02	6.00E+02	F16	1.66E+03	4.10E+03	1.66E+03	F26	2.94E+03	7.82E+03	2.88E+03
	Std	9.23E-01	9.46E+00	6.04E-06		4.79E+01	5.24E+02	7.58E+01		6.09E+01	7.64E+02	4.70E+01
	Best	6.03E+02	6.55E+02	6.00E+02		1.61E+03	3.13E+03	1.60E+03		2.70E+03	6.28E+03	2.80E+03
	Worst	6.06E+02	6.92E+02	6.00E+02		1.75E+03	4.98E+03	1.86E+03		2.99E+03	9.42E+03	2.95E+03

Table 8 (continued)

F	Index	Algorithm			F	Algorithm			F	Algorithm		
		BWO	**OBCWOA**	**HBWO**		**BWO**	**OBCWOA**	**HBWO**		**BWO**	**OBCWOA**	**HBWO**
F7	Mean	7.37E+02	1.26E+03	7.19E+02	F17	1.74E+03	2.74E+03	1.73E+03	F27	3.10E+03	3.44E+03	3.10E+03
	Std	4.69E+00	8.17E+01	3.69E+00		6.58E+00	1.79E+02	1.20E+01		1.90E+00	9.78E+01	2.93E+00
	Best	7.24E+02	1.09E+03	7.14E+02		1.73E+03	2.41E+03	1.71E+03		3.09E+03	3.29E+03	3.10E+03
	Worst	7.43E+02	1.39E+03	7.26E+02		1.75E+03	3.10E+03	1.77E+03		3.10E+03	3.66E+03	3.11E+03
F8	Mean	8.19E+02	1.04E+03	8.09E+02	F18	2.23E+04	4.93E+06	1.89E+03	F28	3.22E+03	3.47E+03	3.55E+03
	Std	3.76E+00	4.24E+01	4.48E+00		1.51E+04	4.44E+06	1.80E+02		1.02E+02	9.28E+01	1.69E+02
	Best	8.11E+02	9.66E+02	8.04E+02		2.77E+03	1.45E+05	1.80E+03		3.12E+03	3.35E+03	3.11E+03
	Worst	8.28E+02	1.13E+03	8.19E+02		4.43E+04	1.66E+07	2.60E+03		3.41E+03	3.73E+03	3.73E+03
F9	Mean	9.18E+02	1.03E+04	9.00E+02	F19	2.83E+03	6.90E+06	1.90E+03	F29	3.18E+03	5.12E+03	3.18E+03
	Std	1.06E+01	3.55E+03	1.52E-09		9.32E+02	6.08E+06	1.25E+00		2.22E+01	6.47E+02	1.51E+01
	Best	9.06E+02	5.56E+03	9.00E+02		2.01E+03	1.94E+04	1.90E+03		3.15E+03	4.00E+03	3.16E+03
	Worst	9.40E+02	1.95E+04	9.00E+02		5.53E+03	2.18E+07	1.91E+03		3.24E+03	6.77E+03	3.23E+03
F10	Mean	1.55E+03	6.59E+03	1.41E+03	F20	2.04E+03	2.82E+03	2.02E+03	F30	1.60E+05	2.74E+07	9.76E+04
	Std	1.18E+02	8.09E+02	1.90E+02		6.72E+00	2.01E+02	7.49E+00		3.16E+05	1.84E+07	1.24E+05
	Best	1.30E+03	5.27E+03	1.03E+03		2.03E+03	2.45E+03	2.00E+03		9.78E+03	2.95E+06	3.71E+03
	Worst	1.79E+03	8.06E+03	1.75E+03		2.05E+03	3.21E+03	2.03E+03		1.20E+06	6.18E+07	4.03E+05

Table 9 Results of HBWO, OBCWOA and BWO for solving 50 dimensional CEC2017 test set

F	Index	Algorithm			F	Algorithm			F	Algorithm		
		BWO	OBCWOA	HBWO		BWO	OBCWOA	HBWO		BWO	OBCWOA	HBWO
F1	Mean	1.01E+11	3.56E+09	2.00E+04	F11	2.12E+04	3.18E+03	1.33E+03	F21	3.16E+03	3.05E+03	2.61E+03
	Std	4.55E+09	1.29E+09	2.05E+04		1.66E+03	4.99E+02	9.51E+01		3.47E+01	1.42E+02	4.33E+01
	Best	9.22E+10	1.67E+09	2.06E+03		1.64E+04	2.61E+03	1.21E+03		3.12E+03	2.79E+03	2.54E+03
	Worst	1.07E+11	6.57E+09	7.18E+04		2.38E+04	4.58E+03	1.65E+03		3.24E+03	3.27E+03	2.69E+03
F2	Mean	NA	NA	NA	F12	5.95E+10	8.70E+08	2.36E+06	F22	1.64E+04	1.37E+04	5.53E+03
	Std	NA	NA	NA		7.50E+09	4.14E+08	1.68E+06		4.13E+02	8.48E+02	4.36E+03
	Best	NA	NA	NA		3.98E+10	3.01E+08	5.48E+05		1.54E+04	1.20E+04	2.30E+03
	Worst	NA	NA	NA		7.38E+10	1.78E+09	6.83E+06		1.73E+04	1.49E+04	1.18E+04
F3	Mean	2.09E+05	2.41E+05	1.14E+04	F13	3.22E+10	3.85E+07	1.47E+04	F23	4.07E+03	3.70E+03	3.11E+03
	Std	2.59E+04	8.22E+04	4.20E+03		6.82E+09	4.31E+07	1.49E+04		7.96E+01	1.69E+02	4.47E+01
	Best	1.60E+05	1.51E+05	5.30E+03		1.68E+10	7.14E+06	5.52E+03		3.95E+03	3.45E+03	3.00E+03
	Worst	2.57E+05	4.18E+05	2.01E+04		4.18E+10	1.95E+08	6.58E+04		4.23E+03	3.97E+03	3.21E+03
F4	Mean	3.13E+04	1.63E+03	5.76E+02	F14	4.07E+07	3.03E+06	6.65E+03	F24	4.38E+03	3.81E+03	3.33E+03
	Std	2.74E+03	3.40E+02	4.65E+01		1.95E+07	1.59E+06	8.05E+03		1.26E+02	1.92E+02	7.99E+01
	Best	2.53E+04	1.06E+03	4.68E+02		8.02E+06	5.73E+05	1.70E+03		4.03E+03	3.48E+03	3.21E+03
	Worst	3.52E+04	2.32E+03	6.27E+02		6.73E+07	7.01E+06	3.37E+04		4.60E+03	4.09E+03	3.51E+03
F5	Mean	1.19E+03	1.05E+03	8.15E+02	F15	5.23E+09	2.46E+06	1.37E+07	F25	1.39E+04	3.72E+03	3.11E+03
	Std	1.71E+01	1.05E+02	3.98E+01		1.20E+09	4.97E+06	6.10E+07		7.66E+02	1.87E+02	2.68E+01
	Best	1.16E+03	8.97E+02	7.39E+02		3.18E+09	1.94E+05	2.78E+03		1.18E+04	3.39E+03	3.06E+03
	Worst	1.22E+03	1.25E+03	8.88E+02		7.38E+09	2.30E+07	2.73E+08		1.49E+04	4.16E+03	3.18E+03
F6	Mean	7.02E+02	6.91E+02	6.20E+02	F16	8.16E+03	5.94E+03	3.48E+03	F26	1.63E+04	1.35E+04	4.51E+03
	Std	2.58E+00	1.37E+01	7.87E+00		6.84E+02	9.22E+02	3.97E+02		4.11E+02	2.01E+03	2.92E+03
	Best	6.93E+02	6.67E+02	6.09E+02		6.86E+03	4.59E+03	2.74E+03		1.54E+04	1.04E+04	2.91E+03
	Worst	7.06E+02	7.17E+02	6.39E+02		9.39E+03	8.02E+03	4.25E+03		1.70E+04	1.73E+04	1.07E+04

Table 9 (continued)

F	Index	Algorithm			F	Algorithm			F	Algorithm		
		BWO	**OBCWOA**	**HBWO**		**BWO**	**OBCWOA**	**HBWO**		**BWO**	**OBCWOA**	**HBWO**
F7	Mean	1.96E+03	1.80E+03	1.23E+03	F17	6.46E+03	4.15E+03	3.05E+03	F27	5.75E+03	4.54E+03	3.96E+03
	Std	3.69E+01	1.14E+02	9.06E+01		1.01E+03	3.56E+02	2.32E+02		4.10E+02	5.04E+02	6.26E+02
	Best	1.86E+03	1.62E+03	1.12E+03		4.60E+03	3.60E+03	2.44E+03		5.03E+03	3.94E+03	3.30E+03
	Worst	2.00E+03	2.06E+03	1.45E+03		8.20E+03	5.09E+03	3.31E+03		6.41E+03	5.51E+03	5.27E+03
F8	Mean	1.49E+03	1.33E+03	1.12E+03	F18	9.97E+07	2.85E+07	2.65E+05	F28	1.18E+04	4.60E+03	3.37E+03
	Std	2.37E+01	8.39E+01	3.65E+01		3.85E+07	1.80E+07	4.55E+05		3.27E+02	2.70E+02	3.47E+01
	Best	1.45E+03	1.23E+03	1.06E+03		4.49E+07	6.04E+06	2.74E+04		1.12E+04	4.23E+03	3.33E+03
	Worst	1.53E+03	1.54E+03	1.18E+03		1.70E+08	7.63E+07	1.74E+06		1.23E+04	5.22E+03	3.46E+03
F9	Mean	3.70E+04	3.17E+04	1.24E+04	F19	2.87E+09	5.86E+06	1.69E+04	F29	1.89E+04	8.76E+03	4.61E+03
	Std	2.98E+03	1.01E+04	3.56E+03		5.69E+08	6.40E+06	6.70E+03		5.30E+03	1.44E+03	3.04E+02
	Best	2.95E+04	1.41E+04	4.30E+03		1.98E+09	7.06E+04	7.43E+03		1.29E+04	6.29E+03	3.91E+03
	Worst	4.01E+04	6.00E+04	1.84E+04		4.43E+09	1.57E+07	3.49E+04		3.42E+04	1.15E+04	5.15E+03
F10	Mean	1.46E+04	1.18E+04	8.53E+03	F20	3.97E+03	3.79E+03	3.19E+03	F30	4.08E+09	2.16E+08	5.33E+06
	Std	4.13E+02	1.10E+03	7.12E+02		1.96E+02	3.76E+02	1.75E+02		1.03E+09	5.80E+07	5.79E+06
	Best	1.38E+04	1.04E+04	6.96E+03		3.62E+03	3.17E+03	2.93E+03		1.68E+09	1.39E+08	1.14E+06
	Worst	1.01E+11	1.48E+04	2.00E+04		4.31E+03	4.57E+03	3.54E+03		5.65E+09	3.33E+08	2.16E+07

Table 10 Results of HBWO, OBCWOA and BWO for solving 100 dimensional CEC2017 test set

F	Index	Algorithm			F	Algorithm			F	Algorithm		
		BWO	OBCWOA	HBWO		BWO	OBCWOA	HBWO		BWO	OBCWOA	HBWO
F1	Mean	2.51E+11	3.74E+10	8.05E+09	F11	2.88E+05	1.86E+05	1.27E+04	F21	4.68E+03	4.20E+03	3.35E+03
	Std	7.89E+09	5.46E+09	3.30E+09		5.85E+04	9.75E+04	7.40E+03		1.06E+02	1.89E+02	7.08E+01
	Best	2.31E+11	2.51E+10	2.70E+09		1.94E+05	8.98E+04	3.80E+03		4.52E+03	3.72E+03	3.21E+03
	Worst	2.59E+11	4.64E+10	1.48E+10		3.93E+05	4.30E+05	3.76E+04		4.88E+03	4.50E+03	3.47E+03
F2	Mean	NA	NA	NA	F12	1.79E+11	7.11E+09	2.09E+08	F22	3.44E+04	2.90E+04	2.55E+04
	Std	NA	NA	NA		1.09E+10	2.83E+09	5.14E+07		5.84E+02	1.92E+03	2.47E+03
	Best	NA	NA	NA		1.58E+11	4.42E+09	1.37E+08		3.33E+04	2.49E+04	1.87E+04
	Worst	NA	NA	NA		1.96E+11	1.51E+10	2.78E+08		3.53E+04	3.17E+04	2.76E+04
F3	Mean	3.49E+05	8.74E+05	1.26E+05	F13	4.10E+10	1.67E+08	4.53E+04	F23	5.94E+03	5.02E+03	4.11E+03
	Std	1.37E+04	1.65E+05	1.25E+04		3.25E+09	7.37E+07	1.26E+04		1.71E+02	2.81E+02	5.68E+02
	Best	3.10E+05	4.16E+05	1.01E+05		3.42E+10	5.91E+07	3.17E+04		5.51E+03	4.59E+03	3.60E+03
	Worst	3.65E+05	1.10E+06	1.55E+05		4.56E+10	3.70E+08	6.71E+04		6.20E+03	5.46E+03	5.88E+03
F4	Mean	9.50E+04	6.90E+03	1.49E+03	F14	6.42E+07	1.05E+07	8.75E+04	F24	8.85E+03	6.52E+03	6.29E+03
	Std	7.35E+03	1.24E+03	2.25E+02		1.42E+07	5.67E+06	5.29E+04		5.04E+02	3.34E+02	1.54E+03
	Best	7.71E+04	4.73E+03	1.16E+03		4.13E+07	4.32E+06	7.53E+03		7.90E+03	5.94E+03	4.36E+03
	Worst	1.09E+05	9.08E+03	2.00E+03		9.05E+07	2.67E+07	2.03E+05		1.00E+04	7.32E+03	8.99E+03
F5	Mean	2.10E+03	1.75E+03	1.47E+03	F15	2.07E+10	2.65E+07	2.01E+04	F25	2.70E+04	6.31E+03	4.23E+03
	Std	1.82E+01	1.02E+02	1.49E+02		2.51E+09	2.24E+07	6.60E+03		1.25E+03	3.80E+02	2.52E+02
	Best	2.04E+03	1.62E+03	1.11E+03		1.36E+10	5.34E+06	8.55E+03		2.31E+04	5.37E+03	3.87E+03
	Worst	2.13E+03	1.97E+03	1.65E+03		2.39E+10	1.08E+08	3.18E+04		2.84E+04	7.04E+03	4.72E+03
F6	Mean	7.12E+02	6.98E+02	6.58E+02	F16	2.13E+04	1.37E+04	7.17E+03	F26	4.93E+04	3.51E+04	2.09E+04
	Std	2.18E+00	1.12E+01	6.47E+00		1.65E+03	2.24E+03	1.13E+03		1.83E+03	4.21E+03	5.43E+03
	Best	7.08E+02	6.78E+02	6.45E+02		1.79E+04	9.86E+03	5.38E+03		4.57E+04	2.61E+04	9.16E+03
	Worst	7.15E+02	7.27E+02	6.71E+02		2.33E+04	1.86E+04	9.40E+03		5.19E+04	4.36E+04	2.74E+04

Table 10 (continued)

F	Index	Algorithm			F	Algorithm			F	Algorithm		
		BWO	**OBCWOA**	**HBWO**		**BWO**	**OBCWOA**	**HBWO**		**BWO**	**OBCWOA**	**HBWO**
F7	Mean	3.83E+03	3.62E+03	2.65E+03	F17	2.92E+06	9.50E+03	5.99E+03	F27	1.18E+04	5.74E+03	3.87E+03
	Std	5.68E+01	1.91E+02	2.75E+02		1.32E+06	1.20E+03	8.58E+02		6.37E+02	9.41E+02	1.82E+02
	Best	3.73E+03	3.24E+03	2.31E+03		5.99E+05	7.38E+03	4.19E+03		1.04E+04	4.38E+03	3.59E+03
	Worst	3.97E+03	4.00E+03	3.47E+03		5.41E+06	1.18E+04	7.32E+03		1.30E+04	8.45E+03	4.26E+03
F8	Mean	2.58E+03	2.23E+03	1.92E+03	F18	1.59E+08	9.52E+06	2.41E+05	F28	2.69E+04	8.62E+03	4.56E+03
	Std	3.00E+01	1.05E+02	1.10E+02		4.02E+07	3.17E+06	1.10E+05		6.27E+02	9.38E+02	3.54E+02
	Best	2.53E+03	2.07E+03	1.56E+03		8.89E+07	4.43E+06	1.06E+05		2.53E+04	7.09E+03	4.05E+03
	Worst	2.63E+03	2.46E+03	2.11E+03		2.25E+08	1.58E+07	5.26E+05		2.78E+04	1.00E+04	5.26E+03
F9	Mean	7.67E+04	7.21E+04	4.31E+04	F19	2.01E+10	6.40E+07	8.89E+04	F29	3.06E+05	1.66E+04	8.96E+03
	Std	3.39E+03	2.43E+04	3.39E+03		2.93E+09	4.70E+07	6.12E+04		1.43E+05	2.26E+03	1.11E+03
	Best	6.69E+04	4.22E+04	3.59E+04		1.31E+10	7.70E+06	2.57E+04		7.70E+04	1.28E+04	6.75E+03
	Worst	8.28E+04	1.26E+05	4.89E+04		2.36E+10	1.77E+08	2.49E+05		6.83E+05	2.13E+04	1.03E+04
F10	Mean	3.17E+04	2.69E+04	2.23E+04	F20	7.55E+03	6.74E+03	5.83E+03	F30	3.51E+10	9.44E+08	4.54E+06
	Std	8.53E+02	2.09E+03	1.98E+03		2.39E+02	6.34E+02	4.91E+02		3.79E+09	3.50E+08	2.56E+06
	Best	3.00E+04	2.37E+04	1.52E+04		7.23E+03	5.44E+03	4.57E+03		2.79E+10	3.82E+08	1.36E+06
	Worst	3.29E+04	3.11E+04	2.46E+04		7.94E+03	7.80E+03	6.62E+03		4.22E+10	1.62E+09	1.07E+07

Table 11 Results of various HBWO on CEC2019

F	Index	Algorithm											
		PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**	**HBWO**
F1	Mean	8.269E+06	1	8.204E+06	**1**	**1**	1	1	7.992E+03	**1**	**1**	**1**	**1**
	Std	9.505E+06	4.651E-15	1.205E+07	0	0	2.010E-12	1.368E-08	3.574E+04	0	0	0	0
	Best	2.469E+05	1	6.605E+00	1	1	1	1	1	1	1	1	1
	Worst	3.718E+07	1	4.881E+07	1	1	1.000E+00	1.000E+00	1.598E+05	1	1	1	1
	Rank	12	7	11	1	1	8	9	10	1	1	1	1
F2	Mean	4.331E+03	4.666E+00	7.376E+03	4.963E+00	5	**4.523E+00**	4.977E+00	8.498E+03	4.997E+00	5	5	4.551E+00
	Std	3.541E+03	5.455E-01	2.771E+03	1.049E-01	0	3.602E-01	8.121E-02	1.882E+03	1.175E-02	1.977E-03	8.662E-06	3.760E-01
	Best	4.822E+02	4.222E+00	3.115E+03	4.569E+00	5	4.217E+00	4.651E+00	5.426E+03	4.947E+00	4.991E+00	5	4.217E+00
	Worst	1.305E+04	6.479E+00	1.543E+04	5	5	5	5	1.206E+04	5	5	5	5
	Rank	10	3	11	4	9	1	5	12	6	7	8	2
F3	Mean	7.968E+00	7.200E+00	3.913E+00	4.097E+00	5.816E+00	2.549E+00	5.556E+00	9.578E+00	4.138E+00	3.582E+00	3.561E+00	**2.143E+00**
	Std	2.390E+00	2.182E+00	2.005E+00	1.318E+00	1.799E+00	9.844E-01	1.835E+00	1.093E+00	8.606E-01	1.386E+00	2.202E+00	6.181E-01
	Best	1.901E+00	3.738E+00	1.485E+00	1.761E+00	1.518E+00	1.409E+00	2.907E+00	7.311E+00	2.729E+00	1.429E+00	1.000E+00	1.296E+00
	Worst	1.071E+01	1.069E+01	7.711E+00	6.429E+00	8.629E+00	5.236E+00	9.760E+00	1.174E+01	5.792E+00	6.237E+00	8.612E+00	3.562E+00
	Rank	11	10	5	6	9	2	8	12	7	4	3	1
F4	Mean	2.528E+01	4.324E+01	5.244E+01	3.968E+01	5.508E+01	4.050E+01	2.733E+01	4.368E+01	5.647E+01	3.786E+01	1.995E+01	**1.034E+01**
	Std	1.085E+01	1.613E+01	2.092E+01	9.130E+00	9.215E+00	1.225E+01	9.203E+00	1.078E+01	6.157E+00	1.236E+01	9.604E+00	3.678E+00
	Best	9.955E+00	1.493E+01	2.395E+01	2.309E+01	3.088E+01	1.794E+01	1.605E+01	2.688E+01	4.320E+01	1.006E+01	7.965E+00	5.029E+00
	Worst	4.659E+01	6.965E+01	9.797E+01	5.595E+01	6.567E+01	6.512E+01	4.595E+01	6.313E+01	6.805E+01	5.579E+01	4.279E+01	1.917E+01
	Rank	3	8	10	6	11	7	4	9	12	5	2	1
F5	Mean	1.578E+01	1.652E+00	2.074E+00	2.056E+00	1.634E+00	6.153E+00	1.860E+00	6.858E+01	6.586E+01	2.024E+00	1.191E+00	**1.004E+00**
	Std	1.208E+01	3.375E-01	4.019E-01	3.427E-01	2.737E-01	6.486E+00	2.274E-01	2.138E+01	2.121E+01	3.171E-01	7.736E-02	1.063E-02
	Best	1.071E+00	1.239E+00	1.420E+00	1.536E+00	1.194E+00	1.153E+00	1.590E+00	2.711E+01	2.326E+01	1.584E+00	1.092E+00	1.000E+00
	Worst	3.920E+01	2.706E+00	3.014E+00	2.767E+00	2.187E+00	2.857E+01	2.644E+00	1.071E+02	1.084E+02	2.714E+00	1.391E+00	1.123E+00
	Rank	10	4	8	7	3	9	5	12	11	6	2	1

Table 11 (continued)

F	Index	Algorithm											
		PSO	SSA	WOA	HHO	DO	SCSO	AO	AOA	BWO	LHHO	LSMA	HBWO
F6	Mean	4.782E+00	6.881E+00	9.165E+00	8.013E+00	8.910E+00	7.093E+00	6.527E+00	1.126E+01	1.099E+01	8.763E+00	4.794E+00	**2.047E+00**
	Std	1.691E+00	1.572E+00	1.262E+00	1.905E+00	2.096E+00	1.713E+00	1.355E+00	1.560E+00	5.897E-01	1.499E+00	1.612E+00	1.048E+00
	Best	1.491E+00	4.125E+00	7.307E+00	5.066E+00	5.516E+00	3.665E+00	4.325E+00	8.417E+00	9.676E+00	5.980E+00	1.389E+00	1.001E+00
	Worst	7.707E+00	9.412E+00	1.236E+01	1.179E+01	1.267E+01	9.779E+00	9.607E+00	1.387E+01	1.222E+01	1.107E+01	7.433E+00	3.832E+00
	Rank	2	5	10	7	9	6	4	12	11	8	3	1
F7	Mean	9.908E+02	1.136E+03	1.220E+03	1.125E+03	1.611E+03	9.988E+02	8.380E+02	1.053E+03	1.653E+03	1.091E+03	8.153E+02	**5.140E+02**
	Std	2.409E+02	2.747E+02	3.528E+02	3.660E+02	3.946E+02	3.416E+02	3.177E+02	2.977E+02	2.099E+02	2.555E+02	2.860E+02	2.256E+02
	Best	6.217E+02	7.492E+02	7.505E+02	3.595E+02	8.104E+02	3.924E+02	3.286E+02	5.270E+02	1.239E+03	6.393E+02	2.698E+02	1.402E+02
	Worst	1.556E+03	1.626E+03	1.961E+03	1.770E+03	2.431E+03	1.711E+03	1.455E+03	1.694E+03	2.012E+03	1.666E+03	1.229E+03	9.344E+02
	Rank	4	9	10	8	11	5	3	6	12	7	2	1
F8	Mean	4.061E+00	4.583E+00	4.526E+00	4.507E+00	4.808E+00	3.964E+00	4.234E+00	4.400E+00	4.517E+00	4.341E+00	3.619E+00	**3.407E+00**
	Std	3.110E-01	2.816E-01	3.428E-01	2.942E-01	2.564E-01	4.520E-01	3.078E-01	3.010E-01	1.215E-01	3.900E-01	5.501E-01	2.421E-01
	Best	3.528E+00	3.871E+00	3.492E+00	3.932E+00	4.287E+00	3.136E+00	3.775E+00	3.565E+00	4.261E+00	3.565E+00	1.934E+00	2.905E+00
	Worst	4.644E+00	5.019E+00	5.043E+00	4.935E+00	5.158E+00	4.593E+00	4.857E+00	4.914E+00	4.705E+00	4.849E+00	4.397E+00	4.153E+00
	Rank	4	11	10	8	12	3	5	7	9	6	2	1
F9	Mean	1.319E+00	1.744E+00	1.515E+00	1.589E+00	1.645E+00	1.386E+00	1.534E+00	1.747E+00	2.014E+00	1.593E+00	1.265E+00	**1.247E+00**
	Std	1.220E-01	1.606E-01	1.572E-01	1.568E-01	1.595E-01	1.429E-01	2.051E-01	1.481E-01	1.382E-01	2.152E-01	8.216E-02	3.834E-02
	Best	1.100E+00	1.436E+00	1.299E+00	1.353E+00	1.404E+00	1.188E+00	1.221E+00	1.443E+00	1.724E+00	1.246E+00	1.137E+00	1.206E+00
	Worst	1.554E+00	2.081E+00	1.877E+00	1.898E+00	1.946E+00	1.686E+00	2.015E+00	2.103E+00	2.296E+00	1.967E+00	1.394E+00	1.348E+00
	Rank	3	10	5	7	9	4	6	11	12	8	2	1

Table 11 (continued)

F	Index	Algorithm											
		PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**	**HBWO**
F10	Mean	21.07486	21.01216	21.18397	21.06706	21.02644	21.09288	21.27971	21.07935	21.41832	21.10596	21.07634	**20.2073**
	Std	9.771E-02	5.828E-02	1.414E-01	6.087E-02	5.503E-02	9.155E-02	1.281E-01	2.768E-02	1.024E-01	1.426E-01	5.656E-02	4.521E+00
	Best	20.1000	20.9978	21.0467	21.0019	20.9997	21.0018	21.0827	21.0441	21.1929	20.9983	21.0015	1.0003
	Worst	21.2659	21.2597	21.5129	21.2196	21.2451	21.2914	21.4906	21.16560	21.5801	21.5156	21.1799	21.3154
	Rank	5	2	10	4	3	8	11	7	12	9	6	1
Average rank		6.4	6.9	9	5.8	7.7	5.3	6	9.8	9.3	6.1	3.1	1.1
Final rank		7	8	10	4	9	3	5	12	11	6	2	1

The bold represents the optimal values of the evaluation indicators

Table 12 Wilcoxon rank sum test results of other algorithms CEC2019 test set based on HBWO

F	Algorithm										
	PSO	**SSA**	**WOA**	**HHO**	**DO**	**SCSO**	**AO**	**AOA**	**BWO**	**LHHO**	**LSMA**
F1	8.00650E-09	0.000066506	8.00650E-09	**NaN**	**NaN**	0.0095844	**0.080631**	1.10490E-06	**NaN**	**NaN**	**NaN**
F2	6.79560E-08	**0.0961960**	6.79560E-08	2.16020E-06	2.99200E-08	**0.568470**	5.88750E-07	6.79560E-08	0.0028787	1.42240E-07	9.31530E-06
F3	3.41560E-07	6.79560E-08	0.0025606	0.000011045	1.57570E-06	**0.180580**	1.65710E-07	6.79560E-08	3.41560E-07	0.0010141	**0.1555700**
F4	4.54010E-06	1.43090E-07	6.79560E-08	6.79560E-08	6.79560E-08	9.17280E-08	1.43090E-07	6.79560E-08	6.79560E-08	3.93880E-07	0.000046804
F5	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08
F6	6.67370E-06	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	7.89800E-08	6.79560E-08	6.79560E-08	6.79560E-08	6.79560E-08	3.06910E-06
F7	3.06910E-06	4.53900E-07	2.95980E-07	3.98740E-06	9.17280E-08	0.000032931	0.0017824	3.49950E-06	6.79560E-08	9.12660E-07	0.0017824
F8	1.2009E-06	6.79560E-08	1.91770E-07	6.79560E-08	6.79560E-08	0.00024706	9.17280E-08	1.65710E-07	6.79560E-08	2.21780E-07	0.023903
F9	**0.0764310**	6.79560E-08	1.43090E-07	7.89800E-08	6.79560E-08	0.00022220	1.37610E-06	6.79560E-08	6.79560E-08	6.01480E-07	**0.8181500**
F10	0.00068682	7.57740E-06	**0.1075100**	0.000011045	5.87360E-06	0.00092091	**0.081032**	2.35570E-06	2.06160E-06	0.0021393	8.5974E-06
+/=/−	0/1/9	0/1/9	0/1/9	0/1/9	0/1/9	0/2/8	0/2/8	0/0/10	0/1/9	0/1/9	0/3/7

The bold represents the optimal values of the evaluation indicators

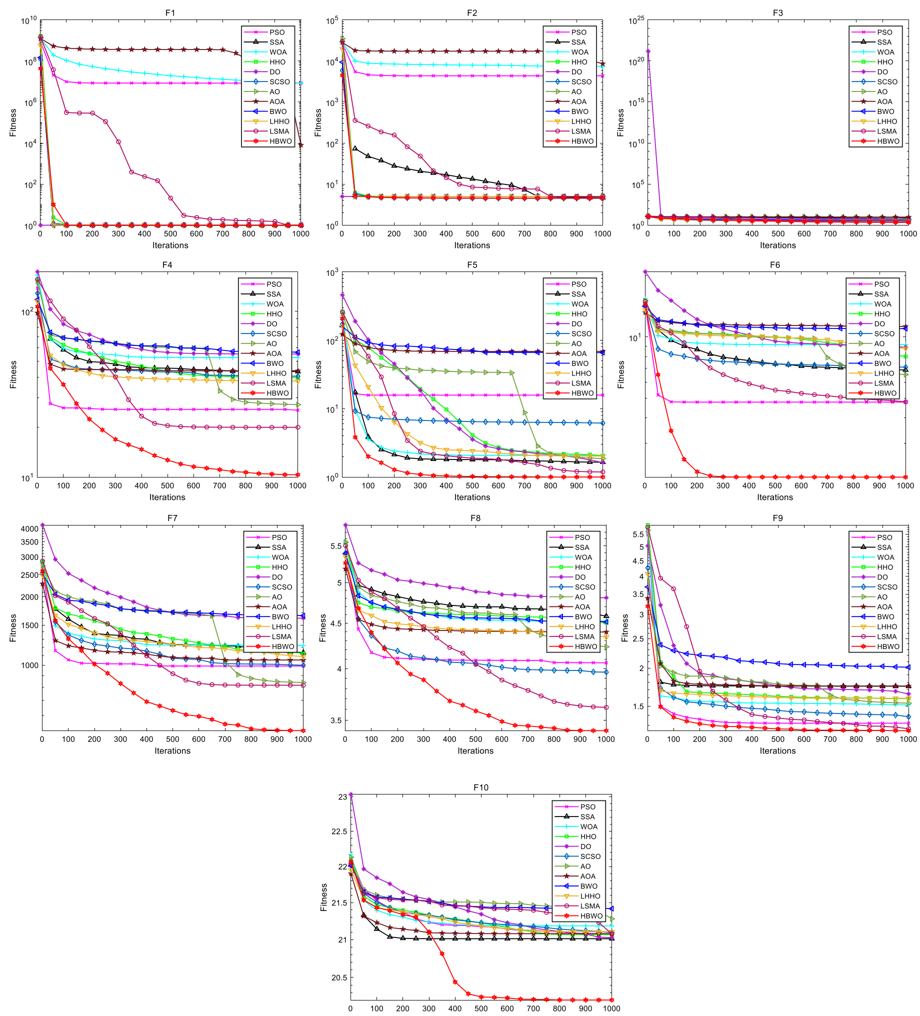

Fig. 8 Convergence curves of HBWO and other algorithms for solving CEC2019 test set

Comparative analysis of time and space complexity

In order to increase the persuasiveness of the comparison experiments and to verify other performances of the proposed HBWO, we present the time and space complexity cases of the comparison algorithms separately. Table 13 provides the time and space complexity cases of the comparison algorithms.

From Table 13, we can find that the complexity of AO, AOA, PSO, WOA, and LSMA are all $O(N_{pop} \times (D \times T_{\max} + 1))$. The reason for this is that the processes of all the above methods are initialization as well as simpler iterative updates. The complexity of the built algorithms compared to these methods differs only in the whale-fall process and the NM search.

Moreover, for the space complexity, the initialization overall can be considered as the maximum amount of space occupied by the optimization method at any time. Therefore, the space complexity of the proposed HBWO is $O(N_{pop} \times D)$. Meanwhile, all the other methods except PSO are $O(N_{pop} \times D)$. PSO has a complexity of $O(2N_{pop} \times D)$ because the fact that the initialization process has to initialize the velocity and position simultaneously.

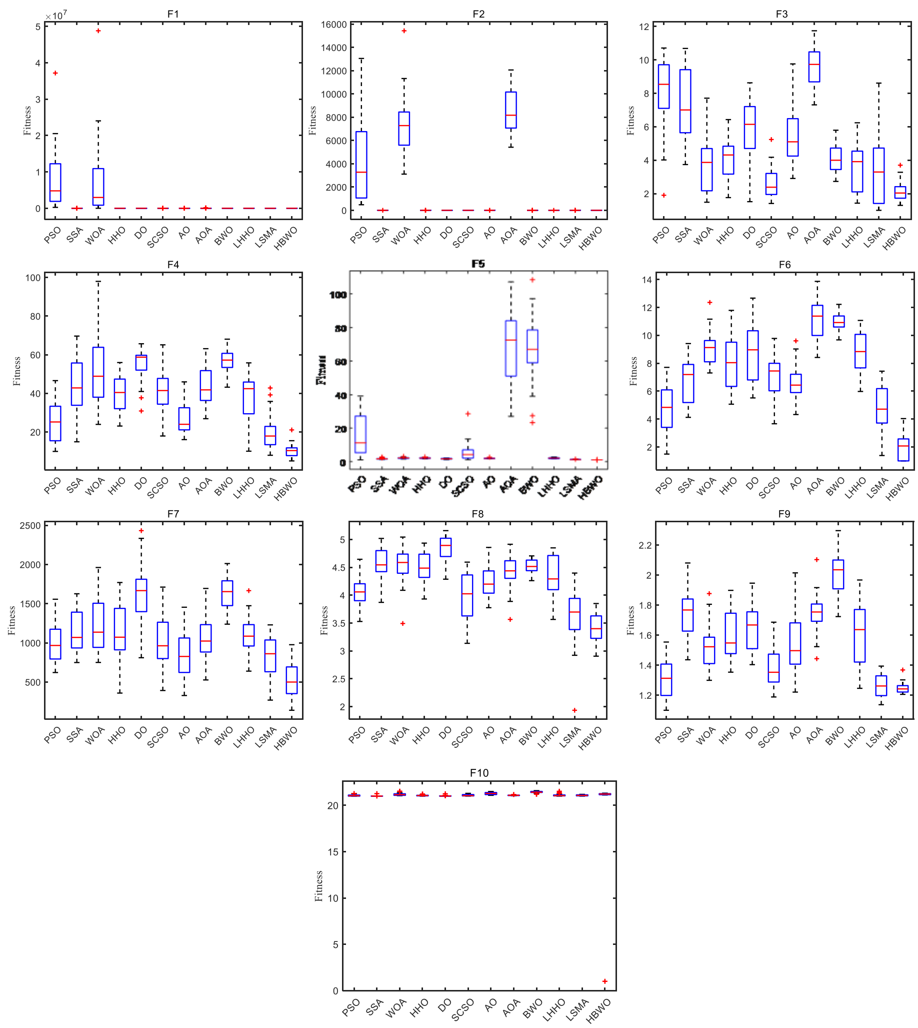

Fig. 9 Box plot of HBWO and other algorithms for solving CEC2019 test set

HBWO for engineering optimization problems

To further verify the merit of HBWO, the proposed method is used to solve six practical engineering design problems. In this section, the population size is $N = 30$ and the maximum number of iterations is $T = 500$.

Corrugated bulkhead design problem

The corrugated bulkhead design problem [60] is to minimize the weight of the wave-trough bulkhead of the chemical tank truck under six constraints, and the design variables are width (w), depth (d), length (l) and plate thickness (t), let $x = (x_1, x_2, x_3, x_4) = (w, d, l, t)$. Then the mathematical model of the corrugated bulkhead design problem is as follows:

$$\min f(x) = \frac{5.885t(w + l)}{w + \sqrt{|l^2 - d^2|}}, \tag{16}$$

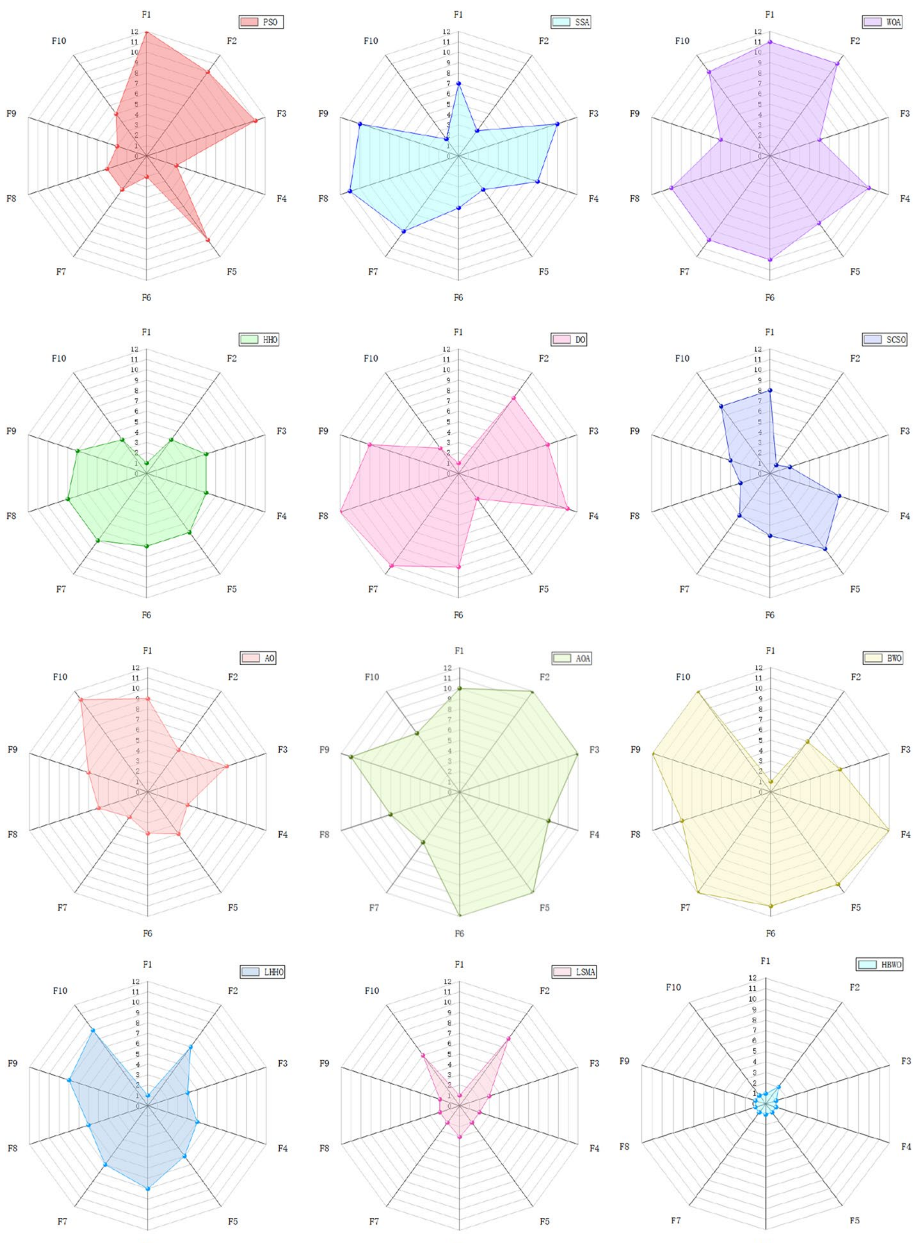

Fig. 10 Radar graph of HBWO and other algorithms for solving CEC2019 test set

subject to:

$$h_1(x) = -td(0.4w + \frac{l}{6}) + 8.94(w + \sqrt{|l^2 - d^2|}) \leq 0,$$

$$h_2(x) = -td^2(0.2w + \frac{l}{12}) + 2.2(8.94(w + \sqrt{|l^2 - d^2|}))^{4/3} \leq 0,$$

$$h_3(x) = -t + 0.0156w + 0.15 \leq 0,$$

Table 13 time and space complexity cases of the comparison algorithms

Methods	Time complexity	Space complexity
PSO	$O(N_{pop} \times (D \times T_{\max} + 1))$	$O(2N_{pop} \times D)$
SSA	$O(N_{pop} \times D + (N_{pop} \times D + N_{pop}) \times T_{\max})$	$O(N_{pop} \times D)$
WOA	$O(N_{pop} \times (D \times T_{\max} + 1))$	$O(N_{pop} \times D)$
HHO	$O(N_{pop} \times (T_{\max} + D \times T_{\max} + 1))$	$O(N_{pop} \times D)$
DO	$O(N_{pop} \times D + N_{pop} \times D \times T_{\max})$	$O(N_{pop} \times D)$
SCSO	$O({N_{pop}}^2 \times T_{\max})$	$O(N_{pop} \times D)$
AO	$O(N_{pop} \times (D \times T_{\max} + 1))$	$O(N_{pop} \times D)$
AOA	$O(N_{pop} \times (D \times T_{\max} + 1))$	$O(N_{pop} \times D)$
BWO	$O(N_{pop} \times (1 + 1.1T_{\max})$	$O(N_{pop} \times D)$
LHHO	$O(N_{pop} \times (T_{\max} + D \times T_{\max} + 1))$	$O(N_{pop} \times D)$
LSMA	$O(N_{pop} \times (D \times T_{\max} + 1))$	$O(N_{pop} \times D)$
HBWO	$O(N_{pop} \times (D + 1.1T_{\max} + 5)$	$O(N_{pop} \times D)$

Table 14 Optimal results of each algorithm for solving corrugated bulkhead design problem

Algorithms	Optimal cost				Optimal cost
	w	***d***	***l***	***t***	
RSA	24.82694	31.28881	49.177087	1.220363	8.753234
GWO	57.616942	34.12353	57.504312	1.050035	6.846144
SCSO	57.503585	34.14618	57.686776	1.050045	6.844509
WOA	57.421771	34.2438	56.698321	1.049997	6.872322
RSO	28.675976	3.27E+01	46.306995	1.061092	7.614702
HHO	57.671873	34.36353	57.692528	1.049998	6.853589
SCA	34.926449	34.02706	57.053581	1.053331	7.063349
AOA	57.493416	38.5876	57.493416	1.074107	7.260199
HGS	54.41391	33.72787	53.563908	1.056595	6.992025
AO	41.822906	34.41305	58.623225	1.069466	7.080754
BWO	54.821252	34.22237	55.238794	1.051405	6.936082
HBWO	57.692156	34.14763	57.692204	1.049998	6.842953

$$h_4(x) = -t + 0.0156l + 0.15 \leq 0,$$

$$h_5(x) = -t + 0.15 \leq 0,$$

$$h_6(x) = -l + d \leq 0,$$

variable range: $0 \leq w, d, l \leq 100, \quad 0 \leq t \leq 5.$

RSA [62], GWO [28], SCSO [55], WOA [61], RSO [63], HHO [32], SCA [65], AOA [59], HGS [64], AO [56], BWO [45], and HBWO were used to solve the problem. The results are shown in Tables 14 and 15; the result of HBWO is 6.842953. The bolded data are the minimum values in each index.

Table 15 Statistical results of corrugated bulkhead design problem solved by each algorithm

Algorithms	Best	Worst	Mean	Std
RSA	8.467677	15.853491	11.482246	1.691024
GWO	6.846688	6.898106	6.861057	0.015189
SCSO	6.844509	18.505374	8.609094	3.504729
WOA	6.872322	8.709423	7.193798	0.428363
RSO	7.614702	17.680705	10.262018	2.200589
HHO	6.853589	8.753881	7.360619	0.514229
SCA	7.063349	9.017586	8.099299	0.649614
AOA	7.260199	11.004275	8.732082	1.130864
HGS	6.992025	24.967670	12.101406	4.687708
AO	7.080754	8.826819	7.636858	0.505590
BWO	6.936082	7.723733	7.265991	0.237218
HBWO	**6.842953**	**6.842953**	**6.842953**	**1.278691E-07**

The bold represents the optimal values of the evaluation indicators

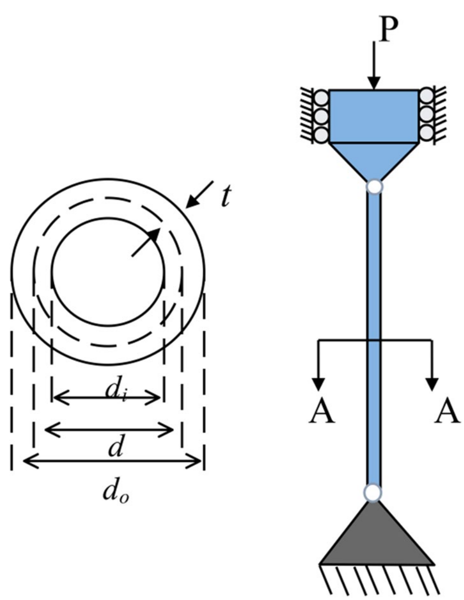

Fig. 11 Tabular column design problem

Tabular column design problem

The tube column design problem [66] is to obtain a uniform tube column. Figure 11 with minimum cost, which can withstand the compression load. The average diameter d and thickness t are varied in the range of [2, 14] and [0.2, 0.8]. The parameters are as follows: yield stress $\sigma_y = 500kgf/cm^2$,modulus of elasticity $E = 0.85 \times 10^6 kgf/cm^2$ and density $\rho = 0.0025kgf/cm^3$. The length L of the column is a $250cm$ vector $x = (x_1, x_2) = (d, t)$.

$$\min f(x) = 9.82dt + 2d, \tag{17}$$

subject to:

Table 16 Optimal results of each algorithm for solving Tabular column design problem

Algorithms	Optimum variables		Optimal cost
	d	***t***	
RSA	5.4130713	0.298899	26.71453
GWO	5.4511627	0.291964	26.5315
SCSO	5.451214	0.291961	26.53139
WOA	5.4596289	0.291509	26.54822
RSO	5.4085484	0.299114	26.70362
HHO	5.4508804	0.292007	26.53228
SCA	5.4541096	0.292089	26.5523
AOA	5.3812627	0.303974	26.82571
SMA	5.4511636	0.291964	26.531295
AO	5.4406886	0.294303	26.60525
BWO	5.5011714	0.290983	26.72166
HBWO	5.451214	0.291961	26.53129

Table 17 Statistical results of Tabular column design problem solved by each algorithm

Algorithms	Best	Worst	Mean	Std
RSA	26.71453	31.97423	28.72993	2.088627
GWO	26.5315	26.54055	26.53606	0.002728
SCSO	26.53139	26.53278	26.53185	0.000405
WOA	26.54822	27.84929	27.07526	0.480596
RSO	26.70362	33.49289	29.11128	1.846825
HHO	26.53228	26.80122	26.60579	0.079761
SCA	26.5523	27.16397	26.70018	0.138264
AOA	26.82571	28.9194	28.05061	0.712523
SMA	26.531295	26.53184	26.53144	1.90E-04
AO	26.60525	27.01874	26.75546	0.10973
BWO	26.72166	28.24725	27.22501	0.405838
HBWO	**26.53129**	**26.53134**	**26.5313**	**9.53E-06**

The bold represents the optimal values of the evaluation indicators

$$h_1(x) = \frac{P}{\pi dt\sigma_y} - 1 \le 0,\ h_2(x) = \frac{8PL^2}{\pi^3 Edt(d^2 + t^2)} - 1 \le 0,$$

$$h_3(x) = \frac{2.0}{d} - 1 \le 0,\ h_4(x) = \frac{d}{14} - 1 \le 0,$$

$$h_5(x) = \frac{0.2}{t} - 1 \le 0,\ h_6(x) = \frac{t}{0.8} - 1 \le 0.$$

where

$$2 \le d \le 14, \quad 0.2 \le t \le 0.8.$$

HBWO is used to solve the tube column design problem, and the obtained results are compared with other optimization algorithms, including RSA [62], GWO [28],

SCSO [55], WOA [61], RSO [63], HHO [32], SCA [65], AOA [59], SMA [67], AO [56], and BWO [45]. The results obtained are shown in Tables 16 and 17. The optimal value of HBWO for solving the design of pipe string is 26.53129, and the result obtained is relatively stable.

Three-bar truss design problem

The three-bar truss design problem [68], as shown in Fig. 12, which is to minimize the value of the total weight of the truss structure while minimizing the volume.

$$\min f(x) = (2\sqrt{2}T_1 + T_2) \times 1,$$

subject to:

$$h_1(x) = \frac{\sqrt{2}T_1 + T_2}{\sqrt{2}T_1^2 + 2T_1T_2}P - \sigma \leq 0,$$

$$h_2(x) = \frac{T_2}{\sqrt{2}T_1^2 + 2T_1T_2}P - \sigma \leq 0,$$

$$h_3(x) = \frac{1}{T_1 + \sqrt{2}T_2}P - \sigma \leq 0,$$

with bounds: $0 \leq T_1, T_2 \leq 1$.

To solve this problem, RSA [62], GWO [28], SCSO [55], WOA [61], RSO [63], HHO [32], SCA [65], AOA [59], HGS [64], AO [56], BWO [45] and HBWO are used. Tables 18 and 19 show the experimental results, from which it can be seen that the result of HBWO solving this problem is 0.012663, indicating that HBWO can achieve good and stable results.

Tension/compression spring design problem

The tension/compression spring design problem [69] (Fig. 13). The problem has three design variables: average coil diameter (D), wire diameter (d), and effective number of coils (N). Let $X = [x_1, x_2, x_3] = [d, D, N]$.

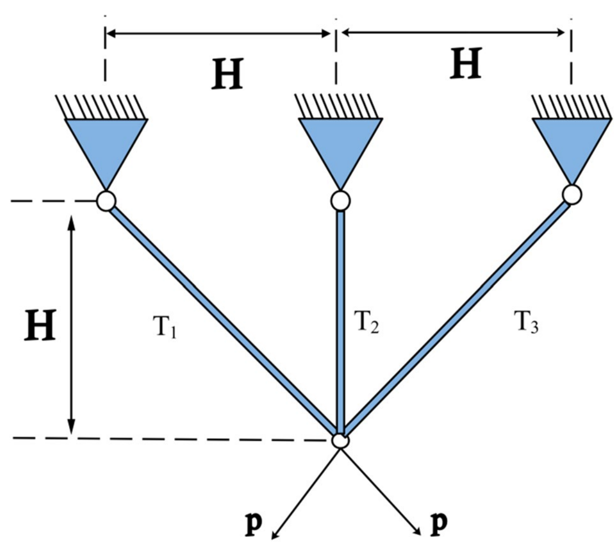

Fig. 12 Three-bar truss design problem

Table 18 Optimal results of each algorithm for solving the three-bar truss design problem

Algorithms	Optimum variables		Optimal cost
	T_1	T_2	
RSA	0.911284	0.088933	1.935997
GWO	0.9138799	0.08632	1.935804
SCSO	0.9145443	0.08565	1.935803
WOA	0.9186338	0.081533	1.936093
RSO	0.8539183	0.13368	2.300392
HHO	0.9144492	0.085745	1.935804
SCA	0.9063154	0.094361	1.939053
AOA	0.9122142	0.088113	1.936051
HGS	0.95406	0.041941	1.969957
AO	0.9119137	0.088341	1.935951
BWO	0.9331996	0.067122	1.94161
HBWO	0.9145443	0.08565	1.935804

Table 19 Statistical results of three bar truss design problem solved by each algorithm

Algorithms	Best	Worst	Mean	Std
RSA	1.935997	4.425519	2.523628	0.807239
GWO	1.935804	1.998754	1.970294	0.031989
SCSO	1.935803	5.351777	2.198185	0.76137
WOA	1.936093	1.998753	1.988659	0.019002
RSO	2.300392	130.445	21.85454	29.68952
HHO	1.935804	1.94926	1.940163	0.005122
SCA	1.939053	1.998756	1.995769	0.013349
AOA	1.936051	1.941958	1.937743	0.001654
HGS	1.969957	22.68553	3.481711	4.766112
AO	1.935951	1.984162	1.951487	0.013188
BWO	1.94161	1.99971	1.971012	0.020467
HBWO	**1.935804**	**1.935818**	**1.935808**	**4.52E-06**

The bold represents the optimal values of the evaluation indicators

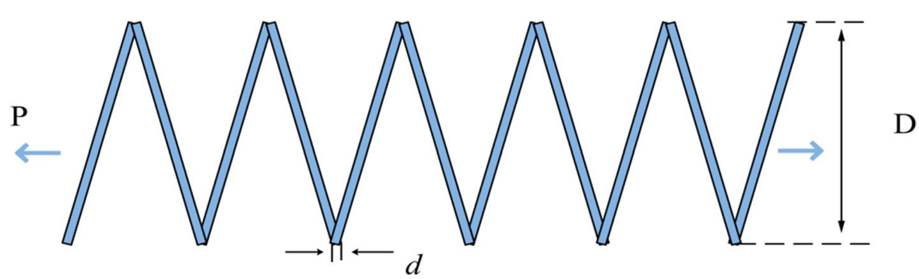

Fig. 13 Tension/compression spring design problem

$$\min f(x) = (N+2)Dd^2,$$

subject to:

Table 20 Optimal results of each algorithm for solving tension/compression spring design problem

Algorithms	Optimum variables			Optimal cost
	d	***D***	***N***	
RSA	0.05	0.31429	14.456237	0.01293
PSO	0.0515523	0.35349	11.478083	0.012664
SCSO	0.0519597	0.36332	10.909539	0.012665
WOA	0.0523329	0.372461	10.419984	0.012671
RSO	0.059779	0.577567	4.7930539	0.014021
HHO	0.0519361	0.362744	10.941707	0.012665
SCA	0.0507414	0.334428	12.763502	0.012716
AOA	0.05	0.310658	15	0.013203
HGS	0.0525589	0.378022	10.146146	0.012684
AO	0.05	0.312914	15	0.013299
BWO	0.05	0.314572	14.629211	0.013078
HBWO	0.0516801	0.35656	11.295501	0.012663

Table 21 Statistical results of pressure tension/compression spring design solved by each algorithm

Algorithms	Best	Worst	Mean	Std
RSA	0.01293	22.82145	3.43483	8.355575
PSO	0.012664	0.01467	0.013247	0.00055
SCSO	0.012665	0.01509	0.01302	0.000664
WOA	0.012671	0.01599	0.013466	0.000864
RSO	0.014021	86.55716	37.61433	36.7287
HHO	0.012665	0.016153	0.01366	0.001056
SCA	0.012716	0.013318	0.013034	0.000177
AOA	0.013203	0.031123	0.014292	0.004006
HGS	0.012684	8.199925	0.426629	1.829681
AO	0.013299	0.027544	0.019613	0.003927
BWO	0.013078	0.021185	0.014592	0.002216
HBWO	**0.012663**	**0.012942**	**0.012699**	**7.20E-05**

The bold represents the optimal values of the evaluation indicators

$$h_1(x) = 1 - \frac{D^3N}{71765d^4} \leq 0,$$

$$h_2(x) = \frac{4D^2-dD}{12566(Dd^3-d^4)} + \frac{1}{5108d^2} - 1 \leq 0,$$

$$h_3(x) = 1 - \frac{140.45d}{D^2N} \leq 0,$$

$$h_4(x) = \frac{D+d}{1.5} - 1 \leq 0,$$

variable range: $0.05 \leq d \leq 2, \quad 0.25 \leq D \leq 1.3, \quad 2 \leq N \leq 15.$

BWO [45], RSA [62], PSO [30], SCSO [55], WOA [61], RSO [63], HHO [32], SCA [65], AOA [59], HGS [64], AO [56], and HBWO are used to solve tension/compression spring design problems. Tables 20 and 21 show that the optimal value of HBWO to solve this problem is 0.012663. HBWO has a strong competitiveness compared with other optimization algorithms.

Heat exchange design problem

The heat exchange design problem [70] is a constrained function with eight variables and six inequalities, and the constrained benchmark minimization problem is found.

$$\min f(x) = x_1 + x_2 + x_3$$

subject to:

$$h_1(x) = 0.0025(x_4 + x_6) - 1 \le 0,$$

$$h_2(x) = 0.0025(x_5 + x_7 - x_4) - 1 \le 0,$$

$$h_3(x) = 1 - 0.01(x_8 - x_5) \ge 0,$$

$$h_4(x) = x_1x_6 - 833.33252x_4 - 100x_1 + 83333.333 \ge 0,$$

$$h_5(x) = x_2x_7 - 1250x_5 - x_2x_4 + 1250x_4 \ge 0,$$

$h_6(x) = x_3x_8 - x_3x_5 + 2500x_5 - 1250000 \ge 0,$
where the range of the variables are

$$100 \le x_1 \le 1000, \quad 1000 \le x_2, x_3 \le 10000, \quad 10 \le x_i \le 1000(i = 4 \sim 8).$$

HBWO is used to solve heat exchange design problems, With RSA [62], GWO [28], SCSO [55], WOA [61], RSO [63], HHO [32], SCA [65], AOA [59], HGS [64], AO [56] and BWO [45] solving results Contrast. The optimal value of HBWO to solve this problem is 7060.574, which can be obtained from Tables 22 and 23. However, the standard deviation of HBWO is still relatively large, and there is still some room for improvement.

Table 22 Optimal results of each algorithm for solving heat exchange design problem

Algorithms	Optimal cost								Optimal cost
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
RSA	591.663	1490.550	5026.956	108.138	265.060	225.697	258.563	383.202	29122.184
PSO	472.316	1439.976	5148.282	172.054	294.074	227.230	277.976	394.072	7060.574
SCSO	162.601	1943.180	5373.106	113.454	285.233	182.703	228.048	385.169	7478.887
WOA	1345.856	4642.143	6910.341	10	229.654	378.034	98.976	327.459	12898.340
RSO	100	1000	7997.630	10	10	165.383	10	167.609	66706.597
HHO	1462.611	4260.920	5727.986	88.675	270.886	139.617	205.876	370.883	11451.517
SCA	332.841	1508.286	8148.095	34.608	216.301	213.628	197.674	314.024	9989.222
AOA	2787.497	9009.821	7901.710	100.887	250.851	134.449	248.985	347.218	19699.029
HGS	100	2252.576	5957.798	12.575	261.694	10	150.844	361.692	8310.3734
AO	1644.461	2427.846	8412.774	113.550	167.128	185.831	149.146	266.092	12485.081
BWO	7168.800	6258.403	7652.393	10	233.035	104.352	100.934	326.145	29122.184
HBWO	531.446	1353.456	5166.177	177.880	293.353	222.120	284.527	393.353	7060.574

Table 23 Statistical results of heat exchange design problem solved by each algorithm

Algorithms	Best	Worst	Mean	Std
RSA	29122.18365	345530.4	162467.6	88661.83
PSO	7060.574402	16262.23	8202.381	2027.299
SCSO	7478.887309	369854.4	37915.28	85630.89
WOA	12898.33983	146090.6	46319.09	41651.46
RSO	66706.59712	301164.4	170316.4	63022.65
HHO	11451.5171	185325.9	64236.2	50454.4
SCA	9989.221815	16996.51	11681.32	1600.421
AOA	19699.02875	207671.2	54094.85	42845.14
HGS	8310.373414	50027.31	20423.13	13485.89
AO	12485.08103	176110.7	58907.96	39985.7
BWO	21079.59656	95453.23	45009.69	20665.01
HBWO	**7051.08035**	**8525.361**	**7246.805**	**350.5593**

The bold represents the optimal values of the evaluation indicators

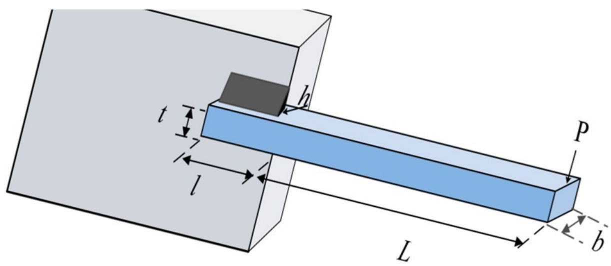

Fig. 14 Welded beam design problem

Welded beam design problem

The welded beam design problem [71] (Fig. 14) is to find the minimum value of the manufacturing cost of the welded beam. The problem contains four design variables, which are: welding thickness h, welding joint length l, beam height t, and beam thickness b. Let $X = [x_1, x_2, x_3, x_4] = [h, l, t, b]$, whose mathematical model is as follows.

$$\min f(X) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$$

$$\begin{aligned}
&h_1(X) = m(x) - m_{\max} \leq 0,\\
&h_2(X) = n(x) - n_{\max} \leq 0,\\
&h_3(X) = p(x) - p_{\max} \leq 0,\\
&h_4(X) = x_1 - x_4 \leq 0,\\
&h_5(X) = B - B_c(x) \leq 0,\\
&h_6(X) = 0.125 - x_1 \leq 0,\\
&h_7(X) = 1.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0,
\end{aligned}$$

where $0.1 \leq x_1 \leq 2$, $0.1 \leq x_2 \leq 10$, $0.1 \leq x_3 \leq 10$, $0.1 \leq x_4 \leq 2$, $L = 14in$, $m_{\max} = 136,000psi, \sigma_{\max} = 36,600psi, B = 6,000lb, E = 30 \times 10^6psi, G = 12 \times 10^6psi,$

$m(X) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}, m' = \frac{B}{\sqrt{2}x_1x_2}, m'' = \frac{AR}{J},$

Table 24 Optimal results of each algorithm for solving welded beam design problem

Algorithms	Optimal cost				Optimal cost
	x_1	x_2	x_3	x_4	
RSA	0.126463	8.849524	9.3452	0.208905	2.302444
GWO	0.205152	3.266731	9.036148	0.205881	1.697297
SCSO	0.206124	3.249979	9.028564	0.206137	1.697076
WOA	0.199333	3.851774	9.038645	0.20572	1.766037
RSO	0.188494	3.854347	8.797637	0.228628	1.879007
HHO	0.180196	3.826716	8.898732	0.212155	1.756419
SCA	0.198486	3.64473	9.089099	0.207635	1.76066
AOA	0.202502	3.191403	10	0.203424	1.827046
HGS	0.198636	3.38609	9.029618	0.206049	1.703834
AO	0.169862	4.018172	9.179303	0.205272	1.761452
BWO	0.22781	3.06293	8.535545	0.231596	1.798354
HBWO	0.20573	3.253115	9.036636	0.20573	1.695252

Table 25 Statistical results of welded beam design problem solved by each algorithm

Algorithms	Best	Worst	Mean	Std
RSA	2.302444	78235.81	3914.288	17493.47
GWO	1.697297	1.704188	1.700271	0.002129
SCSO	1.697076	5.392866	2.369905	1.268177
WOA	1.766037	3.745178	2.484301	0.664593
RSO	1.879007	219313.1	13627.98	49011.31
HHO	1.756419	2.813293	2.030536	0.263279
SCA	1.76066	1.989149	1.882108	0.068482
AOA	1.827046	3.014027	2.422886	0.36323
HGS	1.703834	2.919799	1.958669	0.303923
AO	1.761452	2.428079	2.100519	0.210971
BWO	1.798354	3.220163	2.281636	0.323872
HBWO	**1.695252**	**1.695288**	**1.695269**	**1.08E-05**

The bold represents the optimal values of the evaluation indicators

$$A = P(L + \tfrac{x_2}{2}), R = \sqrt{\tfrac{x_2^2}{4} + (\tfrac{x_1+x_3}{2})^2}, J = 2\sqrt{2}x_1x_2\left[\tfrac{x_2^2}{4} + (\tfrac{x_1+x_3}{2})^2\right],$$
$$n(X) = \tfrac{6PL}{x_4x_3^2},\ p(X) = \tfrac{6BL^3}{Ex_3^2x_4},$$
$$B_c(X) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}(1 - \tfrac{x_3}{2L}\sqrt{\tfrac{E}{4G}}).$$

HBWO, RSA [62], GWO [28], SCSO [55], WOA [61], RSO [63], HHO [32], SCA [65], AOA [59], HGS [64], AO [56] and BWO [45] was used to solve the welding beam design problem. It can be seen from Tables 24 and 25 that the optimal solution of the HBWO solution is 1.695252, indicating the superiority of HBWO.

Conclusion and future

In this paper, we propose an improved beluga whale optimization algorithm (HBWO), which introduces Quasi-opposition-based learning (QOBL), adaptive and spiral predation, and Nelder–Mead simplex search method (NM) into the beluga whale optimization algorithm. The CEC2017 test function and CEC2019 test function are used to test its performance and compared with the Beluga optimization algorithm, classical algorithm, 2022 new algorithm, and improved algorithm. The experimental results show that HBWO has certain advantages. Meanwhile, the experimental results of six engineering cases further verify the high efficiency of HBWO in solving practical problems. Therefore, the introduction of the three strategies enables HBWO to obtain a better initial population while the performance, such as solution accuracy and convergence speed, is substantially improved. However, the experimental results show that in the CEC2017 multimodal test function, the accuracy of HBWO is still poor when facing certain functions, and there is still room for improvement. Meanwhile, HBWO increases some complexity due to adding strategies. Therefore, in future work, the proposed HBWO can be improved by integrating with other algorithms, or some new strategies can be entered into the exploration phase to make the performance of HBWO more recent. The improved algorithm can be used in image segmentation [72, 73], energy problems [74, 75], path optimization [76 ~ 78], feature selection [79, 80], curve and surface optimization [81, 82], and other fields [83 ~ 86, 88–89].

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Author contributions
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Availability of data and materials
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Declarations

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Not applicable.

Consent for publication
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Competing interests
The authors declare that they have no competing interests.

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