Heritage Science

REVIEW

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Application-based principles of islamic geometric patterns; state-of-the-art, and future trends in computer science/technologies: a review

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Abstract

Currently, there is a tendency to use Islamic Geometric Patterns (IGPs) as important identities and cultural elements of building design in the Middle East. Despite high demand, lack of information about the potential of IGPs principles have led to formal inspiration in the design of existing buildings. Many research studies have been carried out on the principles of IGPs. However, comprehensive studies relating to new possibilities, such as structure-based, sustainable-based, and aesthetic-based purposes, developed by computer science and related technologies, are relatively rare. This article reviews the state-of-the-art knowledge of IGPs, provides a survey of the main principles, presents the status quo, and identifies gaps in recent research directions. Finally, future prospects are discussed by focussing on different aspects of the principles in accordance with collected evidence obtained during the review process.

Keywords Islamic geometric patterns, Structural, Sustainable, Aesthetic-based principles, Application-based principles, Computer science

Introduction

Today, there is a tendency to use traditional elements in contemporary Middle Eastern buildings [1]. Islamic geometric patterns are one of the key characteristics of Islamic architecture in many cultural traditions of Islamic countries [2–4]. Patterns derived from the Byzantine and Sassanid eras became a part of Islamic design during the seventh century, which expanded due to the significant growth of science and technology in the Middle East, Iran, and Central Asia during the eighth and ninth centuries [5, 6]. Islamic geometric pattern (s) is a distinctive idiom characterized by networks of interlocking stars and polygons, high levels of symmetry on both local and global scales, and various forms of repetition [7]. They include many simples to the most complex and intricate multi-level designs that emerged in two-dimensional to three-dimensional patterns. IGPs have been used as application-based objectives for centuries, including aesthetics, sustainability, and structure, as shown in Fig. 1.

Due to the underlying principles, IGPs can present an abundant source of possible topologies and geometries that can be explored in the preliminary design development with computer science and creative technology facilities. Recently, discourse on the topic of IGPs has been mainly centered around common and iconic building examples in Islamic countries, usually designed by famous architects such as Jean Nouvel, Zaha Hadid, and

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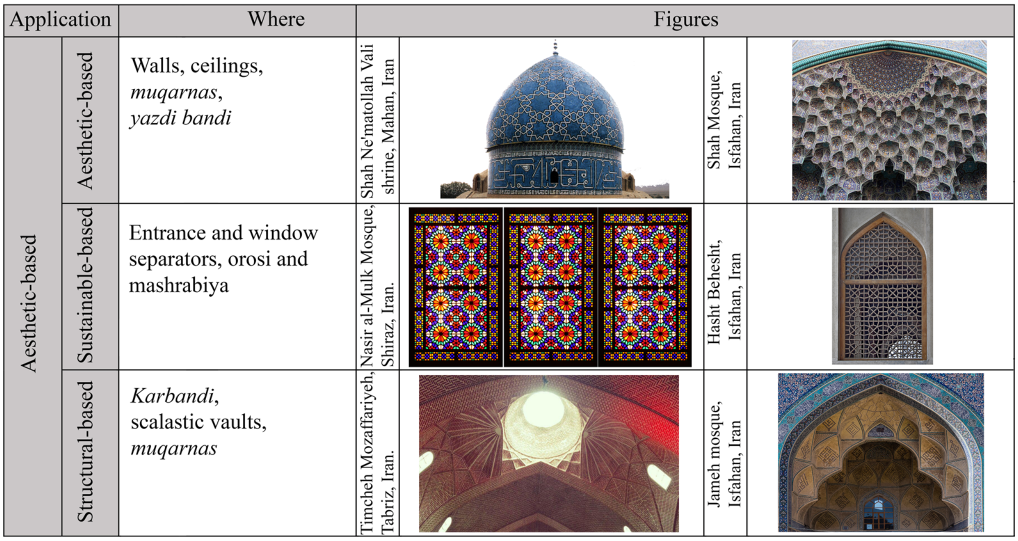


Fig. 1 IGPs' application in history

Norman Foster, as shown in Fig. 2. In these examples, IGPs are performed on the building envelope with formal inspiration, including aesthetic-based, sustainable-based, and rarely structural-based objectives. The inquiry to find new patterns should go beyond existing examples and examine 'the emergence and evolution of architectural forms' [8]. Such an approach provides new research opportunities and re-establishes open-ended research that makes Islamic architecture an active contributor to global architecture. Accomplishing this goal requires understanding the main principles of IGPs and their potential regarding new design developments.

To the best of the authors' knowledge, an overview that summarizes research, design, and development efforts regarding the principles of IGPs is not yet available in the literature. This paper comprehensively reviews and investigates the principles in their traditional applications and new developments to fill this gap. This study establishes a proper definition of IGPs' principles by scrutinizing previous studies and classifying them in terms of their applications in aesthetic, sustainable, and structural-based directions. Then, an overview of the state-of-the-art trends in their principles according to computer science and related technologies is provided. Besides, the review sheds light on the knowledge gap of application-based principles to enhance the potentiality of IGPs for widespread use in contemporary architecture. Finally, this paper proposes some significant recommendations for future research.

Scope and methodology

Three types of art can be recognized as integral aspects of Islamic visual aesthetics: floral design, calligraphy, and geometric patterns. This review focuses on studies that match the Islamic abstract geometric patterns dealing with computer science developments and related technologies concerning their application.

An initial analysis of the selected papers led to utilizing some taxonomies to review the selected articles, as follows:

Application-based classification

- According to Fig. 2. Aesthetic-based, sustainable-based, and structure-based applications are the main embodiment of IGPs in contemporary architecture. Moreover, the initial evaluation of the selected papers showed the main overarching taxonomy criteria that incorporate all of the entities presented in the paper reviews are aesthetic-based, sustainable-based, and structure-based application objectives.
- A literature survey is followed by extracting all principles from the literature, including primary sources. Two-dimensional and three-dimensional IGPs can be seen as an embodiment of the aesthetics of ratio, proportion, symmetric, variations, dimensions, generation methods, and multi-level features. On the other hand, principles like natural light, thermal and visual convenience are categorized as sustainable-based applications. Finally, structure-based applications are

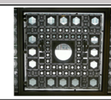
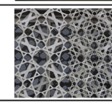
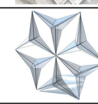
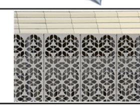
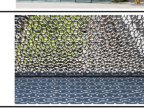
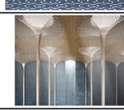
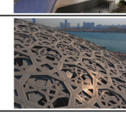
Building	Architect	Design objective(s)	Picture
Arab World Institute	Jean Nouvel, Paris, France 1987	Sustainable-based	
Doha high rise office tower	Jean Nouvel, Doha, Qatar, 2010	Sustainable-based	
Masdar Institute campus	Foster & Partners, 2015	Sustainable-based	
King Abdullah Petroleum center	Zaha Hadid, Riyadh, Saudi Arabia, 2010	Sustainable-based	
Al-Bahar Towers	AHR, Abu Dhabi, UAE, 2012	Sustainable & Structural-based	
Al Janoub Stadium	Al Wakrah, Qatar, 2019	Sustainable-based	
UAE parliament building	Ehrlich Architects, Abu Dhabi, UAE, 2011	Structural-based	
Princess Abdulrahman University	Perkins & Will, Riyadh, Saudi Arabia , 2011	Sustainable-based	
Butterfly Aviary	3deluxe, Sharjah, United Arab, Emirates, 2015	Sustainable & Structural-based	
World Trade Center Souk	Foster & partners, Abu Dhabi, Emirates, 2015	Sustainable-based	
WTC's mosque	Amanda Levete, Abu Dhabi, Emirates, 2015	Sustainable & Structural based	
Da Chang Cultural Center	Architectural Design of Scut, Da Chang, China, 2015	Sustainable-based	
Metro Station	Zaha Hadid, Riyadh, Saudi Arabia, 2017	Sustainable-based	
Louvre Museum	Jean Nouvel, Abu Dhabi, Emirates, 2017	Sustainable-based	
Cambridge Central Mosque	Marks Barfield Architects, Cambridge, England, 2007	Structural-based	

Fig. 2 The new emergence of IGPs in contemporary architecture; all the cases have an aesthetic objective. All the figures adapted in order from: [206–220]

categorized as principles concerning the distribution of forces, self-load bearing, material features, etc.

- The classification per criteria was purified and established during data collection; therefore, the categorization of each criterion is a consequence of the study itself. As it will become clear after all principle-related papers are described, it is impossible to find sub-taxonomies first due to unknown principles.

Computer science and related technologies classification

All the papers studied in the state-of-the-art section must include the category of computer science and associated technologies. The recent developments in computer science and related technologies create new opportunities for developing new IGPs. The following illustrates the basis for related categorizations.

- Many modern approaches for restitution and restoration, like photogrammetry and laser scanning techniques (López et al. 2018), are categorized as digital surveying.
- New fabrication equipment, like robotic fabrication, 3D printing, and 3D projection, are categorized as digital fabrication.
- Many textual programming languages, like Python, Visual Basic, C#, and Rhino Script and visual programming, like Grasshopper, Revit, etc., are categorized as digital modeling. There are more advanced computer science and digital modeling contributors like formal grammar, graph theory, virtual reality technology, augmented reality, optimization, and machine learning.
- This development is also visible in the area of software, allowing for efficient, often semi-automatic, processing of source data and the creation of various types of finished products from them, which are categorized as software tools.

Building and non-building categories

- Most design problems could be investigated by being categorized into non-building and building categories. Building categories can be classified more into building skins and components (roof surfaces, domes, *muqarnases*, columns, etc.).

The research method framework is illustrated in Fig. 3. Numerous publications were analyzed within a broad spectrum of thematic areas by considering the principles found in Sect. "Principles and rules". The research data include academic journals and conference papers collected through Google Scholar, Science Direct, and Web of Science engines. The keywords used to search the relevant references were applications and principles of IGPs, as shown in Table 1. To study the field in-depth, there was no time limitation. The final cut-off date for published

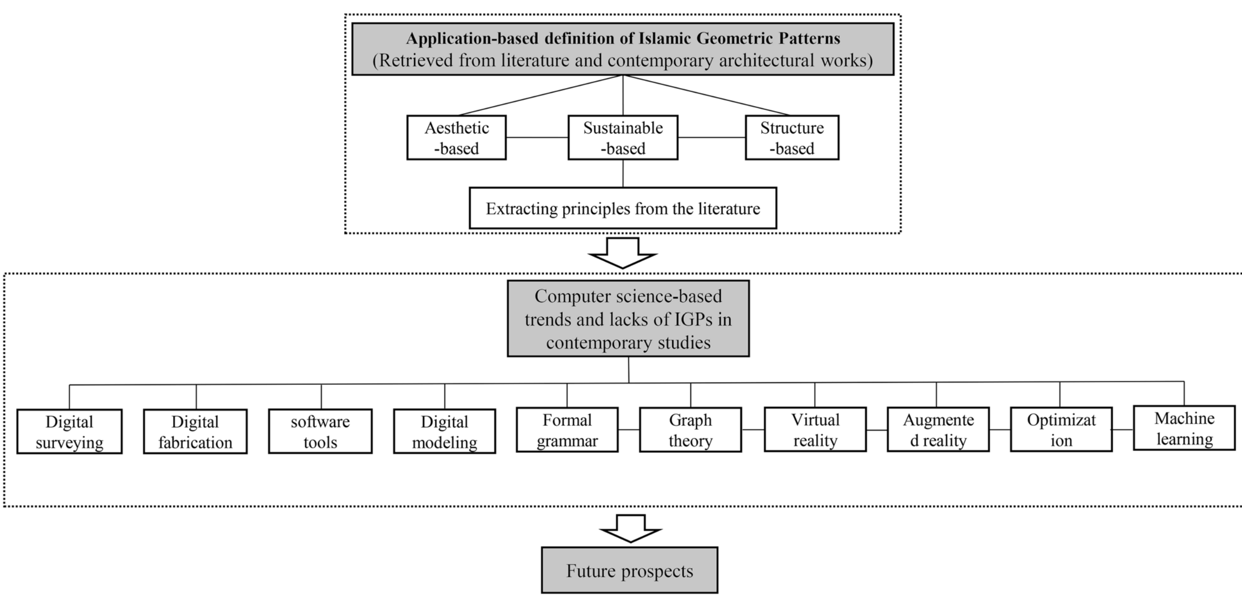


Fig. 3 Research method framework

Table 1 Keywords for searching references

Performance Objectives and Principles	
Aesthetic-Based IGPs	Inspirations, geometric proportions, mathematic, generation methods, scale-dimensionality, double level patterns
Sustainable-Based IGPs	Visual and heat convenience, humidity control, daylight, kinetic façade
Structure-Based IGPs	Structure, material, fabrication,
Computer science and technology-related IGPs	Digital fabrication, Smart restoration, computer science & computer graphic, technology, Cad/Cam applications, visualization, graph theory, formal grammar,

studies was June 30, 2022. The following sections explain each sub-category and its principles in detail.

Principles and rules

Aesthetic-based criteria

Earlier Islamic buildings exhibit extensive geometric patterns, substantiating a mathematical interest in the spatial dimension [9]. Mathematicians who taught practical geometry to artisans played a decisive role in creating IGPs and perhaps in designing the buildings themselves [10–13]. Proportion is a characteristic of IGPs, serving as a tool of the self-guiding process for aesthetically proven design [14]. Many treatises were written by medieval mathematician-astronomers such as Abu Abdallah Muhammad b. Isa b. Ahmad Al-Mahani (d. ca. 884), Abu Al-Wafa' Al-Buzjani, Ibn Al-Haytham, Abu Bakr Al-Khalid Al-Tajir Al-Rasadi, and Umar Khayyam, among others, refer to ratio and proportion. Some researchers, like Broug [3], categorized the circle as the main generator of IGPs, dividing it into four, five, and six segments (proportion) called four-fold, five-fold, and six-fold designs. The underlying ratio/proportion of IGPs also uses the same proportional systems that nature embodies. Thus, the study of geometric proportions has its roots in the survey of nature and matter [15, 16]. For example, the second roots of 2, 3, 5, or golden ratio (Phi, in a square, triangle) [10, 17] are concerned with mathematics and nature even in IGPs.

Modularity is a mathematical principle that produces different geometric star and rosette patterns [18, 19]. In some star-related patterns, earlier geometric designs were created through trial-and-error combinations of cut-tile pieces. This achievement later led to a new level of modularity, generating complex patterns [20, 21].

The concept of **variation** and transformation is essential in the design of IGPs. Transforming a pattern to another can occur by changing some variations, such as angles categorized based on the underlying polygons in four pattern families. Each family's name is differentiated depending upon the contact angle, extracted from the midpoints of the underline's polygonal edges. For the acute family, the angle is 36°; for the median family, the angle is 72°; and for the obtuse family, the angle is 108°. Two-point tilings extract lines from two points at an equal distance. These geometric variations are the main identification for *girih* distinctions named *Tond*, *Shol*, and *Tond-o-Shol* in Persian architecture [22, 23] (Fig. 4). Besides, many strategies have been employed to spread the *girih* over dome surfaces [24, 25] and truncated minarets [26] concerning topological and geometric adaptability. In such a manner, adaptability resulted in a direct relationship between changing curvature and the number of points of star polygons. As curvature increases throughout the surface, the pattern accommodates stars with more number of points [24]. For example, the exterior view of the Friday Mosque's dome at Saveh, Iran, with smoothly transiting zones of adjusted *girih* tiling, is a case in point (Fig. 5). Finally, the curvature and size of *muqarnas* modules are variable [27].

The study of three-dimensional IGPs like domes [25, 28], *muqarnas* [29–31], *yazdibandi* [32, 33], and *rasmibandi* [34] was followed as **dimension** feature, transforming two-dimensional plan drawings. *For instance, muqarnas* is a smooth transition between two levels, two sizes, and or two shapes that combine three-dimensional units arranged at various horizontal levels by applying geometrical rules. The plan drawing has a multifold symmetry with a linear, radial, or grid arrangement of units,

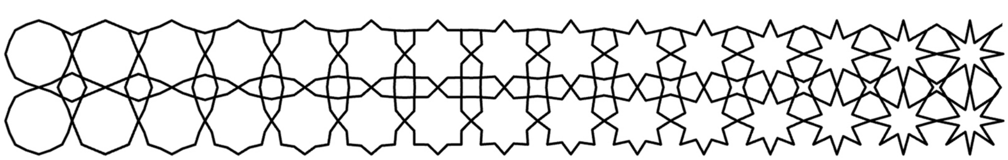


Fig. 4 Changing curvature and number of points of star polygons. Exterior view of the Friday Mosque's dome at Saveh, Iran

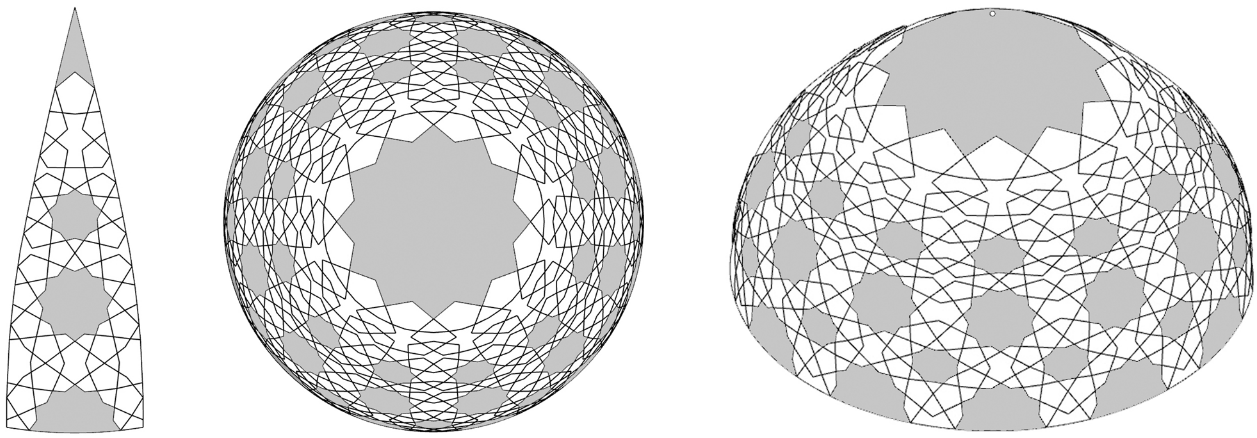


Fig. 5 Variation Principle, based on contact angle

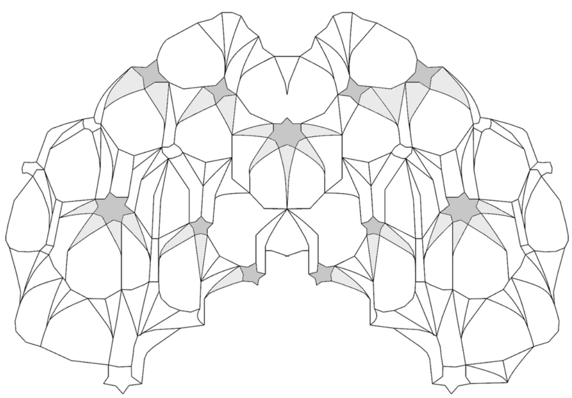


Fig. 6 Various star forms in *muqarnas* of the Shah Mosque in Isfahan, Iran

including focal points. *Muqarnas*, Arabic for stalactite vault, is mainly created from the square and rhombus geometry. Later, especially Safavid examples of *muqarnas* exhibit a more complex geometrical network composed of various star forms [35] (Fig. 6). Al-Kashi, a fifteenth-century Iranian mathematician, was the first scholar who studied *muqarnas*'s principles, including units and curving factors [36]. Al-Kashi classified *muqarnas* according to formal characteristics indicating vertical facets and roofs [37]. In addition, they were classified in three issue, based on the construction types [38], primary innovative features of stone *muqarnas* [39], formative styles [40], and their formal features [41, 42].

Symmetry is a distinguishing feature of IGPs that is directly related to its repeat unit [22] and fills the page through the repetitive use of a single element and repetition structure. The repeat unit is the minimal region containing the basic geometrical composition, polygons such as square, pentagon, hexagon, or their multiplication, that holds the base geometry with the possibility of having several types [43]. furthermore, based on the type of employed extension, IGPs can be classified into periodic and aperiodic symmetry. Periodic patterns cover a plane with two, three, four, and six symmetry systems [44] (Fig. 7, top). According to the crystallographic plane symmetry group, they were classified into seventeen ways: translation, rotation, reflection, and glide reflection dictate the repetitive covering of the two-dimensional plane [29, 44–47].

On the other hand, some studies show that IGPs have all the conceptual elements necessary to produce **quasicrystalline** *girih*, *shah-girih*, and *muqarnas* as aperiodic symmetry [48–50]. They include a five-old and seven-fold symmetry system (Fig. 7, down). Five-fold symmetry consists of a "large regular decagon with an interstitial pentagonal star such that the length of the side of the decagon equals the edge length of the interstitial star". The extant and most well-known examples of patterns generated by aperiodic symmetry are the *Gonbad-e Qabud* tower in *Maragha* during the twelfth century and the *Darb-i Imam* shrine in Isfahan, Iran during the fourteenth and fifteenth centuries [51, 52]. Other studies found some evidence for the presence of quasiperiodic IGPs in western Islamic art, including details [53], octagonal and decagonal types [54–56], *muqarnas* dome [57, 58], and the multigrid generation method [59, 60]. Nevertheless, imputing a nascent quasicrystal theory to medieval mathematicians simply cannot be justified based on historical records [61]. Other views support the presence of self-similar quality in some IGPs [62–65], making it somewhat similar to the structural signature of quasicrystals. Scholars like Cromwell [66] and Chorbachi [10] believe Islamic artists have had the tools to construct quasiperiodic designs without the theoretical framework to appreciate the possibility or significance of doing

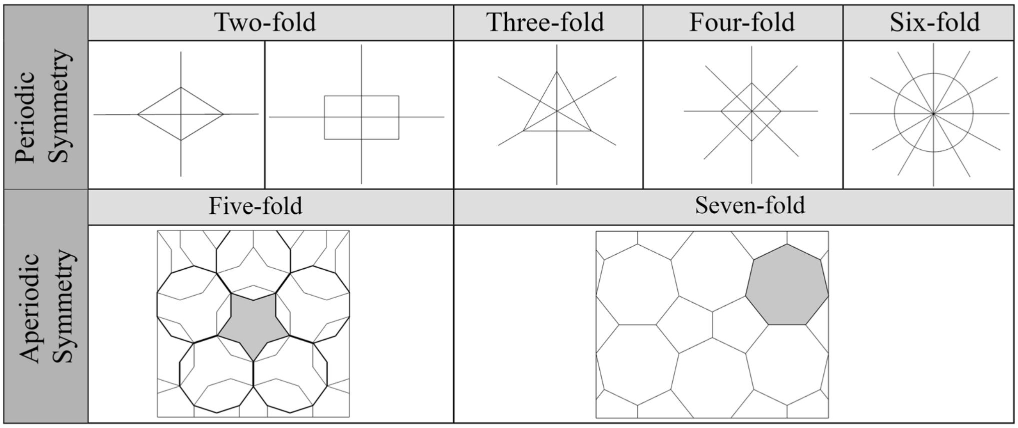


Fig. 7 Symmetry in periodic and aperiodic patterns

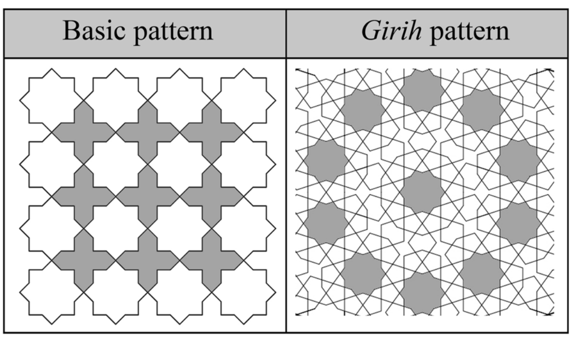


Fig. 8 one level patterns

so. However, Bier [52] believes that folio 180a, a design from Al-Buzjani [67], may have been used as the generative form of a planar pattern of overlapping decagons related to the aperiodic decagonal/pentagonal pattern on the *Gonbad-e Qabud*. The aperiodic tiling method was introduced to the modern world much later by Roger Penrose [68], who discovered quasicrystal tiling systems in the 1970s using non-crystallographic symmetry.

The **multi-layered** [69] feature is divided IGPs into one-level and two-level patterns. For the patterns with one level, some polygons, such as triangles, hexagons, and squares, were used directly to create basic patterns through repetition and symmetry. Moreover, Islamic-era artisans and mathematicians used another type of pattern radiating from different stars [70–76], surrounded by different polygons, found rather ubiquitously across the Islamic world (Fig. 8). As the star pattern progressed, an interlocked pattern emerged named *girih*. The *girih*, coming from the Persian language, equivalent *aqd in* the Arabic language, is a complex pattern with regular structures and defined sets of mathematical elements [29]. The evolution of *girih* followed in dual levels, filling the spaces between the lines of the large-scale pattern with the small-scale pattern. Containing similar generation systems for both patterns of the dual-level designs, such a

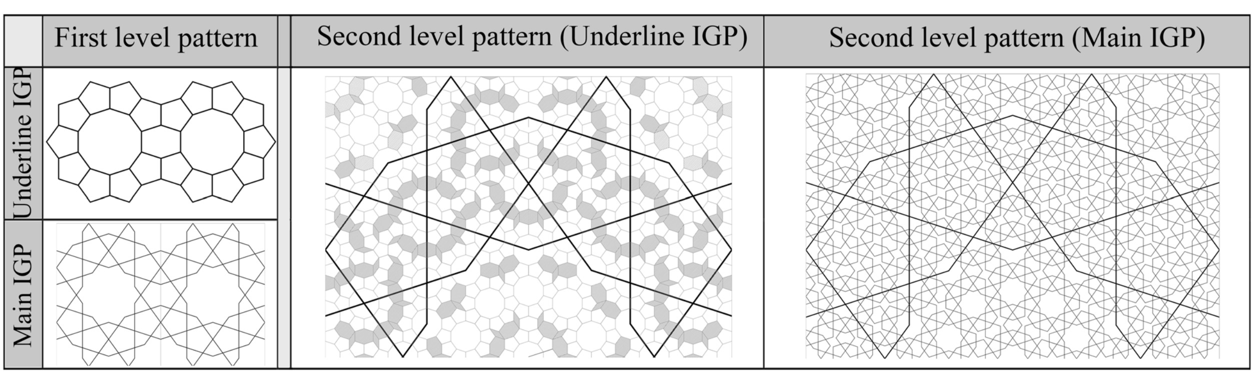


Fig. 9 Multilevel patterns, quasicrystal

pattern illustrated as three types of self-similarity in Bonner [47, 62] (Fig. 9). Differences in the scale of the two levels provide audiences with a progressive appreciation of the primary design from a relatively long distance and the secondary scale upon closer proximity.

The aesthetic character of a given geometric design is greatly determined by **generation methods** [76]. Mathematicians have mainly developed generative methods to provide architects and craftsmen with some methods to be applied at construction sites. Accordingly, the radial *girih* approach and polygon method were historically explained in Al-Buzjain [67], Topkapi, Tashkent, and Mirza Akbar scroll drawings [29] (Fig. 10). Most traditional Islamic artists used the radial *girih* approach, which was implemented by a compass and a straightedge. Therefore, the generating force of patterns lies in the circle's center [22, 29, 43, 77, 78]. The artist first draws a radial generating matrix, and through the points obtained from the intersection of the radii and arcs, the lines and angles of *girih* are drawn. In other words, the starting point in drawing the *girih* is a star polygon, the star inside the circle, which places its centers at a specific distance from each other [79]. Depending on the number of star polygons, particular numbers of rays emanating from the center of stars. The intersections of these rays provide interstitial space, which is filled by different methods and creates various designs. On the other hand, the polygon in contact method is an essential tiling-based approach, underlying the grid system, investigated by many scholars [76, 80–85]. Hankin discovered this method for the first time, providing an excellent starting point for an algorithmic approach [86]. The existence of two design examples from the Topkapi scroll [29, 87] of the late fifteenth century proves the use of this method in the Islamic era. In recent studies, polygon in contact, as a tiling-based approach, and symmetry group methods [45] have been the most employed strategies by scholars.

Finally, IGPs can be identified by interlacing or interwoven features [45, 46, 88, 89] by which pairs of lines cross at various points. The two interlaced patterns distinguish only how two strands cross, on the top in one pattern or at the bottom in the other (Fig. 11). They "enhance the sense of movement according to their direction, breadth, and variation" [90].

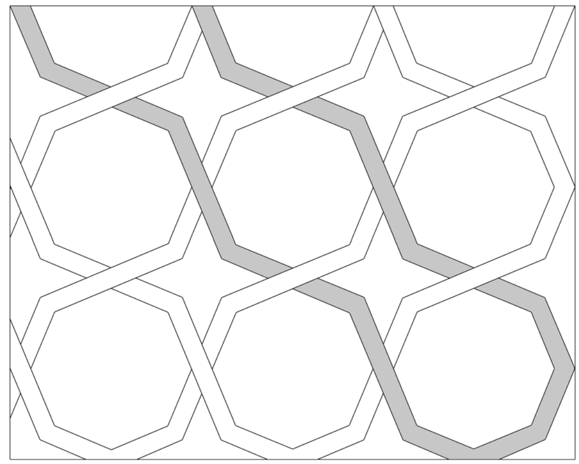


Fig. 11 Friday mosque of Forumad, Iran

Sustainable-based criteria

Islamic architecture coped with environmental constraints in various areas and climates of the Islamic world using sustainable-based elements. Before the age of air conditioning, open plan, and curtain wall, buildings in Islamic countries were constructed of the simplest materials, making them cool in hot and warm in cold weather. However, interest in sustainable architecture has contributed to a revival of the *orosi, mashrabiya,* and other elements of traditional architecture. *Orosi* [91] and *mashrabiya* [92] are identified as important sustainable-based IGPs in which the design of the geometric patterns adheres to the same principles of the general *girih* pattern. They can be applied to shading screens of architectural spaces as an ornamental element and environmental control system, including natural light, thermal, and visual convenience [93, 94]. *Orosi* combines wooden panels with IGPs and colorful glasses that cover

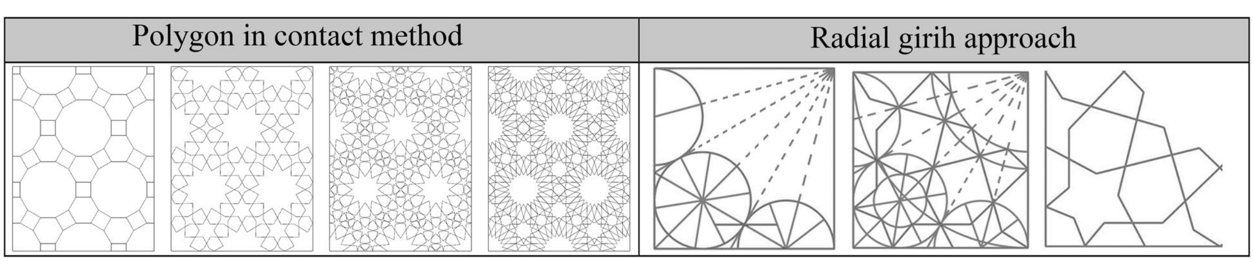


Fig. 10 Generation method in IGPs

different functions. In addition, *mashrabiya* is made of a wooden lattice of cylinders connected with spherical joints. Based on the evaporation's influence, small water jars were positioned through the apertures to cool the air. *Mashrabiya* protects Middle Eastern buildings' unglazed openings. It is a wooden frame covering a window opening and decorating the building façade [95]. *Mashrabiya* is traditionally characterized by its functions, which include the principles mentioned for *orosi* plus the principle of humidity (Fig. 12).

Structural-based criteria

Islamic architecture presents two structural decoration systems inspired by two-dimensional patterns. They are known as *karbandi*, a characteristic Persian ribbed vault, and self-load bearing *muqarnas*.

Karbandi is formed based on two-dimensional star patterns, *shamseh* [96]. It is called *rasmibandi* when it excludes the structural objectives. This transitioned form is evident in the vaults' drawings and domical spaces from Mirza Akbar's scroll in late eighteenth-century Iran. Master Lorzadeh (1906–2004), one of the last of his generation with a link to eighteenth-century traditions of master builders, believed in the drawing of *karbandi* based on the division of semicircle by the radiuses from the center of the span and its extension on the rectangular surface [22]. Other scholars believe that "the plan of an n-sided *karbandi* is on the basis of dividing a circle into n equal sectors and drawing intersecting equal chords between the dividing points [97–99]. Then, the final pattern is formed based on the funicular transforming of any lines of initial patterns, producing lattice structure or ribbed vaults [100]. In addition, the connection of curved elements can improve karbandi's resistance at the top of the structure. *Karbandi* is differentiated by variations, including the angle between lines, the height of elements, and the amount of each element's curvature. There is a relationship between the height of *karbandi* and the size of *shamseh*. [99]. Moreover, there is a correlation and coordination between architectural and structural functions in *karbandi*, causing aesthetic and meaningful spaces with structural performance [101, 102]. Furthermore, the dominant stress distribution in this structure's members as a lattice structure indicates the intelligent construction method [34]. Figure 13 depicts the traditional form generation process as principles.

Self-load-bearing *muqarnas* is differentiated from decorative one through construction method. First, the hung type prevailed in all Islamic lands, like the *muqarnas* of the Hall of the Two Sisters in the Alhambra. Second is the corbelled type, of which there are fewer examples like Thakh-I-Sulayman palace [103], Iran, and the Muslim Ibn Quraysh tomb in Imam Dawr, Iraq. Self-load-bearing *muqarnas* is mainly created from the two-dimensional square and rhombus geometry [104], transforming into a three-dimensional structure through arcs. It consists of niche-like elements arranged in tiers and can be used in

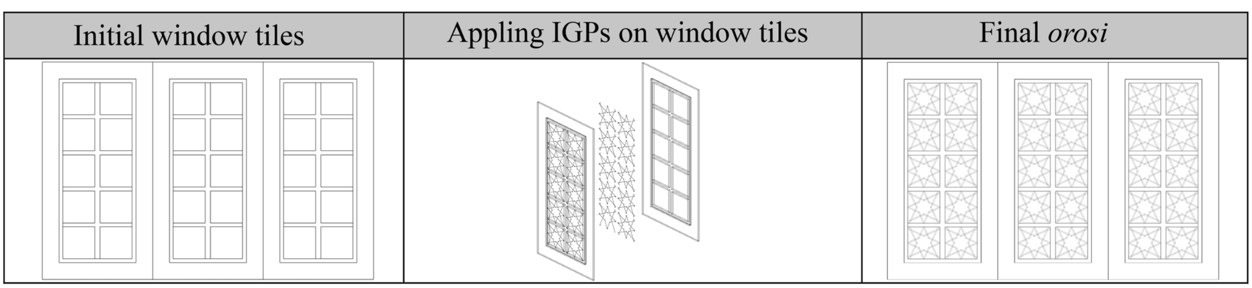


Fig. 12 Component of orosi

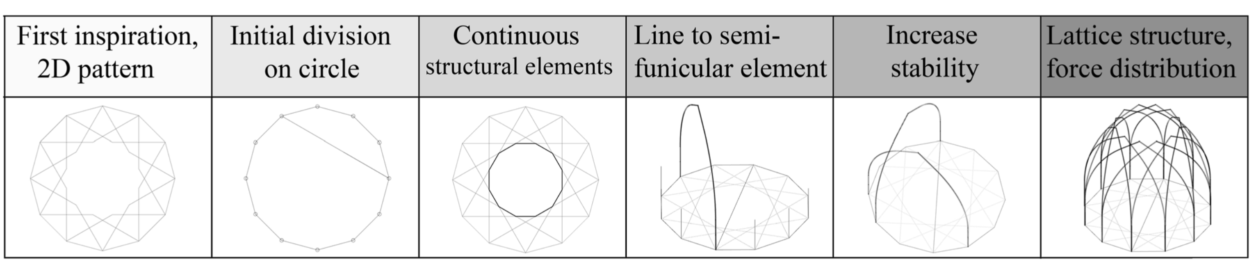


Fig. 13 Traditional 3d form generation of karbandi

domes, niches, or arches. Loads must pass through them as long as they are lower [105]. The main structural feature of form generation is that it breaks double curvature down into single curved surfaces. As a traditional form generation method, this geometric conversion led to unique creativity in the production of structural *muqarnas*. The earliest surviving examples of self-load-bearing *muqarnas* are to be found in the Samanid dynasty, like the Samanid Ismail mausoleum (914–943) at Bukhara [106] and the Arab Ata tomb (977–978) in Tim, Uzbekistan [107]. Squinches structurally helped to enrich the zone of transition from square to the dome, using two flanked half-squinches. Figure 14, depicts the traditional form generation process as principles.

In both described structural systems, self-load bearing and lattice structure, brick, the main compression material [108], serves the central role in structural form generation. The brick's placements align with the force vectors, integrating form and structure. Finally, *karbandi* and self-supporting *muqarnas* have been used as complementary structures, providing support to a structure, concentrating on their role positions as an intermediary between flat walls and vaults, domes, niches, and arches.

State-of-the-art, contemporary directions

Today, the underlying principles of IGPs are a research subject of many scholars focusing on new possibilities and potentials. The state-of-the-art IGPs in contemporary studies are reviewed in this part due to their application objectives concerning computer science and computer graphics. Figure 15. illustrates all the extracted principles from the literature, Sect. "Principles and rules".

Aesthetic-based; generation methods & variations

By focusing on two-dimensional periodic patterns, many studies have used **computer graphic** facilities to generate, study and analyze IGPs through planar symmetry groups [45, 109] or tiling-based methods [44]. Ostromoukhov [110] utilized mathematical tools to analyze and generate two-dimensional IGPs. Rasouli et al. [111] introduced two algorithms for computer-generated IGPs, 8-*Zohreh,* and 8-*Sili*. Aljamali [112] proposed a new method to classify and design star/rosette IGPs using computer software and the implementation of symmetry groups. Khamjane and Benslimane [113] presented a computerized method based on the symmetry groups theory to create periodic Islamic star and rosette patterns. The number of stars/rosettes and their parameters enabled them to develop novel ones. Nadyrshine et al. [114] recently extended these processes, creating the N-angle star pattern that produced more parametric ornaments. Lahcen et al. [115] proposed a new symmetric-based method called *Hasba* to automatically generate a large number of IGPs using a software program. Ouazene et al. [116] studied the traditional geometric

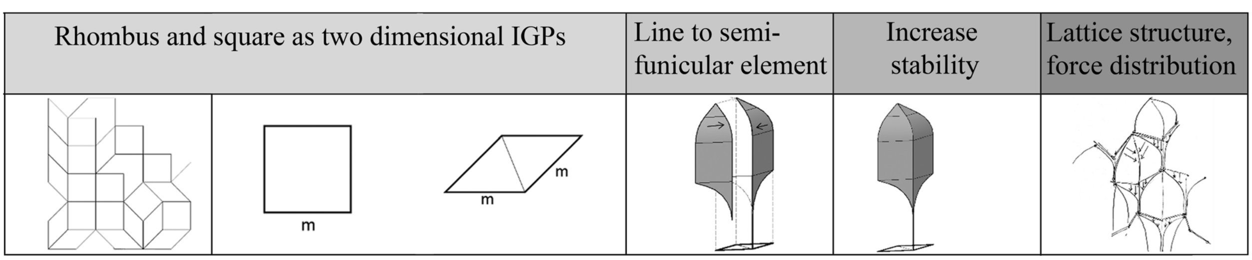


Fig. 14 Traditional 3d form generation of self-load bearing *muqarnas*

Aesthetic-	Sustainable-	Structure -based
Proportion and adaptability	Visual convenience	Self-load bearing
Generation methods/extensions	Heat convenience	Material behaviour
Periodic (tiling-based/polygonal method)/aperiodic	Humidity control	Transform to efficiency
Dimensional approaches/2D to 3D	Day light	Force distribution/Lattice feature
Multi-layers	Airflow	
Interwoven		

Fig. 15 Principles extracted from the literature

patterns from Morocco and Andalusia, showing the relationship between the 8-pointed star and other geometric tiles according to the traditional method called *Tastir* and symmetry groups.

Also, Kaplan and Salesin [117] investigated an approach utilizing a tiling-based system, the inflation process, filled by a parameterized set of stars and rosettes. In another study, the first author [86] addressed Islamic parquet deformations, continuously varying the contact angle of a ray depending on the horizontal position of the ray's starting point. Izadi et al. [118] investigated a computerized algorithm to make traditional and new geometric patterns through tiling-based techniques. Bonner [119] classified IGPs systematically into five groups, including 3–4-6–12 systems of regular polygons, the 4–8 system A, the 4–8 system B, the 5–10 system, and the 7–14 system using the tiling-based polygonal technique. Afterward, Lee et al. [120] introduced a method to efficiently modify and control Islamic star patterns through experiments and computer algorithms, developing a combination of multiple unilateral polygons based on the basic polygon method.

In addition, Lalvani [121] introduced the shape code and morph code, presenting all the information and rules to generate variations of IGPs using symmetry groups. Cenani and Cagdas [122] showed the characteristics, shape grammar rules, and historical background of geometrical ornaments in Islamic culture to point out the possibilities of mathematics of symmetry. Alani and Barrios [123] extended the Lalvani method through the cartesian coordinate system, repeat unit, and fundamental unit. Alani [124] concentrated on the design formalism method that incorporated mathematics and a morphology-based approach, intending to engage the research and design of the IGPs. In another study, the same author [125] focused on the mathematics and relations between design parts through morphological connections of hexagonal-based IGPs beyond the formal dimension. Zahri [126] presented a new mathematical approach for modeling a class of IGPs using the geometrical properties of connected and closed graphs. Nasri and Benslimane [127] automatically generated original and new forms of periodic Moorish geometric patterns utilizing underlying grid-based, symmetry-based approaches and shape grammar formalism. More recently, Refalian et al. [128] introduced a new approach to digital visualization of available IGPs, particularly star patterns, focusing on formal grammar and computer science.

Moreover, many scholars proposed a computational model for patterns' **symmetry features extraction** to analyze and classify simple tile patterns [129], rosette and star patterns [130, 131], and more IGPs [132] utilizing symmetry group theory. Zarghili et al. [133] proposed a new method for indexing the Arabo-Moresque décor database through a geometric-based image retrieval system. Gil et al. [134] presented a methodology for image reconstruction of Alhambra's IGps using a computer tool that allowed automatic image analysis, obtaining information about missing motifs. Al Ajlouni and Justa [135] used this method to reconstruct deteriorated IGPs to predict the disappeared features utilizing an accurate and measurable model. Nasri et al. [136] automatically extracted the basic unit cell of periodic IGPs by detecting the pertinent peaks based on the autocorrelation function. The authors optimized the criterion function using a genetic algorithm. Furthermore, El ouaazizi et al. [137] automatically detected the Islamic geometric rosette's characteristics by its rotational center, order, and symmetry group from a picture using a genetic algorithm. Following this, the authors [138] attempted to find new innovative periodic and aperiodic rosette patterns based on the polygon in contact technique. Albert et al. [139] provided valuable insight into IGPs based on image processing, pattern recognition, and symmetry groups, adding a new object-oriented level of knowledge. In another study, Beatini [140] investigated the conversion of IGPs to kinetic rosette patterns through modular linkages that rely on the same type and number of symmetry operations as the reference models. In addition, Aoulalay et al. classified Islamic Moroccan decorative patterns [141] and more IGPs [142] based on machine learning algorithms through feature extraction. Hajebi and Hajebi [143] proposed a new method for the intelligent restoration of the parametric IGPs. In this study, by accessing the image of the existing patterns, the vanished parts could be reconstructed spontaneously using machine learning. Finally, Baneh et al. [144] applied computer science in BIM to restore and improve two-dimensional IGPs. Accurate geometrical models were developed for the symmetry-based IGPs using photogrammetry, linked to databases carrying information related to similar texture, historical identity, and decoration.

From the viewpoint of the layout, **aperiodic** symmetries regarding two-dimensional patterns, Al Ajlouni [145] proposed a global multi-level hierarchical framework, not tiled-based on a local tiling system, during aperiodic symmetries. This study described the long-range translational and orientational order of aperiodic formations, suggesting the position of geometrical units locally and globally. The author presented infinite eightfold [146], tenfold [145], 12-fold [147], and sevenfold [148] quasi-lattice based on a hierarchical clustering order of quasicrystals, extending the global multi-level framework to another type of aperiodic patterns. These studies used line variations for connecting the same star units. Besides, Bonner and Pelletier [149] extended an aperiodic tessellation in 3 levels of recursion scaling properties of the polygonal modules of both the 5/10 and 7/14 aperiodic systems. The authors used this approach to construct patterns in the traditional Islamic self-similar design conventions developed during the fourteenth and fifteenth centuries. Webster [150] created a wide variety of new fractal Islamic patterns that pursue a different notion of self-similarity, simultaneously incorporating motifs at multiple, often infinite scales within a single pattern by arrangements of {n/2} star.

Additionally, Khamjane and Benslimane [17] investigated IGPs to create new patterns, exploring the golden mean and the fractal geometry in the construction and the distribution of the star/rosettes. In another study, the authors [151] proposed a systematic method for generating new aperiodic patterns inspired by existing Islamic historical patterns based on quasiperiodic tiling and a few intuitive parameters. Also, Khamjane et al. [152] presented a method for constructing IGPs, using unit cells for periodic patterns and mirror reflections for quasiperiodic patterns. Khamjane et al. [153] developed new types of self-similar IGPs, inspiring traditional ones and recent achievements in quasiperiodic science using golden mean triangles. Using computer-aided design, Thalal et al. [154] formulated the method of constructing periodic and aperiodic Moroccan patterns and found new ones based on the symmetry groups missing in Moroccan patterns. Finally, using digital design and fabrication methods, Riether and Baerlecken [155] provided different insights into the relationship between massing and texture on aperiodic patterns in Islamic architecture.

For the sake of **three-dimensional** geometric patterns, Chitchian and Sariyildiz [156] used Maya's programming language to produce three-dimensional IGPs. Sayed et al. [157] introduced a novel approach to generating three-dimensional IGPs utilizing parametric shape grammar. The same authors followed this method in Sayed et al. [158] to construct IGPs using the generated motifs and repeated units. Additionally, Moradzadeh and Nejad Ebrahimi [159] transformed two-dimensional IGPs into a new type of three-dimensional pattern, thanks to mathematics and geometry for contemporary usage. The same authors [160] developed the Islamic geometric patterns from planar coordinates to three dimensions with aperiodic symmetry and high-range variation of patterns using computer-aided design and manufacturing (CAD/CAM) facilities. Agirbas [161] presented the algorithmic decomposition and tessellation resemblance of IGPs in three dimensions, examining those examples in which the third dimension was the dominant feature. Moreover, Marco [162] concentrated on 3D documentation and representation systems for analyzing and cataloging IGPs, developed within virtual models of patterns through polygonal surfaces. The result was obtained from range-based and image-based surveys in Generalife Palace, Alhambra.

From the viewpoint of transforming to a **three-dimensional pattern**, some studies concentrated on *muqarnas*. Al-Asad [163] was the first to use a computer-aided design program to reproduce complex three-dimensional *muqrans* arches based on historical designs. Yaghan [164] created a system of unit set generation by synthesizing available traditional units to develop new types using a computer program. In another study, the same author [31] decoded two historical *muqarnas*-corbel drawings into possible three-dimensional forms by studying projected plans made of the patterned distribution of geometric pieces. Using a computer program, the authors created two new self-supporting systems of *muqarnas*: line- and point-oriented [165]. Dold-Samplonius and Harmsen [166] developed the first algorithmic method for reconstructing the three–dimensional stalactite vaults, *muqarnas*, constructing directed subgraphs for the Seljuk and IL-Khanid ones executed by a software tool. Hamekasi, Samavati, and Nasri [38] presented a new approach to model *muqarnas*, considering it as a transition between layers of different forms. The implementation of the presented methods was made possible by a software tool using b-spline curves. Gherardini and Leali [167] proposed integrating traditional techniques of *muqarnas* design with modern photogrammetric 3D model tools to study and analyze the modularity and elements hierarchy. Guzelci and Alacam [168], after a comprehensive analysis, generated new *muqarnas* patterns using algorithms. In another study, Senhaji and Benslimane, [169] proposed an automatic algorithm to model the Moroccan-Andalusian *muqarnas* patterns from its' two-dimensional plane patterns, suggesting a set of rules. The same authors initially generalized the earlier method, which was limited to rectangular *muqarnas* compositions, covering the construction of *muqarnas* domes through the *Almoravids* period until now [170]. Also, Alacam et al. [171] investigated the performative potential of a specific fold on *muqarnases* and developed computational strategies for form-finding and analysis of vaulting structures. Maarouf and Zeid [172] formulated a parametric generative process for several new *muqarnas* in linear or polar arraying directions. Ferrer-Pérez-Blanco et al. [173] provided a laser scanner to capture new drawings of the *muqarnas*, accurate plans, and elevations for the first time and documented the *muqarnas*' complex shapes from the fourteenth century. Dinçer and Yazar [174] compared different case studies for the digital modeling of *muqarnas* systems in which variations were developed by element-based, tessellation-based, and block-based workflow. More recently, Gokmen et al. [175] focused on 12 unique *muqarnas* structures found in Anatolia that were analyzed through a computational methodology combining photogrammetry, three-dimensional modeling, symmetry, and graph theory. Finally, Agirbas et al. [176] investigated the symmetrical proportion of inequality of star edge lengths in *muqarnas* ground projection plans obtained by 3D laser scanning and examination based on shape grammar. This research was also conducted by Agirbas and Yildiz [1], performing simple parametric *muqarnas* using intersections of the ellipse grid.

Besides, Rasouli and Bastanfard [177] offered an algorithmic method for drawing *yazdibandi* in a two-dimensional scheme, initializing by the star pattern. Mohamadianmansoor and Faramarzi [98] concentrated on the shape-based typology of the *karbandi* generation in a rectangular base. Mohammadi et al. [178] offered a solution to broaden *karbandi*'s use over irregular geometry, showing how to calculate and implement *karbandi* geometry in irregular bases under specific conditions. The same authors [99] discovered geometric relationships and principles of *karbandi* to regulate and facilitate its design process in contemporary architecture. Maleki and Woodbury [34] developed the geometric constraints of *rasmibandi*, representing the location of the defining points of a dome. They presented a goal-seeking algorithm to solve the constraints within a propagation-based parametric modeling system.

Regarding **domes** and **curved surfaces**, Kaplan and Salesin [117] introduced a generalized system within a novel parameterized collection of tiling formations by creating designs on the sphere and hyperbolic plane. Pottman et al. [179] tailored IGPs on geodesic curves of free forms concerning fabrication properties. Shahbazi et al. [180] investigated some IGPs as the curved tessellated surface on domes to achieve more adaptability, changing the number of basic modules and their production parameters. Besides, Bonner [181] introduced new IGPs comprised of primary stars with n-fold rotation symmetry covering the sphere's surface unconventionally. This method expanded on the historical use of polyhedral geometry as an organizing principle for placing IGPs onto the surfaces of domes and domical niches, using geodesic-conforming traditional polyhedral jitterbug. Most studies of *muqarnas*, *karbandi*, and domes are subject to "line to curve" variations.

In addition to these, several studies were associated with three-dimensional IGPs, but are explained in other sections. Riether and Baerlecken [155] considered the aperiodic feature. Kaplan [182] concentrated on the interwoven feature. Agırbas and Basogul [183] considered the lattice and space structure principle.

Regarding **multi-layered** feature, most of the studies regarding aperiodic symmetry patterns are also concerned with multi-layered features, including studies by Al Ajlouni [145–148], Pelletier and Bonner [184], Khamjane and Benslimane [151], and Khamjane et al. [152, 153].

Regarding the **interwoven** feature, Kaplan [182] investigated constructing interwoven two-layer Islamic patterns in the plane and on polyhedral, describing a projection operation that bulged the elements of these designs into undulating dome shapes. In addition to this, several studies were associated with the interwoven feature but are explained in other sections. Agirbas [161] considered the aperiodic feature, while Agirbas and Basogul [183] evaluated the lattice and space structure principle.

Sustainable-based criteria

Concerning sustainable-based criteria, Giovannini et al. [185] focused on a series of analyses that were carried out to explore the lighting and energy performance of adaptive shading of buildings' facades for arid climates, shape variable *mashrabiya*, utilizing Python script. Mohamadin et al. [186] presented optimal IGPs for shading screen design about daylight and energy performance in an existing façade of an educational design studio using generative design and simulation techniques. Emami and Giles. [187] studied the effects of geometric parameters as shading screens on daylighting, using the underlying principles of IGPs. Oghazian et al. [188] investigated the influence of seven types of IGPs and the non-uniform distribution of openings on solar screens installed on a south-facing façade of an office room from a daylighting performance perspective. Koren [189] explained some types of star IGPs used in the Louvre Abu Dhabi, on the dome, optimizing with respect to visual comfort and daylight. The tessellation is true at the apex while it distorts towards the dome's perimeter. The stars are scaled in size and mapped to the further layers of the dome cladding. Furthermore, Tabadkani et al. [190] used the rosette pattern as a sun-responsive shading system, optimizing a list of critical visual comfort preferences that leads to lighting energy performance.

In addition, Rian et al. [191] used *mashrabiya* as visual comfort, daylight, and heat convenience application of IGPs through a barycentric subdivision to a hyperstructure for making the proposed pavilion. They used ropes for weaving the *mashrabiya*-inspired web. Yi et al. [192] investigated the effects of the adaptable auxetic shading structure based on IGPs, applied on building facades to optimize illuminance levels and reduce glare probability. Bagasi and Calautit [193] studied the thermal ventilation performance of buildings' facades incorporated with *mashrabiya* and the potential to enhance its capabilities by combining it with different passive cooling techniques. More recently, Bagasi et al. [194] evaluated *mashrabiya's* performance regarding building ventilation and humidity in hot climates. Hosseini et al. [205] investigated the daylight performance of different IGPs used in orosies with varying thicknesses on the west and south facades to improve indoor daylighting and the visual comfort of occupants. Sabouri Kenarsari et al. [195] conducted a study to develop an innovative approach for the parametric analysis of daylighting and visual comfort through a sun-responsive shading system and proposed a parametric pattern of Persian *girih* IGPs. Mohaghegh et al. [196] extracted the optimal geometric pattern and the appropriate origami stimulus angle in kinetic façades by algorithmically creating several origami patterns and adapting them to daylight at different times of the year. Finally, Maksoud et al. [197] developed a code set to generate different pattern topologies with various porous sizes and followed a vertical gradient, optimizing visual and environmental conditions in corridor's screen walls.

Structure-based criteria

Finally, structure-based criteria are discussed, which have not received much attention in research on IGPs' principles, especially regarding the capabilities of the new material and structural systems. Rezakhani and Kim [198] utilized virtual reality to evaluate the impact of the dispersion of joints on kinetic façade tension. Asefi and Bahremandi [199] generated a barrel vault form using reciprocal frame structures. They served as a revival of *girih,* developing lattice and space structure principles. Agırbas and Basogul [183] created a reciprocal frame structure based on IGPs' interwoven feature. In this study, stripes systematically pass over or under each other in a dome-like design, analyzing their structural performance.

Regarding material behavior, some studies investigated IGPs' formal features regarding new developments in architectured micro-lattices as auxetic material. These new features lack a distinctive principle in traditional architecture. Rafsanjani and Pasini [200] introduced a class of switchable architectured materials that inspired a type of IGPs exhibiting simultaneous auxeticity and structural bistability. Lim [201] established a perfectly auxetic two-dimensional metamaterial using the strain energy approach in terms of the spiral spring stiffness and geometrical parameters. The study was inspired by an eight-point Islamic pattern formed from a circumference of eight squares. This approach has been employed on other IGPs, investigating their material potential, using

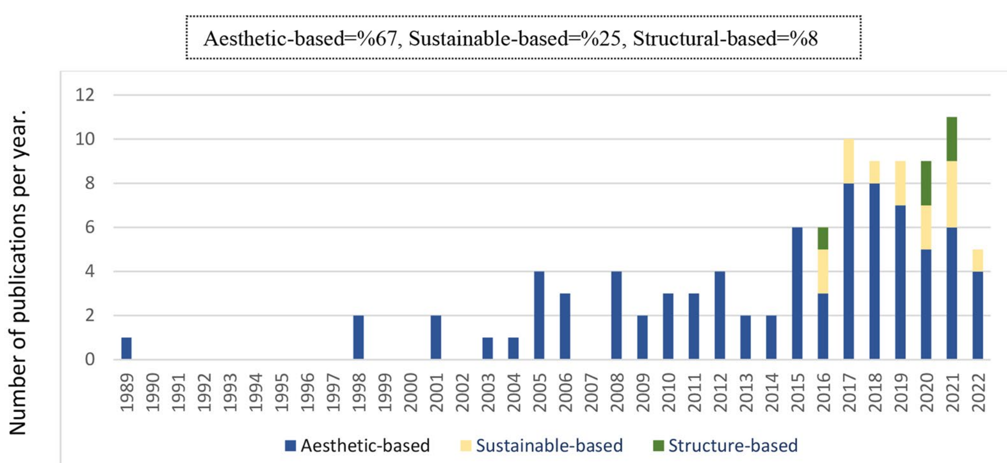


Fig. 16 Application-based distribution in the reviewed literature by date (till 31.06.2022)

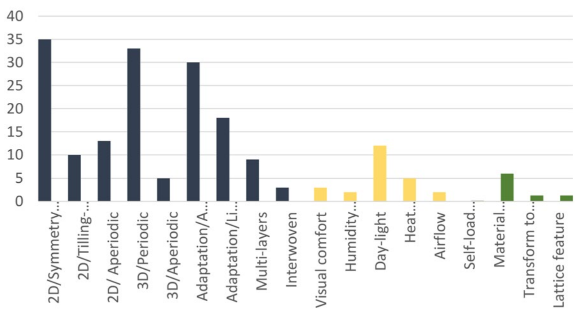


Fig. 17 Distribution of all principles and applications

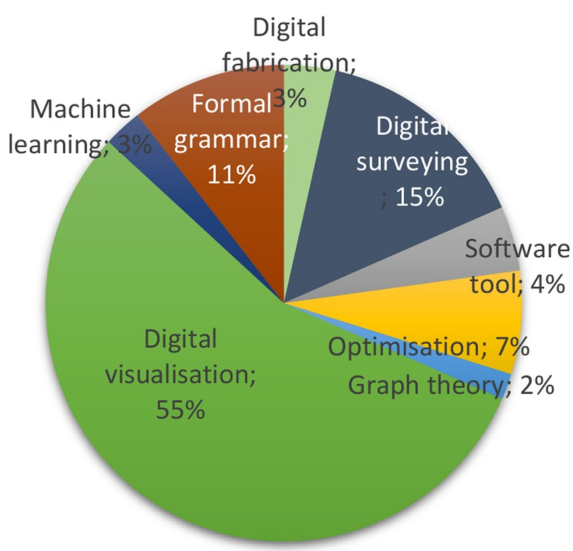


Fig. 18 Distribution of computer science/technologies

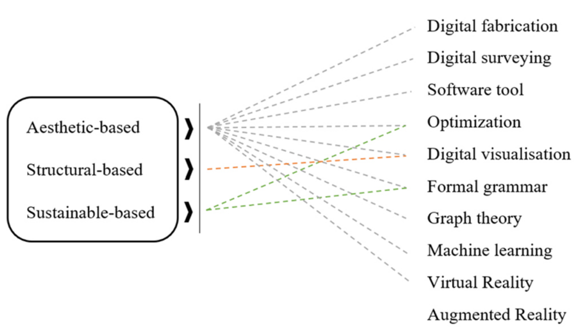


Fig. 19 Relationship between application-based objectives and computer science related technologies

8-pointed Islamic stars [202], 8-pointed rigid stars interspersed with a square array of 4-pointed rigid stars [203], and the Islamic mosaic pattern at the Alhambra Palace [204].

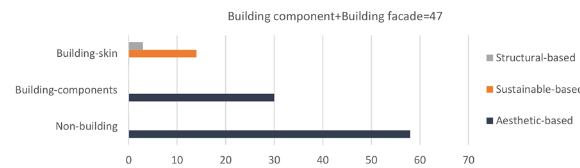


Fig. 20 Distribution of major topics

Review results

To highlight the correspondence between IGPs' principles and the state of art, concerning their application objectives, Table 2 (appendix) is presented. In essence, application-based distribution in the reviewed literature by date and overall, the distribution of each main category, and application objectives, shown in the reviewed documents, are considered herein in Fig. 16. The distribution of each sub-category within the recent literature is displayed in Fig. 17. The total usage amount of computer science/technologies, are shown in Fig. 18. The relationship between the main application-based objectives and computer science-related technologies used for IGPs' developments are listed in Fig. 19. Finally, the total usage of building topics, including building skin, building components, and non-building feature in regard to aesthetic-based, sustainable-based, and structure-based, is listed in Fig. 20.

Assessing these studies allows the extraction of some results as follows:

- From the viewpoint of the main categories, Fig. 5 shows that the aesthetic-based application of IGPs was the most studied topic. The least studied category was the structure-based topic. Overall publications increased during the time. The number of published papers has increased significantly in the last five years. The function-based studies were started in 2016, including structure-based and sustainable-based research.
- From the viewpoint of all the broad sub-categories, 2D/periodic and 3D/periodic were the most dominant principles among all reviewed papers, as can be observed in Fig. 6, while little attention was paid to interwoven/visual convenience/humidity control/airflow/lattice principles/transform to efficiency, and no attention to self-load bearing principle.
- From the viewpoint of the aesthetic-based sub-categories, 2D/periodic and 3D/periodic were the most dominant principle among all the reviewed papers. Little attention was given to 3D/aperiodic and interwoven. In addition, one-layer-based studies were the most dominant rather than multi-layers studies.

- From the point of the sustainable-based category, Fig. 6, daylight was the most studied principle. The least studied sub-category was the humidity control and airflow feature.
- From the point of structure-based category, Fig. 6, material behavior was the most studied principle only in one field, in which formal feature of some IGPs regarding auxeticity was explored. The least studied sub-category was the transform to efficiency and lattice feature. No attention was given to the self-load-bearing principle.
- From the point of adaptation/variation, Fig. 6, angle and line were more frequently studied variations than line to curve.
- As shown in Fig. 7, digital visualization was the topic of most studied computer science/technologies. Little attention was attributed to machine learning and graph theory. One study considered virtual reality. No attention was given to augmented reality.
- Aesthetic-based papers considered a wide range of computer science/technologies, as can be observed in Fig. 8. Simultaneously, sustainable-based papers, including optimization and formal grammar, and structure-based papers, including digital visualization, received little attention.
- As shown in Fig. 9, the major building topic was non-building-related studies. The least studied category was building skin. All the non-building and building component papers considered aesthetic-based principles. All the building skin studies considered performance-based principles, mostly sustainable-based. There are only three structure-based studies on building skin.

Conclusions

This study justified the main principles of IGPs from a traditional application point of view, including aesthetic, structure, and sustainable principles. Then, the principles were reviewed concerning the contemporary literature and recent trends in computer science/technologies, including digital surveying, digital fabrication, digital visualization, software tool, formal grammar, graph theory, machine learning, virtual reality, and augmented reality. The application-based taxonomy provided a framework for associated developments across all principles of IGPs. The studies under review in this paper showed that the widespread and increasing application of IGPs holds the promise of becoming a profound contributor to satisfying our increasingly stricter cultural performance targets. At the same time, our findings also identify lacks and helpful directions for future studies and developments in IGPs. Based on the evidence presented in the results, conclusions were drawn about application objectives and principles, as summarised below.

The review of previous studies showed that most researchers have focused on aesthetic-based principles concerning generation methods of 2D and 3D periodic patterns, leading to new developments in non-building IGPs through computational methods.

Besides, despite the IGPs' structural application in the past, the relatively limited number of documented cases reveals that IGPs cannot be considered a mature concept in new structure-based studies. Information about structure, material, and fabrication principles regarding IGPs' new-generation approaches is rarely mentioned in the literature. Developments in material behavior are only considered in one issue. Force-based and self-load-bearing principles are one of the most crucial parameters used to improve the structure performances of some historical IGPs. In recent studies, it is unexpected and surprising that the mentioned principles have never been investigated concerning the building skin and components.

Further studies on aesthetic-based applications of IGPs are needed for better integration of its principles within the structural and sustainable principles, concerning building facades. It is essential to evaluate adaptation/variation (angle/line, line to curve, and module size), multi-level, and interwoven facilities with self-load, transform to efficiency, lattice features, material behavior, daylight, airflow, etc. In addition, examining these applications using computer science facilities like digital fabrication, optimization, formal programming, graph theory, artificial intelligence, virtual reality, and augmented reality facilities is vital. For instance, optimizing adaptation/variation helps to reach more structure and energy efficiency, developing new function-based patterns as a novel solution for high-performance architecture. Moreover, the appropriate selection of materials for the digital fabrication process could be studied further in terms of structure and sustainable features. Implementing these strategies will be necessary to extend IGP beyond recent formal representation in research and built examples.

Appendix

See Table 2.

Table 2 An overview of the principle in state-of-the-art literature

	Application objectives																		
	Aesthetic-based									Sustainable-based					Structure-based				
	Generation method					Adaptation		-	-	Visual comfort	Humidity control	Daylight	Heat convenience	Airflow	Force-based	Self-load bearing	Material behavior	Transform to efficiency	Lattice feature
	2D/ Symmetry group	2D/ Tilling-based	2D/ Aperiodic	3D/ Periodic	3D/ Aperiodic	Angles and lines	Line to curve	Multi-layers	Interwoven										
(Al-Asad, 1956)				✓															
(Abas and Salman, 1992)	✓																		
(Lalvani, 1989)	✓																		
(Ostromoukhov, 1998)	✓																		
(Zarghili et al. 2001)		✓																	
(M. A. J. Yaghan, 2001)				✓															
(M. A. J. Yaghan, 2003)				✓															
(Kaplan and Salesin, 2004)		✓		✓		✓													
(Kaplan, 2005)		✓				✓													
(M. A. Yaghan, 2005)				✓															
(Valiente et al. 2005)	✓																		
(Dold-Samplonius and Harmsen, 2005)				✓			✓												
(Cenani and Cagdas, 2006)	✓					✓													
(Djibril et al. 2006b)	✓																		

Table 2 (continued)

	Application objectives																		
	Aesthetic-based									Sustainable-based					Structure-based				
	Generation method					Adaptation		-	-	Visual comfort	Humidity control	Daylight	Heat convenience	Airflow	Force-based	Self-load bearing	Material behavior	Transform to efficiency	Lattice feature
	2D/ Symmetry group	2D/ Tilling-based	2D/ Aperiodic	3D/ Periodic	3D/ Aperiodic	Angles and lines	Line to curve	Multi-layers	Interwoven										
(Djibril et al. 2006a)	✓																		
(Rasouli et al. 2008)	✓																		
(Djibril and Thami, 2008)	✓																		
(Chitchian & Sariyildiz, 2008)			✓																
(Maleki and Woodbury, 2008)				✓		✓	✓												
(Aljamali, 2009)	✓																		
(Gil et al., 2009)	✓																		
(Pottmann et al. 2010)				✓			✓												
(Izadi et al. 2010)	✓	✓				✓													
(Rasouli and Bastanfard, 2010)				✓															
(Mohamadi-anmansoor and Fara-marzi, 2011)				✓			✓												
(R. Al Ajlouni and Justa, 2011)	✓					✓													
(Hamekasi et al. 2011)				✓	✓														
(R. A. Al Ajlouni, 2012)			✓			✓		✓											

Table 2 (continued)

	Application objectives																		
	Aesthetic-based									Sustainable-based					Structure-based				
	Generation method					Adaptation		-	-	Visual comfort	Humidity control	Daylight	Heat convenience	Airflow	Force-based	Self-load bearing	Material behavior	Transform to efficiency	Lattice feature
	2D/ Symmetry group	2D/ Tilling-based	2D/ Aperiodic	3D/ Periodic	3D/ Aperiodic	Angles and lines	Line to curve	Multi-layers	Interwoven										
(Riether and Baerlecken, 2012)					✓	✓													
(Bonner and Pelletier, 2012)			✓					✓											
(Pelletier and Bonner, 2012)			✓					✓											
(R. Al Ajlouni, 2013)			✓			✓		✓											
(Webster, 2013)				✓															
(Nasri et al. 2014)	✓																		
(Bonner, 2014)	✓	✓																	
(Lee et al. 2015)		✓				✓													
(Giovannini et al. 2015)						✓						✓	✓						
(Alani and Barrios, 2015)	✓					✓													
(Albert et al. 2015)	✓																		
(Sayed et al. 2015)				✓		✓													
(El ouaazizi et al. 2015)	✓																		
(Sayed et al. 2016)				✓		✓													

Table 2 (continued)

	Application objectives																		
	Aesthetic-based									Sustainable-based					Structure-based				
	Generation method					Adaptation		-	-	Visual comfort	Humidity control	Daylight	Heat convenience	Airflow	Force-based	Self-load bearing	Material behavior	Transform to efficiency	Lattice feature
	2D/ Symmetry group	2D/ Tilling-based	2D/ Aperiodic	3D/ Periodic	3D/ Aperiodic	Angles and lines	Line to curve	Multi-layers	Interwoven										
(Moham-madin et al. 2016)		✓										✓	✓						
(Emami and Giles, 2016)												✓							
(Gherardini and Leali, 2016)							✓												
(Alani, 2016)	✓						✓												
(Rafsanjani and Pasini, 2016)																	✓		
(Nasri et al. 2017)	✓		✓																
(Nasri and Benslimane, 2017)	✓																		
(Shahbazi et al. 2017)				✓			✓												
(R. Ajlouni, 2017)			✓			✓		✓											
(Beatini, 2017)	✓																		
(Alacam et al. 2017)				✓			✓												
(Koren, 2017)				✓			✓		✓	✓		✓							
(Oghazian et al. 2017)												✓							
(Khamjane and Bensli-mane, 2017)			✓																
(Kaplan, 2017)				✓		✓													

Table 2 (continued)

	Application objectives																		
	Aesthetic-based									Sustainable-based					Structure-based				
	Generation method					Adaptation		-	-	Visual comfort	Humidity control	Daylight	Heat convenience	Airflow	Force-based	Self-load bearing	Material behavior	Transform to efficiency	Lattice feature
	2D/ Symmetry group	2D/ Tilling-based	2D/ Aperiodic	3D/ Periodic	3D/ Aperiodic	Angles and lines	Line to curve	Multi-layers	Interwoven										
(Alani, 2018)	✓					✓													
(Tabadkani et al., 2018)												✓							
(Thalal et al. 2018)	✓	✓																	
(Khamjane and Bensli-mane, 2018b)			✓			✓		✓											
(Khamjane and Bensli-mane, 2018a)	✓					✓													
(R. A. Ajlouni, 2018)			✓					✓											
(Mohammadi et al. 2018)				✓		✓	✓												
(Bonner, 2018)				✓		✓													
(De Marco, 2019)	✓			✓	✓														
(Senhaji and Benslimane, 2019)				✓			✓												
(ZAHRI, 2019)	✓					✓													
(Maarouf and Zeid, 2019)				✓															
(Amjad Mohammadi et al. 2019)				✓		✓	✓												

Table 2 (continued)

	Application objectives																		
	Aesthetic-based									Sustainable-based					Structure-based				
	Generation method					Adaptation		-	-	Visual comfort	Humidity control	Daylight	Heat convenience	Airflow	Force-based	Self-load bearing	Material behavior	Transform to efficiency	Lattice feature
	2D/ Symmetry group	2D/ Tilling-based	2D/ Aperiodic	3D/ Periodic	3D/ Aperiodic	Angles and lines	Line to curve	Multi-layers	Interwoven										
(Guzelci and Alacam, 2019)				✓			✓												
(Khamjane et al., 2019)		✓	✓					✓											
(Yi et al. 2019)										✓		✓							
(Rian et al. 2019)										✓		✓	✓						
(Ferrer-Pérez-Blanco et al. 2019)				✓	✓														
(Khamjane et al., 2020)			✓			✓		✓											
(Rezakhani and Kim, 2020)																		✓	
(Moradzadeh and Nejad Ebrahimi, 2020a)				✓															
(Hosseini et al. 2020)												✓							
(Moradzadeh and Nejad Ebrahimi, 2020b)					✓	✓													
(Agirbas, 2020)				✓					✓										
(Bagasi and Calautit, 2020)											✓		✓	✓					
(Lim, 2020)																	✓		

Table 2 (continued)

	Application objectives																		
	Aesthetic-based									Sustainable-based					Structure-based				
	Generation method					Adaptation		-	-	Visual comfort	Humidity control	Daylight	Heat convenience	Airflow	Force-based	Self-load bearing	Material behavior	Transform to efficiency	Lattice feature
	2D/ Symmetry group	2D/ Tilling-based	2D/ Aperiodic	3D/ Periodic	3D/ Aperiodic	Angles and lines	Line to curve	Multi-layers	Interwoven										
(Aoulalay et al. 2020)	✓	✓																	
(Ouazene et al. 2021)	✓																		
(Bagasi et al. 2021)											✓			✓					
(Nadyrshine et al. 2021)	✓																		
(Sabouri Kenarsari et al. 2021)										✓		✓							
(Hajebi and Hajebi, 2021)	✓		✓	✓		✓													
(Agirbas and Yildiz, 2021)				✓			✓												
(Wafaei Baneh et al. 2021)	✓																		
(Agirbas and Basogul 2021)				✓					✓								✓		
(Mohaghegh et al. 2021)												✓							
(Lahsen et al., 2021)	✓																		
(Asefi and Bahrem andi Tolou, 2021)									✓										✓
(Lim, 2021a)						✓											✓		
(Lim, 2021b)						✓											✓		
(Lim, 2021c)						✓											✓		

Table 2 (continued)

	Application objectives																		
	Aesthetic-based									**Sustainable-based**					**Structure-based**				
	Generation method					**Adaptation**		**-**	**-**	**Visual comfort**	**Humidity control**	**Daylight**	**Heat convenience**	**Airflow**	**Force-based**	**Self-load bearing**	**Material behavior**	**Transform to efficiency**	**Lattice feature**
	2D/ Symmetry group	**2D/ Tilling-based**	**2D/ Aperiodic**	**3D/ Periodic**	**3D/ Aperiodic**	**Angles and lines**	**Line to curve**	**Multi-layers**	**Interwoven**										
(Dinçer and Yazar, 2021)				✓			✓												
(Refalian et al. 2021)	✓					✓													
(Aoulalay et al. 2022)	✓																		
(Senhaji and Benslimane 2022)				✓			✓												
(Gokmen et al. 2022)				✓			✓												
(Maksoud et al. 2022)												✓	✓						
(Agirbas et al.. 2022)				✓			✓												

	Trends in Computer science/technologies									Topic		
	Digital fabrication	**Digital surveying**	**Software tool**	**Optimization**	**Digital visualisation**	**Formal grammar**	**Graph theory**	**Machine learning**	**Virtual reality**	**Building facade**	**Building element**	**Non-Building**
(Al-Asad, 1956)					×						+	
(Abas & Salman, 1992)					×							+
(Lalvani, 1989)					×	×						+
(Ostromoukhov, 1998)					×							+
(Zarghili et al., 2001)		×										+
(M. A. J. Yaghan, 2001)					×						+	
(M. A. J. Yaghan, 2003)					×						+	
(Kaplan & Salesin, 2004)			×									+
(Kaplan, 2005)					×							+

Table 2 (continued)

	Trends in Computer science/technologies									Topic		
	Digital fabrication	Digital surveying	Software tool	Optimization	Digital visualisation	Formal grammar	Graph theory	Machine learning	Virtual reality	Building facade	Building element	Non-Building
(M. A. Yaghan, 2005)					×						+	
(Valiente et al., 2005)		×										+
(Dold-Samplonius & Harmsen, 2005)			×				×				+	
(Cenani & Cagdas, 2006)						×						+
(Djibril et al., 2006b)		×										+
(Djibril et al., 2006a)		×										+
(Rasouli et al., 2008)		×										+
(Djibril & Thami, 2008)		×										+
(Chitchian & Sari-yildiz, 2008)					×							+
(Maleki & Woodbury, 2008)					×						+	
(Aljamali, 2009)			×									+
(Gil et al., 2009)		×									+	
(Pottmann et al., 2010)	×										+	
(Izadi et al., 2010)					×							+
(Rasouli & Bastan-fard, 2010)					×							+
(Mohamadianman-soor & Faramarzi, 2011)					×						+	
(R. Al Ajlouni & Justa, 2011)		×									+	
(Hamekasi et al., 2011)			×								+	
(R. A. Al Ajlouni, 2012)					×							+
(Riether & Baer-lecken, 2012)	×					×					+	
(Bonner & Pelletier, 2012)					×							+

Table 2 (continued)

	Trends in Computer science/technologies									Topic		
	Digital fabrication	Digital surveying	Software tool	Optimization	Digital visualisation	Formal grammar	Graph theory	Machine learning	Virtual reality	Building facade	Building element	Non-Building
(Pelletier & Bonner, 2012)					×							+
(R. Al Ajlouni, 2013)					×							+
(Webster, 2013)					×							+
(Nasri et al., 2014)		×		×								+
(Bonner, 2014)					×							+
(Lee et al., 2015)					×							+
(Giovannini et al., 2015)					×					+		
(Alani & Barrios, 2015)						×						+
(Albert et al., 2015)				×								+
(Sayed et al., 2015)						×						+
(El ouaazizi et al., 2015)		×		×								+
(Sayed et al., 2016)						×						+
(Mohammadin et al., 2016)				×						+		
(Emami & Giles, 2016)					×					+		
(Gherardini & Leali, 2016)				×							+	
(Alani, 2016)						×						+
(Rafsanjani & Pasini, 2016)					×							+
(Nasri et al., 2017)		×		×								+
(Nasri & Benslimane, 2017)						×						+
(Shahbazi et al., 2017)					×							+
(R. Ajlouni, 2017)					×							+
(Beatini, 2017)					×							+
(Alacam et al., 2017)					×						+	
(Koren, 2017)					×					+		

Table 2 (continued)

	Trends in Computer science/technologies									Topic		
	Digital fabrication	Digital surveying	Software tool	Optimization	Digital visualisation	Formal grammar	Graph theory	Machine learning	Virtual reality	Building facade	Building element	Non-Building
(Oghazian et al., 2017)					×					+		
(Khamjane & Benslimane, 2017)					×							+
(Kaplan, 2017)					×							+
(Alani, 2018)						×						+
(Tabadkani et al., 2018)					×					+		
(Thalal et al., 2018)					×							+
(Khamjane & Benslimane, 2018b)					×							+
(Khamjane & Benslimane, 2018a)					×							+
(R. A. Ajlouni, 2018)					×							+
(Mohammadi et al., 2018)					×						+	
(Bonner, 2018)					×						+	
(De Marco, 2019)		×									+	
(Senhaji & Benslimane, 2019)					×						+	
(ZAHRI, 2019)					×							+
(Maarouf & Zeid, 2019)					×						+	
(Amjad Mohammadi et al., 2019)					×						+	
(Guzelci & Alacam, 2019)					×						+	
(Khamjane et al., 2019)					×							+
(Yi et al., 2019)				×						+		
(Rian et al., 2019)					×					+		
(Ferrer-Pérez-Blanco et al., 2019)		×									+	
(Khamjane et al., 2020)					×							+

Table 2 (continued)

	Trends in Computer science/technologies									Topic		
	Digital fabrication	Digital surveying	Software tool	Optimization	Digital visualisation	Formal grammar	Graph theory	Machine learning	Virtual reality	Building facade	Building element	Non-Building
(Rezakhani & Kim, 2020)									×	+		
(Moradzadeh & Nejad Ebrahimi, 2020a)					×							+
(Hosseini et al., 2020)					×					+		
(Moradzadeh & Nejad Ebrahimi, 2020b)	×				×							+
(Agirbas, 2020)					×							+
(Bagasi & Calautit, 2020)					×					+		
(Lim, 2020)					×							+
(Aoulalay et al., 2020)								×			+	
(Ouazene et al., 2021)					×							+
(Bagasi et al., 2021)					×					+		
(Nadyrshine et al., 2021)					×							+
(Sabouri Kenarsari et al., 2021)										+		
(Hajebi & Hajebi, 2021)		×			×			×				+
(Agirbas & Yildiz, 2021)		×				×					+	
(Wafaei Baneh et al., 2021)		×									+	
(Agirbas & Basogul, 2021)							×			+		
(Mohaghegh et al., 2021)					×					+		
(Lahsen et al., 2021)			×									+
(Asefi & Bahremandi Tolou, 2021)				×						+		
(Lim, 2021a)					×							+
(Lim, 2021b)					×							+

Table 2 (continued)

	Trends in Computer science/technologies									Topic		
	Digital fabrication	**Digital surveying**	**Software tool**	**Optimization**	**Digital visualisation**	**Formal grammar**	**Graph theory**	**Machine learning**	**Virtual reality**	**Building facade**	**Building element**	**Non-Building**
(Lim, 2021c)					×							+
(Dinçer & Yazar, 2021)					×						+	
(Refalian et al., 2021)	×					×						+
(Aoulalay et al., 2022)								×			+	
(Senhaji & Benslimane, 2022)					×						+	
(Gokmen et al., 2022)					×		×				+	
(Maksoud et al., 2022)					×					+		
(Agirbas et al., 2022)		×				×					+	

Acknowledgements
Not applicable

Author contributions
MR. Conceptualization, Methodology, Writing—original draft. MB, MM, AK, and JK Supervision, Writing—review editing. All authors read and approved the final manuscript.

Funding
This research received no external funding.

Availability of data and materials
The data supporting this study's findings are available from the corresponding author or first author, upon reasonable request.

Declarations

Ethics approval and consent for publication
Not applicable.

Consent to participate
Not applicable.

Competing interests
We have no conflicts of interest to disclose.

Received: 22 September 2022 Accepted: 26 December 2022
Published online: 01 February 2023

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