SpringerPlus

RESEARCH

Open Access

CrossMark

An improved generalized Newton method for absolute value equations

Jingmei Feng[1,2*] and Sanyang Liu[1]

*Correspondence:
fengjingmeilq@hotmail.com
[1] School of Mathematics
and Statistics, Xidian
University, Xi'an 710126,
China
Full list of author information
is available at the end of the
article

Abstract

In this paper, we suggest and analyze an improved generalized Newton method for solving the NP-hard absolute value equations $Ax - |x| = b$ when the singular values of A exceed 1. We show that the global and local quadratic convergence of the proposed method. Numerical experiments show the efficiency of the method and the high accuracy of calculation.

Keywords: Absolute value equations, Generalized Newton's method, Global and local convergence

Background

We consider the absolute value equations (AVEs):

$$Ax - |x| = b, \tag{1}$$

where $A \in R^{n \times n}$, $b \in R^n$, and $|x|$ denotes a vector in R^n, whose i-th component is $|x_i|$. A more general form of the AVEs, $Ax + B|x| = b$, was introduced by Rohn (2004) and researched in a more general context in Mangasarian (2007a). Hu et al. (2011) proposed a generalized Newton method for solving absolute value equation $Ax + B|x| = b$ associated with second order cones, and showed that the method is globally linearly and locally quadratically convergent under suitable assumptions. As was shown in Mangasarian and Meyer (2006) by Mangasarian, the general NP-hard linear complementarity problems (LCPs) (Cottle and Dantzing 1968; Chung 1989; Cottle et al. 1992) subsume many mathematical programming problems such as absolute value equations (AVEs) (1), which own much simpler structure than any LCP. Hence it has inspired many scholars to study AVEs. And in Mangasarian and Meyer (2006) the AVEs (1) was investigated in detail theoretically, the bilinear program and the generalized LCP were prescribed there for the special case when the singular values of A are not less than 1. Based on the LCP reformulation, sufficient conditions for the existence and nonexistence of solutions are given in this paper. Mangasarian also has used concave minimization model (Mangasarian 2007b), dual complementarity (Mangasarian 2013), linear complementarity (Mangasarian 2014a), linear programming (Mangasarian 2014b) and a hybrid algorithm (Mangasarian 2015) to solve AVEs (1). Hu and Huang reformulated a system of absolute value equations as a standard linear complementarity problem without any

assumption and gave some existence and convexity results for the solution set of the AVEs (1) in Hu and Huang (2010). Paper Zhang et al. (2015) presented a new algorithm which relaxes the absolute value equations into a convex optimization problem, Zhang et al. found the sparsest solution of AVEs by the minimum l_0-norm. Caccetta et al. proposed a globally and quadratically convergent method for AVEs in Caccetta et al. (2011). Rohn et al. gave an iterative method for AVEs (1) and analyzed the sufficient conditions of unique solvability by Rohn et al. (2014), Uniqueness is always a hot spot (Wu and Guo 2015), and Moosaei et al. gave the minimum norm solution of absolute value equations $Ax - |x| = b$ which has multiple solutions (at most 2^n) via simulated annealing algorithm in Moosaei et al. (2014). Salkuyeh (2014) put forward a hybrid algorithm which combined with skew-Hermitian and Picard-Hermitian splitting iteration method for solving AVEs (1), and gave the convergence analysis of the hybrid algorithm. Furthermore, Mangasarian (2009) clearly showed that generalized Newton method is a very effective method by solving some high dimensional examples in very few iterations. Haghani (2015) proposed an improved Newton method with two-step form, called Traub's method, whose effectiveness is better than that of Mangasarian's. Iqbal et al. (2015) proposed a Levenberg–Marquardt method for AVEs (1), which is the combination of steepest descent and the Gauss–Newton method. Paper Cruz et al. (2016) raised an inexact semi-smooth Newton algorithm for AVEs (1), and proved that the method is globally convergent. So, how to solve absolute value equations based on the classic Newton algorithm have been received many of the concerns. That have motivated us for trying to improve Newton method as the main aim of the present paper.

Now we describe our notation. The scalar product of two vectors x and y in the n-dimensional real space will be denoted by $\langle x, y \rangle$. For $x \in R^n$, the norm $\|x\|$ will denote the two-norm $(x^T x)^{1/2}$, and $sign(x)$ will denote a vector with components equal to $+1$, 0 or -1, depending on whether the corresponding component of x is positive, zero or negative, respectively. In addition, diag($sign(x)$) will denote a diagonal matrix corresponding to sign (x).

In "Preliminary" section of the present work we give the notations and preliminary notes about AVEs. "The improved generalized Newton method" section gives an improved generalized Newton iterative algorithm which is globally and locally quadratic convergent under certain assumptions. In "Computational results" section some numerical reports show the efficiency of the proposed scheme. "Conclusions" section gives some concluding remarks to end the paper.

Preliminary

We begin by defining the piece-wise linear vector function $f(x)$ specified by the AVEs (1) as follows:

$$f(x) = Ax - |x| - b. \tag{2}$$

A generalized Jacobian of f at x is

$$\partial f(x) = Ax - D(x), \tag{3}$$

where $D(x) = \partial |x| = diag(sign(x))$.

To solve (1), the iterative computational method of Mangasarian (2009) is as follows:

$$x^{k+1} = (A - D(x^k))^{-1}b, \quad k = 0, 1, 2, \dots. \tag{4}$$

Another method proposed by Haghani (2015) is as follows:

$$\begin{aligned} y^k &= (A - D(x^k))^{-1}b, \\ x^{k+1} &= y^k - (A - D(x^k))^{-1}((A - D(y^k))y^k - b), \quad k = 0, 1, 2, \dots. \end{aligned} \tag{5}$$

It has been shown that both the sequences $\{x^k\}$ generated by Eqs. (4) and (5) converge linearly to the true solution $\bar{x}$ of AVEs (1) when the singular values of A are exceed 1. However, the computational time of (5) is a little less than that of (4), with a higher residual error precision than (4).

The improved generalized Newton method

Ostrowski (1960) and Traub (1964) presented a modified Newton's iteration for solving nonlinear equation in real space R, which give us some inspiration. We will promote this idea to the n-dimensional space. The iterative method is as follows:

$$\begin{aligned} y^k &= x^k - (A - D(x^k))^{-1}f(x^k), \\ a^k &= \frac{\|f(y^k)\|}{\|2f(y^k) - f(x^k)\|}, \\ x^{k+1} &= y^k - a^k(y^k - x^k), \quad k = 0, 1, 2, \dots. \end{aligned} \tag{6}$$

We can simplify (6) as much as possible to obtain the follwing form of improved generalized Newton method for AVEs

$$x^{k+1} = x^k + (1 - a^k)d^k, \quad k = 0, 1, 2, \dots, \tag{7}$$

where $d^k := -(A - D(x^k))^{-1}(Ax^k - |x^k| - b)$. It is clear that our method is a Newton method with a specific liner search.

Algorithm 1 An improved generalized Newton method

Step 0. Choose an initial vector $x \in R^n$ to AVE (1), set $k = 0$.
Step 1. If $\|Ax^k - |x^k| - b\| = 0$, stop.
Step 2. Compute x^{k+1} by
$$x^{k+1} = x^k + (1 - a^k)d^k,$$
where $d^k := -(A - D(x^k))^{-1}(Ax^k - |x^k| - b), a^k = \frac{\|f(y^k)\|}{\|2f(y^k) - f(x^k)\|}$.
Step 3. Set $k = k + 1$, go to Step 1.

We shall need a few theoretical results to establish convergence of Algorithm 1, we first quote the following two results from Mangasarian and Meyer (2006).

Lemma 1 *The singular values of the matrix $A \in R^{n \times n}$ exceed 1 if and only if the minimum eigenvalue of $A'A$ exceeds 1.*

Lemma 2 *If the singular values of $A \in R^{n \times n}$ exceed 1 for the method* (6), *then $(A - D)^{-1}$ exists for any diagonal matrix D whose diagonal elements are ± 1 or 0.*

Therefore, the sequence of vector iterates form (6) is well defined no matter how to choose the initial vector x^0 in R^n.

We now prove the proposed Newton direction d^k of (7) is a descent direction for the objective function $\|f(x)\|^2$.

Lemma 3 *If the singular values of $A \in R^{n \times n}$ exceed 1, then the proposed Newton direction d^k of* (7) *is a descent direction for the objective function $F(x) = \frac{1}{2}\|f(x)\|^2$.*

Proof Since $f(x) = Ax - |x| - b$, $\partial f(x) = A - D(x)$, and $(A - D(x))^{-1}$ exists for any diagonal matrix D whose diagonal elements are ± 1 or 0.

In addition, we know

$$d^k = -(\partial f(x))^{-1} f(x) = -(A - D(x^k))^{-1}(Ax^k - |x^k| - b).$$

and

$$\begin{aligned}
x^{k+1} &= y^k - a^k(y^k - x^k) \\
&= (1 - a^k)y^k + a^k x^k \\
&= (1 - a^k)(x^k + d^k) + a^k x^k \\
&= x^k + (1 - a^k)d^k.
\end{aligned}$$

Moreover, $F(x) = \frac{1}{2}\|f(x)\|^2$, then $\partial F(x) = \partial f(x)f(x)$.

So

$$\begin{aligned}
\langle \partial F(x), d^k \rangle &= \langle \partial f(x)f(x), -(\partial f(x))^{-1}f(x) \rangle \\
&= -(f(x))^T (\partial f(x))^T (\partial f(x))^{-1}f(x) \\
&= -\|f(x)\|^2 < 0.
\end{aligned}$$

Consequently, d^k is a descent direction of $F(x)$. $\qquad\square$

Lemma 4 *Let the singular values of A exceed 1, then the sequence $\{x_k\}$ generated by the improved generalized Newton method* (6) *is bounded, and there exists an accumulation point $\bar{x}$ such that $(A - \bar{D})\bar{x} = b + \bar{a}f(\bar{x})$ for some diagonal matrixes $\bar{D}$ with diagonal elements of ± 1 or 0.*

Proof Suppose that the sequence $\{x^k\}_{k=0}^{\infty}$ is unbounded. Then there exists a subsequence $\{x^{k_j+1}\} \to \infty$ with nonzero x^{k_j+1}, such that $D(x^{k_j}) = \widetilde{D}$, where $\widetilde{D}$ is a assured diagonal matrix with diagonal elements equal to ± 1 or 0 extracted from the finite number of possible configurations for $D(x^k)$ in the sequence $\{D(x^k)\}$, and such that the bounded subsequence $\left\{ \frac{x^{k_j+1}}{\|x^{k_j+1}\|} \right\}$ converges to $\widetilde{x}$. By Eq. (6), we have

$$
\begin{aligned}
y^k &= x^k - (A - D(x^k))^{-1} f(x^k) \\
&= x^k - (A - D(x^k))^{-1} (Ax^k - |x^k| - b) \\
&= x^k - (A - D(x^k))^{-1} ((A - D(x^k))x^k - b) \\
&= x^k - (A - D(x^k))^{-1} (A - D(x^k))x^k + (A - D(x^k))^{-1} b \\
&= x^k - x^k + (A - D(x^k))^{-1} b \\
&= (A - D(x^k))^{-1} b. \\
x^{k+1} &= y^k - a^k (y^k - x^k) \\
&= (A - D(x^k))^{-1} b + a^k (A - D(x^k))^{-1} f(x^k) \\
&= (A - D(x^k))^{-1} (b + a^k f(x^k)).
\end{aligned}
$$

So, $(A - D(x^k))x^{k+1} = b + a^k f(x^k)$, thus, $(A - D(x^{k_j})) \frac{x^{k_j+1}}{\|x^{k_j+1}\|} = \frac{b + a^{k_j} f(x^{k_j})}{\|x^{k_j+1}\|}$.

Due to the following equation:

$$
\|a^{k_j} \cdot f(x^{k_j})\| = \frac{\|f(y^{k_j})\| \cdot \|f(x^{k_j})\|}{\|2f(y^{k_j}) - f(x^{k_j})\|}, \tag{8}
$$

and Lemma 3, the Newton direction d^k in (7) is a descent direction for the objective function $\|f(x)\|^2$. We get $\|a^{k_j} \cdot f(x^{k_j})\| \to 0$, as $j \to \infty$.

Now, $j \to \infty$ gives us:

$$
(A - \widetilde{D})\widetilde{x} = 0, \quad \|\widetilde{x}\| = 1,
$$

since $x^{k_j+1} \to \infty$. This is a contradiction with the nonsingularity of $(A - D)$ which follows from Lemma 2. Hence, the vector sequence $\{x^k\}$ is bounded and there exists an accumulation point $(\bar{D}, \bar{x})$ of $\{(D(x^k), x^{k+1})\}$ such that

$$
\bar{x} = (A - \bar{D})^{-1}(b + \bar{a} f(\bar{x})).
$$

The proof is complete. $\qquad\square$

Theorem 1 (Global linear convergence) *If $\|(A - D)^{-1}\| < \frac{1}{3}$ for any diagonal matrix D with diagonal elements of ± 1 or 0, then the improved generalized Newton method (6) converges linearly from any starting point x^0 to a solution $\bar{x}$ for any solvable AVEs (1).*

Proof Suppose that $\bar{x}$ is a solution of the AVE (1). Noting that $D(\bar{x})\bar{x} = |\bar{x}|$ and $D(x^k)x^k = |x^k|$, for convenience, let $\bar{D} = D(\bar{x})$, $D^k = D(x^k)$. Subtracting $(A - D(\bar{x}))\bar{x} = b$ from $(A - D(x^k))x^{k+1} = b + a^k f(x^k)$, we get

$$
\begin{aligned}
A(x^{k+1} - \bar{x}) &= D^k x^{k+1} - \bar{D}\bar{x} + a^k f(x^k) \\
&= D^k (x^{k+1} - x^k + x^k) - \bar{D}\bar{x} + a^k f(x^k) \\
&= |x^k| - |\bar{x}| + D^k (x^{k+1} - x^k) + a^k f(x^k) \\
&= |x^k| - |\bar{x}| + D^k (x^{k+1} - \bar{x} + \bar{x} - x^k) + a^k f(x^k) \\
&= |x^k| - |\bar{x}| + D^k (x^{k+1} - \bar{x}) + D^k (\bar{x} - x^k) + a^k f(x^k).
\end{aligned}
$$

So,

$$(A - D^k)(x^{k+1} - \bar{x}) = |x^k| - |\bar{x}| + D^k(\bar{x} - x^k) + a^k f(x^k).$$

i.e.,

$$(x^{k+1} - \bar{x}) = (A - D^k)^{-1}(|x^k| - |\bar{x}| + D^k(\bar{x} - x^k) + a^k f(x^k)).$$

From Mangasarian (2009, Lemma 5), we know that for $\forall x, y \in R^n$, $\||x| - |y|\| \le 2\|x - y\|$.
  Thus,

$$
\begin{aligned}
\|x^{k+1} - \bar{x}\| &= \|(A - D^k)^{-1}(|x^k| - |\bar{x}| + D^k(\bar{x} - x^k) + a^k f(x^k))\| \\
&\le \|(A - D^k)^{-1}\|(\||x^k| - |\bar{x}|\| + \|D^k(\bar{x} - x^k)\| + \|a^k f(x^k))\|) \\
&\le \|(A - D^k)^{-1}\|(2\|x^k - \bar{x}\| + \|\bar{x} - x^k\| + \|a^k f(x^k))\|) \\
&\le \|(A - D^k)^{-1}\|(3\|x^k - \bar{x}\| + \|a^k f(x^k))\|) \\
&< \|x^k - \bar{x}\| + \frac{1}{3}\|a^k f(x^k))\|.
\end{aligned}
$$

Since, $\|a^k \cdot f(x^k)\| \to 0$, as $k \to \infty$, so the sequence$\{\|x^k - \bar{x}\|\} \to 0$, as $k \to \infty$. Consequently, $\{x^k\}$ converges to $\bar{x}$.
  The proof is complete. □

Theorem 2 *If* $\|A^{-1}\| < \frac{1}{4}$ *and* $D(x^k) \ne 0$ *for any diagonal matrix* $D(x^k)$ *with diagonal elements of* ± 1 *or* 0, *then the Algorithm 1 converges linearly from any starting point* x^0 *to a solution* $\bar{x}$ *for any solvable AVEs* (1).

Proof The proof directly from Mangasarian (2009, Proposition 7). It is hence omitted.

  In the following, we use Eq. (7) to prove the locally quadratic convergence of the Algorithm 1. □

Lemma 5 *If* $A - D$ *is nonsingular for any diagonal matrix* D *with diagonal elements of* ± 1 *or* 0, *then the Algorithm 1 is approximately Newton's method.*

Proof Taking into account the step length

$$1 - a^k = \frac{\|f(y^k) - f(x^k) + f(y^k)\| - \|f(y^k)\|}{\|2f(y^k - f(x^k))\|} \le \frac{\|f(y^k) - f(x^k)\|}{\|2f(y^k) - f(x^k)\|}.$$

When x^k is in a neighborhood of the solution $\bar{x}$ of AVEs (1), $f(y^k)$ is close to $f(x^k)$, we perform a Newton step from x^k to y^k, for larger k, $f(y^k) = O(\|y^k - \bar{x}\|^2)$, thus

$$
\begin{aligned}
f(y^k) - f(x^k) &= Ad^k + |x^k| - |x^k + d^k| \\
&= Ad^k - D(x^k)d^k + O(\|d^k\|^2) \\
&= -f(x^k) + O(\|d^k\|^2).
\end{aligned}
$$

So, $2f(y^k) - f(x^k) = f(y^k) - f(x^k) + O(\|d^k\|^2)$, when k is sufficiently large and $(A - D(x^k)$ is uniformly bounded $(|A - D(x^k)| < |A| + |D(x^k)| < |A| + 1)$, $d^k = O(f(x^k))$. Then, $f(y^k) - f(x^k) = -f(x^k) + O(\|f(x^k)\|^2)$, we also have

$\|f(x^k)\| = \|(A - D(\overline{x}))(x^k - \overline{x})\| = O(\|x^k - \overline{x}\|) =: O(\|c^k\|)$, where $c^k := x^k - \overline{x}$. Thus, $f(y^k) = O(\|c^k\|^2)$, and

$$1 - a^k = 1 - \frac{O(\|c^k\|^2)}{O(\|c^k\|)} = 1 - O(\|c^k\|).$$

Hence, our method is approximately the generalized Newton's method. $\square$

Theorem 3 (Locally quadratic convergence) *If $A - D$ is nonsingular for any diagonal matrix D with diagonal elements of ± 1 or 0, then the sequence $\{x^k\}$ from improved generalized Newton's method* (7) *converges to $\bar{x}$ and $\|x^{k+1} - \bar{x}\| = O(\|x^k - \bar{x}\|^2)$.*

Proof This theorem can be proved in a similar way as Theorem 2 by Qi and Sun (1993). We omit it here. $\square$

Computational results

To illustrate the implementation and efficiency of the proposed method, we test the following two examples. All the experiments are performed by MATLAB R2010a. In comparisons, NM, TM and INM denote the generalized Newton's method (4), the generalized Traub's method (5) and the improved generalized Newton's method (6), respectively.

Example 1 We choose randomly matrix A according to the following formula:

$$A = rand(n * (eye(n, n) - 0.02 * (2 * rand(n, n) - 1))).$$

Then, we choose a random $x \in R^n$, compute $b = Ax - |x|$, and denote the problem size by n. To ensure that the minimum singular value of each A exceeds 1, we compute the minimum singular value of A and reseal A by the minimum singular value multiply by a random number in the interval $[1, 2]$, The results are shown in Table 1.

Table 1 The comparison of NM, TM and INM in Example 1

Dim	NM			TM			INM		
	K	ACC	T	K	ACC	T	K	ACC	T
100	3	1.6125×10^{-10}	0.0036	3	1.8416×10^{-11}	0.0031	3	1.6168×10^{-11}	0.0027
200	3	6.0215×10^{-10}	0.0097	3	7.0496×10^{-11}	0.0095	3	6.2749×10^{-11}	0.0084
300	3	1.2931×10^{-9}	0.0216	3	1.6284×10^{-10}	0.0071	3	4.5510×10^{-10}	0.0217
400	3	2.5705×10^{-9}	0.0567	3	2.6186×10^{-10}	0.0517	3	7.6406×10^{-10}	0.0321
500	3	4.3078×10^{-8}	0.0831	3	4.7102×10^{-10}	0.0944	3	3.5408×10^{-10}	0.0512
600	4	6.3700×10^{-8}	0.1365	3	5.5877×10^{-10}	0.1576	3	4.1326×10^{-10}	0.0906
700	4	7.0085×10^{-8}	0.2227	3	1.0945×10^{-9}	0.2304	3	8.8369×10^{-10}	0.1278
800	4	2.1838×10^{-7}	0.3766	4	2.0634×10^{-9}	0.3103	4	2.5631×10^{-9}	0.1788
900	4	3.6958×10^{-7}	0.4706	4	6.9784×10^{-9}	0.4173	4	2.9899×10^{-9}	0.2325
1000	5	6.3484×10^{-7}	0.6973	4	1.7392×10^{-8}	0.5221	4	6.3255×10^{-9}	0.3063

Example 2 The matrix A of which all the singular values are greater than 1 is generated by the following MATLAB procedure:
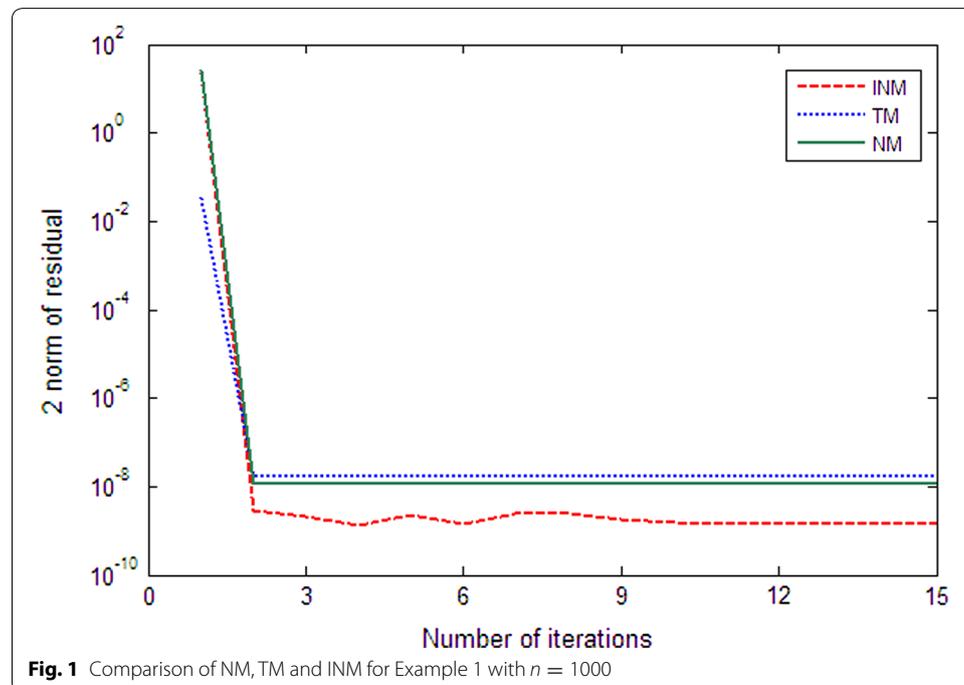
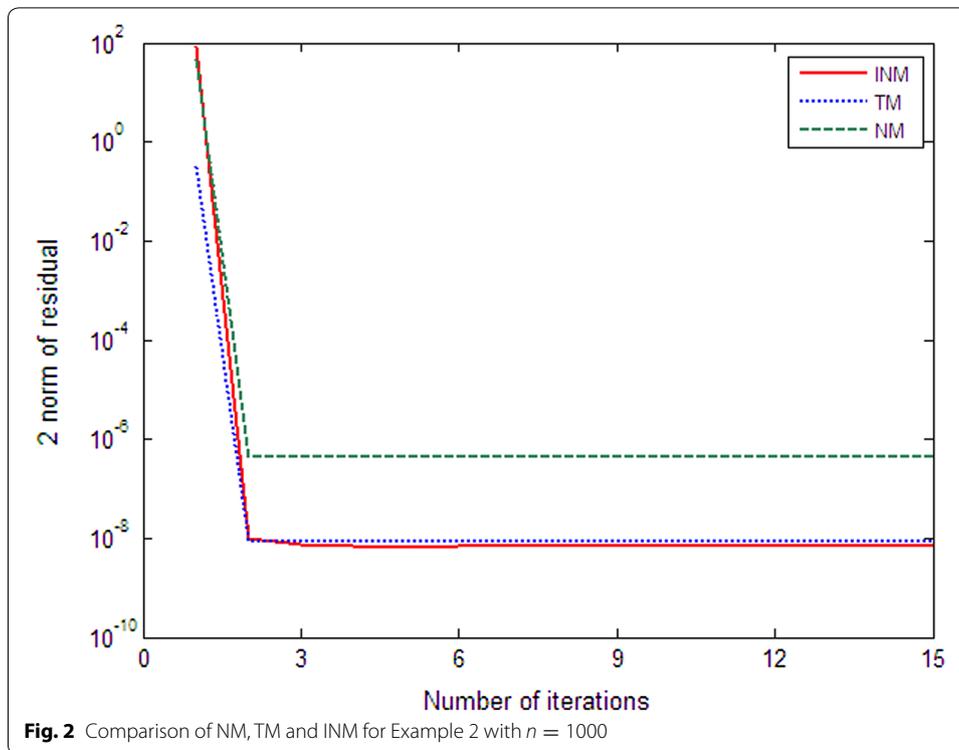$rand(\text{'state'}, 0); R = rand(n, n); A = R' * R + n * eye(n); b = (A - eye(n, n)) * ones(n, 1).$

And, the results are shown in Table 2.

In Tables 1 and 2, Dim, K, ACC and T denote the dimensions of the problem, the number of iterations, $\|Ax^k - |x^k| - b\|_2$ and times(s), respectively. It is evident from Tables 1 and 2 that the improved generalized Newton's method is very effective for solving large problems.

Table 2 The comparison of NM, TM and INM in Example 2

Dim	NM			TM			INM		
	K	ACC	T	K	ACC	T	K	ACC	T
100	3	1.7851×10^{-11}	0.0042	3	1.1360×10^{-11}	0.0333	3	1.2528×10^{-11}	0.0029
200	3	3.3603×10^{-10}	0.0146	3	8.0510×10^{-11}	0.0132	3	6.7278×10^{-11}	0.0069
300	3	3.9958×10^{-9}	0.0392	3	2.8067×10^{-10}	0.0301	3	2.6026×10^{-10}	0.0203
400	3	7.3587×10^{-9}	0.0857	3	6.8738×10^{-10}	0.0681	3	5.7847×10^{-10}	0.0444
500	3	2.1626×10^{-9}	0.1411	3	1.1259×10^{-9}	0.1085	3	1.1792×10^{-9}	0.0679
600	4	1.1403×10^{-8}	0.2356	3	2.1310×10^{-9}	0.1595	3	2.4023×10^{-9}	0.1055
700	4	4.6354×10^{-8}	0.3993	3	2.9880×10^{-9}	0.2681	3	3.6051×10^{-9}	0.1704
800	4	5.7742×10^{-8}	0.4829	4	5.1658×10^{-9}	0.3543	3	3.8475×10^{-9}	0.2239
900	5	6.4563×10^{-8}	0.5749	4	6.9541×10^{-9}	0.4665	3	7.0228×10^{-9}	0.3076
1000	5	8.6322×10^{-8}	0.7589	4	8.8558×10^{-9}	0.6615	3	7.3382×10^{-9}	0.4127

Fig. 1 Comparison of NM, TM and INM for Example 1 with $n = 1000$

Fig. 2 Comparison of NM, TM and INM for Example 2 with $n = 1000$

We give below the convergence curves of three algorithms for solving Examples 1, 2 by Figs. 1 and 2. We can see that the convergence of the INM is better than NM's and TM's.

Conclusions

In this paper, we have proposed the generalized Newton's method with special search direction for solving the NP-hard absolute value equations under certain assumptions on A. The method have some nice convergence properties and calculation results. Further work is to find more effective methods for AVEs.

Authors' contributions

Analysis is done by FJM and LSY. Algorithm design and data simulation are carried out by FJM. Manuscript is drafted by FJM with contribution from LSY. Both authors read and approved the final manuscript.

Author details

[1] School of Mathematics and Statistics, Xidian University, Xi'an 710126, China. [2] Department of Engineering Management, Shaanxi Radio and TV University, Xi'an 710119, China.

Competing interests

Both authors declare that they have no competing.

References

Caccetta L, Qu B, Zhou G-L (2011) A globally and quadratically convergent method for absolute value equations. Comput Optim Appl 48:45–58

Chung S-J (1989) NP-completeness of the linear complementarity problem. J Optim Theory Appl 60(3):393–399

Cottle RW, Dantzing G (1968) Complementary pivot theory of mathematical programming. Linear Algebra Appl 1(1):103–125

Cottle RW, Pang J-S, Stone RE (1992) The linear complementarity problem. Academic Press, New York

Cruz JYB, Ferreira OP, Prudente LF (2016) On the global convergence of the inexact semi-smooth Newton method for absolute value equation. Comput Optim Appl. doi:10.1007/s10589-016-9837-x

Haghani FK (2015) On generalized Traub's method for absolute value equations. J Optim Theory Appl 166(2):619–625

Hu S-L, Huang Z-H (2010) A note on absolute value equations. Optim Lett 4(3):417–424

Hu S-L, Huang Z-H, Zhang Q (2011) A generalized Newton method for absolute value equations associated with second order cones. J Comput Appl Math 235(5):1490–1501

Iqbal J, Iqbal A, Arif M (2015) Levenberg–Marquardt method for solving systems of absolute value equations. J Comput Appl Math 2820:134–138

Mangasarian OL (2007a) Absolute value programming. Comput Optim Appl 36(1):43–53

Mangasarian OL (2007b) Absolute value equations solution via concave minimization. Optim Lett 1(1):3–8

Mangasarian OL (2009) A generalized Newton method for absolute value equations. Optim Lett 3(1):101–108

Mangasarian OL (2013) Absolute equation solution via dual complementarity. Optim Lett 7:625–630

Mangasarian OL (2014a) Linear complementarity as absolute value equations solution. Optim Lett 8:1529–1534

Mangasarian OL (2014b) Absolute value equations solution via linear programming. J Optim Theory Appl 16:1870–1876

Mangasarian OL (2015) A hybrid algorithm for solving the absolute value equation. Optim Lett 9(7):1–6

Mangasarian OL, Meyer RR (2006) Absolute value equations. Linear Algebra Appl 419(2):359–367

Moosaei H, Ketabchi S, Jafari H (2014) Minimum norm solution of the absolute value equations via simulated annealing algorithm. Afr Mat 26:1–8

Ostrowski AM (1960) Solution of equation in Euclidean and Bananch spaces. Academic Press, New York

Qi L-Q, Sun J (1993) A nonsmooth version of Newton's method. Math Program 58:353–367

Rohn J (2004) A theorem of the alternatives for the equation $|A| + |B||x| = b$. Linear Multilinear Algebra 52(6):421–426

Rohn J, Hooshyarbakhsh V, Farhadsefat R (2014) An iterative method for solving absolute value equations and sufficient conditions unique solvability. Optim Lett 8:35–44

Salkuyeh DK (2014) The Picard-HSS iteration method for absolute value equations. Optim Lett 8(8):2191–2202

Traub JF (1964) Iterative methods for the solution of equations. Prentice-Hall, London

Wu S-L, Guo P (2015) On the unique solvability of the absolute value equation. J Optim Theory Appl 169(2):705–712

Zhang M, Huang Z-H, Li Y-F (2015) The sparsest solution to the system of absolute value equations. J Oper Res Soc China 3(1):31–51