Stijepic *Economic Structures (2017) 6:19*
DOI 10.1186/s40008-017-0078-7

Journal of Economic Structures

RESEARCH Open Access

Positivistic models of long-run labor allocation dynamics

Denis Stijepic*

*Correspondence:
denis.stijepic@fernuni-hagen.de
Fernuniversität in Hagen,
Universitätsstrasse 41,
58084 Hagen, Germany

Abstract

The three-sector framework (relating to agriculture, manufacturing, and services) is one of the major concepts for studying the long-run dynamics of the economic structure. We summarize the empirical/theoretical literature consensus by formulating 'economic laws' of long-run structural change in the three-sector framework. Based on these laws, we derive the (qualitative/geometrical) properties of the dynamic system describing the structural change and apply the standard concepts/theorems of dynamic systems analysis (e.g., omega limit sets) to derive the implications of these laws for the future (transitional and limit) structural dynamics in developed and developing countries. The advantage of this system theoretical approach is that it minimizes the dependence of the predictions on theoretical/ideological arguments, which are often criticized in economics.

Keywords: Structural change, Labor, Re-allocation, Sectors, Agriculture, Manufacturing, Services, Long run, Dynamics, Trajectory, Geometry, Simplex, Dynamic systems

JEL Classification: C61, C65, O41, O14

1 Background

A great part of long-run economic dynamics literature seeks to identify regularities (e.g., dynamic patterns) in empirical data and construct theoretical/intuitive explanations of these regularities. In general, an economic model is regarded as an explanation of an empirical regularity if the model can reproduce the observed regularity under reasonable parameter restrictions. If economic models generally predict that an observed regularity is persistent across time and across countries (under reasonable parameter settings), we state that this regularity is an economic law.[1]

Since economic laws are statements (about the properties of economic variables), we can combine different laws and use logical operations to derive their direct implications. These implications can be regarded as predictions of economic dynamics based on

[1] While there are different definitions of economic law in the literature (see, e.g., Jackson and Smith 2005; Reutlinger et al. 2015), we focus on this (working) definition in our paper, i.e., we name an empirical regularity that is supported by theoretical models' predictions an economic law.

Springer Open

relatively general and widely accepted economic laws.[2] As we will see in our paper, we can go much further: Since the laws are statements about the dynamic properties of variables, which can be translated into geometrical/topological properties of development paths, we can use the geometrical/topological concepts and theorems of mathematical dynamic systems analysis to derive predictions of dynamics on the basis of these laws.

In general, this 'positivistic approach'[3] of deriving predictions of economic dynamics on the basis of widely accepted economic laws can be interpreted as a meta-modeling approach, since it relies on the laws that are supported by different economic models (following different theoretical doctrines). Thus, in general, the predictions derived in such a way are less ideological or theoretical than the predictions of standard economic models and represent the theoretical consensus to some extent.

In our paper, we use the positivistic approach described above to analyze structural change in the three-sector framework. In particular, we collect widely accepted laws of structural change and use them to predict the transitional and limit dynamics of long-run labor re-allocation across agriculture, manufacturing, and services in developed and developing countries.[4] The application of the positivistic approach for structural change modeling seems interesting for two reasons. First, structural change is one of the most persistent long-run phenomena of economic development having characteristics that are easily identifiable and stable across countries and time. The latter aspect is one of the core characteristics of an economic law, as stated above. Second, structural change in the three-sector framework can be modeled by a continuous trajectory on a bounded subset of a plane (cf. Stijepic 2015). Such a dynamical system is easily predictable due to its topological properties (cf. Guckenheimer and Holmes 1990, p. 42f.). In particular, many concepts and theorems (e.g., the Poincaré–Bendixson theory) of dynamic systems analysis are applicable to this type of dynamical system.

Since as always in the empirical sciences, neither empirical evidence nor economic models imply unambiguously that an empirical observation is a regularity or even a law (because of, e.g., different data sources, measurement problems, and dissent between different schools of economic thought), it is debatable which of the empirical observations can be regarded as laws. Thus, we present different models based on different sets of laws, such that the readers can choose the models that correspond to their ideology. In particular, our set of models encompasses a conservative model (which is only based on the most accepted and least disputable laws) and several less conservative models (which rely on more disputable laws).

Our results differ significantly from the results of the standard structural change literature (cf. the papers listed in Sect. 4.1.3). In particular, our results cover a wider range of

---

[2] Some of our predictions are solely based on observable laws, and some are based on the additional assumption that the long-run dynamics can be modeled by using continuous or differentiable functions (implying continuous or differentiable functional forms describing the economic relations), which is characteristic for the long-run economic dynamics modeling literature (cf., e.g., the models listed in Sect. 4.1.3).

[3] For an overview of the use of the term 'positivism' in sciences and a discussion of positivism as a methodological approach, see, e.g., Jackson and Smith (2005). On positive economic modeling and the role of ('unnecessary') *theoretical* assumptions in economics, see, e.g., Friedman (1953), Archibald et al. (1963), Milgrom and Roberts (1994), p. 442, and Jensen (2017).

[4] For an overview of the structural change literature, see, e.g., Schettkat and Yocarini (2006), Krüger (2008), Silva and Teixeira (2008), Stijepic (2011, Chapter IV), and Herrendorf et al. (2014). Recent papers modeling structural change in the three-sector framework are, e.g., Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Uy et al. (2013), and Stijepic (2015, 2016).

possible structural change scenarios than the standard structural change literature does, since our predictions are less restricted by ideological assumptions.[5] For a summary of our models' forecasts, see Sect. 5.

The rest of the paper is set up as follows. In the next section, we discuss briefly the mathematical prerequisites and, in particular, the geometrical aspects of structural change analysis. In Sect. 3, we summarize the empirical regularities of structural change. In Sect. 4, we formulate different models (i.e., sets of laws) on the basis of the regularities postulated in Sect. 3, study their transitional and limit dynamics, and use these results to predict structural change in developed and developing countries. Concluding remarks are provided in Sect. 5.

2 Mathematical and conceptual prerequisites

In this section, we define the terms 'labor allocation' and 'structural change' and discuss their geometrical interpretation via simplexes and trajectories. Furthermore, we discuss the characterization of structural change by referring to the geometrical properties of trajectories and different horizons of analysis (limit vs. transitional dynamics).

While there are different mathematical notational conventions, we choose the following notation for reasons of simplicity: small letters (e.g., x), bold small letters (e.g., $\mathbf{x}$), capital letters (e.g., X), and Greek letters (e.g., α) denote scalars, vectors/points, sets, and angles, respectively. A dot indicates a derivative with respect to time (e.g., $\dot{x}$ is the derivative of x with respect to time).

2.1 Mathematical definition of labor allocation and structural change

As noted in Sect. 1, we study structural change in the three-sector framework, which is a widespread choice for analyzing structural change empirically and theoretically (cf. footnote 4). The three-sector framework refers to three sectors: primary or agricultural sector (which we name sector 1), secondary or manufacturing sector (which we name sector 2), and tertiary or services sector (which we name sector 3). Let $y_{1c}(t)$, $y_{2c}(t)$, and $y_{3c}(t)$ denote the employment in sector 1, 2, and 3 at time t in country c, respectively. Furthermore, let $y_c(t) := y_{1c}(t) + y_{2c}(t) + y_{3c}(t)$ denote the aggregate employment at time t in country c. The *employment share of sector i at time t in country c* is defined as follows: $x_{ic}(t) := y_{ic}(t)/y_c(t)$ for $i = 1, 2, 3$, for all t, and for all c. Since employment cannot be negative and $y_c(t) := y_{1c}(t) + y_{2c}(t) + y_{3c}(t)$ for all t and for all c, the following statements are true:

$$\forall i \in \{1, 2, 3\}\, \forall t\, \forall c \quad 0 \leq x_{ic}(t) \leq 1 \tag{1}$$

$$\forall t\, \forall c \quad x_{1c}(t) + x_{2c}(t) + x_{3c}(t) = 1 \tag{2}$$

According to these definitions, the vector $\mathbf{x}_c(t)$, which is defined by (3), represents the *labor allocation* across agriculture, manufacturing, and services at time t in country c.

$$\forall t\, \forall c \quad \mathbf{x}_c(t) := (x_{1c}(t), x_{2c}(t), x_{3c}(t)) \tag{3}$$

[5] In general, standard structural change models (e.g., the papers listed in Sect. 4.1.3) predict that in the long run, the labor allocation converges to a steady state allocation dominated by the services sector (cf. Stijepic 2015).

The term 'structural change' refers to the long-run changes in the labor allocation $\mathbf{x}_c(t)$. Thus, according to our definition of the term labor allocation, '*structural change in country c*' means that at least some of the employment shares $x_{1c}(t)$, $x_{2c}(t)$, and $x_{3c}(t)$ are not constant in the long run in country *c*. For example, $x_{1c}(t)$ may grow over time, $x_{2c}(t)$ may decrease over time, and $x_{3c}(t)$ may be constant over time in country *c*.

Definition 1 summarizes this discussion, where we do not implement mathematically the fact that structural change refers to the long run, since a mathematical formulation of the notion of the long run is not necessary for deriving our results; by omitting such a formulation, the mathematical expressions are significantly abbreviated. Of course, when necessary, we will emphasize that our statements refer to the long-run dynamics.

Definition 1 The term 'structural change (over the period $[a, b]$) in country *c*' refers to the long-run dynamics of the labor allocation $\mathbf{x}_c(t)$ (over the period $[a, b]$).

Simply speaking, Definition 1 states that structural change takes place in country *c* if $\mathbf{x}_c(t)$ is not constant in the long run.

2.2 Geometrical interpretation of labor allocation and structural change

In this section, we recapitulate some geometrical concepts for structural change analysis as discussed by Stijepic (2015, 2016).

Consider the Cartesian coordinate system (x_1, x_2, x_3). We can identify any point in the three-dimensional real space $(R \times R \times R)$ with its Cartesian coordinates (x_1, x_2, x_3). Furthermore, let us define the following set of points (in the Cartesian coordinate system):

$$S := \{\mathbf{x} \equiv (x_1, x_2, x_3) \in R \times R \times R : \, x_1 + x_2 + x_3 = 1 \wedge \forall i \in \{1, 2, 3\} \; 0 \le x_i \le 1\} \tag{4}$$

It is well known that (a) *S* is a two-dimensional standard simplex (henceforth, 2-simplex), (b) the 2-simplex is a triangle, and (c) the Cartesian coordinates of its vertices are:

$$(1, 0, 0) =: \mathbf{v}_1 \quad (0, 1, 0) =: \mathbf{v}_2 \quad (0, 0, 1) =: \mathbf{v}_3 \tag{5}$$

For an illustration, see Figs. 1 and 2, where we omit the coordinate axes in Fig. 2.

According to (4), all the points (x_1, x_2, x_3) that satisfy the conditions $x_1 + x_2 + x_3 = 1$ and $\forall i \in \{1, 2, 3\}\; 0 \le x_i \le 1$ are located on the 2-simplex *S*, i.e., on the triangle depicted in Figs. 1 and 2. These facts and our definitions of labor allocation and structural change (cf. Sect. 2.1) imply that we can depict the labor allocation $\mathbf{x}_c(t)$ and its dynamics (i.e., structural change) on the 2-simplex [cf. (1)–(4)], as explained in the following.

In Sect. 2.1, we have implicitly assumed that the labor allocation in country *c* ($\mathbf{x}_c(t)$) is a function of time [cf. (3)]. Now, we make this assumption explicit by stating that

$$\mathbf{x}_c(t) : D \times C \to S \tag{6a}$$

$$\mathbf{x}_c(t) : (t, c) \mapsto (x_1, x_2, x_3) \tag{6b}$$

$$t \in D \subseteq R \wedge c \in C \subset N \tag{6c}$$

where *D* is a time interval, i.e., a connected subset of real numbers (*R*), and *C* is the set of countries, which are indexed by natural numbers (*N*). (6) states that the function $\mathbf{x}_c(t)$

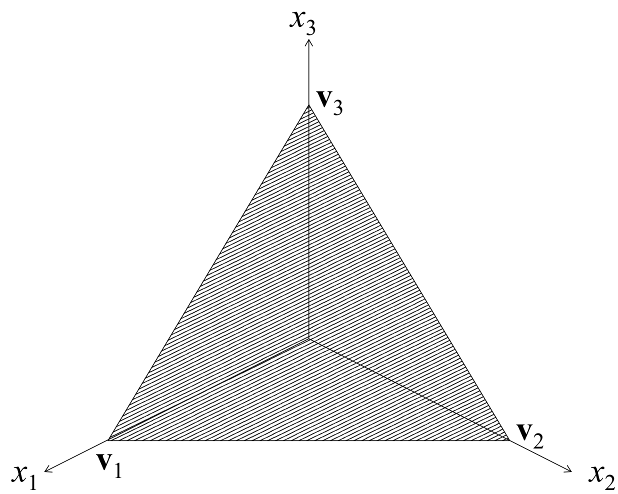

Fig. 1 2-simplex in the Cartesian coordinate system (x_1, x_2, x_3)

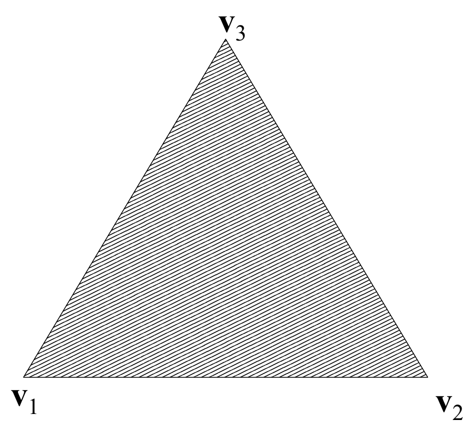

Fig. 2 2-simplex (without coordinate axes)

maps time t and the country index c to the 2-simplex. In particular, for a given $c \in C$, the function $\mathbf{x}_c(t)$ assigns a point on the 2-simplex S, which is located in the coordinate system (x_1, x_2, x_3), to each time point $t \in D$. Note that due to (1)–(4), we know that the function $\mathbf{x}_c(t)$ has values in the set S and not elsewhere in $R \times R \times R$.

This discussion and Sect. 2.1 imply the following geometrical interpretation of the term 'labor allocation.' The labor allocation in the three-sector framework ($\mathbf{x}_c(t)$) can be represented by a point on the 2-simplex. This 2-simplex contains all the points that satisfy the definition of the term labor allocation [cf. (1)–(3)]. Two different points on the 2-simplex represent two different labor allocations. Thus, if, e.g., $\mathbf{x}_c(1) \neq \mathbf{x}_c(2)$ [cf. (3)], where $\mathbf{x}_c(1), \mathbf{x}_c(2) \in S$, then in country c, the labor allocation at $t = 2$ is not the same as the labor allocation at $t = 1$, i.e., structural change took place over the time period (1, 2) in country c (cf. Definition 1).

Overall, per Definition 1, we can derive all the information about structural change by studying the properties of the labor allocation function $\mathbf{x}_c(t)$, which is defined in Sect. 2.1 and by (6). For the most part, we focus on the geometrical properties of the image of the labor allocation function, which can be analyzed by using the concept of the (image of a) trajectory (T_c), which we define as follows:

$$\forall c \in C \quad T_c(G) := \{\mathbf{x}_c(t) \in S : t \in G\}, \quad \text{where } G \subseteq D \tag{7}$$

In fact, $T_c(G)$ is the (image of the) trajectory describing the dynamics of country $c \in C$ over the interval $G \subseteq D$. In other words, $T_c(G)$ is simply the set of states (or: labor allocations) that the economy experiences (or: realizes) over the time period G. Geometrically speaking, economy c moves along $T_c(G)$ over the time period G. Note that (7) implies that the labor allocation trajectory $T_c(G)$ is always located on the 2-simplex S, i.e., S is the *domain* of $T_c(G)$.

Figure 3 depicts an example of a trajectory given by (6) and (7), where we assume that $\mathbf{x}_c(t)$ is continuous in t.

Note that the arrow in Fig. 3 indicates the direction of the movement along the trajectory. Let $\mathbf{x}_c(a) \equiv (x_{1c}(a), x_{2c}(a), x_{3c}(a))$ denote the initial point and $\mathbf{x}_c(b) \equiv (x_{1c}(b), x_{2c}(b), x_{3c}(b))$ be the end-point of the trajectory depicted Fig. 3. Obviously, Fig. 3 shows that these points differ. Thus, the trajectory in Fig. 3 depicts structural change (cf. Definition 1). In more detail, by recalling the position of the 2-simplex in the Cartesian coordinate system (x_1, x_2, x_3) (cf. Fig. 1), we can see that the trajectory in Fig. 3 implies that $x_{1c}(a) > x_{1c}(b)$, $x_{2c}(a) < x_{2c}(b)$, and $x_{3c}(a) < x_{3c}(b)$. That is, x_{1c} decreased and x_{2c} and x_{3c} increased over the time period $[a, b]$.

2.3 Geometrical characterization of trajectories

Trajectories can be characterized by using the concepts of closeness (to the vertices of the simplex), continuity, monotonicity, and self-intersection. In Sects. 3 and 4, we use these concepts to characterize the empirically observable trajectories and to formulate economic laws and models based on evidence.

The intuitive/geometrical notion of continuity and self-intersection is more or less obvious. For a *continuous* and *non-self-intersecting* trajectory, see, e.g., Fig. 3; in contrast,

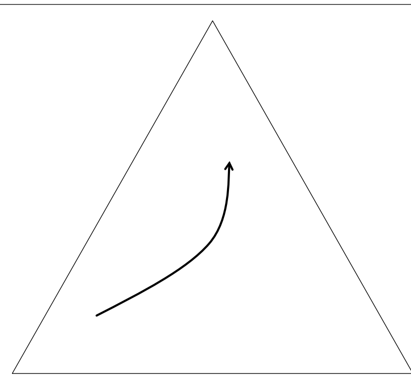

Fig. 3 Example of a (continuous) trajectory on the 2-simplex

Figs. 4 and 5 depict examples of non-continuous and self-intersecting trajectories, respectively.

In our paper, we apply the following formal definitions of continuity and non-self-intersection (cf. Stijepic 2016).

Definition 2 The trajectory (7) is continuous on S (for a given $c \in C$) if the corresponding function $\mathbf{x}_c(t)$ [cf. (6)] is continuous (in t) on the interval G (for the given c).

Definition 3 The (continuous) trajectory (7) is non-self-intersecting (for a given $c \in C$) if $\nexists (t_1, t_2, t_3) \in G \times G \times G: \ t_1 < t_2 < t_3 \wedge \mathbf{x}_c(t_1) = \mathbf{x}_c(t_3) \neq \mathbf{x}_c(t_2)$.

Note that per Definition 3, a self-intersection requires that the economy leaves the point $\mathbf{x}_c(t_1)$ at least for some instant of time (t_2) before it returns to it (at t_3). Thus, per Definition 3, a self-intersection does not occur if the economy reaches some point on S (in finite time) and stays there forever.

Later, we will need some notion of closeness to the vertices of the 2-simplex. We use the following definition.

Definition 4 A point $\mathbf{x}_c(t) \equiv (x_{1c}(t), x_{2c}(t), x_{3c}(t)) \in S$ is close to the vertex $\mathbf{v}_i$ [cf. (5)] if and only if $x_{ic}(t) > 0.5$, where $i \in \{1, 2, 3\}$, $c \in C$, and $t \in D$.

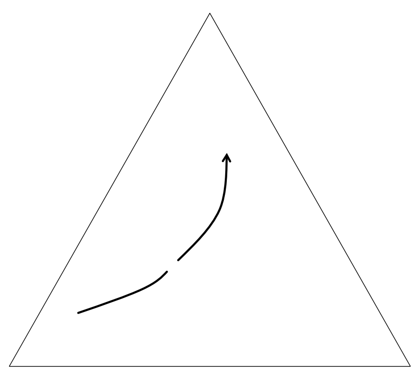

Fig. 4 Example of a non-continuous trajectory on the 2-simplex

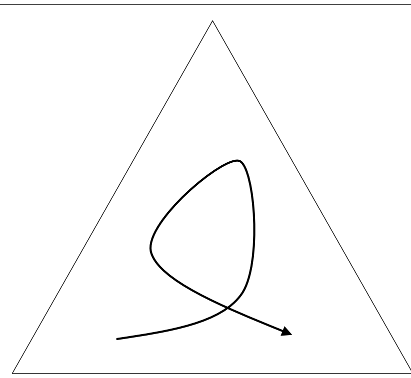

Fig. 5 Example of a self-intersecting (and continuous) trajectory on the 2-simplex

Note that (4) and Definition 4 imply that a point can be close to only one of the three vertices of the 2-simplex. That is, a point can never be close to two or more vertices at the same time. A geometrical interpretation of Definition 4 is given by the following partitioning of the simplex S (cf. Fig. 6):

$$\forall i \in \{1,2,3\} \quad S_i := \{(x_1, x_2, x_3) \in S : x_i > 0.5\} \tag{8a}$$

$$S_0 := S \backslash (S_1 \cup S_2 \cup S_3) \tag{8b}$$

Definition 4 and (8) imply the following statements: A point is close to the vertex $\mathbf{v}_1$ if and only if it is located in the partition S_1; a point is close to the vertex $\mathbf{v}_2$ ($\mathbf{v}_3$) if and only if it is located in the partition S_2 (S_3); a point is not close to any of the vertices if and only if it is located in the partition S_0 (cf. Fig. 6).

To economically interpret the notion of closeness given by Definition 4, recall that x_1, x_2, and x_3 stand for the employment shares of agriculture, manufacturing, and services, respectively [cf. Sect. 2.1 and (6)]. Thus, according to Definition 4, if the labor allocation in country c is represented by a point close to the vertex $\mathbf{v}_i$, sector i employs more than 50% of the country c labor force, i.e., country c is dominated by sector i. For example, if the labor allocation at time t in country c is represented by a point ($\mathbf{x}_c(t)$) close to the vertex $\mathbf{v}_3$, country c is dominated by services at time t, i.e., $x_{3c}(t) > 0.5 > x_{2c}(t) + x_{1c}(t)$ [cf. (1)–(3)].

Definition 5 Let the function (6) be differentiable with respect to time for $t \in G \subseteq D$ and $c \in C$, and let $\mathbf{r}$(t) be the tangential vector associated with the (regular) point $\mathbf{x}_c(t) \equiv (x_{1c}(t), x_{2c}(t), x_{3c}(t)) \in T_c(G) \subset S$, where $\dot{\mathbf{x}}_c(t) \neq 0$, $t \in G \subseteq D$, and $c \in C$ [cf. (7)]. The vector angle $\alpha(t)$ is the angle between $\mathbf{r}(t)$ and the simplex edge $\mathbf{v}_1\mathbf{v}_2$ [cf. (5) and Fig. 7], i.e., $\alpha(t) := \angle(\mathbf{r}(t), \overline{\mathbf{v}_1\mathbf{v}_2})$.

We can use Definition 5 to geometrically interpret *monotonous* dynamics of sectors, as shown in the following three properties of the 2-simplex.

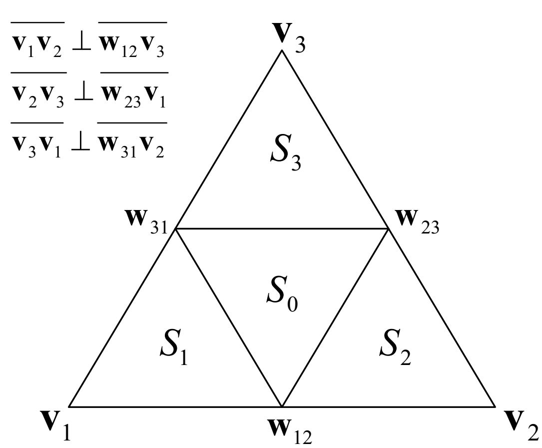

Fig. 6 Partitioning of the 2-simplex according to Definition 4

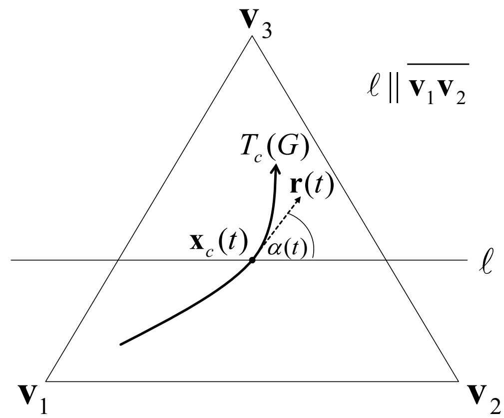

Fig. 7 Vector angle *α*

Property 1 (cf. Definition 5) (*a*) $\dot{x}_{1c}(t) > 0 \Leftrightarrow 120° < \alpha(t) < 300°$. (*b*) $\dot{x}_{1c}(t) < 0 \Leftrightarrow 0° < \alpha(t) < 120° \vee 300° < \alpha(t) < 360°$. (*c*) $\dot{x}_{1c}(t) = 0 \Leftrightarrow \alpha(t) \in \{120°, 300°\}$.

Property 1 becomes evident when studying the 2-simplex in the Cartesian coordinate system (cf. Fig. 1). For example, according to Property 1, the employment share of the agricultural sector decreases monotonously along the trajectory (7) if each of the tangential vectors associated with the trajectory (7) is characterized by a vector angle between 0° and 120° or between 300° and 360°. Thus, the employment share of the agricultural sector decreases strictly monotonously along the trajectory depicted in Fig. 3. The following Property 2 and Property 3 are analogous to Property 1. For a detailed discussion of the economic interpretation of the tangential vector angles associated with labor allocation trajectories, see Stijepic (2015).

Property 2 (cf. Definition 5) (*a*) $\dot{x}_{2c}(t) > 0 \Leftrightarrow 0° < \alpha(t) < 60° \vee 240° < \alpha(t) < 360°$. (*b*) $\dot{x}_{2c}(t) < 0 \Leftrightarrow 60° < \alpha(t) < 240°$. (*c*) $\dot{x}_{2c}(t) = 0 \Leftrightarrow \alpha(t) \in \{60°, 240°\}$.

For example, according to Property 2, the employment share of the manufacturing sector is constant along the trajectory (7) if each of the tangential vectors associated with the trajectory (7) is characterized by a vector angle of 60° or 240°. Thus, the employment share of the manufacturing sector is constant along the trajectory that is linear and parallel to the $\mathbf{v}_3\mathbf{v}_1$-edge of the 2-simplex (cf. Fig. 1).

Property 3 (cf. Definition 5) (*a*) $\dot{x}_{3c}(t) > 0 \Leftrightarrow 0° < \alpha(t) < 180°$. (*b*) $\dot{x}_{3c}(t) < 0 \Leftrightarrow 180° < \alpha(t) < 360°$. (*c*) $\dot{x}_{3c}(t) = 0 \Leftrightarrow \alpha(t) \in \{0°, 180°\}$.

For example, according to Property 3, the employment share of the services sector increases monotonously along the trajectory (7) if each of the tangential vectors associated with the trajectory (7) is characterized by a vector angle between 0° and 180°. Thus, the employment share of services increases along the trajectory depicted in Fig. 3.

2.4 Horizons of analysis

When characterizing the labor allocation dynamics of a country, we distinguish between limit dynamics (and set of attraction) and transitional dynamics (and range of fluctuation). In this section, we recapitulate these notions briefly, since they are an integral part of our argumentation in Sect. 4.2.

The term '*limit dynamics*' refers to the dynamics for $t \to \infty$. A standard concept for studying and describing the limit dynamics is the 'omega limit set.'

Definition 6 Let the function $\mathbf{x}_c(t)$ satisfy the conditions (1)–(3), (6), and $D \supseteq [0, \infty)$. The point $\mathbf{x}_c^*$ is an omega limit point of the trajectory $T_c([0, \infty))$ [cf. (7)] if there exists a sequence of time points t_k (where $k = 0, 1, 2, \ldots$) that satisfies two conditions: (a) t_k converges to infinity (i.e., $t_k \to \infty$ for $k \to \infty$), and (b) the corresponding sequence $\mathbf{x}_c(t_k)$ converges to $\mathbf{x}_c^*$ (i.e., $\mathbf{x}_c(t_k) \to \mathbf{x}_c^*$ as $t_k \to \infty$). The omega limit set $O(T_c([0, \infty)))$ of the trajectory $T_c([0, \infty))$ is the union of all the omega limit points of the trajectory $T_c([0, \infty))$.

For a discussion and explanation of the omega limit set, see, e.g., Andronov et al. (1987, p. 353f.), Walter (1998, p. 322), and Hale (2009, p. 46f.). The (type of the) limit dynamics of an economy that moves along the trajectory T_c is indicated by the omega limit set of the trajectory T_c. Intuitively speaking, in the cases discussed by us, the omega limit set $O(T_c([0, \infty)))$ is the set to which the labor allocation in country c ($\mathbf{x}_c(t)$) converges along the trajectory $T_c([0, \infty))$ for $t \to \infty$. The omega limit set may consist of only one point, i.e., a fixed point ($\mathbf{x}_c^*$); in this case, the labor allocation in economy c converges along the trajectory T_c to the fixed point $\mathbf{x}_c^*$ (i.e., the labor allocation converges to a 'steady state' labor allocation) for $t \to \infty$; the proof of existence of such steady states in long-run labor allocation models is interesting, since structural change is transitory if the labor allocation converges to a steady state. Moreover, the omega limit set may be more complex; e.g., it may consist of (the image of) a Jordan curve, such that the labor allocation converges to a limit cycle, i.e., the labor allocation dynamics are cyclical for $t \to \infty$. Overall, the concept of the omega limit set allows us to describe the *type* of the labor allocation dynamics (or: their dynamic pattern) as time goes to infinity.

In general, a model (and, in particular, each of the six models discussed in Sect. 4.2) generates different trajectories depending on the initial conditions and the model parameters. We define *a model's* '*set of attraction*' as the union of the omega limit sets of all the trajectories generated by the model. That is, the set of attraction refers to all the possible 'end-states' predicted by a model (i.e., the states to which the economy may converge for $t \to \infty$ according to the model). As demonstrated in Sect. 4.2, the size of the set of attraction allows us to estimate the (potential) *strength* of structural change as time goes to infinity.

While the concepts of limit dynamics and set of attraction refer to the dynamics for $t \to \infty$, *transitional dynamics* refers to the dynamics over the period $[0, \infty)$; i.e., this concept does not refer only to the limit ($\lim_{t \to \infty}$), but to the time 'before the limit.' In general, the transitional dynamics can be characterized by the shape of the trajectory, as we will see in Sect. 4.2.

To express the strength of transitional dynamics (but also the strength of limit dynamics), we use the concept of *range of fluctuation*.

Definition 7 Assume that Model w generates different functions $x_{ic}^{j}(t)$ (indicating the employment share of sector i at time t in country c) on the interval $G \subseteq D$, which are indexed by $j \in J$ and satisfy (1) and (2). Let $K_i^w(G) := \bigcup_{j \in J} \bigcup_{t \in G} x_{ic}^{j}(t) \subseteq [0, 1]$ denote the set of all values $x_{ic}^{j}(t)$ generated by Model w over the period G [cf. (1)]. The potential range of fluctuation of the employment share of sector i over the period G in Model w is defined as $M_i^w(G) := [\min(\mathrm{cl}(K_i^w(G))), \max(\mathrm{cl}(K_i^w(G)))]$ where $\mathrm{cl}(K_i^w(G))$ denotes the closure of the set $K_i^w(G)$. The potential strength of fluctuation of the employment share of sector i in Model w is defined as the length $|M_i^w(G)| := \max(\mathrm{cl}(K_i^w(G))) - \min(\mathrm{cl}(K_i^w(G)))$ of the interval $M_i^w(G)$.

Although Definition 7 refers to 'fluctuation,' $|M_i^w(G)|$ is also defined for non-cyclical and, in particular, monotonous functions $x_{ic}(t)$. If $|M_i^w(G)|$ is small, the strength of fluctuation (or the strength of monotonous dynamics) of sector i's employment share over the period G cannot be great, where $i \in \{1, 2, 3\}$. Obviously, $|M_i^w(G)|$ is a relatively crude index: It represents the upper limit of the strength of structural change (with respect to sector i) in Model w over the period G. Nevertheless, it proves useful in Sect. 4.2, since it allows us to compare different countries and different models based on qualitative empirical information.

Both, the limit dynamics and the transitional dynamics, are important, since a priori it cannot be decided whether an economy is close to its dynamic equilibrium (and, thus, limit dynamics are prevalent) or not (and, thus, transitional dynamics are prevalent).

3 Empirical regularities of structural change

In this paper, we focus on the following empirical regularities (or stylized facts) of structural change.

Regularity 1 *In the early phases of development, the agricultural employment share* (x_1) *is greater than* 0.5.

Regularity 2 *In the later phases of development, the employment share of services* (x_3) *becomes greater than* 0.5.

Regularity 3 *The employment share of agriculture decreases monotonously in the long run.*

Regularity 4 *The employment share of services increases monotonously in the long run.*

Regularity 5 *The employment share of manufacturing increases in the early phases of development* ('*industrialization phases*') *and decreases in the later phases of development* ('*de-industrialization phases*').

Regularity 6 (Stijepic 2016) (*a*) *The long-run dynamics of labor allocation can be represented by non-self-intersecting trajectories.* (*b*) *The empirically observable self-intersections of labor allocation trajectories are of short-run nature, i.e., there are no long-run trajectory loops* (*covering long periods of time*).

Regularities 1–4 are well known; evidence on them can be found in much of the previous literature, e.g., Maddison (1995), Kongsamut et al. (2001), Herrendorf et al. (2014), and Stijepic (2016). Recent structural change contributions have focused on *Regularity 5* (see Sect. 4.1.3 for references); corresponding empirical evidence is provided by, e.g., Maddison (1995), Herrendorf et al. (2014), and Stijepic (2016). Evidence on Regularity 6 is presented and discussed by Stijepic (2016). For a brief overview of the theoretical contributions seeking to model and explain Regularities 1–6, see Sect. 4.1.3. Moreover, we discuss the Regularities 1–6 in Sect. 4.1.2.

4 Positivistic models based on the empirical regularities

In this section, we formulate models based on Regularities 1–6. While Regularities 1 and 2 may be regarded as 'robust,' Regularities 3–6 may be regarded as controversial to some extent, since not all countries are characterized by them over all periods of time (see the references listed in Sect. 3 for empirical evidence). In general, different readers may find different regularities controversial. Thus, it makes sense to generate different models based on different regularities. Model 1, which is our most conservative model, relies only on Regularities 1 and 2, i.e., the regularities that are the least controversial. Models 2–4 combine the remaining regularities in different ways such that the readers can choose the models that correspond to their ideology.

First, we discuss the axioms that represent the assumptions that are not empirically founded in our modeling framework (cf. Sect. 4.1.1). Then, we formulate Laws 1–6 on the basis of Regularities 1–6 (cf. Sect. 4.1.2) and show that Laws 1–6 are theoretically founded (cf. Sect. 4.1.3). Finally, in Sect. 4.2, we formulate the models and discuss their predictions.

4.1 Basic axioms and laws

In this section, we formulate the axioms and laws that we use in Sect. 4.2 to define structural change models, where each model uses a different set of axioms and laws. Furthermore, we provide references on the theoretical foundations of the laws.

4.1.1 Axioms

First, we formulate *axioms* by using the concepts introduced in Sect. 2.1. In our paper, the axioms represent all the assumptions that are not empirically founded. Although we try to minimize the use of such assumptions, our models, like all other models of empirical sciences, cannot be formulated without using a minimum of not empirically founded assumptions.

Axiom 1 *The long-run labor allocation dynamics of country* $c \in C$ *over the period* $t \in D$ *are described by the function* $\mathbf{x}_c(t) \equiv (x_{1c}(t), x_{2c}(t), x_{3c}(t))$ *as defined by* (1)–(3) *and* (6). $x_{1c}(t)$, $x_{2c}(t)$, *and* $x_{3c}(t)$ *represent the employment share of agriculture, manufacturing, and services at time t in country c, respectively.* $D \equiv (d, d^{+}) \supset [0, \infty)$ *is the time interval to which the model applies. C is the set of countries to which the model applies.*

Axiom 1 is a standard for modeling structural change in the three-sector framework (see Stijepic (2015) for a discussion and the papers listed in Sect. 4.1.3 for some examples of models based on Axiom 1). In Sect. 4.2, we use Axiom 1 in all our models, while the following two axioms are only used in some of our models. Without loss of generality, Axiom 1 extends the modeling horizon d^+ to infinity such that we can exploit the maximum prediction range of our models. However, most of the following discussion does not require this extension.

Axiom 2 $\forall t \in D\ \forall c \in C$, $\mathbf{x}_c(t)$ *is continuous in t.*

The continuity axiom (i.e., Axiom 2) is a typical (long-run) modeling convention in development and growth theory. The models presented by Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Foellmi and Zweimüller (2008), and Boppart (2014) are typical examples of multi sector models satisfying Axiom 2.

Axiom 3 $\forall t \in D\ \forall c \in C$, $\mathbf{x}_c(t)$ *is differentiable with respect to t.*

Axiom 3 is not necessary for formulating our assumptions, describing the empirical evidence, deriving the predictions in Sect. 4.2, or any of our other results. That is, we could write an alternative version of our paper that does *not* rely on differentiable functions or the notion of the derivative and generates the same results. However, the use of derivatives abbreviates, among others, the formulation of Properties 1–3, Definition 5, as well as Laws 3 and 4, significantly. Therefore, we rely on Axiom 3.

4.1.2 Laws

In this section, we translate the verbal statements of Regularities 1–6 into mathematical statements, which we name laws and, in particular, Laws 1–6. As we will see in Sect. 4.1.3, Laws 1–6 are supported by the theoretical literature. Thus, their naming ('laws') is consistent with the definition of the term 'law' discussed in Sect. 1.

In general, a law is defined as a regularity that is valid across *time* and *space*.[6] The fact that laws are valid across *time* (cf. *D*) means, among others, that they are valid in future to some extent; thus, we can use them for prediction; as we will see later, there are different ways to extend laws across time (i.e., across the time interval (*D*) to which our models refer). '*Space*' refers here to countries, where we can distinguish between general laws (i.e., laws that are valid across all countries) and ceteris paribus laws, which are valid only for some countries (see Stijepic 2016, p. 20ff. for a discussion). This distinction is, however, not important in our paper, since our mathematical/logical derivations are the same irrespective of the type of law to which they refer (general vs. ceteris paribus law). Therefore, we assume, henceforth, that the laws are valid for the country set *C*. This set may represent all countries of the world or only a subset of them. The readers may decide whether they consider the laws discussed in our paper as general laws or as ceteris paribus laws and, thus, whether our results/predictions are valid for all countries or only for some subset of countries. Overall, Laws 1–6 are statements that are valid across time (*D*) and space (*C*). We let the readers decide to which countries ('space') the laws apply and focus now on the discussion of the period *D*.

[6] For a discussion of laws in economics and natural sciences, see, e.g., Jackson and Smith (2005) and Reutlinger et al. (2015). Stijepic (2016, p. 20ff.) discusses the application of the term 'law' in structural change modeling.

In the following, we rely on Axiom 1 (and Axioms 2 and 3, in part) and the concepts of elementary calculus and set theory to translate the verbal statements of Regularities 1–6 into Laws 1–6. Recall that Axiom 1 refers to the long run; thus, all the statements of Laws 1–6 are statements about the long-run dynamics. As we will see, while Laws 1 and 2 describe the state of a country $c \in C$ (at the time points a_c and b_c), Laws 3–6 describe the (transitional) dynamics of the country $c \in C$. The notation used in Laws 1–6 and the verbal statements of Regularities 1–6 jointly imply the following interpretation of the time points a_c, b_c, and z_c: 'a_c' is an 'early point in development' of country $c \in C$; 'b_c' is a 'later point in development' of country $c \in C$; and 'z_c' is the turning point in manufacturing sector dynamics (from the industrialization period to the de-industrialization period) of country $c \in C$. We start with the translation of Regularity 1 into Law 1.

Law 1 (*cf. Regularity 1 and Axiom 1*) $\forall c \in C\, \exists a_c \in D : \forall t \in (d, a_c]\; x_{1c}(t) > 0.5$.

Law 1 states that each country belonging to the group C is an agricultural economy (i.e., is characterized by $x_1 > 0.5$) over the period of time $(d, a_c]$. As we can see, Law 1 extends to the lower limit (d) of the time period considered (D) (cf. Axiom 1). Thus, Law 1 states that primitive economies are agricultural economies. This fact may also be relevant for long-run predictions where the backward extension of the trajectory, i.e., $\{\mathbf{x}_c(t) \in S: d \leq t < a_c\}$, is relevant (cf. Stijepic (2015, p. 81)).

Note that the period $(d, a_c]$ represents (a part of) the 'early development phase' (cf. Regularity 1 and Sect. 4.2.1) of country c, where a_c is indexed by $c \in C$. In other words, Law 1 allows for differences in the duration of the phase $(d, a_c]$ across countries $c \in C$. This makes sense, since different countries overcome the early development phase at different points of time (see the references listed in Sect. 3 for empirical evidence).

Law 1 is formulated by using the expression $\forall c \in C$. This reflects our discussion of the fact that (our) laws are valid for a group of countries (C) and not only for one country (c). We adhere to this view when formulating Laws 2–6.

Law 2 (*cf. Regularity 2 and Axiom 1*) (*a*) $\forall c \in C\, \exists b_c \in D : x_{3c}(b_c) > 0.5 \wedge b_c > a_c$. (*b*) $\forall c \in C\, \exists b_c \in D : \forall t \in [b_c, d^+)\; x_{3c}(t) > 0.5 \wedge b_c > a_c$.

Law 2 states that each country c belonging to the country group C is a services economy (i.e., is characterized by $x_3 > 0.5$) at the time point $b_c > a_c$. Thus, b_c represents a point in the later phases of development of country c (cf. Regularity 2).

In Law 2, we do not explicitly define the point a_c, since we use Law 2 only in conjunction with Law 1 when modeling structural change in Sect. 4.2 (and Law 1 has already defined the point a_c). Laws 1 and 2 jointly state that each country from the country group C is, first, an agricultural economy (at time a_c) and, later, a services economy (at time b_c).

Note that Law 2 allows that the point b_c differs across countries $c \in C$, since b_c is indexed by c. This is consistent with the empirical evidence, which shows that some countries reach the status of a services economy earlier than others (see the references listed in Sect. 3 for empirical evidence).

The difference between Laws 2a and 2b is simple: Law 2b states that country c is a services economy at time b_c and continues to be a services economy for the rest of the time period D; in contrast, Law 2a does not state what happens after the time point b_c (i.e., economy c may be a services economy or not for $t > b_c$). This fact is of importance for the predictions of the limit dynamics of labor allocation, as we will see in Sect. 4.2.

Law 3 (*cf. Regularity 3 and Axioms 1–3*) $\forall c \in C\, \exists p_c \in D : \forall t \in [p_c, d^+)\; \dot{x}_{1c}(t) \leq 0$.

Law 3 states that in each country $c \in C$, there exists a period $[p_c, d^+)$ of monotonously decreasing agricultural share ($\dot{x}_{1c} \leq 0$), where d^+ is defined by Axiom 1. In particular, Law 3 extends to the upper limit (d^+) of the time period considered (D) and over all countries belonging to the group C. Moreover, Law 3 allows for cross-country differences in the starting point of the period $[p_c, d^+)$, since p_c is indexed by the country index $c \in C$.

Law 4 (*cf. Regularity 4 and Axioms 1–3*) $\forall c \in C\, \exists q_c \in D : \forall t \in [q_c, d^+)\; \dot{x}_{3c}(t) \geq 0$.

The discussion of Law 4 is analogous to the discussion of Law 3. Law 4 states that each of the countries belonging to the group C is characterized by a monotonously growing services share over the period $[q_c, d^+)$. The starting point of the period $[q_c, d^+)$ may differ across countries.

Law 5 (*cf. Regularity 5 and Axioms 1–3*) $\forall c \in C\, \exists z_c \in (a_c, b_c) : (\forall t \in [a_c, z_c)\; \dot{x}_{2c}(t) \geq 0))$ $\wedge (\forall t \in (z_c, b_c]\; \dot{x}_{2c}(t) \leq 0)$.

Law 5 states that the dynamics of each country c are characterized by a 'turning point' z_c, where $c \in C$. Per Law 5, this 'turning point' partitions the period $[a_c, b_c]$ into a phase of monotonously increasing manufacturing share and a phase of monotonously decreasing manufacturing share. z_c may differ across countries $c \in C$, since z_c is indexed by the country index c. Law 5 refers to the points a_c and b_c without defining them explicitly, since we use Law 5 only in conjunction with Laws 1 and 2, which define the points a_c and b_c.

Law 6 (*cf. Regularity 6 and Axiom 1*) $\forall c \in C\, \nexists (t_{1c}, t_{2c}, t_{3c}) \in D \times D \times D : t_{1c} < t_{2c}$ $< t_{3c} \wedge \mathbf{x}_c(t_{1c}) = \mathbf{x}_c(t_{3c}) \neq \mathbf{x}_c(t_{2c})$.

Law 6 refers to the time period D and states that in this period, each country $c \in C$ does not have a point of self-intersection (cf. Definition 3). Law 6 extends over all the countries belonging to the group C and over the whole period D.

4.1.3 The theoretical foundations of Laws 1–6

The theoretical foundations of Laws 1–4 and 6 are provided by, e.g., Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Stijepic (2011), and Herrendorf et al. (2014); these papers present models that generate structural change trajectories that have the characteristics described in Laws 1–4 and 6 and can, therefore, be regarded as intuitive/theoretical explanations of these laws. The theoretical foundations of Law 5 are provided by, e.g., Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), Stijepic (2011), Uy et al. (2013), and Herrendorf et al. (2014); these papers focus among others on the explanation of the hump-shaped manufacturing sector dynamics. The theoretical foundations and explanations of Law 6 are extensively discussed by Stijepic (2015, 2016).

4.2 Long-run models of structural change

In this section, we formulate models of structural change on the basis of the laws and axioms formulated in Sect. 4.1. Laws 1–6 and Axioms 1–3 are logical statements; we can perform logical operations on them to derive their implications. Each of our models (i.e., Models 1–4) assumes that a subset of Laws 1–6 and Axioms 1–3 is true; each model's predictions (i.e., the statements that refer to future labor allocation dynamics) are the implications of the laws and axioms that are assumed to be valid within the model.

We start with the most conservative model (i.e., Model 1), which is only based on Axiom 1 and Laws 1 and 2. The subsequent models (i.e., Models 2–4) rely on the more controversial laws (i.e., Laws 3–6) and axioms (i.e., Axioms 2 and 3). For each of the models, we discuss the transitional and limit dynamics (as usual in growth theory), the set of attraction, and the strength of fluctuation (cf. Sect. 2.4) and derive the predictions of structural change for developing and developed countries. Of course, we cannot discuss here all the possible combinations of Axioms 1–3 and Laws 1–6 due to space restrictions. Therefore, we only focus on some examples, which demonstrate the capabilities of the positivistic approach and the implications of the laws.

In all the models of Sect. 4.2, we assume that today's labor allocation is given and we aim to predict the future dynamics. We define the corresponding time points as follows.

Definition 8 The time point $t = 0$ stands for the present and $\mathbf{x}_c(0) \equiv (x_{1c}(0), x_{2c}(0), x_{3c}(0))$ stands for the present-day allocation in country c (cf. Axiom 1). The future is represented by the time interval $(0, \infty)$ and the future labor allocation dynamics in country c are indicated by $\mathbf{x}_c(t)$ for $t \in (0, \infty)$.

4.2.1 Model 1: the implications of Laws 1 and 2a or 2b

Model 1 is relatively rudimentary; its predictions follow almost directly from its assumptions (i.e., laws and axioms). Nevertheless, it makes sense to discuss these predictions, since they seem to be the most reliable predictions that we can make. In some sense, this model elucidates what we 'really know' about the future structural change in developing and developed countries. The predictions of Models 2–4 require more mathematics; at the same time, they are more controversial due to the many additional assumptions they require.

We distinguish between two versions of Model 1 (Model 1a and Model 1b) depending on whether Law 2a or Law 2b is assumed to be true.

4.2.1.1 Assumptions of Model 1a Assume that country c belongs to the group C and satisfies Axiom 1 and Laws 1 and 2a. We are interested in predicting the future dynamics of country c (cf. Definition 8 and Axiom 1).

4.2.1.2 Predictions of Model 1a If country c is relatively underdeveloped at the present (cf. Definition 8), i.e., if

$$a_c \geq 0 \text{ (cf. Law 1 and Definition 8)} \tag{9}$$

is true, the following predictions (of the dynamics for $t > 0$) can be made based on Model 1a.

Law 1 and (9) imply that at the present (cf. Definition 8), country c is in the early development phase $(d, a_c]$. Thus, per Law 2a, there exists a future time point (cf. Definition 8) $b_c > 0$ that is characterized by $x_{3c}(b_c) > 0.5$. In other words, (9) and Laws 1 and 2a imply that the country will become a services economy in future. This is all we can say about the *transitional dynamics* (cf. Sect. 2.4) of Model 1a. Any imaginable transitional behavior (e.g., non-continuous, erratic, cyclical) is possible in Model 1a as long as economy c reaches at least temporarily the state of $x_3 > 0.5$ in finite time.

Similarly, we cannot say anything about the *limit dynamics* (cf. Sect. 2.4) of economy c based on Model 1a, since Model 1a does not state much about the nature of the function $\mathbf{x}_c(t)$. That is, the labor allocation in country c may converge to a fixed point (steady state) or to a limit cycle, or may exhibit any other imaginable limit dynamics (e.g., resulting from some sort of chaotic behavior) on S. Obviously, Model 1a's *set of attraction* (cf. Sect. 2.4) cannot be greater than S (cf. Axiom 1) and $O(T_c([0, \infty))) \subseteq S$ (cf. Definition 6 and Axiom 1).

If we replace (9) by

$$x_{3c}(0) > 0.5 \tag{10}$$

reflecting the initial state of a developed economy, we cannot say anything about the future (cf. Definition 8) dynamics of economy c, except that $O(T_c([0, \infty))) \subseteq S$ (cf. Axiom 1 and Definition 6).

4.2.1.3 Application of Model 1a Obviously, these results can be used for predicting the future labor allocation dynamics of today's *developing countries*, which satisfy condition (9). Model 1a implies that these countries will become services economies at some time in future. Afterward, everything can happen according to Model 1a, i.e., the economies may become agricultural or manufacturing economies again or remain services economies forever. In general, Model 1a may be regarded as an 'optimistic' model, since it states that all economies (belonging to the group C) will become services economies at some point in time.

On the basis of Model 1a, we cannot make any predictions of future structural change in today's *developed countries*, which satisfy (10). Note, however, that in contrast to the standard theoretical literature (cf. Sect. 4.1.3), Model 1a allows for strong structural change in the future of today's developed economies: They may become manufacturing or agricultural economies again or stay services economies forever, i.e., they may reach any point on S in future.

4.2.1.4 Assumptions of Model 1b Assume that country c belongs to the group C and satisfies Axiom 1 and Laws 1 and 2b. That is, in contrast to Model 1a, Model 1b assumes that Law 2b is valid. We are interested in predicting the future dynamics of country c (cf. Definition 8 and Axiom 1).

4.2.1.5 Predictions of Model 1b The time period to which Models 1a and 1b apply can be divided into two subperiods: (d, b_c) and $[b_c, \infty)$ (cf. Law 2 and Axiom 1). While Model 1a's and Model 1b's predictions of the dynamics over the period (d, b_c) do not differ, Model 1b provides interesting predictions of the dynamics over the period $[b_c, \infty)$. Therefore, we focus on this period.

Law 2b and (8) imply that $\forall t \in [b_c, \infty)\ \mathbf{x}_c(t) \in S_3$; Axiom 1 [and, in particular, (1) and (2)] and (8) imply that $\mathbf{x}_c(t) \in S_3 \Rightarrow x_{1c}(t) \in [0, 0.5) \wedge x_{2c}(t) \in [0, 0.5) \wedge x_{3c}(t) \in (0.5, 1]$; thus, the following statement is true:

$$\forall t \in [b_c, \infty) \quad x_{1c}(t) \in [0, 0.5) \wedge x_{2c}(t) \in [0, 0.5) \wedge x_{3c}(t) \in (0.5, 1] \tag{11}$$

The assumptions of Model 1b do not impose any further restrictions on the dynamics over the period $[b_c, \infty)$, i.e., economy c may experience any imaginable sort of labor allocation dynamics (on S_3) over the period $[b_c, \infty)$, e.g., transitory, non-continuous, erratic, cyclical. This fact and (11) imply that over the *transitional* phase ($[b_c, \infty)$) and in the *limit* ($\lim_{t\to\infty}$) (cf. Sect. 2.4), economy c may experience any imaginable sort of labor allocation dynamics on S_3, where the employment shares may change (or fluctuate) strongly over time and, in particular, the potential ranges of fluctuation of the agricultural, manufacturing, and services shares in Model 1b are $M_1^{1b}([b_c, \infty)) \subseteq [0, 0.5]$, $M_2^{1b}([b_c, \infty)) \subseteq [0, 0.5]$, and $M_3^{1b}([b_c, \infty)) \subseteq [0.5, 1]$, respectively [cf. (11) and Definition 7]. Thus, each of the employment shares may change or fluctuate by 0.5 over the transitional period $[b_c, \infty)$ and in the limit, i.e., the potential strength of fluctuation of the agricultural, manufacturing, and services shares in Model 1b is given by: $|M_1^{1b}([b_c, \infty))| \leq 0.5$, $|M_2^{1b}([b_c, \infty))| \leq 0.5$, and $|M_3^{1b}([b_c, \infty))| \leq 0.5$. Thus, the structural change predicted by Model 1b can be relatively strong in comparison with the structural change observed in the past: For example, the agricultural employment shares in France, Germany, Netherlands, and UK have decreased by less than 0.5 since 1870 (cf. Maddison 1995).

This discussion implies that Model 1b's *set of attraction* (cf. Sect. 2.4) is a subset of S_3 [cf. (8)]. In contrast, Model 1a's set of attraction is a subset of S, as shown above. Thus, Model 1b allows us to specify the set of attraction much more precisely than Model 1a does; S_3 covers only 25% of the area of S [cf. (4), (8), and Fig. 6].

Moreover, this discussion, Definition 6, and the definition of the set of attraction (cf. Sect. 2.4) imply, obviously, that the *omega limit set* of a trajectory generated by Model 1b is located in the partition S_3, i.e., $O(T_c([0, \infty))) \subseteq S_3$.

4.2.1.6 Application of Model 1b These results can be used for predicting the future structural change dynamics of today's *developing countries*, which satisfy condition (9). Like Model 1a, Model 1b predicts that these countries will become services economies at some time in future (b_c). Moreover, Model 1b predicts that from then on (i.e., for $t > b_c$) the dynamics of these economies will be the same as the future dynamics of today's developed countries (see below for a discussion of Model 1b's predictions of the future dynamics of developed economies). In general, Model 1b is even more 'optimistic' than Model

1a is: It does not only state that all economies (belonging to the group C) will become services economies at some point in time but also that they will be able to sustain this development (i.e., stay services economies forever).

Furthermore, Model 1b can be used to predict the future dynamics of today's *developed economies*, which are, of course, services economies and satisfy (10). Model 1b predicts that today's developed economies will remain services economies forever and may, nevertheless, experience strong structural change in future: Each of their sectoral employment shares may fluctuate by 0.5 over time (even in the limit), which is comparable to the magnitude of the structural change over the last 150 years in today's highly developed countries. In contrast to the standard literature (cf. Sect. 4.1.3), Model 1b states that structural change does not necessarily come to a halt (in highly developed economies).

4.2.2 Model 2: the implications of Laws 3 or 4

We distinguish between two versions of Model 2 (Model 2a and Model 2b) depending on whether Law 3 or Law 4 is true.

4.2.2.1 Assumptions of Model 2a Assume that country c belongs to the group C and satisfies Axioms 1–3 and Law 3 (where $p_c < 0$). We are interested in predicting the future dynamics of country c (cf. Definition 8 and Axiom 1).

4.2.2.2 Predictions of Model 2a We begin with the *transitional dynamics* (cf. Sect. 2.4) for $t > 0$. Obviously, the employment share of agriculture (x_1) is constant or decreases, according to Law 3. The services and manufacturing employment shares may be increasing, decreasing, constant, or non-monotonous (e.g., cyclical), as long as their sum ($x_2 + x_3$) is constant (if x_1 is constant) or increases over time (if x_1 decreases over time), since, otherwise, (1)–(2) is violated (cf. Axiom 1). The (regular) vector angles of the trajectory generated by Model 2a are stated in Property 1b/c. Moreover, if the agricultural employment share decreases strictly monotonously over time, the trajectory does not intersect itself according to Definition 3 (cf. Stijepic 2016, p. 27, 2015, p. 82f.). Examples of transitional dynamics consistent with Model 2a are depicted in Figs. 3 and 9. The trajectories depicted in Figs. 4 and 5 are not consistent with Model 2a.

Law 3 and $p_c < 0$ imply that $x_{1c}(t) \in [0, x_{1c}(s)]$ for $t \in [s, \infty)$, where $s \in [0, \infty)$. Thus, Definition 7 implies $M_1^{2a}([t, \infty)) \subseteq [0, x_{1c}(t)]$ for $t \in [0, \infty)$. Therefore, Axiom 1 [and, in particular, (1) and (2)] and Definition 7 imply $M_2^{2a}([t, \infty)) \subseteq [0, 1]$ and $M_3^{2a}([t, \infty)) \subseteq [0, 1]$ for $t \in [0, \infty)$. Thus, $|M_1^{2a}([t, \infty))| \leq x_{1c}(t)$, $|M_2^{2a}([t, \infty))| \leq 1$, and $|M_3^{2a}([t, \infty))| \leq 1$. Overall, Model 2a allows for stronger future fluctuations of the manufacturing and services share than Model 1b (cf. Sect. 4.2.1) does [cf. $|M_2^{2a}([t, \infty))|$, $|M_3^{2a}([t, \infty))|$, $|M_2^{1b}([b_c, \infty))|$, and $|M_3^{1b}([b_c, \infty))|$].

The *limit dynamics* (cf. Sect. 2.4) are relatively easy to predict in Model 2a, as shown in the following proposition.

Proposition 1 *Assume that Axioms 1–3 and Law 3 are valid (where $p_c < 0$). Then,* $\lim_{t\to\infty} x_{1c}(t) \equiv f \in [0, x_{1c}(0)]$ *and* $O(T_c([0, \infty))) \subseteq S^f := \{(x_1, x_2, x_3) \in S : x_1 = f\}$

$\subseteq S^{\mathrm{A2a}} := \{(x_1, x_2, x_3) \in S : 0 \le x_1 \le x_{1c}(0)\}$ [*cf.* (7) *and Definition* 6]. *Among others,* $O(T_c([0,\infty)))$ *can consist of only one point, i.e.,* $O(T_c([0,\infty))) \equiv \{\mathbf{x}_c^*\} \subset S^{\mathrm{A2a}}$.

Proof The following fact is implied by the monotone convergence theorem: if $\dot{x}_{1c}(t) \le 0$ for $t \in [0, \infty\}$ (cf. Law 3) and $x_{1c}(t) \ge 0$ for $t \in [0, \infty)$ [cf. (1)], then $\lim_{t\to\infty} x_{1c}(t) \equiv f \in [0, x_{1c}(0)]$, i.e., $x_{1c}(t)$ converges to a fixed point f (recall Axioms 2 and 3). The set of all points on S that satisfy the condition $x_{1c} = f$ is S^f, which is defined in Proposition 1. These facts imply that $\mathbf{x}_c(t) \equiv (x_{1c}(t), x_{2c}(t), x_{3c}(t))$ converges to some subset of S^f. In other words, the omega limit set (cf. Definition 6) of the trajectory associated with $\mathbf{x}_c(t)$ [cf. (7)] is a subset of S^f. On the one hand, this set (i.e., $O(T_c([0,\infty)))$) can consist of only one point (i.e., a fixed point) given the assumptions of Proposition 1; in this case, $\lim_{t\to\infty} \mathbf{x}_c(t) \equiv \mathbf{x}_c^* \in S$, i.e., country c converges to a fixed point ($\mathbf{x}_c^*$), as proven by the following example: assume that $\forall t \in [0,\infty), x_{1c}(t) = 7/9\exp(-2t)$, $x_{2c}(t) = 1/9$, and $x_{3c}(t) = 8/9 - x_{1c}(t)$; it is easy to prove that these equations satisfy the assumptions of Proposition 1 (i.e., Axioms 1–3 and Law 3) and imply that economy c converges to the fixed point $\mathbf{x}_c^* = (0, 1/9, 8/9)$ for $t \to \infty$. On the other hand, given the assumptions of Proposition 1, $O(T_c([0,\infty)))$ can contain more than only one point, as proven by the following example: assume that $\forall t \in [0,\infty), x_{1c}(t) = 0.1, x_{2c}(t) = 1/4\sin(t) + 0.4, x_{3c}(t) = 0.9 - x_{2c}(t)$; it is easy to prove that these equations satisfy the assumptions of Proposition 1 (i.e., Axioms 1–3 and Law 3), while the omega limit set (cf. Definition 6) of the corresponding trajectory is $\{(x_1, x_2, x_3) \in S : x_1 = 0.1 \wedge 0.15 \le x_2 \le 0.65 \wedge x_3 = 0.9 - x_2\}$, i.e., the omega limit set is equal to (the set of points on) a line segment parallel to the $\mathbf{v}_2\mathbf{v}_3$-edge of the 2-simplex. The fact that $S^{\mathrm{A2a}} \supseteq S^f$ for $f \in [0, x_{1c}(0)]$ follows immediately from the definitions of the sets S^{A2a} and S^f (cf. the assumptions of Proposition 1). □

In fact, Proposition 1 states that as time goes to infinity, the labor allocation in the economy described by Model 2a converges to (i) a fixed point or (ii) a line segment (on S^f) parallel to the $\mathbf{v}_2\mathbf{v}_3$-edge of the simplex S. (The latter fact follows from Proposition 1 and Property 1, where the latter implies that $x_1 = f =$ const. only if the economy moves along a line segment parallel to the $\mathbf{v}_2\mathbf{v}_3$-edge of the simplex S.) The fixed point or line segment must be located in S^{A2a}, which is defined in Proposition 1 and depicted in Fig. 8. If economy c converges to a fixed point (case i), structural change comes to a halt (in the limit), i.e., the labor allocation converges to a steady state allocation ($\mathbf{x}_c^*$). Figure 3 represents an example of these dynamics where the trajectory end represents the fixed point. If economy c converges to a line segment parallel to the $\mathbf{v}_2\mathbf{v}_3$-edge (case ii), structural change has a cyclical component (in the limit) where only the employment shares of manufacturing (x_2) and services (x_3) behave cyclically, while x_1 decreases monotonously to its fixed-point value f. For an example of such a cyclical trajectory, see Fig. 9.

Overall, while Model 2a allows for a much exacter prediction of the limit dynamics than Model 1b does, it does not necessarily allow for an exacter specification of the *set of attraction* (cf. Sect. 2.4): as shown in Sect. 4.2.1, Model 1b implies that for $t \to \infty$, economy c is located in a subset of S_3; Model 2a (i.e., Proposition 1) predicts that for $t \to \infty$, economy c is located in a subset of S^{A2a}; S^{A2a} may be larger (i.e., may cover a larger area of the simplex S) than S_3, depending on the initial value ($x_{1c}(0)$) of the agricultural share.

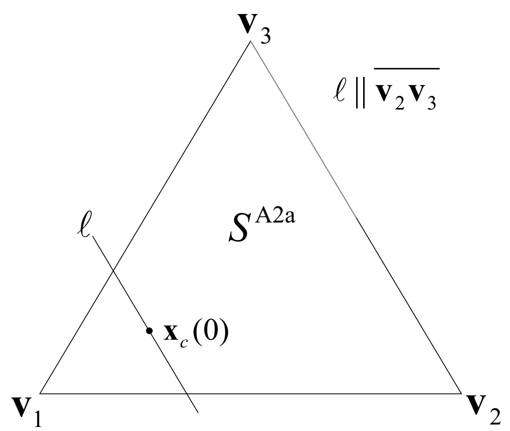

Fig. 8 An example for the set S^{A2a}

Analogously, while Model 2a implies that the employment share of agriculture is fixed in the limit (i.e., $\lim_{t\to\infty} x_{1c}(t) \equiv f \in [0, x_{1c}(0)]$), it allows for stronger *limit* fluctuations of the services and manufacturing shares than Model 1b does (see the discussion of the potential ranges of fluctuation $|M_2^{2a}([t, \infty))|$, $|M_3^{2a}([t, \infty))|$, $|M_2^{1b}([b_c, \infty))|$, and $|M_3^{1b}([b_c, \infty))|$).

4.2.2.3 Application of Model 2a The statements of Model 2a are equally applicable to *developed and developing countries*. Model 2a allows for cyclical behavior of the manufacturing and services employment shares even in the limit, i.e., the labor allocation does not necessarily converge to a fixed labor allocation but may be characterized by cyclical behavior of the manufacturing and services employment shares in the limit (i.e., 'forever'). Thus, Model 2a (like Models 1a and 1b) allows for the possibility that structural change never comes to a halt, neither in *developed* nor in *developing economies*.

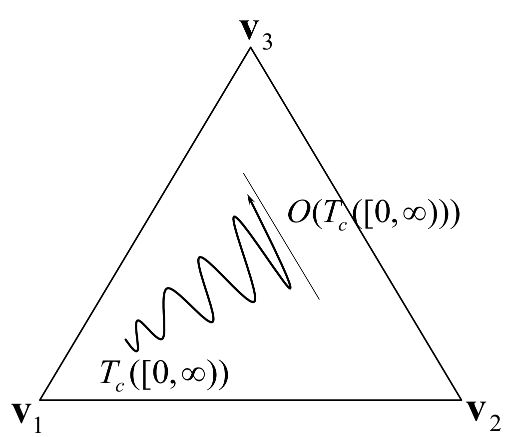

Fig. 9 An example of a trajectory and its omega limit set generated by Model 2a

This is relevant for the prediction of structural change in today's *highly developed countries* (e.g., the USA), where today's structural change is relatively slow, the services employment share is relatively great, and the agricultural and manufacturing shares are relatively small. Model 2a states that these economies may experience strong structural change in future and, in particular, they may re-industrialize.[7] Since Model 2a allows for even stronger (limit) structural change than Model 1b does, it implies that the future structural change (i.e., the changes in the manufacturing and services shares) in developed economies may be stronger than the structural change that they experienced over the last 150 years (cf. Sect. 4.2.1).

Furthermore, in contrast to Model 1, Model 2a does not state that all countries (belonging to the group C) must become services economies at some point in time, since Law 3 does not state that the agricultural employment share must decrease below 0.5. Thus, Model 2a is consistent with the pessimistic view that some *developing economies* may never develop beyond the agricultural stage.

4.2.2.4 Assumptions of Model 2b Assume that country c belongs to the group C and satisfies Axioms 1–3 and Law 4 (where $q_c < 0$). That is, in contrast to Model 2a, Model 2b assumes that Law 4 and not Law 3 is true. We are interested in predicting the future dynamics of country c (cf. Definition 8 and Axiom 1).

4.2.2.5 Predictions of Model 2b It can be shown that the most results of Model 2b are analogous to the results of Model 2a. In particular, the following statements are true for $t > 0$: (a) the services employment share (x_3) increases monotonously over time and converges to its steady state value; (b) the agricultural and manufacturing employment shares may exhibit any type of smooth (transitional) dynamics as long as their sum ($x_1 + x_2$) decreases monotonously over time; and (c) cyclical limit dynamics of the manufacturing and agricultural shares are possible, i.e., structural change does not necessarily come to a halt in the limit. We omit a detailed discussion of these aspects, since their proofs and the mathematical techniques used are analogous to the proofs and the techniques used in the discussion of Model 2a. Rather, we focus on the key difference between Models 2a and 2b, namely the strength of structural change over the transitional period and in the limit.

Law 4 and $q_c < 0$ imply that $x_{3c}(t) \in [x_{3c}(s), 1]$ for $t \in [s, \infty)$, where $s \in [0, \infty)$. Thus, Definition 7 implies: $M_3^{2b}([t, \infty)) \subseteq [x_{3c}(t), 1]$ for $t \in [0, \infty)$. This fact, Definition 7, and Axiom 1 [and, in particular, (1) and (2)] imply: $M_1^{2b}([t, \infty)) \subseteq [0, (1-x_{3c}(t))]$ and $M_2^{2b}([t, \infty)) \subseteq [0, (1-x_{3c}(t))]$ for $t \in [0, \infty)$. Thus, according to Definition 7, the following is true:

$$\begin{aligned} \forall t \in [0, \infty) \quad & \left| M_1^{2b}([t, \infty)) \right| \leq 1-x_{3c}(t) \wedge \left| M_2^{2b}([t, \infty)) \right| \\ & \leq 1-x_{3c}(t) \wedge \left| M_3^{2b}([t, \infty)) \right| \leq 1-x_{3c}(t) \end{aligned} \tag{12}$$

[7] The situation in highly developed economies described by Model 2a is as follows (cf. Sect. 3): (a) x_2 and x_1 are relatively small and cannot fall below 0 [cf. (1)]; (b) x_1 cannot increase (cf. Law 3); and (c) x_3 is relatively great and cannot grow beyond 1 [cf. (1)]. In other words, the economy is located close to vertex $\mathbf{v}_3$ (since x_3 is very great); thus, it cannot move toward vertex $\mathbf{v}_3$ much (i.e., it cannot increase the services share significantly); moreover, the economy cannot move toward vertex $\mathbf{v}_1$ (i.e., it cannot increase the agricultural share) due to Law 3 (cf. Fig. 2). Thus, the only way to achieve a strong labor re-allocation is to move toward vertex $\mathbf{v}_2$ (i.e., to increase the manufacturing share) and away from vertex $\mathbf{v}_3$ (i.e., to decrease the services share). This process may be described as *re-industrialization*.

(12) implies that $|M_1^{2b}([0,\infty))|$, $|M_2^{2b}([0,\infty))|$, and $|M_3^{2b}([0,\infty))|$ are very small if $x_{3c}(0)$ is very great; that is, according to Model 2b, structural change is very weak in future (i.e., for $t \in [0,\infty)$) if today's services share ($x_{3c}(0)$) is very great (cf. Definitions 7 and 8). We can apply this result as follows.

4.2.2.6 Application of Model 2b Model 2b predicts that in the highly *developed economies*, which are characterized by a very great services share (x_3) at the present, structural change will be relatively weak in future; in particular, Model 2b predicts that these economies will not be able to re-industrialize significantly (cf. footnote 7).[8] These predictions contradict the predictions of Models 1a, 1b, and 2a, where the latter state that in future, the developed economies may experience changes/fluctuations of the sectoral employment shares that are comparable to or even much stronger than the changes that they experienced over the last 150 years (cf. Sect. 4.2.1).

Furthermore, like Model 2a, Model 2b does not state that all countries (belonging to the group *C*) must become services economies at some point in time, since Law 4 does not state that the services employment share must grow above 0.5. Thus, Model 2b is consistent with the pessimistic view that some *developing economies* may never become services economies.

4.2.3 Model 3: the implications of Laws 3 and 4

4.2.3.1 Assumptions of Model 3 Assume that country *c* belongs to the group *C* and satisfies Axioms 1–3 and Laws 3 and 4 (where $p_c < 0$ and $q_c < 0$). We are interested in predicting the future dynamics of country *c* (cf. Definition 8 and Axiom 1).

4.2.3.2 Predictions of Model 3 The *limit dynamics* (cf. Sect. 2.4) are easy predictable in Model 3 (cf. Proposition 2).

Proposition 2 Assume that Axioms 1–3 and Laws 3 and 4 are valid (where $p_c < 0$ and $q_c < 0$). Then, $O(T_c([0,\infty))) \equiv \left\{\mathbf{x}_c^*\right\} \subseteq S^{A3} := \{(x_1,x_2,x_3) \in S : 0 \leq x_1 \leq x_{1c}(0) \wedge x_{3c}(0) \leq x_3 \leq 1\}$, i.e., country c's labor allocation converges to a steady state allocation $\mathbf{x}_c^*$.

Proof The assumptions of Proposition 2 imply that $\lim_{t\to\infty} x_{1c}(t) \equiv f \in [0, x_{1c}(0)]$, i.e., $x_{1c}(t)$ converges to the fixed point f (cf. Proof of Proposition 1). Analogously, it can be shown that $\lim_{t\to\infty} x_{3c}(t) \equiv g \in [x_{3c}(0),1]$, i.e., $x_{3c}(t)$ converges to the fixed point g. Since $\forall t\, x_{2c}(t) = 1 - x_{1c}(t) - x_{3c}(t)$ [cf. (2)], these two facts imply that $\lim_{t\to\infty} x_{2c}(t) = 1 - f - g \equiv h \in [0,(1 - x_{3c}(0))]$, i.e., $x_{2c}(t)$ converges to the fixed point h. These facts, Axiom 1, and (4) imply that $\mathbf{x}_c(t) \equiv (x_{1c}(t), x_{2c}(t), x_{3c}(t))$ converges to the fixed point $(f,g,h) \equiv \mathbf{x}_c^* \in S^{A3}$, where S^{A3} is defined in Proposition 2. The rest of the proof follows from (7) and Definition 6.□

Thus, Model 3 is much more specific about the limit dynamics of labor allocation than Models 1 and 2 are. Model 3 excludes, e.g., cyclical limit dynamics and predicts that structural change is transitory, i.e., comes to a halt (in the limit).

[8] Recall that Law 4 states that the services share (x_3) cannot start decreasing at some time in future; i.e., if it is very great at the present, it remains very great forever.

Now, we turn to the *transitional dynamics* (cf. Sect. 2.4). Laws 3 and 4 imply that if the economy develops (i.e., if the labor allocation is not constant), then the employment share of agriculture decreases over time and/or the services employment share grows over time. Note that Laws 3 and 4 are consistent with the scenario where the economy does not change for all $t > 0$, i.e., no-structural-change scenario. This scenario may reflect a development trap or a very mature economy that has converged close to its steady state. Overall, Model 3 is consistent with the following scenarios of transitional dynamics:

$$\dot{x}_{1c} = \dot{x}_{2c} = \dot{x}_{3c} = 0. \tag{13}$$

$$\dot{x}_{1c} < 0 \wedge \dot{x}_{2c} = 0 \wedge \dot{x}_{3c} > 0. \tag{14}$$

$$\dot{x}_{1c} < 0 \wedge \dot{x}_{2c} > 0 \wedge \dot{x}_{3c} = 0. \tag{15}$$

$$\dot{x}_{1c} = 0 \wedge \dot{x}_{2c} < 0 \wedge \dot{x}_{3c} > 0. \tag{16}$$

$$\dot{x}_{1c} < 0 \wedge \dot{x}_{2c} > 0 \wedge \dot{x}_{3c} > 0. (\Rightarrow |\dot{x}_{1c}| > |\dot{x}_{3c}|) \tag{17}$$

$$\dot{x}_{1c} < 0 \wedge \dot{x}_{2c} < 0 \wedge \dot{x}_{3c} > 0. (\Rightarrow |\dot{x}_{1c}| < |\dot{x}_{3c}|) \tag{18}$$

Exactly speaking, Model 3 predicts that at any point of time $t \in [0, \infty)$, one (and only one) of the statements (13)–(18) is true. Otherwise, one of the axioms or laws of Model 3 is violated. Of course, the dynamics over the period $[0, \infty)$ can be a mixture of the archetypes (13)–(18). For example, there may exist a $z_c \in (0, \infty)$ such that $\forall t \in [0, z_c)$ statement (17) is true, at $t = z_c$ statement (14) is true, and $\forall t \in (z_c, \infty)$ statement (18) is true.

The transitional dynamics predicted by Model 3 cover different structural change models: the Kongsamut et al. (2001) model predicts that (14) is true for all t; the Ngai and Pissarides (2007) model generating hump-shaped manufacturing dynamics (cf. Law 5) predicts dynamics that first follow (17) and then (18).

Properties 1–3 can be used to geometrically summarize the transitional dynamics scenarios (14)–(18): Each of these scenarios is covered by the vector angle condition $0° \leq \alpha(t) \leq 120°$. For example, scenario (14) can be represented by a linear trajectory that is characterized by the angle $\alpha(t) = 60° \ \forall t \in [0, \infty)$ and represents a movement away from the vertex $\mathbf{v}_1$. Obviously, the vector angle condition $\forall t \in [0, \infty) \ 0° \leq \alpha(t) \leq 120°$ implies that there are no self-intersections (cf. Definition 3). Furthermore, due to Axioms 2 and 3, there is no 'erratic' (exactly speaking, discontinuous) behavior in the long run.

The strength of structural change over the transitional period in Model 3 can be studied as follows. It can be shown that $M_1^3([t, \infty)) \subseteq [0, x_{1c}(t)]$, $M_2^3([t, \infty)) \subseteq [0, (1 - x_{3c}(t))]$, and $M_3^3([t, \infty)) \subseteq [x_{3c}(t), 1]$ for $t \in [0, \infty)$. The proof of this fact is the same as the proof of $M_1^{2a}([t, \infty))$, $M_2^{2b}([t, \infty))$, and $M_3^{2b}([t, \infty))$ (cf. Sect. 4.2.2). These facts imply: $|M_1^3([t, \infty))| \leq x_{1c}(t)$, $|M_2^3([t, \infty))| \leq 1 - x_{3c}(t)$, and $|M_3^3([t, \infty))| \leq 1 - x_{3c}(t)$ for $t \in [0, \infty)$. Thus, the results of Model 3 regarding the potential strength of future fluctuation of the agricultural share (the manufacturing and services shares) are the same as the results of Model 2a (Model 2b).

Finally, we turn to the *set of attraction* (cf. Sect. 2.4) of Model 3. As implied by Proposition 1 (Proposition 2), the set of attraction of Model 2a (Model 3) is a subset of S^{A2a} (S^{A3}). In particular, Propositions 1 and 2 imply that if $x_{1c}(0)$ is assumed to be equal in Models 2a and 3, then S^{A3} is not larger and can be smaller than S^{A2a} (where we define the size of a set as the area of S that it covers). That is, Model 3 allows for an exacter specification of the set of attraction. This is not surprising, since the set of restrictions/laws imposed on the dynamics by Model 3 is greater than the set of restrictions/laws imposed on the dynamics by Model 2a.

4.2.3.3 Application of Model 3 Model 3 implies that the structural change in *developed economies* (belonging to the group C) is close to the end, i.e., developed economies will not experience significant (long-run) labor re-allocation in future. The reason for this fact is that in the highly developed countries (e.g., in the USA), the services (agricultural) employment share has already reached a very high (low) level and, therefore, cannot grow (decrease) much anymore, where Law 4 (Law 3) prohibits a decrease (an increase) in the services (agricultural) share. In other words, according to Law 4 (Law 3), the services (agricultural) share must grow (decrease) or be constant; it cannot grow (decrease) significantly, since it is restricted by its upper (lower) limit of 1 (0) [cf. (1)]; thus, it must be approximately constant. Due to Axiom 1 [and, in particular, (1)], the manufacturing employment share cannot change significantly if the agricultural or services share does not change significantly. Overall, Model 3 does not allow for (significant) structural change in the highly developed economies.

Moreover, Model 3 implies that the *developing countries* (belonging to the group C) may experience structural change or not. Particularly, scenario (13) represents a stagnating labor allocation (for all future time points). Even if the economy develops, it does not necessarily become a services economy in future (but may remain an agricultural or become a manufacturing economy); that is, Model 3 is not as optimistic as Model 1 is. If the economy develops, its long-run dynamics are relatively smooth and monotonous: the employment share of services grows and/or the agricultural share shrinks; the manufacturing employment share may exhibit any sort of (smooth and) monotonous or non-monotonous dynamics (e.g., the 'hump-shaped' dynamics described by Law/Regularity 5 or transitory cyclical dynamics). Nevertheless, Model 3 predicts that structural change is transitory; thus, according to Model 3, the labor allocation in developing countries converges to a fixed labor allocation ('steady state').

Since we have shown that Model 3 can generate the hump-shaped dynamics of the manufacturing sector postulated in Law 5, we do not dedicate a model to Law 5, but go on with Law 6 in Sect. 4.2.4.

4.2.4 Model 4: the implications of Laws 1, 2a, and 6

We present now a model of non-self-intersecting trajectories.

4.2.4.1 Assumptions of Model 4 Assume that country c belongs to the group C and satisfies Axioms 1 and 2 and Laws 1, 2a, and 6. We are interested in predicting the future dynamics of country c (cf. Definition 8 and Axiom 1).

4.2.4.2 Predictions of Model 4 If country c is relatively underdeveloped at the present (cf. Definition 8), i.e., if (9) is true, the following predictions (of the dynamics for $t > 0$) can be made based on Model 4.

Law 1 and (9) imply that at the present (cf. Definition 8), country c is in the early development phase $(d, a_c]$. Thus, per Law 2a there exists a future time point (cf. Definition 8) $b_c > 0$ that is characterized by $x_{3c}(b_c) > 0.5$. In other words, (9) and Laws 1 and 2a imply that country c will become a services economy in future. Furthermore, the *transitional dynamics* (cf. Sect. 2.4) over the time period $[0, b_c]$ can be described by a non-self-intersecting trajectory (cf. Definition 3 and Law 6). In general, this does not mean much, since such a trajectory can represent very different types of dynamics. However, interesting statements can be made about the *transitional dynamics* after b_c, i.e., after the country has become a services economy, since in this case, a sort of path dependency arises, which can be used to reduce the number of feasible structural change scenarios, as discussed by Stijepic (2015) in detail.

Regarding the *limit dynamics* (cf. Sect. 2.4), we can say that neither a fixed point nor a limit cycle is ruled out by the assumptions of Model 4. A limit cycle does not represent a self-intersection and, thus, does not violate Law 6, since, in the case of a limit cycle, the trajectory only converges to the image of a Jordan curve and never becomes a Jordan curve. While an omega limit set consisting of a fixed point means that structural change comes to a halt (in the limit), a limit cycle means that labor allocation dynamics are cyclical in the limit, where the employment shares of all sectors ($i = 1, 2, 3$) behave cyclically in the limit. An approach toward a more precise specification of the limit dynamics in Model 4 could be based on the Poincaré–Bendixson theory, as discussed in the following.

The Poincaré–Bendixson theory (henceforth, 'PB theory') is a well-known mathematical result applying to smooth autonomous differential equation systems in (a bounded subset of) the plane. It lists the *limit dynamics* scenarios that can arise in such a smooth system (among others, a fixed point and a limit cycle may arise).[9] Since smooth autonomous differential equation systems generate *continuous* and *non-self-intersecting* trajectories (see Stijepic (2015, p. 84f.) for a brief discussion and references from the mathematical literature) as does Model 4 (cf. Axiom 2 and Law 6), the PB theory seems to be the most natural extension of Model 4. In other words, the application of the Poincaré–Bendixson theory in structural change modeling is supported by Model 4/Law 6 and seems to be useful, since it allows us to elaborate all the possible limit dynamics of structural change. However, the laws, axioms, and methodology discussed in Sects. 4.2.1, 4.2.2, 4.2.3, and 4.2.4 are not sufficient to fully justify the assumption that structural change is representable by smooth autonomous differential equation systems. Thus, for applying the Poincaré–Bendixson theory, we have to introduce a very strong axiom postulating that structural change can be represented by smooth autonomous differential equation systems and, thus, (a) deviate strongly from the positivistic nature of our approach or (b) carry out lengthy empirical and methodological research aiming at

[9] For a discussion of the Poincaré–Bendixson theory, see, e.g., Andronov et al. (1987, p. 351ff.), Guckenheimer and Holmes (1990, p. 45), Hale (2009, p. 55) (in particular, Theorem 1.5), and Teschl (2012, Chapter 7.3) (in particular, Theorem 7.16).

justifying this axiom, which is far beyond the scope of this paper. For these reasons, we leave the detailed discussion of the application and the applicability of the Poincaré–Bendixson theory in structural change modeling for forthcoming papers.

4.2.4.3 Application of Model 4 Stijepic (2015) uses a version of Model 4 for predicting the future labor allocation dynamics of *developed economies*, showing that their future dynamics can be characterized by only *three scenarios* ('relative de-industrialization,' 'relative industrialization,' and 'remaining a services economy'). For predicting the future labor allocation dynamics of *developing economies* by using Model 4, we must distinguish between two phases of their development: the phase before b_c and the phase after b_c (cf. Law 2a). The key Model 4 predictions regarding the labor allocation dynamics in developing economies until they become services economies (i.e., *until* b_c) are the same as the corresponding predictions of Model 1a (cf. Sect. 4.2.1). The results derived by Stijepic (2015) can be used for predicting the dynamics *after* b_c (i.e., the *three scenarios* mentioned above apply here).

5 Concluding remarks

5.1 Discussion of the method

Standard quantitative approaches for prediction of economic dynamics rely heavily on: (a) theoretical information, which is ideological in great part, as in the case of predictions based on theoretical models; (b) complex quantitative empirical relationships, which are difficult to interpret intuitively, as in the case of, e.g., vector auto-regressions or nonlinear regressions; (c) oversimplifying (e.g., linear) estimation equations, which are ideological, yet often loosely related to theoretical arguments, as in the case of linear regression; or (d) in general, quantitative statements that are often restricted in validity to relatively small country groups. In contrast, a great deal of economic knowledge ('economic laws') is rather of qualitative or nonlinear nature. In particular, many economic phenomena seem to follow qualitative economic laws that are relatively robust in the sense that they are persistent across time and space. This is particularly true for many topics associated with long-run dynamics and, in particular, long-run labor allocation dynamics. Thus, the idea of our paper is to try to (a) use only such robust (qualitative) information for predicting labor allocation dynamics and (b) reduce the extent of ideological information used, which seems to be a valuable directive (cf. Sect. 1). Of course, in economics, it is not possible to make predictions without relying on ideological information and to find laws that are true for *all* countries and for *all* time periods. Nevertheless, the reader may agree that there are 'more' ideological statements and 'less' ideological statements as well as more reliable regularities and less reliable regularities. In Sect. 4.2, we pay tribute to this fact by suggesting not only one model but a set of models, where the models differ by the number of axioms (which represent merely ideological information) and the sets of laws (which represent the empirical information and differ by 'reliability') that they assume to be true.

Mathematics provides us with many tools and concepts (e.g., set theory and predicate logic) that can be used to derive statements/predictions on the basis of qualitative information (on empirical regularities). For using these concepts, we must translate the observed regularities/laws, which are statements that refer to the labor allocation dynamics, into geometrical and topological notions by using the concepts of trajectory and its domain. Then, we can use logic and set theory to perform logical operations on these transformed statements and, thus, derive implications, which can be interpreted as predictions of future labor allocation dynamics. In this sense, the predictions made in Sect. 4.2 are logical implications of the observable laws. Each of our models focuses on two or more laws and derives more or less their direct implications. Thus, the readers of this paper, who have their own opinion on the reliability/validity of the different laws discussed in Sect. 4.1.2, can use this paper to identify the direct implications of their preferred laws for future dynamics in developed and developing economies. Of course, these implications are based on ideological information (cf. Axioms 1–3). However, we tried to minimize the use of this type of information (e.g., the predictions of Model 1 do not depend on Axioms 2 and 3) and formulated the axioms such that they do not differ from the ideological assumptions of standard structural change, growth, and, in general, long-run dynamic models. Thus, our models seem to be less ideological or at least not more ideological than the standard (empirical and theoretical) dynamic models.

5.2 Summary of the predictions

In general, we have shown in our paper that simple statements (such as 'the services employment share increases monotonously over time') can have interesting implications in the three-sector framework, which can be used for prediction of future structural change if they are regarded as economic laws. In particular, we can specify the type of transitional and limit dynamics (e.g., steady state, limit cycle, or chaotic dynamics), the potential strength of structural change over the transitional period and in the limit, and the location of the economy in its dynamic equilibrium (i.e., the set of attraction).

Moreover, we have shown that apparently very similar statements/laws can have very different implications. For example, as shown in Sect. 4.2.2, the statement 'the agricultural share decreases monotonously over time' (cf. Law 3) implies that highly developed countries may experience very strong structural change in future, while the statement 'the services share grows monotonously over time' (cf. Law 4) implies that the highly developed economies will not experience any significant structural change in future. In general, our models generate very different predictions of structural change, as discussed in the following.

Our results regarding the *strength of future structural change in today's developed economies* cover a wide range of predictions: while Models 2b and 3 predict that developed economies will not experience significant structural change in future, Model 1b predicts that the potential for future labor re-allocation/fluctuation in developed economies is comparable to the cumulative amount of labor re-allocated in these countries over the last 150 years; the remaining models allow for much stronger future structural change in developed economies.

Moreover, the *type of predicted structural change* differs significantly across models. For example, Model 3 predicts that structural change comes to a halt in the limit, i.e., structural change is transitory; Model 2a (Model 4) allows, additionally, for cyclical limit dynamics of the manufacturing and services sectors (of all sectors); and in Model 1a, irregular dynamics (erratic dynamics or chaos) may arise.

In general, we show that the empirical regularities ('stylized facts') of structural change do not necessarily imply that the structural change in *today's developed economies* is near to its end: Some of our models imply that the developed economies may re-industrialize significantly or may be characterized by limit fluctuations of sectoral employment shares in future.

Our predictions of structural change in *today's developing economies* range from pessimistic to optimistic predictions stating that today's developing economies (a) may never become services economies, (b) may not sustain their development (i.e., become agricultural economies again), or (c) develop as today's developed economies.

5.3 Topics for further research

The six models of labor allocation dynamics presented in Sect. 4.2 are only examples of models that can be formulated on the basis of Axioms 1–3 and Laws 1–6; they are aimed to demonstrate some major implications of each of the laws and the range of the mathematical methods that are applicable when the positivistic approach to structural change modeling is taken. Further research could study other combinations of Axioms 1–3 and Laws 1–6 and their implications. Of course, alternative laws and axioms could be formulated (referring to long-run labor allocation dynamics) and models could be based on them.

The (limit) fluctuations in the employment shares seem to be an interesting topic. The empirical evidence shows that the employment shares fluctuate in the short run (cf., e.g., Stijepic (2016)). Although our paper does not focus on explaining short-run employment share dynamics, we have shown that (a) some of our models allow for such fluctuations over the transitional period and in the limit, (b) some of our models allow only for transitional fluctuations, and (c) Model 3 does not allow for any fluctuations. Further research could focus on these aspects.

While our paper focuses on labor allocation dynamics, other types of structural change could be studied by using the method and the techniques discussed in our paper. For some examples of the topics that are covered by our method, see Stijepic (2016).

Last not least, our discussion of the applicability of the Poincaré–Bendixson theory (cf. Sect. 4.2.4) implies many interesting (yet lengthy) empirical and methodological research topics.

These topics are left for further research.

Acknowledgements
I would like to thank Damir Stijepic for valuable comments.

Competing interests
No competing interests.

Availability of data and materials
Not applicable (data has not been used).

Consent for publication
Not applicable.

Ethics approval and consent to participate
Not applicable.

Funding
No funding sources.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 29 May 2017 Accepted: 15 August 2017
Published online: 06 September 2017

References

Acemoglu D, Guerrieri V (2008) Capital deepening and non-balanced economic growth. J Polit Econ 116(3):467–498
Andronov AA, Vitt AA, Khaikin SE (1987) Theory of oscillators. Dover Publications, Mineola
Archibald GC, Simon HA, Samuelson PA (1963) Discussion. Am Econ Rev 53(2):227–236
Boppart T (2014) Structural change and the Kaldor facts in a growth model with relative price effects and non-Gorman preferences. Econometrica 82(6):2167–2196
Foellmi R, Zweimüller J (2008) Structural change, Engel's consumption cycles and Kaldor's facts of economic growth. J Monet Econ 55(7):1317–1328
Friedman M (1953) Essays in positive economics. University of Chicago Press, Chicago
Guckenheimer J, Holmes P (1990) Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer, New York
Hale JH (2009) Ordinary differential equations. Dover Publications, Mineola
Herrendorf B, Rogerson R, Valentinyi Á (2014) Growth and structural transformation. In: Aghion P, Durlauf SN (eds) Handbook of economic growth, vol 2B. Elsevier, Amsterdam, pp 855–941
Jackson F, Smith M (eds) (2005) The Oxford handbook of contemporary philosophy. Oxford University Press, New York
Jensen MK (2017) Distributional comparative statics. Rev Econ Stud (forthcoming)
Kongsamut P, Rebelo S, Xie D (2001) Beyond balanced growth. Rev Econ Stud 68(4):869–882
Krüger JJ (2008) Productivity and structural change: a review of the literature. J Econ Surv 22(2):330–363
Maddison A (1995) Monitoring the world economy 1820–1992. OECD Development Centre, Paris
Milgrom P, Roberts J (1994) Comparing equilibria. Am Econ Rev 84(3):441–459
Ngai RL, Pissarides CA (2007) Structural change in a multisector model of growth. Am Econ Rev 97(1):429–443
Reutlinger A, Schurz G, Hüttemann A (2015) Ceteris paribus laws. In: Zalta EN (ed) The Stanford encyclopedia of philosophy, fall 2015 edition. http://plato.stanford.edu/archives/fall2015/entries/ceteris-paribus/
Schettkat R, Yocarini L (2006) The shift to services employment: a review of the literature. Struct Change Econ Dyn 17(2):127–147
Silva EG, Teixeira AAC (2008) Surveying structural change: seminal contributions and a bibliometric account. Struct Change Econ Dyn 19(4):273–300
Stijepic D (2011) Structural change and economic growth: analysis within the partially balanced growth-framework. Südwestdeutscher Verlag für Hochschulschriften, Saarbrücken. An older version is available online: http://deposit.fernuni-hagen.de/2763/
Stijepic D (2015) A geometrical approach to structural change modelling. Struct Change Econ Dyn 33:71–85
Stijepic D (2016) A topological approach to structural change analysis and an application to long-run labor allocation dynamics. MPRA Working Paper No. 74568. Available via MPRA. https://mpra.ub.uni-muenchen.de/74568/1/MPRA_paper_74568.pdf
Teschl G (2012) Ordinary differential equations and dynamical systems. American Mathematical Society, Providence
Uy T, Yi K-M, Zhang J (2013) Structural change in an open economy. J Monet Econ 60(6):667–682
Walter W (1998) Ordinary differential equations. Springer, New York