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High-precision full quaternion based finite-time cascade attitude control strategy considering a class of overactuated space systems

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Abstract

A high-precision full quaternion based finite-time three-axis cascade attitude control strategy is considered in the present research with respect to state-of-the-art to deal with a class of overactuated space systems. The main idea behind the subject is to design a new quaternion based proportional derivative approach, which is realized along with the linear quadratic regulator method. In a word, the control technique proposed here is organized based upon an inner closed loop control to handle the angular rates in the three axes and the corresponding outer closed loop to handle the rotational angles in the same three axes, as well. It aims us to cope with the present complex and complicated systems, in the productive and constructive manner, in a number of programmed space missions such as orbital, communicational, thermal and so on maneuvers. It can be shown that the proposed cascade control strategy is organized in association with a set of pulse-width pulse-frequency modulators to drive a number of on-off reaction thrusters. It should be noted that these ones could significantly be increased w. r. t. the investigated control efforts, in order to provide overall accurate performance of the present space systems. There is currently a control allocation realization to complete the process of the approach presentation and organization. At last, the investigated results are presented in comparison with some potential benchmarks to guarantee and verify the approach performance.

Keywords: High-precision full quaternion based control strategy, Proportional derivative linear quadratic regulator approach, Overactuated space systems, Control allocation, Pulse-width pulse-frequency modulator

Background

With the development of space technologies and with the rapidly growing information available on the related literatures, proposing the new insights in the area of system modeling and control with respect to state-of-the-art are a challenging issue for potential researchers. As is the case, the present research attempts to consider the new solutions regarding a class of overactuated space systems for the purpose of making the new contribution in this area with a focus on system modeling and control. With this purpose, at first, a cascade control strategy including two closed loops is considered to

be designed based upon the full quaternion based three-axis finite-time attitude control approach. It should be noted that the first one as outer closed control loop is realized along with a new quaternion based PD approach, organized based upon the LQR technique as QPDLQR approach to handle the rotational angles in the three axes, while the corresponding inner closed loop control is realized to handle the angular rates in the same three axes for the purpose of driving the present complicated space system, in a better performance. The proposed strategy is investigated in association with a set of PWPF modulators to handle a number of on-off thrusters, where these ones could significantly be increased w. r. t. the resulted control efforts to provide overall accurate system performance. The proposed control technique can now be completed provided that the control allocation is realized to finalize the process of the approach organization.

Regarding the background of the research, in their brief forms, Zheng et al. suggest an autonomous attitude coordinated control for a space system [1]. Yang et al. propose nonlinear attitude tracking control for space system [2]. In the Du et al. research, an attitude synchronization control for a class of flexible space system is proposed to deal with the problem of attitude synchronization for a class of flexible space system [3]. Lu et al. research is to deal with an adaptive attitude tracking control for rigid space system with finite-time convergence [4]. Yang et al. review space system attitude determination and control using quaternion based method [5]. Zou et al. work is presented based upon an adaptive fuzzy fault-tolerant attitude control of space system [6]. Cai et al. work is to deal with the leader-following attitude control of multiple rigid space system systems [7]. Hereinafter, Kuo et al. work is presented in the area of attitude dynamics and control of miniature space system via pseudo-wheels, once Zhang et al. research is given in attitude control of rigid space system with disturbance generated by time varying exo-systems [8, 9]. Katzakis et al. illustrate extending plane-casting for the purpose of dealing with a six-DOF system [10]. Erdong et al. propose robust decentralized attitude coordination control of space system formation [11]. Lu et al. have proposed a design of control approach for rigid space system attitude tracking with actuator saturation, where Pukdeboon et al. have suggested an optimal sliding mode controller for attitude tracking of space system via Lyapunov function [12, 13]. Afterwards, time-varying sliding mode control in the area of rigid space system attitude tracking is presented by Yongqiang et al., while adaptive sliding mode control with its application to six-DOF relative motion of space system under input constraint is given by Wu et al. [14, 15]. Furthermore, the realization of attitude control of space system is presented by Butyrin et al. [16].

Regarding the control allocation research, Johansen et al. present a survey to address this issue [17]. Zaccarian has proposed dynamic allocation for input redundant control systems [18]. Servidia's research is to deal with control allocation for gimbaled/fixed thrusters [19]. Yeh presents an approach to sliding-mode adaptive attitude controller design with its application to space systems with thrusters [20].

As are obvious, the whole of above-referenced investigations along with other related potential ones are all tried to address some efficient methods to deal with this complicated space system. In the same way, the proposed control approach is now made another new effort, while its main differences w. r. t. these considered methods are given in the approach's structure and integration as well as their corresponding results.

The rest of the manuscript is organized as follows: the proposed cascade attitude control strategy is first given in "[The proposed cascade attitude control strategy](#)" section.

The simulation results are then given in “The simulation results” section. Finally, the research concludes in “Conclusion” section.

The proposed cascade attitude control strategy

The schematic diagram of the proposed high-precision control strategy is first illustrated in Fig. 1. This cascade attitude control approach is organized based upon two closed loops including the inner and the outer loops. As are obvious, the inner loop consists of (1) the LPC approach, (2) the PWPF modulator, (3) the CA and finally (4) the dynamics of the space system. Hereinafter, the outer loop consists of (1) the QPDLQR approach and (2) the kinematics of the space system including the QMG, the QMG2QV and finally the QV2RA, respectively. These ones are designed to present the quaternion vector regarding the system under control in the form of three-axis rotational angles to be used in the process of referenced commands tracking. Also, the rest of the modules employed in the strategy consists of the DCM, the 3DRG, the DCM and finally the UADG.

In one such case, the DCM module is realized to convert the referenced commands information from the degree to its radian form, while the iDCM module is correspondingly realized to convert the present information from radian to its degree form. The 3DRG module is also designed to apply to the approach as the desired referenced commands inputs and finally the UADG module is employed to be able to consider the approach performance, in such real situations, in the presence of uncertainties and disturbances. Some of the subsystems are now presented in the proceeding sub-sections.

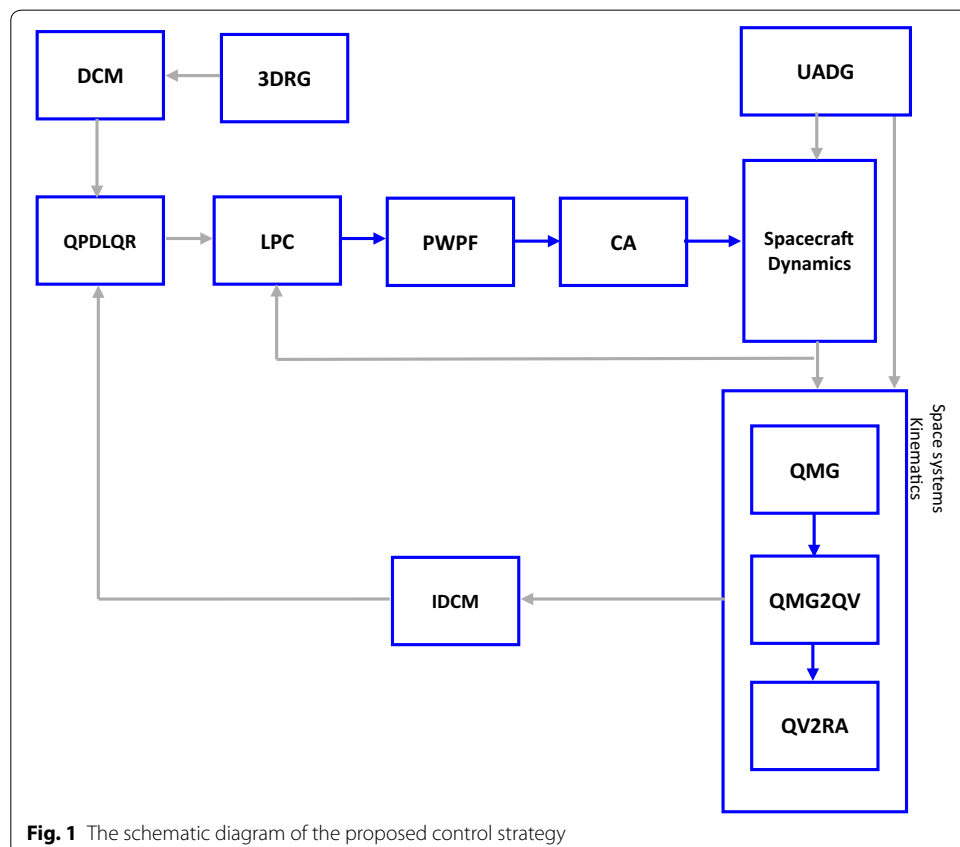


Fig. 1 The schematic diagram of the proposed control strategy

The QPDLQR and LPC approaches

Regarding the QPDLQR approach, the space system under control that is represented in the proceeding subsection can be dealt with in the outer loop to track the rotational angles, i.e. φ_s, θ_s and ψ_s based upon its referenced rotational angles, i.e. φ_r, θ_r and ψ_r , respectively. The whole of control coefficients are acquired via the well-known LQR technique to optimize its performance index. Here, the QPDLQR approach is realized along with the linear state space model of the present system, given by the following

$$\begin{cases} \dot{X} = AX + Bu, & u = \frac{\tau_i}{I_i} \\ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, & B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{cases} \quad (1)$$

where X is taken as the state vector and u is organized based upon τ_i and I_i , i.e. the i th; $i = x, y$ and z -axis torque and the corresponding moments of inertial regarding the same space system. In this way, the performance index is realized as

$$\begin{cases} V = \int_0^{\infty} \left(x_1^2 + x_2^2 + \frac{1}{c^2} u^2 \right) \\ Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & R = \frac{1}{c^2} \end{cases} \quad (2)$$

The three-axis control efforts concerning the QPDLQR approach; $u = -KX = -k_{p_i}(x_1 - x_0) - k_{d_i}x_2$, are designed to optimize the present performance index. In one such case, by supposing P as positive definite matrix, the Riccati equation, i.e. $A^*P + PA - PBR^{-1}B^*P + Q = 0$ can be dealt with to calculate

$$P = \begin{bmatrix} \sqrt{1 + \frac{2}{c}} & \frac{1}{c} \\ \frac{1}{c} & \frac{1}{c} \sqrt{1 + \frac{2}{c}} \end{bmatrix}. \text{ Now, the QPDLQR approach coefficients may be resulted}$$

through $K = R^{-1}B^*P = [c \sqrt{c^2 + 2c}]$. Subsequently, the QPDLQR approach coefficients are resulted through

$$k_{p_i} = c \frac{I_i}{\tau_i}, k_{d_i} = \sqrt{c^2 + 2c} \frac{I_i}{\tau_i}; \quad i = x, y, z \quad (3)$$

Finally, the control efforts concerning the QPDLQR approach are finally rewritten based upon the quaternion errors, i.e. $q_{\mu e}$; $\mu = 1, 2, 3$ in association with the angular rates in the three axes, i.e. ω_{s_i} ; $i = x, y$ and z by the following

$$\begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{pmatrix} -T(k_{px}q_{1e} + k_{dx}\omega_{sx}) \\ -T(k_{py}q_{2e} + k_{dy}\omega_{sy}) \\ -T(k_{pz}q_{3e} + k_{dz}\omega_{sz}) \end{pmatrix} \quad (4)$$

where by using $q_e = q_r q_s$, its expanded form can be written by

$$\begin{bmatrix} q_{1e} \\ q_{2e} \\ q_{3e} \\ q_{4e} \end{bmatrix} = \begin{bmatrix} q_{4r} & q_{3r} & -q_{2r} & -q_{1r} \\ -q_{3r} & q_{4r} & q_{1r} & -q_{2r} \\ q_{2r} & -q_{1r} & q_{4r} & -q_{3r} \\ q_{1r} & q_{2r} & q_{3r} & q_{4r} \end{bmatrix} \begin{bmatrix} q_{1s} \\ q_{2s} \\ q_{3s} \\ q_{4s} \end{bmatrix} \quad (5)$$

Moreover, it is needed to note that the parameter T as the thruster's level is discussed in the proceeding sub-section entitled *The CA scheme realization*. Moreover, regarding the LPC approach, the outcomes are the same as the QPDLQR approach, while the three-axis derivative control terms can completely be ignored to calculate, in its brief form.

The PWPF realization

The PWPF modulator is employed, in so many environments, such as space system. It is realized due to its advantages over other types of modulators. It consists of a first order lag filter along with a Schmitt trigger inside a negative feedback loop. The various modulation methods are used to relate between the level of required torque, the width and the frequency of pulses, due to the fact that reaction control approaches do not possess the linear relationship between the input to the control approach and its output torque. It can be shown that in order to shape the non-linear output of on-off thrusters into linear request output, a set of thruster control methods can be exploited. The most frequently used method is known as the PWPF modulator. Others like Schmitt trigger control, pseudo rate modulator, integrated pulse frequency modulator and pulse width modulator are also realized to shape the output of constant thrusters. A deep consideration can be performed to find the relationships between the static characteristics of the PWPF modulator along with its parameters selection.

The CA scheme realization

The torque in the three axes including τ_x , τ_y and τ_z and the corresponding thruster's level including T_i ; $i = 1, 2, \dots, n$ can first be presented by the following

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} = \mathbf{E}^+ \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{6}$$

In such a case, the relation between \mathbf{E} and \mathbf{E}^+ can easily be presented through $\mathbf{E}^+ = \mathbf{E}^T(\mathbf{E}\mathbf{E}^T)^{-1}$, as well. Now, by supposing the number of thrusters to be eight, the following above-mentioned matrices could be resulted

$$\mathbf{E} = \begin{bmatrix} 0 & 0 & 0 & 0 & -R & -R & R & R \\ -R & 0 & R & 0 & 0 & L & 0 & -L \\ 0 & R & 0 & -R & L & 0 & -L & 0 \end{bmatrix} \tag{7}$$

Here, R and L are taken as thruster's arm and its thruster's length, respectively. Due to the fact that T_i ; $i = 1, 2, \dots, n$ in Eq. (6) are in need of a sequence of binary information, a relay, i.e. $f_{on/off}$ could be realized. In one such case, the produce of binary information for the whole of on-off thrusters are truly guaranteed, although the parameters τ_x , τ_y and τ_z may be changed to τ_{x_e} , τ_{y_e} and τ_{z_e} , namely efficient torques. The relation of the present torques in the three axes and its efficient ones is presented by

$$\begin{bmatrix} \tau_{x_e} \\ \tau_{y_e} \\ \tau_{z_e} \end{bmatrix} = \mathbf{E}f_{on/off} \left(\mathbf{E}^+ \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \right) \tag{8}$$

It should be noted that this $f_{on/off}$ relay hysteresis; ε , could be optimized, in order to present the efficient thrusts in association with the corresponding ones.

The dynamics and kinematics of the space systems

Regarding the dynamics of the space systems, according to the Newton’s second law, the summation of the external moments acting on the body can be equal to the time rate of change of the angular momentum in the inertial frame ($D^I(h_B^{BI}) = m_B$); Now, transferring the rotational time derivative to the body frame B can be written

$$D^I(I_B^B \omega^{BI}) + \Omega^{BI} I_B^B \omega^{BI} = \sum m_B \tag{9}$$

where I_B^B is taken as space system’s moment of inertia, ω^{BI} is taken as space system’s angular rate, relative to the inertial coordinate system and Ω^{BI} is taken as its skew symmetric matrix. Picking body coordinate]^B, the closed-form results can be presented by the following

$$[I_B^B]^B \left[\frac{d\omega^{BI}}{dt} \right]^B + [\Omega^{BI}]^B [I_B^B]^B [\omega^{BI}]^B = \left[\sum m_B \right]^B \tag{10}$$

Now, the quaternion feedback method can be realized in the attitude dynamics, once its time derivative ones are presented as

$$\{\dot{q}\} = \frac{1}{2} \left\{ \begin{array}{cc} 0 & -[\overline{\omega^{BE}}] \\ [\overline{\omega^{BE}}] & -[\overline{\Omega^{BE}}] \end{array} \right\} \{q\} \tag{11}$$

where $\{q\} = \{q_0 [\dot{q}]^T\}^T$ is taken as an attitude quaternion that represents the attitude of the space system, relative to the local-level coordinate system.

Regarding the kinematics of the space systems, the angular rates in the three axes are taken as $p = \omega_{sx}$, $q = \omega_{sy}$, $r = \omega_{sz}$, while $\varphi_s, \theta_s, \psi_s$ are correspondingly taken as the rotational angles (Euler angles). And also τ_i, I_{ii} ; $i = x, y, z$ are taken as the system torque inputs and the moments of inertia, respectively, in the same axes. Subsequently, the following nonlinear state space model of the system is resulted by the following

$$\left\{ \begin{array}{l} \dot{p} = \frac{\tau_x}{I_{xx}} - \frac{(I_{zz} - I_{yy})}{I_{xx}} qr \\ \dot{q} = \frac{\tau_y}{I_{yy}} - \frac{(I_{xx} - I_{zz})}{I_{yy}} pr \\ \dot{r} = \frac{\tau_z}{I_{zz}} - \frac{(I_{yy} - I_{xx})}{I_{zz}} pq \\ \dot{\varphi}_s = p + (\tan \theta_s \sin \varphi_s) q + (\tan \theta_s \cos \varphi_s) r \\ \dot{\theta}_s = (\cos \varphi_s) q - (\sin \varphi_s) r \\ \dot{\psi}_s = \left(\frac{\sin \varphi_s}{\cos \theta_s} \right) q + \left(\frac{\cos \varphi_s}{\cos \theta_s} \right) r \end{array} \right. \tag{12}$$

The simulation results

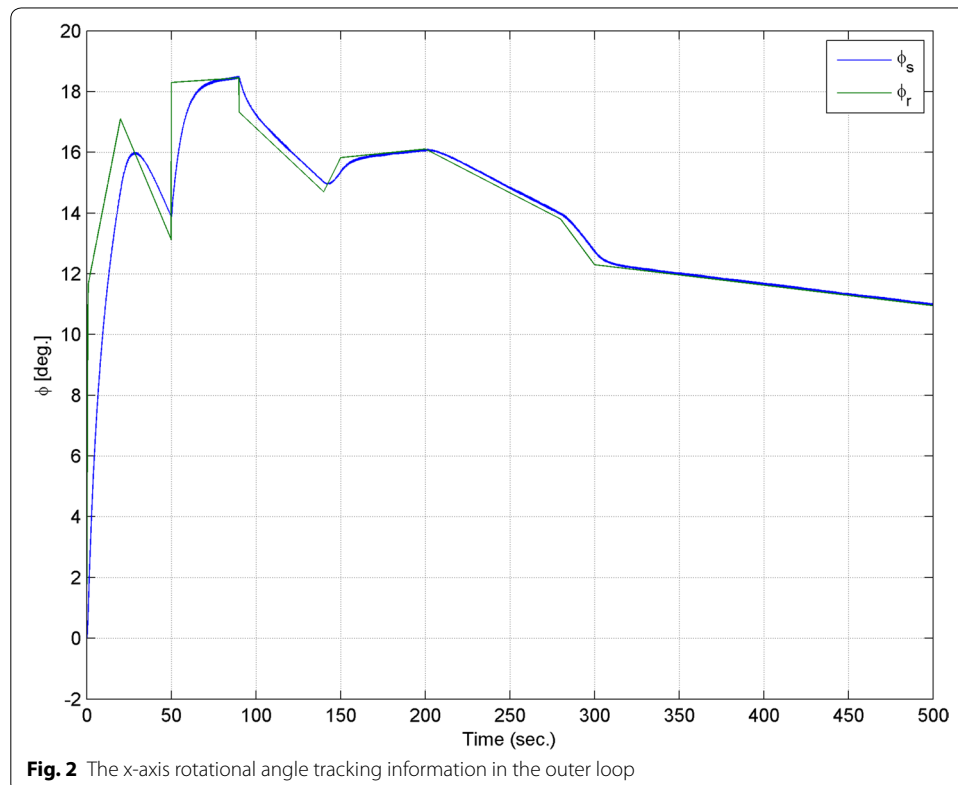
The outcomes acquired through a number of simulation programs are presented in this section to consider the applicability of the strategy investigated here. The information regarding the space system and also both control loops are now tabulated in Table 1.

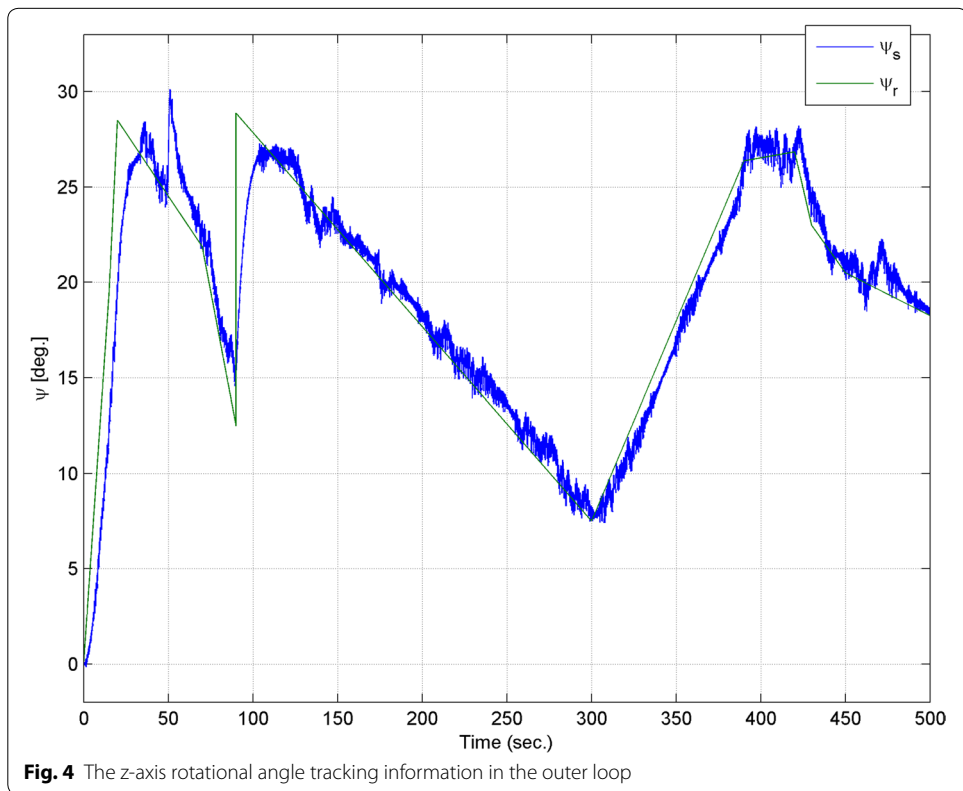
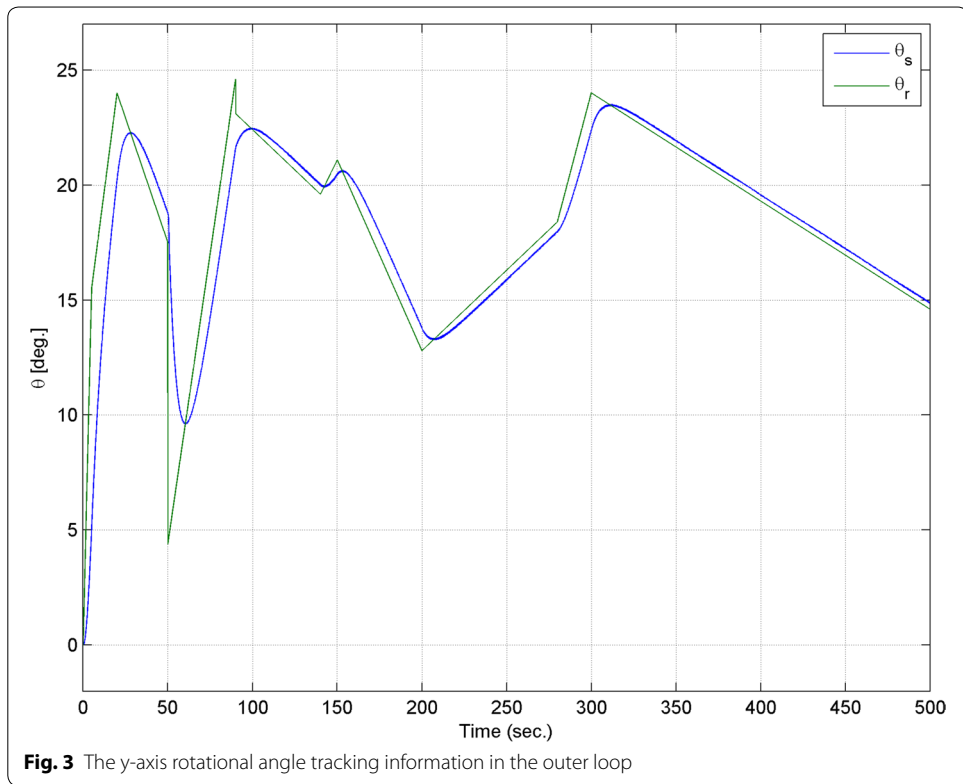
Table 1 The parameters regarding the proposed control strategy

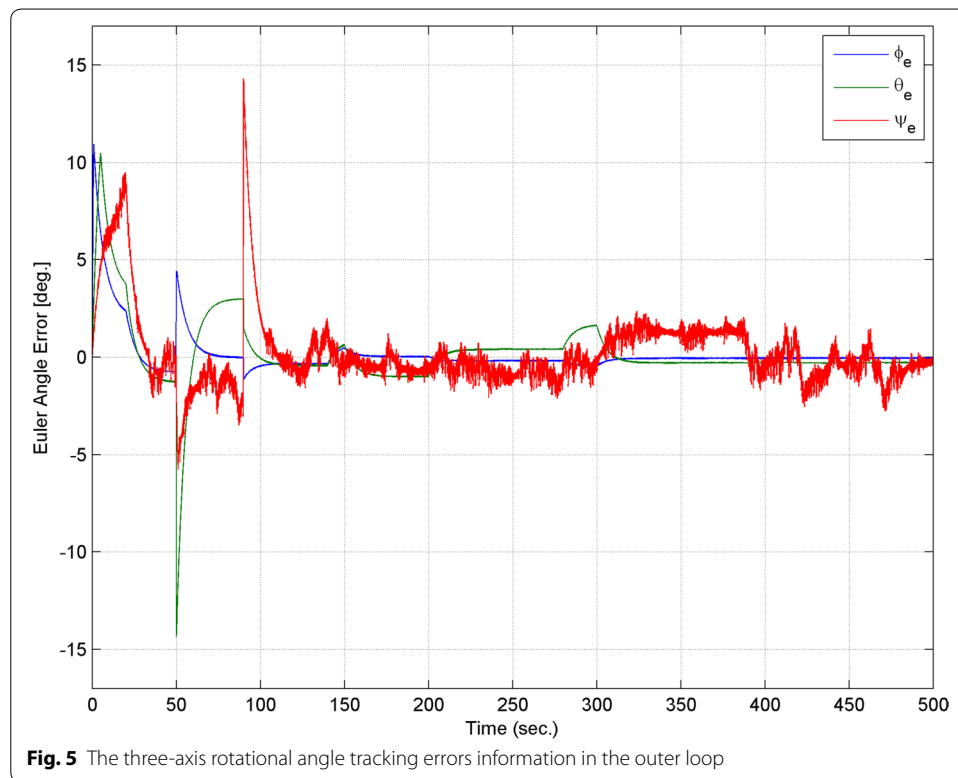
	The parameters	The values
1	Space system moments of inertia	$\begin{cases} I_x = 15.95 \\ I_y = 72.19 \\ I_z = 72.19 \end{cases}$
2	Thruster's level	$T = 15.0$
3	The LPC coefficients in the inner loop	$\begin{cases} k_{px} = 15.0 \\ k_{py} = 15.0 \\ k_{pz} = 15.0 \end{cases}$
4	The QPDLQR coefficients in the outer loop	$\begin{cases} k_{px} = 72.0 \\ k_{py} = 72.0 \\ k_{pz} = 72.0 \\ k_{dx} = 200.0 \\ k_{dy} = 200.0 \\ k_{dz} = 200.0 \end{cases}$
5	Thruster's configurations	$\begin{cases} L = 0.22 \\ R = 0.45 \end{cases}$
6	Relay hysteresis	$\varepsilon = 0.1$

The outer loop results

In such a case, the tracking of three-axis rotational angles are illustrated in Figs. 2, 3 and 4, while the corresponding tracking errors are illustrated in Fig. 5, respectively. In one such case, the initial three-axis attitude of the system is given as 0, 0 and 0 deg., respectively, where the referenced commands are abruptly varied w. r. t. time, respectively. These results indicate that the strategy proposed here is able to control the three-axis







rotational angles at each instant of time, where each one of them is behaved, in its different way.

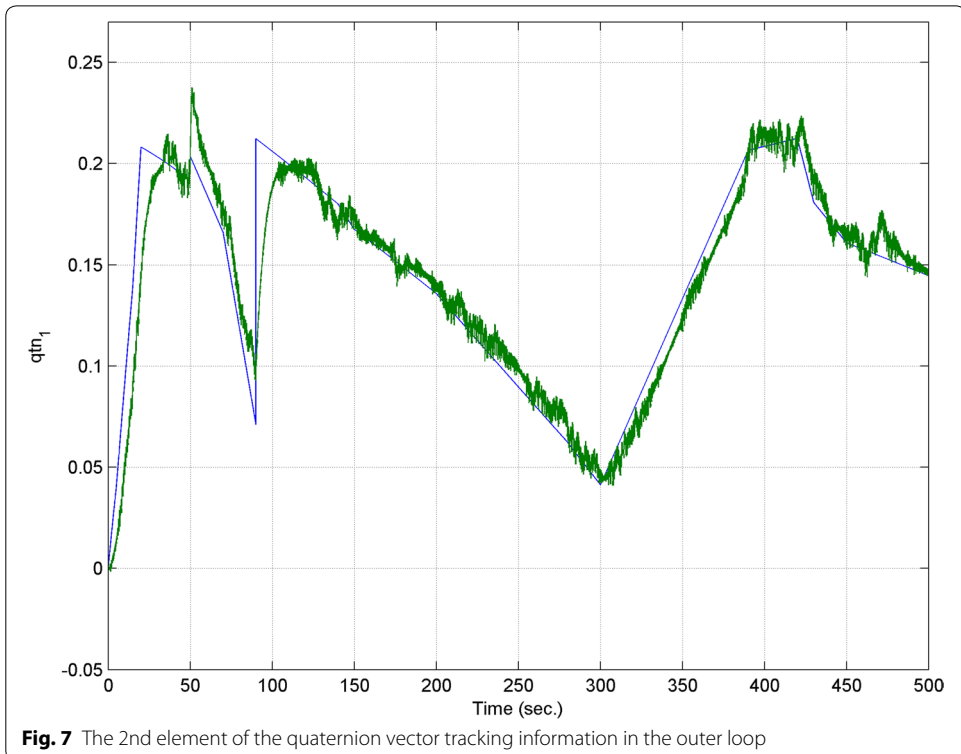
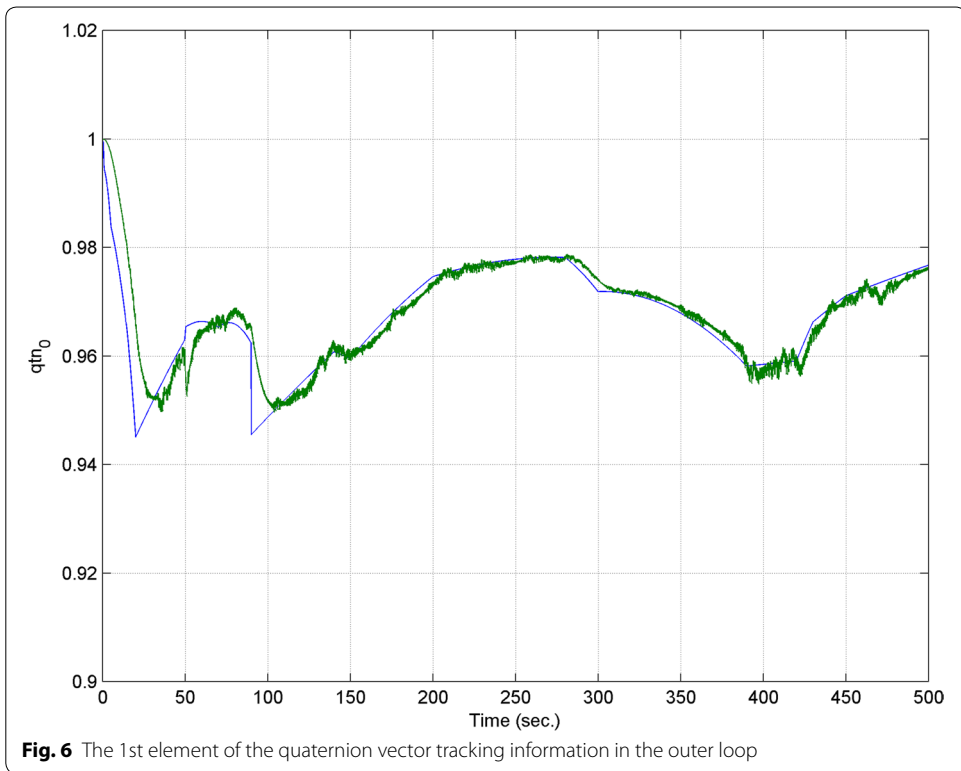
The quaternion vector tracking information is illustrated in Figs. 6, 7, 8 and 9, respectively. The significance of these outcomes is the same as the tracking information illustrated in the three-axis rotational angles, correspondingly.

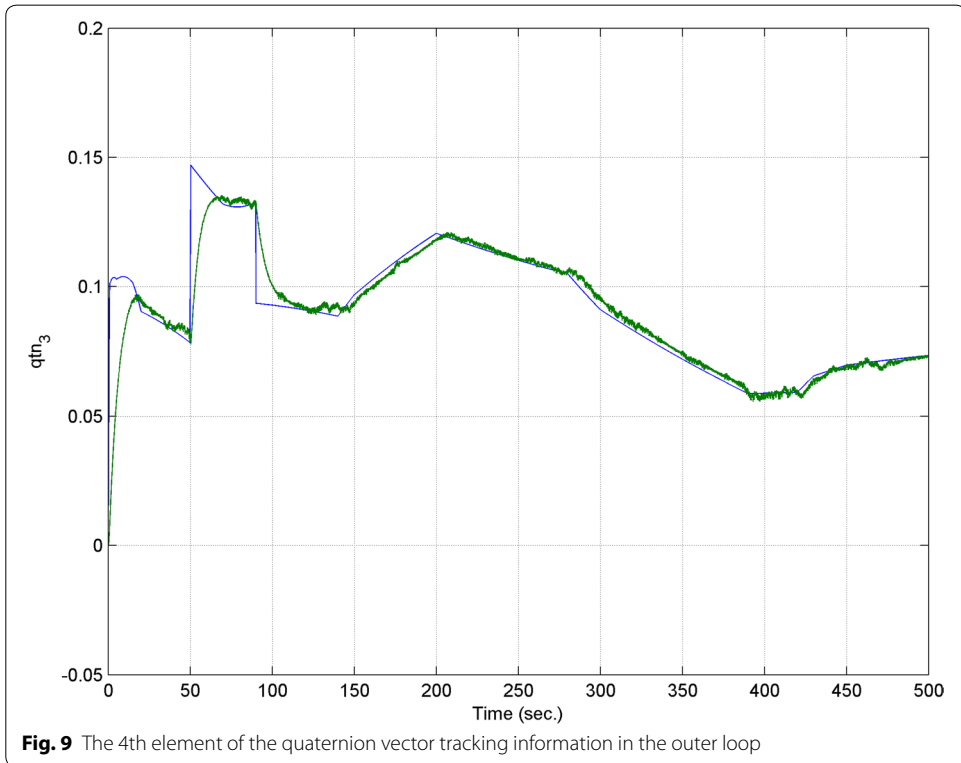
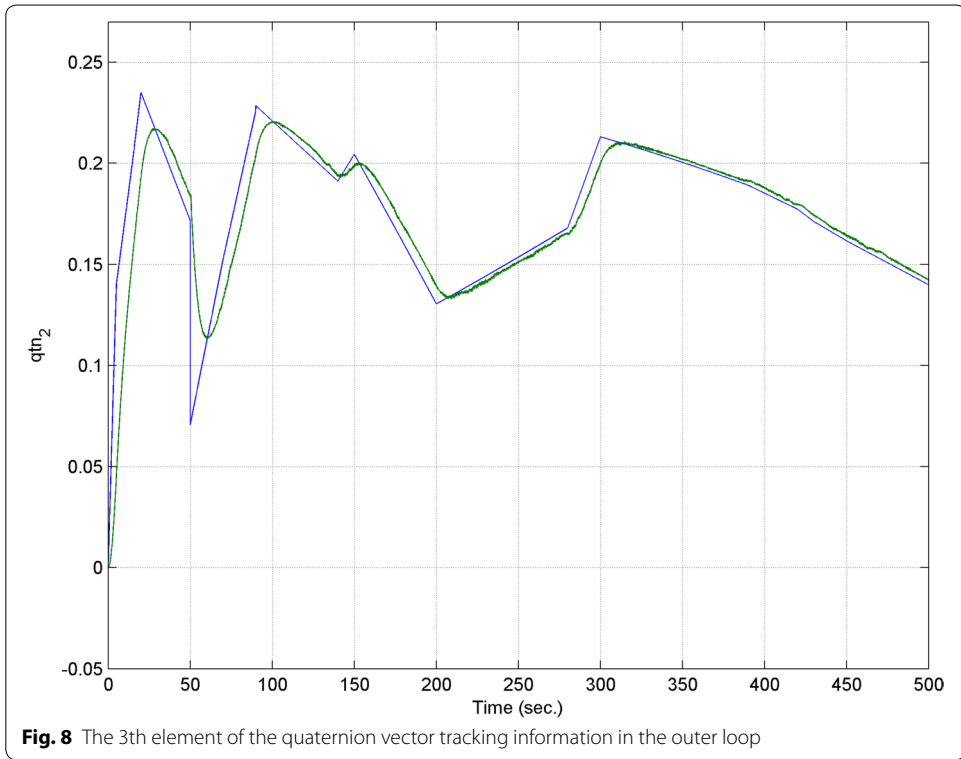
The inner loop results

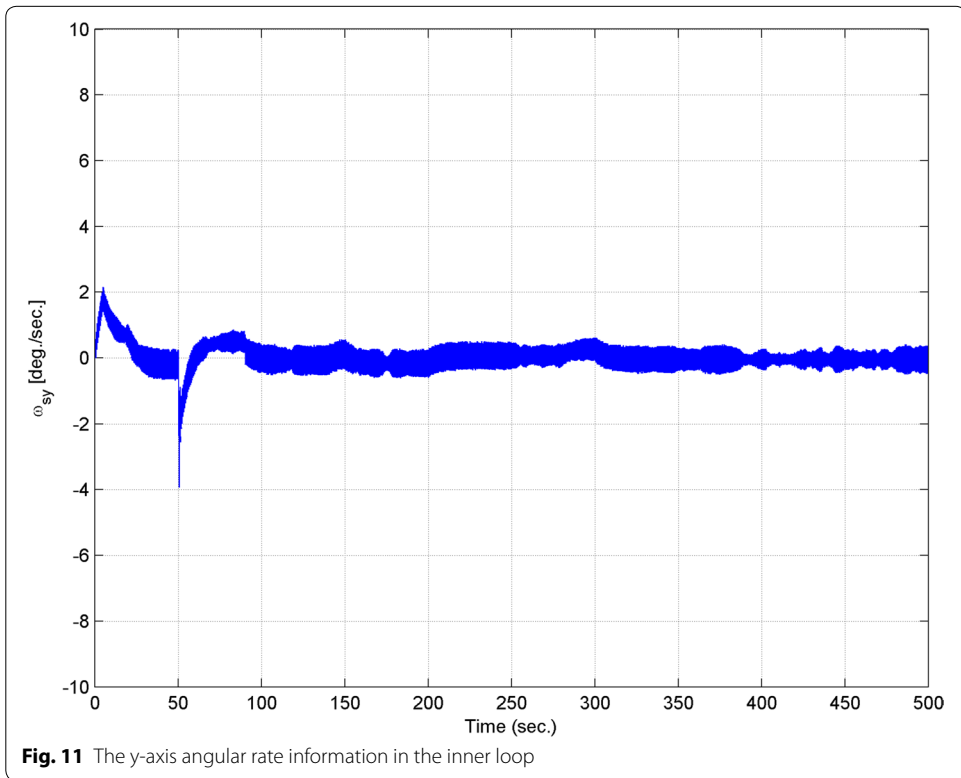
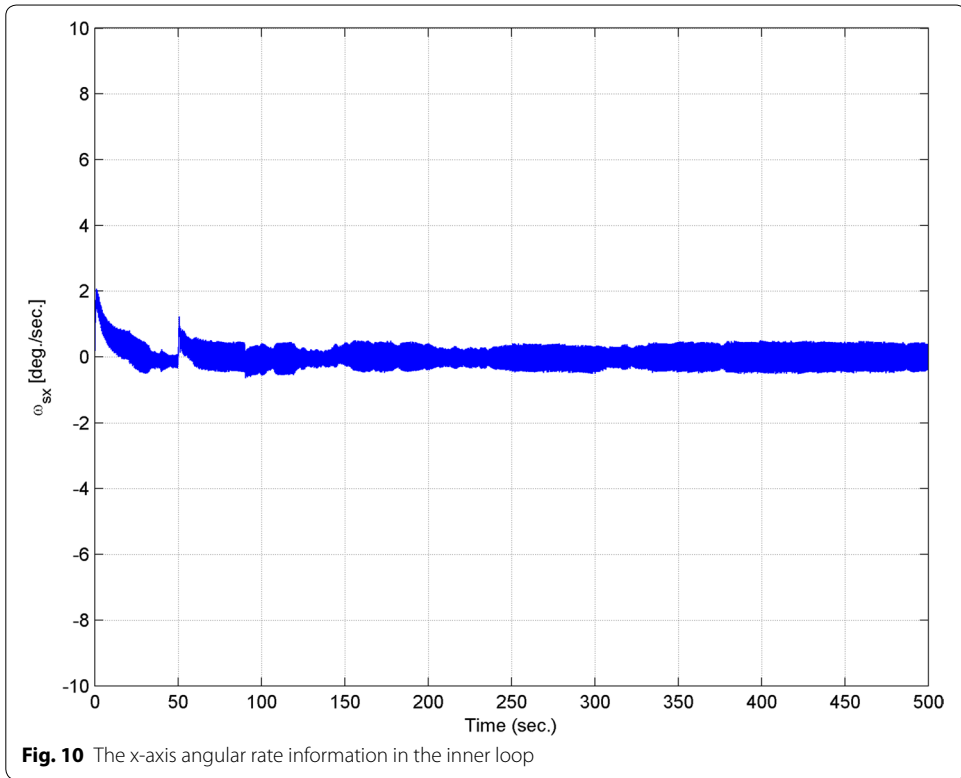
The angular rate information is presented in Figs. 10, 11 and 12, respectively. These results are meaningful versus the three-axis tracking information, illustrated as rotational angles. The outcomes indicate that the strategy proposed here is well behaved to deal with the whole of angular rates in the three-axes to approach to be zero in the small amount of time in correspondence with the three-axis rotational angles.

The verification of the results

The verification of the investigated outcomes is finally analyzed by considering two potential benchmarks, published in recent years. There are the following criteria to be considered in Table 2 including (1) the maximum three-axis rotational angles errors in steady state, (2) the maximum three-axis angular rates errors in steady state and finally (3) the trajectory convergence time. As a deduction matter, the results indicate that the proposed approach is now well behaved in line with both benchmarks concerning the items (1) and (3), while the Butyrin approach is well behaved regarding the item (2), as well.







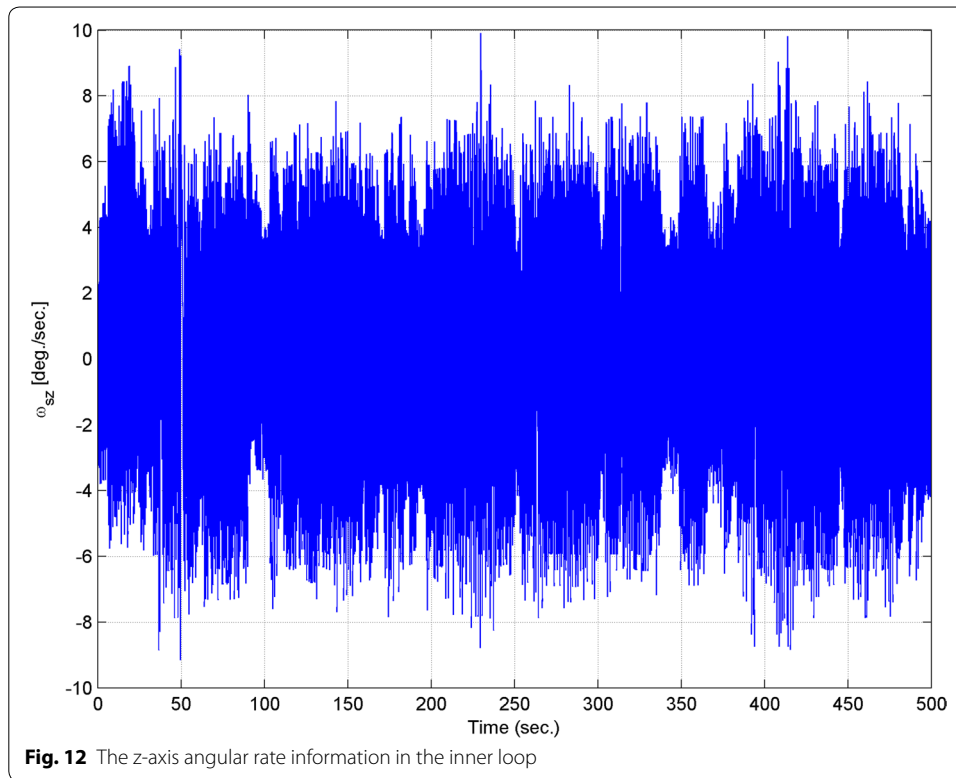


Table 2 The verification of the proposed control strategy performance w. r. t. the corresponding benchmarks

	The approach titles	Maximum three-axis rotational angles errors in steady state (deg)	Maximum three-axis angular rates errors in steady state (deg/s)	Trajectory convergence time (s)
1	The proposed approach	Less than 3	Less than 4	Less than 10
2	The Wu approach [15]	Less than 5	Less than 6	Less than 25
3	The Butyrin approach [16]	Less than 4	Less than 3	Less than 15

Conclusion

The present research addresses the new insights concerning a class of overactuated space systems to make the new contribution in this area with a focus on system modeling and control. It introduces a new high-precision cascade control strategy including the inner and the corresponding outer loops that are handled via the LPC and the QPDLQR approaches, respectively. It is shown that the inner closed loop of the proposed control strategy is designed based upon a set of pulse-width pulse-frequency modulator to deal with a number of on-off thrusters as system actuators for the purpose of handling the rotational angles of the system under control in the three axes. The outer closed control loop of the proposed control strategy is also designed to drive the angular rates in the same three axes for the purpose of dealing with the present complicated space system, in a better performance. The acquired results and the structure of the proposed control strategy are taken into consideration as the state-of-the-art outcomes. Moreover, the investigated results are completely considered to be verified through a number of potential benchmarks, employed in this research. In the sequel,

the research is useful to organize space programmed mission including orbital, communication, thermal and other related ones maneuvers in the real situations.

Abbreviations

QMG: quaternion matrix generation; QMG2QV: quaternion matrix generation conversion to the corresponding quaternion vector; QV2RA: quaternion vector conversion to rotational angles; QPDLQR: quaternion based proportional derivative linear quadratic regulator; PD: proportional derivative; LPC: linear proportional control; PWWF: pulse-width pulse-frequency; CA: control allocation; 3DRG: three-axis desired referenced commands generator; DCM: data conversion module; iDCM: inverse data conversion module; UADG: uncertainties and disturbances generator; DOF: degrees of freedom.

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Compliance with ethical guidelines

Competing interests

The authors declare that they have no competing interests.

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