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An essential remark on fixed point results on multiplicative metric spaces

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Abstract
In this short note, we announce that all the presented fixed point results in the setting of multiplicative metric spaces can be derived from the corresponding existing results in the context of standard metric spaces in the literature.

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Keywords: multiplicative metric; fixed point

1 Introduction and preliminaries

Recently, Bashirov *et al.* [1] announced multiplicative distance as a new distance notion. Following these initial papers, several authors have reported some fixed point results in the framework of multiplicative metric spaces (see *e.g.* [2–7] and related references therein).

Definition 1.1 Let X be a non-empty set. A mapping $d^* : X \times X \to [0, \infty)$ is said to be a multiplicative metric if it satisfies the following conditions:

(i)* $d^*(x, y) = 1$ for all $x, y \in X$,
(ii)* $d^*(x, y) = 1$ if and only if $x = y$,
(iii)* $d^*(x, y) = d^*(y, x)$ for all $x, y \in X$,
(iv)* $d^*(x, z) \leq d^*(x, y) \cdot d^*(y, z)$ for all $x, y, z \in X$ (multiplicative triangle inequality).

Also, (X, d^*) is called a multiplicative metric space.

For the sake of completeness, we shall present the definition of the (standard) metric.

Definition 1.2 Let X be a non-empty set. A mapping $d : X \times X \to [0, \infty)$ is said to be a (standard) metric if it satisfies the following conditions:

(i) $d(x, y) = 1$ for all $x, y \in X$,
(ii) $d(x, y) = 1$ if and only if $x = y$,
(iii) $d(x, y) = d(y, x)$ for all $x, y \in X$,
(iv) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$ (standard triangle inequality).

Also, (X, d) is called a (standard) metric space.

Although the multiplicative metric was announced as a new distance notion, we note that composition of the multiplicative metric with a logarithmic function yields a standard metric. Hence, all fixed point results in the context of multiplicative metric spaces can easily be concluded from the corresponding existing famous fixed point results in the context of the standard metric.

2 Main results

Theorem 2.1 *Let X be a non-empty set. A mapping $d^* : X \times X \to [0,\infty)$ is said to be a multiplicative metric. Then the mapping $d : X \times X \to [0,\infty)$ with $d(x,y) = \ln(d^*(x,y))$ forms a metric.*

Proof By using $d(x,y) = \ln(d^*(x,y))$, the first three assumptions of Definition 1.2 are obtained trivially. Since a logarithmic function is non-decreasing, (iv)* yields

$$\begin{aligned} d(x,y) &= \ln\bigl(d^*(x,z)\bigr) \\ &\leq \ln\bigl(d^*(x,y)\cdot d^*(y,z)\bigr) = \ln\bigl(d^*(x,y)\bigr) + \ln\bigl(d^*(y,z)\bigr) \\ &= d(x,y) + d(y,z). \end{aligned} \tag{2.1}$$

□

It is clear that all topological notions (convergence, Cauchy, completeness) for multiplicative metric space are consequences of the standard topology of metric space.

Abbas *et al.* [7] published the following result.

Theorem 2.2 [7] *Let (X,d^*) be a complete multiplicative metric space and $f : X \to X$. Suppose that*

$$\psi\bigl(d^*(fx,fy)\bigr) \leq \frac{\psi(M^f_{d^*}(x,y))}{\varphi(M^f_{d^*}(x,y))} \tag{2.2}$$

for any $x,y \in X$, where

$$M^f_{d^*}(x,y) = \bigl\{d^*(x,y), d^*(fx,x), d^*(y,fy), \bigl(d^*(fx,y)d^*(x,fy)\bigr)^{\frac{1}{2}}\bigr\} \tag{2.3}$$

and $\psi : [1,\infty) \to [1,\infty)$ is continuous, non-decreasing, $\psi^{-1}(\{1\}) = \{1\}$, and $\varphi : [1,\infty) \to [1,\infty)$ is lower semi-continuous and $\varphi^{-1}(\{1\}) = \{1\}$. Then f has a unique fixed point.

Dorić [8] reported the following extension of the Banach contraction principle.

Theorem 2.3 *Let (X,d) be a complete metric space and let $f : X \to X$ be a mapping such that for each pair of points $x,y \in X$,*

$$\psi\bigl(d(fx,fy)\bigr) \leq \psi\bigl(M^f(x,y)\bigr) - \varphi\bigl(M^f(x,y)\bigr), \tag{2.4}$$

where

$$M^f(x,y) = \left\{d(x,y), d(fx,x), d(y,fy), \frac{1}{2}\bigl[d(fx,y) + d(x,fy)\bigr]\right\} \tag{2.5}$$

and $\psi : [0,\infty) \to [0,\infty)$ is continuous, non-decreasing, $\psi^{-1}(\{0\}) = \{0\}$, and $\varphi : [0,\infty) \to [0,\infty)$ is lower semi-continuous and $\varphi^{-1}(\{0\}) = \{0\}$. Then F has a unique fixed point.

Theorem 2.4 *Theorem* 2.2 *is a consequence of Theorem* 2.3.

Proof By using $d(x,y) = \ln(d^*(x,y))$, we easily see that equation (2.3) yields (2.5). Hence, the inequalities (2.2) implies (2.4). Consequently, Theorem 2.3 provides the existence and uniqueness of the fixed point of f. □

It is clear that one can easily derive the other fixed results in [2–7] from the relevant existing results in the literature. Regarding the analogy, we shall not list the other results.

3 Conclusion

Some authors misuse the notion of the multiplicative calculus since they misunderstand the place and role of this calculus like other non-Newtonian calculuses. Indeed, it represents the same system of knowledge, only different by the presentation of them with respect to so-called reference function. Notice that in Newtonian calculus, the reference function is linear, whereas the reference function for multiplicative calculus is exponential. Consequently, every definition and also every theorem of Newtonian calculus has an analog in multiplicative calculus and vice versa. Therefore, ordinary and multiplicative fixed point theorems are applicable to the same class of functions. In this paper, we only underline these facts in the framework of fixed point theory. It would be possible to approach the problem globally by the use of the preceding discussion.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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