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The influence of partial closure for the populations to a non-selective harvesting Lotka–Volterra discrete amensalism model

Qianqian Su[1*] and Fengde Chen[2]

*Correspondence:
1051818898@qq.com
[1]General Education Center, Zhengzhou Business University, Gongyi, P.R. China
Full list of author information is available at the end of the article

Abstract

In this paper, a non-selective harvesting Lotka–Volterra amensalism discrete model incorporating partial closure for the populations is proposed and studied. By applying the relevant conclusions of difference inequality and some calculation technique, sufficient conditions are obtained to ensure the permanence and extinction of the system. By constructing a suitable Lyapunov function, sufficient conditions that ensure the global attractivity of the system are obtained. Finally, numerical simulations show the feasibility of our results.

Keywords: Amensalism; Harvesting; Permanence; Extinction; Globally attractive

1 Introduction

During the last decade, the study of dynamic behaviors of the amensalism model has become one of the most important research topics, see [1–12]; here, amensalism means that a species inflicts harm on other species without any costs or benefits received by the other. Such topics as the stability of the equilibrium [1, 3–5, 8], the existence of the positive periodic solution [2, 9, 11], the extinction of the species [8, 10], the influence of the cover [8, 12], the influence of the functional response [10], etc. have been extensively studied. Recently, Xiong et al. [1] proposed the following amensalism model:

$$\begin{cases} \frac{dN_1}{dt} = r_1 N_1 (1 - \frac{N_1}{P_1} - u\frac{N_2}{P_1}), \\ \frac{dN_2}{dt} = r_2 N_2 (1 - \frac{N_2}{P_2}), \end{cases} \tag{1.1}$$

where u, r_i, P_i, $i = 1, 2$, are all positive constants. They investigated the local stability property of the equilibria of system (1.1).

On the other hand, as was pointed out by Chakraborty et al. [13], the study of resource management, including fisheries, forestry, and wildlife management, is very important. They argued that it is necessary to harvest the population, but harvesting should be regulated so that both the ecological sustainability and conservation of the species can be implemented in a long run. Already, they proposed a non-selective harvesting predator-prey system incorporating partial closure for the populations, they investigated the local and global stability property of the system, and some interesting results related to the optimal harvesting were obtained. Recently, Chen [3] proposed the following non-selective harvesting Lotka–Volterra amensalism model incorporating partial closure for the populations:

$$\begin{cases} \frac{dN_1}{dt} = r_1 N_1 (1 - \frac{N_1}{P_1} - u\frac{N_2}{P_1}) - q_1 EmN_1, \\ \frac{dN_2}{dt} = r_2 N_2 (1 - \frac{N_2}{P_2}) - q_2 EmN_2. \end{cases} \tag{1.2}$$

They investigated the local and global stability of the boundary and interior equilibria. They proved that depending on the fraction of the stock available for harvesting, the system maybe extinction, partial survival, or two species may coexist in a stable state.

As we all know, though most dynamic behaviors of population models are based on the continuous models governed by differential equations, the discrete time models governed by difference equation are more appropriate than the continuous ones when the size of the population is rarely small or the population has non-overlapping generations. It has been found that the dynamic behaviors of the discrete system is rather complex and contains richer dynamics than the continuous ones [14]. Recently, more and more scholars pay attention to studying the discrete population models (see [14–19] and the references cited therein).

However, to the best of our knowledge, to this day, seldom did scholars propose and consider the influence of harvesting on the discrete amensalism model. This motivates us to propose and study the discrete system of (1.2). The aim of this paper is to investigate the permanence, extinction, and global attractivity of the following system:

$$\begin{cases} x_1(n+1) = x_1(n)\exp\{r_1(n)(1 - \frac{x_1(n)}{p_1(n)} - \frac{\mu(n)}{p_1(n)}x_2(n)) - q_1(n)Em\}, \\ x_2(n+1) = x_2(n)\exp\{r_2(n)(1 - \frac{x_2(n)}{p_2(n)}) - q_2(n)Em\}, \end{cases} \tag{1.3}$$

where $x_1(n)$, $x_2(n)$ denote the population densities of the two species at any time n. $r_i(p_i)$ represents the intrinsic growth rate (environmental carrying capacity) of the ith species, q_i is the catchability co-efficient of the two species. E is the combined fishing effort used to harvest, and m ($0 < m < 1$) is the fraction of the stock available for harvesting. One could refer to [1, 13, 20] for more background and the adjustment of system (1.3). Throughout this paper, we assume that $\{\mu(n)\}$, $\{r_i(n)\}$, $\{p_i(n)\}$, $\{q_i(n)\}$ are bounded non-negative almost sequences such that

$$\begin{aligned} &0 < \mu^l \le \mu(n) \le \mu^u, \qquad 0 < r_i^l \le r_i(n) \le r_i^u, \\ &0 < p_i^l \le p_i(n) \le p_i^u, \qquad 0 < q_i^l \le q_i(n) \le q_i^u, \quad i = 1, 2. \end{aligned} \tag{H}$$

Here, for any bounded sequence $\{a(n)\}$, $a^u = \sup_{n\in N}\{a(n)\}$, $a^l = \inf_{n\in N}\{a(n)\}$.

From the point of view of biology, we assumed that $x_i(0) > 0$, ($i = 1, 2$). Then it is easy to see that the solutions of (1.3) with the above initial condition remain positive for all $n \in N^+ = \{0, 1, 2, \ldots\}$.

The organization of this paper is as follows. In Sect. 2, we give some useful lemmas. Sufficient conditions for the permanence and extinction of (1.3) are given in Sect. 3 and Sect. 4. Then, in Sect. 5, we establish sufficient conditions for the global attractivity of (1.3). Some examples together with their numeric simulations are presented in Sect. 6. We end this paper with a brief discussion.

2 Preliminaries

In this section, we will introduce several useful lemmas.

Lemma 2.1 ([21]) *Assume that* $\{x(k)\}$ *satisfies* $x(k) > 0$ *and*

$$x(k+1) \le x(k)\exp\bigl\{a(k) - b(k)x(k)\bigr\}$$

for $k \in N$, *where* $a(k)$ *and* $b(k)$ *are non-negative sequences bounded above and below by positive constants. Then*

$$\limsup_{k\to+\infty} x(k) \le \frac{1}{b^l}\exp\bigl(a^u - 1\bigr).$$

Lemma 2.2 ([22]) *Assume that* $\{x(k)\}$ *satisfies*

$$x(k+1) \ge x(k)\exp\bigl\{a(k) - b(k)x(k)\bigr\}, k \ge N_0,$$

$\limsup_{k\to+\infty} x(k) \le x^*$ *and* $x(N_0) > 0$, *where* $a(k)$ *and* $b(k)$ *are non-negative sequences bounded above and below by positive constants and* $N_0 \in N$. *Then*

$$\liminf_{k\to+\infty} x(k) \ge \min\left\{\frac{a^l}{b^u}\exp\bigl(a^l - b^u x^*\bigr), \frac{a^l}{b^u}\right\}.$$

3 Permanence

Theorem 3.1 *Assume that*

$$m < \min\left\{\frac{r_1^l p_1^l - r_1^u \mu^u M_2}{q_1^u p_1^l E}, \frac{r_2^l}{q_2^u E}\right\}. \tag{H_1}$$

Then system (1.3) *is permanent.*

Proof From the equations of system (1.3), it follows that

$$x_i(n+1) \le x_i(n)\exp\left\{r_i(n) - \frac{r_i(n)x_i(n)}{p_i(n)}\right\}, \quad (i = 1, 2).$$

It follows from Lemma 2.1 that

$$\limsup_{n\to+\infty} x_i(n) \le \frac{p_i^u}{r_i^l}\exp\bigl(r_i^u - 1\bigr) \triangleq M_i \quad (i = 1, 2).$$

So, for small enough $\varepsilon > 0$, there exists $n_1 > 0$, for all $n > n_1$, we have

$$\begin{aligned} &r_1^l p_1^l - r_1^u \mu^u (M_2 + \varepsilon) > q_1^u p_1^l E m, \\ &x_2(n) \le M_2 + \varepsilon, \qquad r_2^l - q_2^u E m > 0. \end{aligned}$$

Then, for $n > n_1$, we have

$$x_1(n+1) \ge x_1(n)\exp\left\{r_1(n) - \frac{r_1(n)}{p_1(n)}x_1(n) - \frac{\mu(n)r_1(n)}{p_1(n)}(M_2 + \varepsilon) - q_1^u E m\right\}.$$

From Lemma 2.2 and letting $\varepsilon \to 0$, we have

$$\liminf_{n\to+\infty} x_1(n) \geq m_1,$$

where

$$m_1 = \min\left\{\Delta \exp\left\{r_1^l - \frac{r_1^u \mu^u M_2}{p_1^l} - q_1^u Em - \frac{r_1^u M_1}{p_1^l}\right\}, \Delta\right\},$$
$$\Delta = \frac{r_1^l p_1^l - \mu^u r_1^u M_2 - p_1^l q_1^u Em}{r_1^u}.$$

From the second equation of system (1.3) it follows that

$$x_2(n+1) \geq x_2(n) \exp\left\{r_2^l - \frac{r_2^u}{p_2^l} x_2(n) - q_2^u Em\right\}.$$

From Lemma 2.2 we have

$$\liminf_{n\to+\infty} x_2(n) \geq m_2,$$

where

$$m_2 = \min\left\{\frac{r_2^l p_2^l - p_2^l q_2^u Em}{r_2^u} \exp\left\{r_2^l - q_2^u Em - \frac{r_2^u M_2}{p_2^l}\right\}, \frac{r_2^l p_2^l - p_2^l q_2^u Em}{r_2^u}\right\}.$$

So the proof of Theorem 3.1 is completed. □

4 Extinction

Theorem 4.1 *Assume that*

$$m > \max\left\{\frac{r_1^u}{q_1^l E}, \frac{r_2^u}{q_2^l E}\right\}, \tag{H_2}$$

let $(x_1(n), x_2(n))^T$ *be any positive solution of system* (1.3), *then*

$$\lim_{n\to\infty} x_i(n) = 0, \quad i = 1, 2.$$

Proof From (1.3) we have

$$x_i(n+1) \leq x_i(n) \exp\left\{r_i^u - q_i^l Em\right\}, \quad i = 1, 2. \tag{4.1}$$

By using (4.1), we get

$$\prod_{p=0}^{n-1} x_i(p+1) \leq \prod_{p=0}^{n-1} x_i(p) \exp\left\{r_i^u - q_i^l Em\right\}, \quad i = 1, 2.$$

That is,

$$x_i(n) \leq x_i(0) \exp\left\{n\left(r_i^u - q_i^l Em\right)\right\}, \quad i = 1, 2.$$

For $x_i(0) > 0$, $i = 1,2$. So it is easy to get that $x_i(n) > 0$, $i = 1,2$. Since (H_2) holds, we have

$$\lim_{n\to\infty} x_i(n) = 0, \quad i = 1,2.$$

The proof of Theorem 4.1 is completed. □

In this section, we will use the analysis technique of [14].

Theorem 4.2 *Assume that*

$$\begin{aligned} &r_2^l - q_2^u Em > 0,\\ &r_1^u - q_1^l Em > 0,\\ &\frac{r_1^u - q_1^l Em}{r_2^l - q_2^u Em} < \frac{r_1^l \mu^l p_2^l}{r_2^u p_1^u} \end{aligned} \tag{H_3}$$

holds, let $(x_1(n), x_2(n))^T$ be any positive solution of system (1.3), then the species x_2 is permanent, while x_1 will be driven to extinction.

Proof By (H_3) we can choose positive constants α and β such that

$$\frac{r_1^u - q_1^l Em}{r_2^l - q_2^u Em} < \frac{\beta}{\alpha} < \frac{r_1^l \mu^l p_2^l}{r_2^u p_1^u}. \tag{4.2}$$

Thus

$$\begin{aligned} &\alpha\left(r_1^u - q_1^l Em\right) < \beta\left(r_2^l - q_2^u Em\right),\\ &\frac{\beta r_2^u}{p_2^l} < \frac{\alpha r_1^l \mu^l}{p_1^u}, \end{aligned} \tag{4.3}$$

and there exists $\delta > 0$

$$\beta\left(r_2^l - q_2^u Em\right) - \alpha\left(r_1^u - q_1^l Em\right) > \delta > 0. \tag{4.4}$$

Let $(x_1(n), x_2(n))^T$ be any positive solution of system (1.3). For any $k \in N$, we can get

$$\begin{aligned} \ln\frac{x_1(k+1)}{x_1(k)} &= r_1(k)\left(1 - \frac{x_1(k)}{p_1(k)} - \frac{\mu(k)}{p_1(k)}x_2(k)\right) - q_1(k)Em\\ &\le r_1^u - \frac{r_1^l}{p_1^u}x_1(k) - \frac{r_1^l\mu^l}{p_1^u}x_2(k) - q_1^l Em, \end{aligned} \tag{4.5}$$

$$\begin{aligned} \ln\frac{x_2(k+1)}{x_2(k)} &= r_2(k)\left(1 - \frac{x_2(k)}{p_2(k)}\right) - q_2(k)Em\\ &\ge r_2^l - \frac{r_2^u}{p_2^l}x_2(k) - q_2^u Em. \end{aligned} \tag{4.6}$$

Then, inequalities (4.3)–(4.6) lead to

$$\begin{aligned}
&\alpha \ln \frac{x_1(k+1)}{x_1(k)} - \beta \ln \frac{x_2(k+1)}{x_2(k)} \\
&\quad \le \alpha\bigl(r_1^u - q_1^l Em\bigr) - \alpha \frac{r_1^l}{p_1^u} x_1(k) - \alpha \frac{r_1^l \mu^l}{p_1^u} x_2(k) - \beta\bigl(r_2^l - q_2^u Em\bigr) + \beta \frac{r_2^u}{p_2^l} x_2(k) \\
&\quad = \bigl[\alpha\bigl(r_1^u - q_1^l Em\bigr) - \beta\bigl(r_2^l - q_2^u Em\bigr)\bigr] + \left(\frac{\beta r_2^u}{p_2^l} - \frac{\alpha r_1^l \mu^l}{p_1^u}\right) x_2(k) - \alpha \frac{r_1^l}{p_1^u} x_1(k) \\
&\quad < -\delta \\
&\quad < 0. \qquad (4.7)
\end{aligned}$$

Summating both sides of (4.7) from 0 to $n-1$, we obtain

$$\alpha \ln \frac{x_1(n)}{x_1(0)} - \beta \ln \frac{x_2(n)}{x_2(0)} < -n\delta. \qquad (4.8)$$

Then

$$x_1(n) < x_1(0) \left(\frac{x_2(n)}{x_2(0)}\right)^{\frac{\beta}{\alpha}} \exp\left\{-\frac{n}{\alpha}\delta\right\}. \qquad (4.9)$$

Theorem 3.1 implies that $x_2(n)$ is bounded eventually. Then the above inequality (4.9) shows that $\lim_{n\to\infty} x_1(n) = 0$. Since $r_2^l - q_2^u Em > 0$, then the species x_2 is permanent.

The proof of Theorem 4.2 is completed. □

5 Globally attractive

Theorem 5.1 *Assume that* $m > \frac{r_2^u}{q_2^l E}$ (H_4) *holds and there exists a positive constant* $\eta > 0$ *such that*

$$\min\left\{\frac{r_1^l}{p_1^u}, \frac{2}{M_1} - \frac{r_1^u}{p_1^l}\right\} > \eta \qquad (H_5)$$

holds, then species x_1 *is globally attractive while* x_2 *will be driven to extinction.*

Proof Suppose that $(x_1(n), x_2(n))^T$, $(x_1^*(n), x_2^*(n))^T$ are any two positive solutions of system (1.3). Under the assumption condition (H_4), it follows from Theorem 4.1 that $\lim_{n\to+\infty} x_2(n) = 0$. Since $\limsup_{n\to\infty} x_1(n) < M_1$, then for small enough $\varepsilon > 0$, there exists $N_0 > 0$, for all $n > N_0$, we have

$$x_1(n) < M_1 + \varepsilon, \qquad x_2(n) < \varepsilon, \qquad \min\left\{\frac{r_1^l}{p_1^u}, \frac{2}{M_1+\varepsilon} - \frac{r_1^u}{p_1^l}\right\} > \eta.$$

To end the proof of Theorem 5.1, it is enough to show that $\lim_{n\to+\infty}(x_1(n) - x_1^*(n)) = 0$.

Let $V(n) = |\ln x_1(n) - \ln x_1^*(n)|$. From (1.3) we have

$$\begin{aligned}
\bigl|\ln x_1(n+1) - \ln x_1^*(n+1)\bigr| = \biggl|&\ln x_1(n) - \ln x_1^*(n) - \frac{r_1(n)}{p_1(n)}\bigl(x_1(n) - x_1^*(n)\bigr) \\
&- \frac{\mu(n) r_1(n)}{p_1(n)}\bigl(x_2(n) - x_2^*(n)\bigr)\biggr|
\end{aligned}$$

$$\begin{aligned}&\leq \left|\ln x_1(n) - \ln x_1^*(n) - \frac{r_1(n)}{p_1(n)}\big(x_1(n) - x_1^*(n)\big)\right| \\ &\quad + \frac{\mu(n) r_1(n)}{p_1(n)}\big|x_2(n) - x_2^*(n)\big|.\end{aligned}$$

Since $\ln x_1(n) - \ln x_1^*(n) = \frac{1}{\xi_1(n)}(x_1(n) - x_1^*(n))$, where $\min\{x_1(n), x_1^*(n)\} \leq \xi_1(n) \leq \max\{x_1(n), x_1^*(n)\} \leq M_1 + \varepsilon$. So we can get

$$\begin{aligned}\Delta V(n) &\leq -\left(\frac{1}{\xi_1(n)} - \left|\frac{1}{\xi_1(n)} - \frac{r_1(n)}{p_1(n)}\right|\right)\big|x_1(n) - x_1^*(n)\big| \\ &\quad + \frac{\mu^u r_1^u}{p_1^l}\big|x_2(n) - x_2^*(n)\big| \\ &\leq -\min\left\{\frac{r_1^l}{p_1^u}, \frac{2}{M_1 + \varepsilon} - \frac{r_1^u}{p_1^l}\right\}\big|x_1(n) - x_1^*(n)\big| + 2\frac{\mu^u r_1^u}{p_1^l}\varepsilon \\ &\leq -\eta\big|x_1(n) - x_1^*(n)\big| + 2\frac{\mu^u r_1^u}{p_1^l}\varepsilon.\end{aligned}$$

Letting $\varepsilon \to 0$, it follows that

$$\Delta V(n) \leq -\eta\big|x_1(n) - x_1^*(n)\big|,$$

then

$$\sum_{p=N_0}^{n}\big(V(p+1) - V(p)\big) \leq -\eta\sum_{p=N_0}^{n}\big|x_1(p) - x_1^*(p)\big|,$$

that is,

$$V(n+1) - V(N_0) \leq -\eta\sum_{p=N_0}^{n}\big|x_1(p) - x_1^*(p)\big|,$$

therefore

$$\sum_{p=N_0}^{n}\big|x_1(p) - x_1^*(p)\big| \leq \frac{V(N_0)}{\eta} < +\infty.$$

So it is easy to know that $\lim_{n\to+\infty}(x_1(n) - x_1^*(n)) = 0$.

The proof of Theorem 5.1 is completed. □

Similarly, we can get the following theorem.

Theorem 5.2 *Assume that there exists a positive constant $\gamma > 0$ such that*

$$\min\left\{\frac{r_2^l}{p_2^u}, \frac{2}{M_2} - \frac{r_2^u}{p_2^l}\right\} > \gamma \qquad (H_6)$$

holds, then the species x_2 is globally attractive.

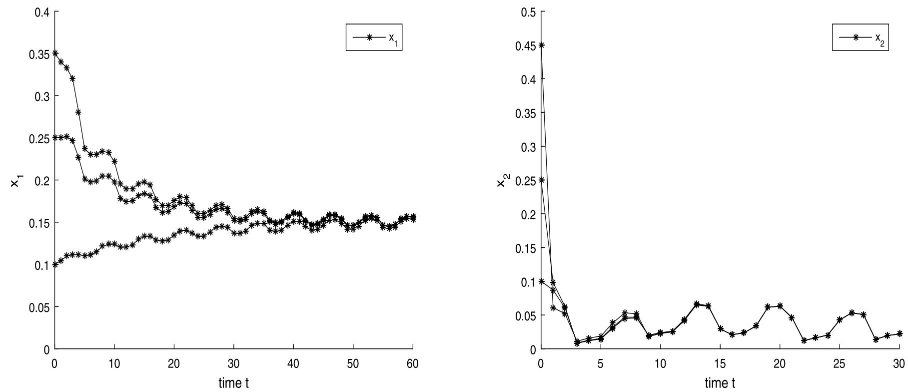

Figure 1 Dynamic behaviors of system (6.1) with initial condition $(x_1(t), x_2(t))^T = (0.1, 0.45)^T, (0.25, 0.25)^T, (0.35, 0.1)^T$

6 Examples and numeric simulations

The following examples lend credence to the plausibility of the main results.

Example 6.1 Corresponding to system (1.3), we assume that

$$\begin{aligned} &r_1(n) = 0.05(\sin^2 n + 1), \qquad p_1(n) = 0.1(\sin n + 2), \\ &q_1(n) = 0.05(\cos^2 n + 1), \qquad r_2(n) = 0.8\cos^2 n + 0.2, \\ &p_2(n) = 0.04(\cos n + 1.5), \qquad q_2(n) = 2(\cos n + 1.5), \\ &m = 0.1, \qquad \mu(n) = 0.05, \qquad E = 1. \end{aligned} \tag{6.1}$$

It is easy to see that $\frac{r_2^l}{q_2^u E} = 0.4$, $M_2 = \frac{p_2^u}{r_2^l}\exp(r_2^u - 1) = 0.5$, $\frac{r_1^l p_1^l - r_1^u \mu^u M_2}{q_1^u p_1^l E} = 0.25$, $m = 0.1 < \min\{0.4, 0.25\}$. Then the conditions of Theorem 3.1 are satisfied (see Fig. 1).

Example 6.2 Corresponding to system (1.3), we assume that

$$\begin{aligned} &r_1(n) = 0.05(\sin^2 n + 1), \qquad p_1(n) = p_2(n) = 1, \\ &q_1(n) = 0.05(\cos^2 n + 1), \qquad \mu(n) = 0.05, \\ &r_2(n) = 0.5\cos^2 n + 0.2, \qquad m = 0.8, \\ &q_2(n) = 0.2(\cos n + 1.5), \qquad E = 10. \end{aligned} \tag{6.2}$$

It is easy to see that $\frac{r_1^u}{q_1^l E} = 0.2$, $\frac{r_2^u}{q_2^l E} = 0.7$, $m = 0.8 > \max\{0.2, 0.7\}$. Then the conditions of Theorem 4.1 are satisfied (see Fig. 2).

Example 6.3 Corresponding to system (1.3), we assume that

$$\begin{aligned} &r_1(n) = 0.05(\sin^2 n + 1), \qquad p_1(n) = 0.004(\cos n + 1.5), \\ &q_1(n) = 0.02(\cos^2 n + 1), \qquad r_2(n) = 0.8\cos^2 n + 0.3, \\ &q_2(n) = 0.2(\cos n + 1.5), \qquad p_2(n) = \sin n + 3, \\ &m = 0.5, \qquad \mu(n) = 1, \qquad E = 1. \end{aligned} \tag{6.3}$$

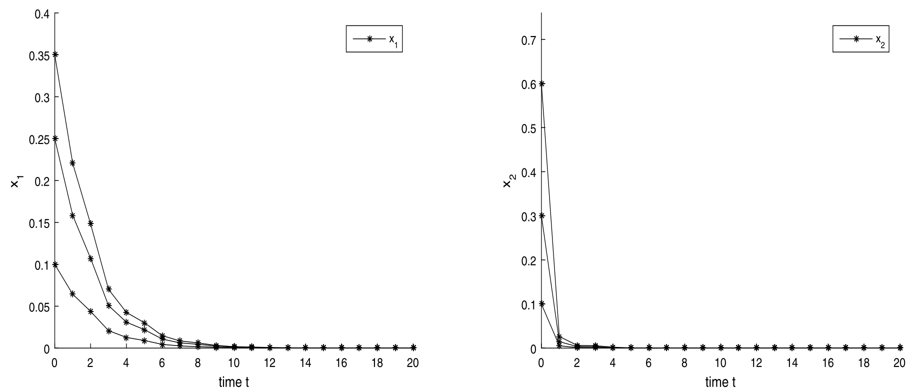

Figure 2 Dynamic behaviors of system (6.2) with initial condition $(x_1(t), x_2(t))^T = (0.1, 0.3)^T, (0.25, 0.6)^T, (0.35, 0.1)^T$

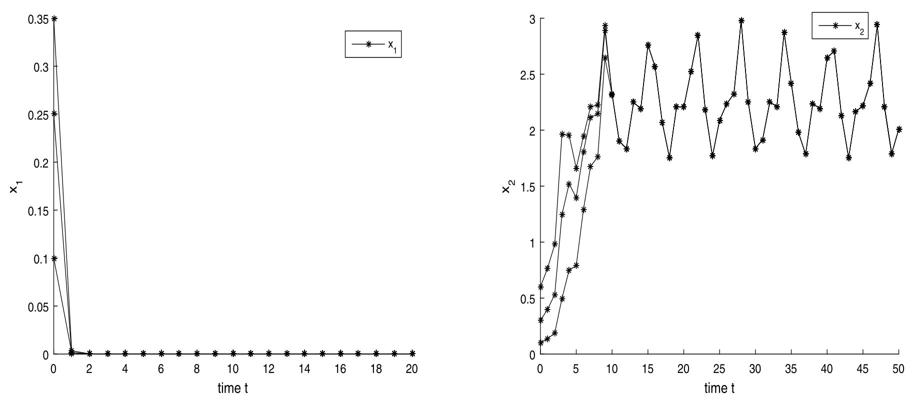

Figure 3 Dynamic behaviors of system (6.3) with initial condition $(x_1(t), x_2(t))^T = (0.1, 0.3)^T, (0.25, 0.6)^T, (0.35, 0.1)^T$

It is easy to see that $r_1^u - q_1^l Em = 0.09 > 0$, $r_2^l - q_2^u Em = 0.05 > 0$, $\frac{r_1^u - q_1^l Em}{r_2^l - q_2^u Em} = 1.8 < \frac{r_1^l \mu^l p_2^l}{r_2^u p_1^u} \approx 9.09$. Then the conditions of Theorem 4.2 are satisfied (see Fig. 3).

Example 6.4 Corresponding to system (1.3), we assume that

$$\begin{aligned} &r_1(n) = p_1(n) = p_2(n) = 1, \qquad q_1(n) = 0.05\bigl(\cos^2 n + 1\bigr), \\ &r_2(n) = 0.5\cos^2 n + 0.2, \qquad q_2(n) = 0.2(\cos n + 1.5), \\ &\mu(n) = 1, \qquad m = 0.8, \qquad E = 10. \end{aligned} \tag{6.4}$$

It is easy to see that $m = 0.8 > \frac{r_2^u}{q_2^l E} = 0.7$, $\min\{\frac{r_1^l}{p_1^u}, \frac{2}{M_1} - \frac{r_1^u}{p_1^l} = \min\{1, 1\}\} > 0$, $M_1 = \frac{p_1^u}{r_1^l}\exp(r_1^u - 1) = 1$. Then the conditions of Theorem 5.1 are satisfied (see Fig. 4).

7 Discussion

With the aim of the ecological sustainability and conservation of the species to be implemented in a long run, in this paper, we have attempted to study the dynamic behaviors of a non-selective harvesting Lotka–Volterra discrete amensalism model. We have proved that

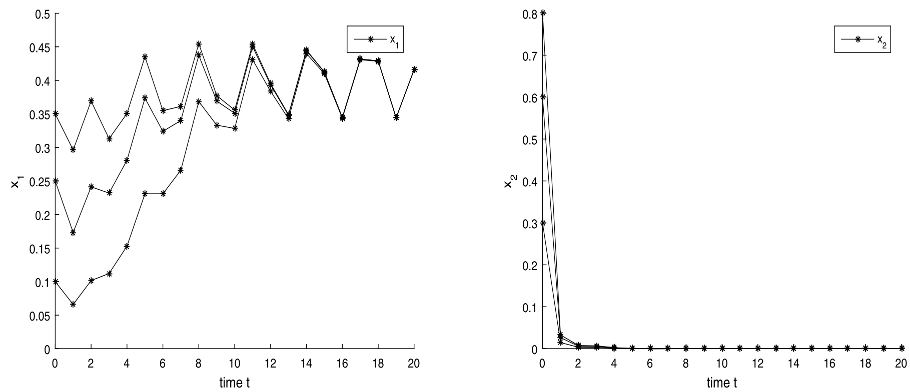

Figure 4 Dynamic behaviors of system (6.4) with initial condition $(x_1(t), x_2(t))^T = (0.1, 0.8)^T, (0.25, 0.6)^T, (0.35, 0.3)^T$

if (H_1) holds, then the system is permanent, which means that if m, which is the fraction of the stock available for harvesting, is small enough, the system will coexist. Theorem 4.1 implies that if m is large enough, then the system will be driven to extinction. Theorem 4.2 gives some threshold on m, which ensures that the species x_2 is permanent while x_1 will be driven to extinction. In Sect. 5, sufficient conditions for the global attractivity of (1.3) are given, which means that if m is larger than a certain value and satisfies (H_5), then the species x_1 is globally attractive while x_2 will be driven to extinction. The results obtained in this paper maybe useful in designing the natural protection area.

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We agree to publish in your journal.

Authors' contributions
QQS is a major contributor in writing the manuscript. FDC is responsible for numerical simulation and drawing. All authors read and approved the final manuscript.

Author details
[1]General Education Center, Zhengzhou Business University, Gongyi, P.R. China. [2]College of Mathematics and Computer Science, Fuzhou University, Fuzhou, P.R. China.

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