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Cluster linear generalized outer synchronization in community networks via pinning control with two different switch periods

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Abstract

This study investigates the problem of cluster generalized outer synchronization in community networks via pinning control with two different switch periods. Several pinning controllers have been designed to achieve linear generalized outer synchronization. Using Lyapunov stability theory, sufficient linear generalized outer synchronization criteria for community networks are derived. Finally, numerical examples are presented to demonstrate the effectiveness of the theoretical results.

Keywords: community networks; linear generalized outer synchronization; pinning control

1 Introduction

Recently, complex networks have drawn increasing attention from researchers and engineers in virtue of its wide applications in many fields, such as World Wide Web, communication networks, social networks, neural networks, epidemic networks, traffic networks, etc. Lots of network models, such as weighted networks [1, 2], directed networks [3, 4], hierarchical networks [5], community networks [6–8] are introduced to explore the potential applications better. As is well known, the research on network synchronization is very important due to its potential applications in many fields including secure communication, laser transmission, image identification, information science, and so on [9–14]. In recent years, much literature reported the research results of network synchronization, and it has become a frontier issue [15–19]. As a result, different types of network synchronization have been put forward, for example, complete synchronization [20–22], phase synchronization [23, 24], projective synchronization [25, 26] and cluster synchronization [27, 28].

Furthermore, many real complex networks cannot synchronize themselves or synchronize with the desired orbits. Therefore, proper controllers should be designed to achieve the goals by adopting some control schemes, such as adaptive control [29], feedback control [30], observer-based control [31], impulsive control [32], intermittent control [33–35], pinning control [36, 37] and so on. As a matter of fact, there are many examples of relationships between different networks, which indicates that it is necessary and significant

to investigate the dynamical systems between different networks. Recently, [38] investigated the synchronization between two unidirectionally coupled complex networks with identical topological structures. [39] discussed the synchronization between two complex dynamical networks with non-identical topological structures via using adaptive control method. [40] discussed adaptive projective synchronization between two complex networks with time-varying coupling delay. In the above papers, it is assumed that each node in drive-response networks has identical dynamics. Later, [41] studied the problem of generalized outer synchronization between two complex dynamical networks with different topologies and diverse node dynamics. Reference [42] discussed the linear generalized synchronization between two complex networks with the non-delay coupling and the same topological structure, each network has identical dynamics. However, detailed analysis of the linear generalized synchronization between two networks of different topological structures and time-varying coupling delay has not been attempted in [42].

Motivated by the above discussions, this paper investigates the problem of cluster linear generalized outer synchronization (CLGOS) in community networks via pinning control with two different switch periods. Several pinning controllers have been designed to achieve linear generalized outer synchronization. Using Lyapunov stability theory, sufficient linear generalized outer synchronization criteria for community networks are derived. Finally, numerical examples are presented to demonstrate the effectiveness of the theoretical results. Contributions of this paper can be summarized as follows:

- By adding adaptive semi-periodically intermittent controllers to a small fraction of nodes in response network, several sufficient conditions are derived based on the Lyapunov stability theory and strict mathematical proofs.
- Both community networks with identical nodes and non-identical nodes are investigated. Therefore, our proposed control schemes are more applicable technically.

The rest of the current paper is organized as follows. Section 2 introduces the problem formulation and some necessary definitions, lemmas, and hypotheses. Some sufficient conditions for the linear generalized outer synchronization are obtained in Section 3. Section 4 gives some numerical examples to demonstrate the effectiveness of our main results. Finally, Section 5 draws the conclusion.

Notation The superscripts T and (-1) stand for matrix transposition and matrix inverse, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; I_l means the l-dimensional identity matrix. The notation $X > Y$ ($X \geq Y$), where X, Y are symmetric matrices, means that $X - Y$ is positive definite (positive semidefinite). $*$ denotes the term that is induced by symmetry. $\|\xi\|$ indicates the 2-norm of a vector ξ, *i.e.*, $\|\xi\| = \xi^T\xi$. $\text{col}\{x_1, x_2, \ldots, x_n\}$ means $[x_1^T, x_2^T, \ldots, x_n^T]^T$ and $\text{Sym}\{X\}$ means $X + X^T$. The shorthand notation $\text{diag}\{M_1, M_2, \ldots, M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $M_1, M_2, \ldots, M_n$. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the smallest and largest eigenvalue of $\cdot$. The symbol $\otimes$ denotes the Kronecker product. Matrices, if their dimensions are not explicitly stated, are assumed to have appropriate dimensions for algebraic operations.

2 Problem formulation and preliminaries

Consider the following complex networks with time-varying coupling delay consisting of N nodes and s communities with $2 \leq s < N$

$$\dot{x}_i(t) = f_{\varphi i}\big(x_i(t)\big) + c \sum_{k=1}^{s} \sum_{j \in V_k} a_{ij} \Gamma x_j(t)$$

$$+ c \sum_{k=1}^{s} \sum_{j \in V_k} b_{ij} \Gamma x_j\big(t - \tau(t)\big), \quad i \in 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state variables of node i in networks X. $f_{\varphi i}(\cdot) : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a continuously differentiable nonlinear function. c is the coupling strength, and Γ is an inner-coupling matrix. $\tau(t)$ is the time-varying coupling delay satisfying $0 \leq \dot{\tau}(t) \leq \mu < 1$. $A = (a_{ij})_{N \times N} \in \mathbb{R}^{N \times N}$ and $B = (b_{ij})_{N \times N} \in \mathbb{R}^{N \times N}$ are the outer-coupling matrices with the sum of each row being zero. If there is a connection from node i to node j ($j \neq i$), then the coupling $a_{ij}(b_{ij}) \neq 0$; otherwise, $a_{ij}(b_{ij}) = 0$ ($j \neq i$), and the diagonal elements of matrix are defined as $a_{ii} = -\sum_{j=1, j \neq i}^{N} a_{ij}$ or $b_{ii} = -\sum_{j=1, j \neq i}^{N} b_{ij}$. The function φ is defined as φ: $\{1, 2, \ldots, N\} \rightarrow \{1, 2, \ldots, s\}$; if a node $i \in V_k$, then $\varphi_i = k$; V_k ($k = 1, 2, \ldots, s$) denotes the set of all nodes belong to the sth community.

Consider the controller response complex dynamical network as follows:

$$\dot{y}_i(t) = \tilde{f}_{\varphi i}\big(y_i(t)\big) + c \sum_{k=1}^{s} \sum_{j \in V_k} a_{ij} \Gamma y_j(t)$$

$$+ c \sum_{k=1}^{s} \sum_{j \in V_k} b_{ij} \Gamma y_j\big(t - \tau(t)\big) + u_i(t), \quad i \in 1, 2, \ldots, N, \tag{2}$$

where $y_i \in \mathbb{R}^n$ is the response state variables of node i in networks Y. $\tilde{f}_{\varphi i}(\cdot) : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a continuously differentiable nonlinear function. $u_i(t) \in \mathbb{R}^n$ ($i = 1, 2, \ldots, N$) are the nonlinear controllers ro be designed later, and the other parameters involved in system (2) all have the same meaning with the corresponding parameters in system (1).

Remark 2.1 The nonlinear vector-valued functions $f_{\varphi i}$ and $\tilde{f}_{\varphi i}$ can be identical or non-identical.

Remark 2.2 There are no limitations for the division of the clusters, the number of nodes in each cluster and the connections between nodes.

Remark 2.3 All nodes within a cluster have the same dynamics, and the dynamics of the nodes in different clusters can be different.

Remark 2.4 The proposed approach on the case with undirected topology is similar to the one that on the case with directed topology. So in this paper the underlying topology is assumed to be undirected.

Suppose that the networks (2) will be controlled onto some desired inhomogeneous state as $\{y_1(t), \ldots, y_{m_1}(t)\} \rightarrow \phi_1(t)$, $\{y_{m_1+1}(t), \ldots, y_{m_2}(t)\} \rightarrow \phi_2(t)$, $\ldots$, $\{y_{m_{s-1}+1}(t), \ldots, y_{m_s}(t)\} \rightarrow$

$\phi_s(t)$, i.e., $\mathcal{M} = \{\{\phi_1(t), \ldots, \phi_1(t)\}, \{\phi_2(t), \ldots, \phi_2(t)\}, \ldots, \{\phi_s(t), \ldots, \phi_s(t)\}\} \in \mathbb{R}^{n \times N}$ is desired cluster synchronization pattern under the pinning control.

Definition 2.1 Let $\phi_{\varphi_i}(t) = Px_i(t) + Q : \mathbb{R}^n \to \mathbb{R}^n$ $(i = 1, 2, \ldots, N)$ be continuously differentiable vector maps. If $\varphi_i \neq \varphi_j$, $\phi_{\varphi_i} \neq \phi_{\varphi_j}$. Generalized outer synchronization between the drive-response networks are achieved if

$$\lim_{t \to \infty} \|e_i(t)\| = \lim_{t \to \infty} \|y_i(t) - \phi_{\varphi_i}(t)\| = 0, \quad i \in 1, 2, \ldots, N, \tag{3}$$

where P and Q are constant matrices with proper dimension.

Assumption 2.1 *Assuming that there are positive constants $L, \tilde{L}$ such that f and $\tilde{f}$ satisfy the following inequalities*:

$$
\begin{aligned}
\left(y_i(t) - \phi_{\varphi_i}(t)\right)^T \left(f\left(y_i(t)\right) - f\left(\phi_{\varphi_i}(t)\right)\right) &\leq L\left(y_i(t) - \phi_{\varphi_i}(t)\right)^T \Gamma\left(y_i(t) - \phi_{\varphi_i}(t)\right), \\
\left(y_i(t) - \phi_{\varphi_i}(t)\right)^T \left(\tilde{f}\left(y_i(t)\right) - \tilde{f}\left(\phi_{\varphi_i}(t)\right)\right) &\leq \tilde{L}\left(y_i(t) - \phi_{\varphi_i}(t)\right)^T \tilde{\Gamma}\left(y_i(t) - \phi_{\varphi_i}(t)\right),
\end{aligned}
\tag{4}
$$

where Γ and $\tilde{\Gamma}$ are positive definite matrix, $i = 1, 2, \ldots, N$. Here, x and y are time-varying vectors.

Lemma 2.1 *For a diagonal matrix $D = \text{diag}\{\underbrace{d_1, d_2, \ldots, d_l}_{i = \{1, 2, \ldots, l\} \subseteq \bar{V}_{\varphi i}} 0, 0, \ldots, 0\}$ with $d_i > 0$, $(i = 1, 2, \ldots, l; 1 \leq l \leq N)$ and a symmetric matrix $M \in \mathbb{R}^{N \times N}$, let $M - D = \begin{bmatrix} E - \bar{D} & S \\ * & M_l \end{bmatrix}$, where M_l is the minor matrix of M by removing its first l $(1 \leq l \leq N)$ row-column pairs, E and S are matrices with appropriate dimensions, $\bar{D} = \text{diag}\{d_1, d_2, \ldots, d_l\}$. If $d_i > \lambda_{\max}(E - SM_l^{-1}S^T)$, then $M - D < 0$ is equivalent to $M_l < 0$.*

Proof Let $\bar{D} = \text{diag}\{\underbrace{d_1, d_2, \ldots, d_l}_{i = \{1, 2, \ldots, l\} \subseteq \bar{V}_{\varphi i}}\}$. Using matrix decomposition, $M \in \mathbb{R}^{N \times N}$, let $M - D = \begin{bmatrix} E - \bar{D} & S \\ * & M_l \end{bmatrix}$, where M_l is the minor matrix of M by removing its first l $(1 \leq l \leq N)$ row-column pairs, E and S are matrices with appropriate dimensions.

Using the Schur complement, it is easy to see that $M - D < 0$ is equivalent to $M_l < 0$. We only need to prove that if $M_l < 0$, then $M - D < 0$. When $d_i > 0$ $(i = 1, 2, 3, \ldots, l)$ are sufficiently large such that $d_i > \lambda_{\max}(E - SM_l^{-1}S^T)$ hold, it is easy to see that $E - \bar{D} - SM_l^{-1}S^T < 0$. Then, using the Schur complement, we can conclude that $M - D < 0$, so the proof is finished. $\square$

Lemma 2.2 ([37]) *Assume that A, B are N by N Hermitian matrices. Let $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_N$, $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_N$ and $\gamma_1 \geq \gamma_2 \geq \cdots \gamma_N$ be eigenvalues of A, B and $A + B$, respectively. Then one has $\alpha_i + \beta_N \leq \gamma_i \leq \alpha_i + \beta_1$, $i = 1, 2, \ldots, N$.*

3 Main results

In this section, the CLGOS of the drive-response community networks (1) and (2) will be investigated in three cases.

Case I. Assuming that the nodes dynamics in both community networks are identical, *i.e.*, nonlinear vector-functions $f_{\varphi i} = \tilde{f}_{\varphi i} = f$ for all $1 \le i \le N$. Then the drive-response networks (1) and (2) can be written as

$$\dot{x}_i(t) = f\big(x_i(t)\big) + c \sum_{k=1}^{s} \sum_{j \in V_k} a_{ij} \Gamma x_j(t)$$

$$+ c \sum_{k=1}^{s} \sum_{j \in V_k} b_{ij} \Gamma x_j\big(t - \tau(t)\big), \quad i \in 1, 2, \dots, N, \tag{5}$$

$$\dot{y}_i(t) = f\big(y_i(t)\big) + c \sum_{k=1}^{s} \sum_{j \in V_k} a_{ij} \Gamma y_j(t)$$

$$+ c \sum_{k=1}^{s} \sum_{j \in V_k} b_{ij} \Gamma y_j\big(t - \tau(t)\big) + u_i(t), \quad i \in 1, 2, \dots, N. \tag{6}$$

In this subsection, the intermittent control with two different switched periods is considered. The sketch of such control strategy is given by Figure 1. As shown in Figure 1, T_1 and T_2 are two periods appearing alternately. η_1 ($0 < \eta_1 < 1$) and η_2 ($0 < \eta_2 < 1$) are called the rates of control duration in each control period. $(1 - \eta_1)T_1$ and $(1 - \eta_2)T_2$ are called non-feedback control widths in control periods T_1 and T_2, respectively. The rates of control duration may be different, *i.e.*, $\eta_1 \ne \eta_2$, while they are assumed to be the same in [21]. In this regards, the semi-periodically intermittent control scheme considered here is more general than [21].

We denote $\Xi_1^m = [mT, mT + \eta_1 T_1]$ is the control width in period T_1, $\Xi_2^m = [mT + \eta_1 T_1, mT + T_1]$ is the non-feedback control width in period T_1, $\Xi_3^m = [mT + T_1, mT + T_1 + \eta_2 T_2]$ is the control width in period T_2, $\Xi_4^m = [mT + T_1 + \eta_2 T_2, (m+1)T]$ the non-feedback control width in period T_2, where $m = 0, 1, 2, \dots$.
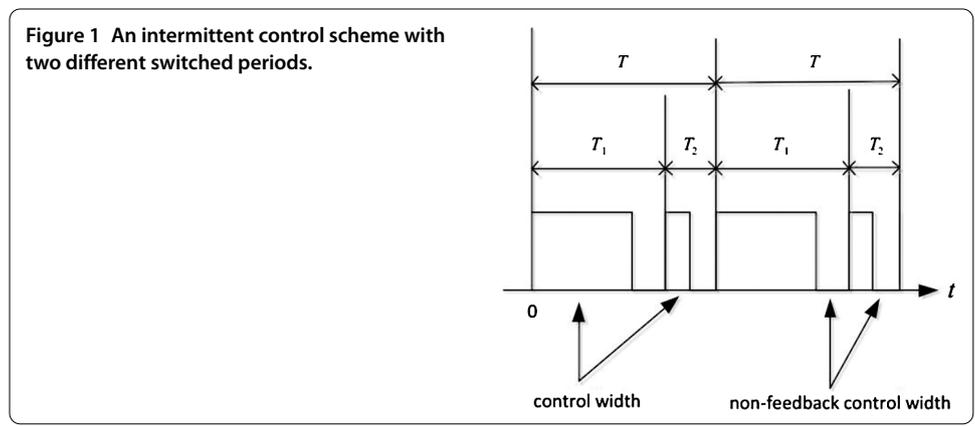
Figure 1 An intermittent control scheme with two different switched periods.

The adaptive semi-periodically intermittent controllers are defined as follows:

$$
u_i(t) = \begin{cases}
Pf\big(x_i(t)\big) - f\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) - c\sum_{j=1}^{N} b_{ij}\Gamma e_j\big(t - \tau(t)\big) \\
\quad - ck_i(t)\Gamma e_i(t), \quad i \in \bar{V}_{\varphi i}, t \in \Xi_1^m \cup \Xi_3^m, \\
Pf\big(x_i(t)\big) - f\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) - c\sum_{j=1}^{N} b_{ij}\Gamma e_j\big(t - \tau(t)\big), \\
\quad i \in \bar{V}_{\varphi i}, t \in \Xi_2^m \cup \Xi_4^m, \\
Pf\big(x_i(t)\big) - f\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) - c\sum_{j=1}^{N} b_{ij}\Gamma e_j\big(t - \tau(t)\big), \\
\quad i \subsetneq \bar{V}_{\varphi i}, t \in \Xi_1^m \cup \Xi_3^m, \\
Pf\big(x_i(t)\big) - f\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) - c\sum_{j=1}^{N} b_{ij}\Gamma e_j\big(t - \tau(t)\big), \\
\quad i \subsetneq \bar{V}_{\varphi i}, t \in \Xi_2^m \cup \Xi_4^m,
\end{cases}
\tag{7}
$$

where $\bar{V}_{\varphi i}$ denotes the set of the nodes in the φ_ith community which have direct connections to the nodes in other communities and the updating laws

$$
\dot{\tilde{p}}_{ij} = -e_i^T(t)\Gamma y_i\big(t - \tau(t)\big), \quad i, j = 1, 2, 3, \ldots, N,
\tag{8}
$$

and

$$
\dot{k}_i(t) = \begin{cases}
\varepsilon_i e^{\beta t} e_i^T(t)\Gamma e_i(t), & t \in \Xi_1^m \cup \Xi_3^m, \\
0, & t \in \Xi_2^m \cup \Xi_4^m,
\end{cases}
\tag{9}
$$

where ε_i ($i \in \bar{V}_{\varphi i}$) and β are positive constants.

According to the definition of the coupling matrix A, B, it is easy to see that

$$
\begin{cases}
c\sum_{j=1}^{N} a_{ij}\Gamma Q = 0, \\
c\sum_{j=1}^{N} b_{ij}\Gamma Q = 0.
\end{cases}
\tag{10}
$$

Let $e_i(t) = y_i(t) - \phi_{\varphi_i}(t)$, $g(e_i(t)) = f(y_i(t)) - f(\phi_{\varphi i}(t))$, $g(\cdot): \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a continuously differentiable nonlinear function. With the aid of equations (5)-(10), the error systems can be rewritten as

$$
\begin{cases}
\dot{e}_i(t) = g\big(e_i(t)\big) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) - ck_i(t)\Gamma e_i(t), \\
\quad i \in \bar{V}_{\varphi i}, t \in \Xi_1^m \cup \Xi_3^m, \\
\dot{e}_i(t) = g\big(e_i(t)\big) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big), \\
\quad i \in \bar{V}_{\varphi i}, t \in \Xi_2^m \cup \Xi_4^m, \\
\dot{e}_i(t) = g\big(e_i(t)\big) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big), \\
\quad i \subsetneq \bar{V}_{\varphi i}, t \in \Xi_1^m \cup \Xi_3^m, \\
\dot{e}_i(t) = g\big(e_i(t)\big) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big), \\
\quad i \subsetneq \bar{V}_{\varphi i}, t \in \Xi_2^m \cup \Xi_4^m.
\end{cases}
\tag{11}
$$

Theorem 3.1 *Suppose that Assumption* 2.1 *holds. Using the adaptive controllers and updated laws* (7)-(9), *then the response networks* (6) *can linear generalized synchronize with*

the drive networks (5) *if there exist positive constants $\alpha > \beta > 0$ such that the following conditions are satisfied*:

$$c < -\frac{\kappa}{\lambda_{\max}(\hat{A}_l)}, \tag{12}$$

$$L\Gamma + \frac{\beta - \alpha}{2}I_N + \frac{1}{2(1-\mu)}I_N < 0, \tag{13}$$

$$\beta - \alpha\theta > 0, \tag{14}$$

$$\alpha\rho - a_* > 0, \tag{15}$$

where

$$\kappa = \frac{1}{\lambda_{\min}(\Gamma)}\left(L\|\Gamma\| + \frac{\beta}{2} + \frac{1}{2(1-\mu)}\right),$$

$$\theta = (1-\eta_1)\frac{T_1}{T} + (1-\eta_2)\frac{T_2}{T}, \qquad \rho = \frac{\eta_1 T_1 + \eta_2 T_2}{T},$$

$$\hat{A} = \frac{\mathrm{Sym}\{A\}}{2}, \qquad a_* = \alpha - \beta,$$

$\hat{A}_l$ *is the minor matrix of A by removing its first l $(1 \le l \le N)$ row-column pairs.*

Proof Construct the following Lyapunov function candidate:

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{16}$$

where

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t),$$

$$V_2(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} c\tilde{p}_{ij}^2,$$

$$V_3(t) = \frac{1}{2}\sum_{i\in\bar{V}_{\varphi i}} \exp(-\beta t)\frac{c(k_i(t)-k_i^*)^2}{\varepsilon_i},$$

$$V_4(t) = \frac{1}{2(1-\mu)}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} \tilde{e}_j^T(\theta)\tilde{e}_j(\theta)\,d\theta,$$

where k_i^* are sufficiently large positive constant to be determined. We write $\tilde{e}_j(\theta) = (\tilde{e}_{1j}(\theta), \tilde{e}_{2j}(\theta), \dots, \tilde{e}_{Nj}(\theta))^T$. $\dot{\tilde{p}}_{ij} = -e_i^T(t)\Gamma y_i(t-\tau(t))$.

When $t \in \Xi_1^m \cup \Xi_3^m$, differentiating $V(t)$ with respect to time along the solution of (11) yields

$$
\dot{V}_1(t) = \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t)
$$

$$
= \sum_{i=1}^{N} e_i^T(t)\left[g\big(e_i(t)\big) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t-\tau(t)\big) \right]
$$

$$
- c\sum_{i\in\bar{V}_{\varphi i}} e_i^T(t)k_i(t)\Gamma e_i(t), \tag{17}
$$

$$
\dot{V}_2(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} c\tilde{p}_{ij}\dot{\tilde{p}}_{ij} = -c\sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{p}_{ij}e_i^T(t)\Gamma y_i\big(t-\tau(t)\big), \tag{18}
$$

$$
\dot{V}_3(t) = c\sum_{i\in\bar{V}_{\varphi i}} \big(k_i(t)-k_i^*\big)e_i^T(t)\Gamma e_i(t) - \frac{\beta}{2}e^{-\beta t}\sum_{i\in\bar{V}_{\varphi i}} \frac{c(k_i(t)-k_i^*)^2}{\varepsilon_i}, \tag{19}
$$

$$
\dot{V}_4(t) = \frac{1}{2(1-\mu)}\sum_{i=1}^{N} \tilde{e}_j^T(t)\tilde{e}_j(t) - \frac{1-\dot{\tau}(t)}{2(1-\mu)}\tilde{e}_j^T\big(t-\tau(t)\big)\tilde{e}_j\big(t-\tau(t)\big). \tag{20}
$$

From (17)-(20), it is easy to see that

$$
\dot{V}(t) = \sum_{i=1}^{4} \dot{V}_i(t)
$$

$$
= \sum_{i=1}^{N} e_i^T(t)\left[g\big(e_i(t)\big) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i\big(t-\tau(t)\big) \right]
$$

$$
- c\sum_{i\in\bar{V}_{\varphi i}} e_i^T(t)k_i^*\Gamma e_i(t) - c\sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{p}_{ij}e_i^T(t)\Gamma y_i\big(t-\tau(t)\big)
$$

$$
- \frac{\beta}{2}e^{-\beta t}\sum_{i\in\bar{V}_{\varphi i}} \frac{c(k_i(t)-k_i^*)^2}{\varepsilon_i}
$$

$$
+ \frac{1}{2(1-\mu)}\sum_{i=1}^{N} \tilde{e}_j^T(t)\tilde{e}_j(t) - \frac{1-\dot{\tau}(t)}{2(1-\mu)}\tilde{e}_j^T\big(t-\tau(t)\big)\tilde{e}_j\big(t-\tau(t)\big)
$$

$$
\leq \sum_{i=1}^{N} e_i^T(t)(L\Gamma + c\hat{A}\Gamma)e_i(t) + \frac{1}{2(1-\mu)}\sum_{i=1}^{N} \tilde{e}_j^T(t)\tilde{e}_j(t)
$$

$$
- \frac{1-\dot{\tau}(t)}{2(1-\mu)}\tilde{e}_j^T\big(t-\tau(t)\big)\tilde{e}_j\big(t-\tau(t)\big)
$$

$$
- c\sum_{i\in\bar{V}_{\varphi i}} e_i^T(t)k_i^*\Gamma e_i(t) - \beta V_3(t)
$$

$$
= \sum_{i=1}^{N} e_i^T(t)\left(L\Gamma + c\hat{A}\Gamma + \frac{\beta}{2}I_N - cK^*\Gamma \right)e_i(t) + \frac{1}{2(1-\mu)}\sum_{i=1}^{N} \tilde{e}_j^T(t)\tilde{e}_j(t)
$$

$$
- \frac{1-\dot{\tau}(t)}{2(1-\mu)}\tilde{e}_j^T\big(t-\tau(t)\big)\tilde{e}_j\big(t-\tau(t)\big) - \beta\big(V_1(t) + V_3(t)\big)
$$

$$\leq e^T(t)\left(L\Gamma + \frac{\beta}{2}I_N + \frac{1}{2(1-\mu)}I_N\right)e(t) + e^T(t)\left((c\hat{A} - cK^*)\otimes\Gamma\right)e(t) - \beta V(t)$$

$$\leq e^T(t)\left((\kappa I_N + c\hat{A} - cK^*)\otimes\Gamma\right) - \beta V(t), \tag{21}$$

where

$$e(t) = \text{col}\{e_1(t), e_2(t), \dots, e_n(t)\},$$

$$K^* = \text{diag}\{\underbrace{k_1^*, k_2^*, \dots, k_l^*}_{i=\{1,2,3,\dots,l\}\subseteq\bar{V}_{\varphi i}}, 0, \dots, 0\},$$

$$\kappa = \frac{1}{\lambda_{\min}(\Gamma)}\left(L\|\Gamma\| + \frac{\beta}{2} + \frac{1}{2(1-\mu)}\right).$$

Let $\Omega = \kappa I_N + c(\hat{A} - K^*) = \begin{bmatrix} E-K^{**} & S \\ * & \Omega_l \end{bmatrix}$, in which Ω_l is the minor matrix of Ω by removing its $l(l \in \bar{V}_{\varphi i})$ row-column pairs, E and S are matrices with appropriate dimensions, $K^{**} = \text{diag}\{\underbrace{ck_1, ck_2, \dots, ck_l}_{i=\{1,2,3,\dots,l\}\subseteq\bar{V}_{\varphi i}}\}$. It is obvious that Ω is symmetric. According to Lemma 2.1, we know that if one can select $k_i > \lambda_{\max}(E - S\Omega_l^{-1}S^T)$, then $\Omega < 0$ is equivalent to $\Omega_l < 0$. Based on Lemma 2.2 and the condition (12), we have $\lambda_{\max}(\Omega_l) \leq \kappa + c\lambda_{\max}(A_l) < 0$, which implies that $\Omega_l < 0$. Then we obtain

$$\dot{V}(t) \leq -\beta V(t). \tag{22}$$

When $t \in \Xi_2^m \cup \Xi_4^m$, differentiating $V(t)$ with respect to time along the solution of (11) and using the condition in (13) yields

$$\dot{V}(t) = \sum_{i=1}^{4} \dot{V}_i(t)$$

$$= \sum_{i=1}^{N} e_i^T(t)\left[g(e_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + c\sum_{j=1}^{N} \tilde{p}_{ij}\Gamma y_i(t - \tau(t))\right]$$

$$- c\sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{p}_{ij}e_i^T(t)\Gamma y_i(t - \tau(t)) - \frac{\beta}{2}e^{-\beta t}\sum_{i\in\bar{V}_{\varphi i}} \frac{c(k_i(t) - k_i^*)^2}{\varepsilon_i}$$

$$+ \frac{1}{2(1-\mu)}\sum_{i=1}^{N} \tilde{e}_j^T(t)\tilde{e}_j(t) - \frac{1-\dot{\tau}(t)}{2(1-\mu)}\tilde{e}_j^T(t - \tau(t))\tilde{e}_j(t - \tau(t))$$

$$\leq e^T(t)\left(L\Gamma + \frac{\beta-\alpha}{2}I_N + \frac{1}{2(1-\mu)}I_N\right)e(t) + e^T(t)(c\hat{A}\otimes\Gamma)e(t)$$

$$- \frac{\beta-\alpha}{2}\left(\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i\in\bar{V}_{\varphi i}} \exp(-\beta t)\frac{c(k_i(t) - k_i^*)^2}{\varepsilon_i}\right)$$

$$\leq (\alpha - \beta)(V_1(t) + V_3(t))$$

$$\leq (\alpha - \beta)V(t) = a_*V(t). \tag{23}$$

Then we have

$$\dot{V}_i(t) = \begin{cases} -\beta V(t), & t \in \Xi_1^m \cup \Xi_3^m, \\ a_* V(t), & t \in \Xi_2^m \cup \Xi_4^m. \end{cases} \tag{24}$$

From (24), it is easy to see that:

When $t \in \Xi_1^m$, i.e., $\frac{t - \eta_1 T_1}{T} < m \le \frac{t}{T}$

$$\begin{aligned} V(t) &\le V(mT) \exp\big(-\beta(t - mT)\big) \\ &\le V(0) \exp\big(-\beta mT + m\alpha\big((1 - \eta_1)T_1 + (1 - \eta_2)T_2\big)\big) \\ &\le V(0) \exp\big((-\beta + \alpha\theta)t + \beta\eta_1 T_1\big). \end{aligned} \tag{25}$$

When $t \in \Xi_2^m$, i.e., $\frac{t - T_1}{T} < m \le \frac{t - \delta_1 T_1}{T}$,

$$\begin{aligned} V(t) &\le V(mT + T_1) \exp\big(t - (mT + T_1)\big) \\ &\le V(0) \exp\big((\alpha - \beta)(mT + T_1) - \alpha\big((m + 1)\eta_1 T_1 + m\eta_2 T_2\big)\big) \\ &\le V(0) \exp\bigg((a_* - \alpha\rho)t - \alpha(\eta_1 - \eta_2)\frac{T_1 T_2}{T} + a_*(1 - \eta_1)T_1\bigg). \end{aligned} \tag{26}$$

When $t \in \Xi_3^m$, i.e., $\frac{t - T_1 - \eta_2 T_2}{T} < m \le \frac{t - T_1}{T}$,

$$\begin{aligned} V(t) &\le V(mT + T_1) \exp\big(-\beta\big(t - (mT + T_1)\big)\big) \\ &\le V(0) \exp\big(-\beta(mT + T_1) + \alpha\big((m + 1)(1 - \eta_1)T_1 + m(1 - \eta_2)T_2\big)\big) \\ &\le V(0) \exp\bigg((-\beta + \alpha\theta) + \beta\eta_2 T_2 + \alpha\frac{\eta_2 - \eta_1}{T}T_1 T_2\bigg). \end{aligned} \tag{27}$$

When $t \in \Xi_4^m$, i.e., $\frac{t}{T} < m + 1 \le \frac{t + T - T_1 - \eta_2 T_2}{T}$,

$$\begin{aligned} V(t) &\le V\big((m + 1)T\big) \exp\big(a_*\big(t - (m + 1)T\big)\big) \\ &\le V(0) \exp\big(a_*(m + 1)T - (m + 1)\alpha(\eta_2 T_2 + \eta_1 T_1)\big) \\ &\le V(0) \exp\big((a_* - \alpha\rho)t + a_*(1 - \eta_2)T_2\big). \end{aligned} \tag{28}$$

Therefore, when $t \in \Xi_1^m \cup \Xi_3^m$, if $\beta - \alpha\theta > 0$ is satisfied, one has $\lim_{t \to \infty} V(t) = 0$; when $t \in \Xi_2^m \cup \Xi_4^m$, if $\alpha\rho - a_* > 0$ is satisfied, one has $\lim_{t \to \infty} V(t) = 0$. The conclusion of Theorem 3.1 holds. This completes the proof. $\qquad\square$

Case II. Assume that the nodes dynamics in both community networks are non-identical; in view of the special property, the adaptive semi-periodically intermittent con-

trollers are defined as follows:

$$
u_i(t) =
\begin{cases}
Pf_{\varphi i}\big(x_i(t)\big) - \tilde{f}_{\varphi i}\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N}\tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) \\
\qquad - c\sum_{j=1}^{N}b_{ij}\Gamma e_j\big(t - \tau(t)\big) - ck_i(t)\Gamma e_i(t), \quad i \in \bar{V}_{\varphi i}, t \in \Xi_1^m \cup \Xi_3^m, \\
Pf_{\varphi i}\big(x_i(t)\big) - \tilde{f}_{\varphi i}\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N}\tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) \\
\qquad - c\sum_{j=1}^{N}b_{ij}\Gamma e_j\big(t - \tau(t)\big), \quad i \in \bar{V}_{\varphi i}, t \in \Xi_2^m \cup \Xi_4^m, \\
Pf_{\varphi i}\big(x_i(t)\big) - \tilde{f}_{\varphi i}\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N}\tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) \\
\qquad - c\sum_{j=1}^{N}b_{ij}\Gamma e_j\big(t - \tau(t)\big), \quad i \subsetneqq \bar{V}_{\varphi i}, t \in \Xi_1^m \cup \Xi_3^m, \\
Pf_{\varphi i}\big(x_i(t)\big) - \tilde{f}_{\varphi i}\big(\phi_{\varphi_i}(t)\big) + c\sum_{j=1}^{N}\tilde{p}_{ij}\Gamma y_i\big(t - \tau(t)\big) \\
\qquad - c\sum_{j=1}^{N}b_{ij}\Gamma e_j\big(t - \tau(t)\big), \quad i \subsetneqq \bar{V}_{\varphi i}, t \in \Xi_2^m \cup \Xi_4^m,
\end{cases}
\tag{29}
$$

$$
\dot{k}_i(t) =
\begin{cases}
\varepsilon_i e^{\beta t} e_i^T(t)\Gamma e_i(t), & t \in \Xi_1^m \cup \Xi_3^m, \\
0, & t \in \Xi_2^m \cup \Xi_4^m.
\end{cases}
\tag{30}
$$

Theorem 3.2 *Suppose that Assumption* 2.1 *holds. Using the adaptive controllers and up-dated laws* (29)-(30), *then the response networks* (2) *can linear generalized synchronize with the drive networks* (1) *if there exist positive constants* $\alpha > \beta > 0$ *such that the following conditions are satisfied*:

$$
c < -\frac{\tilde{\kappa}}{\lambda_{\max}(\hat{A}_l)},
\tag{31}
$$

$$
\tilde{L}\tilde{\Gamma} + \frac{\beta - \alpha}{2}I_N + \frac{1}{2(1-\mu)}I_N < 0,
\tag{32}
$$

$$
\beta - \alpha\theta > 0,
\tag{33}
$$

$$
\alpha\rho - a_* > 0,
\tag{34}
$$

where

$$
\tilde{\kappa} = \frac{1}{\lambda_{\min}(\tilde{\Gamma})}\left(\tilde{L}\|\tilde{\Gamma}\| + \frac{\beta}{2}I_N + \frac{1}{2(1-\mu)}I_N\right),
$$

$$
\theta = (1 - \eta_1)\frac{T_1}{T} + (1 - \eta_2)\frac{T_2}{T}, \qquad \rho = \frac{\eta_1 T_1 + \eta_2 T_2}{T},
$$

$$
\hat{A} = \frac{\mathrm{Sym}\{A\}}{2}, \quad a_* = \alpha - \beta,
$$

$\hat{A}_l$ *is the minor matrix of* A *by removing its first* l $(1 \le l \le N)$ *row-column pairs.*

Proof The proof is omitted here, as it is similar to that of Theorem 3.1. $\qquad\square$

4 Numerical examples and simulation

In this section, two numerical examples will be provided to verify and demonstrate the effectiveness of the proposed method.

Example 1 Theorem 3.1 is verified. The node dynamics of the first community are chosen as the well-known Lorenz system:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -10x_1 + 10x_2 \\ 28x_1 - x_2 - x_1x_3 \\ -\frac{8}{3}x_3 + x_1x_2 \end{bmatrix},
$$

and the node dynamics of the second community are chosen as the well-known Chen system:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -35x_1 + 35x_2 \\ -7x_1 + 28x_2 - x_1x_3 \\ -3x_3 + x_1x_2 \end{bmatrix},
$$

and the node dynamics of the third community are chosen as the well-known Lv system:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -36x_1 + 36x_2 \\ 15x_2 - x_1x_3 \\ -3x_3 + x_1x_2 \end{bmatrix}.
$$

Similar to the discussion of the Chen system in [24], one can discuss the Lorenz and Lv systems and choose the positive constant $L = 65$ such that Assumption 2.1 holds.

For simplicity, in the numerical simulations, assuming $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $T_1 = 0.2$, $T_2 = 0.1$, $\eta_1 = 0.9$, $\eta_2 = 0.8$, the time-varying $\tau(t) = 1 - 0.5e^{-t}$, then $\dot{\tau}(t) = 0.5e^{-t} \in (0, 0.5] \leq \frac{1}{2} \doteq \mu < 1$. A complex network consisting of 19 nodes with three communities is shown in Figure 2 ($A = B$).

The feedback control gain is $k_i = 20$ for $i \in \bar{V}_{\varphi i}$. By simple calculation, $\theta = \frac{1}{15}$, $\rho = 0.867$, for Theorem 3.1, one can choose $\alpha = 155$, $\beta = 22$, and $c < -\frac{\kappa}{\lambda_{\max}(\tilde{A}_l)}$ such that conditions (13)-(15) hold. Therefore, the CLGOS can be achieved for any initial values. Figures 3-5 show the orbits of linear generalized outer synchronization errors.

Figure 2 A complex network consisting of 19 nodes with three communities where $a_{ij} = a_{ji} = 1$ if there exists a connection between nodes i and j ($j \neq i$), otherwise $a_{ij} = a_{ji} = 0$.
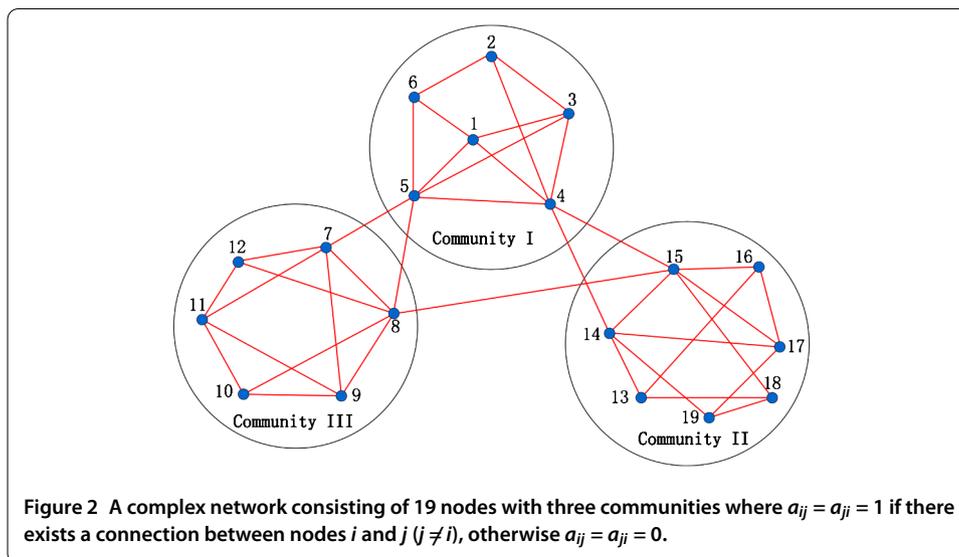
Figure 3 The orbits of linear generalized outer synchronization errors in the first community.
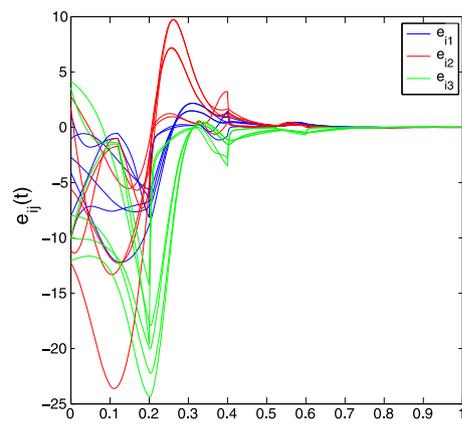
Figure 4 The orbits of linear generalized outer synchronization errors in the second community.
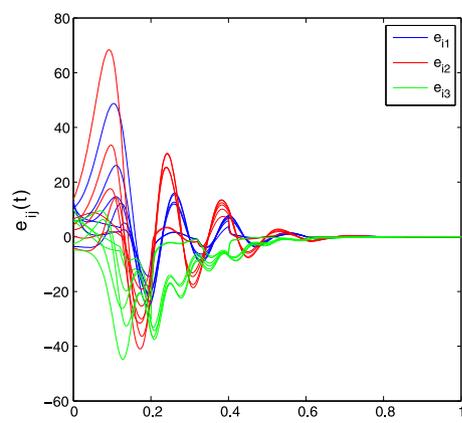
Figure 5 The orbits of linear generalized outer synchronization errors in the third community.
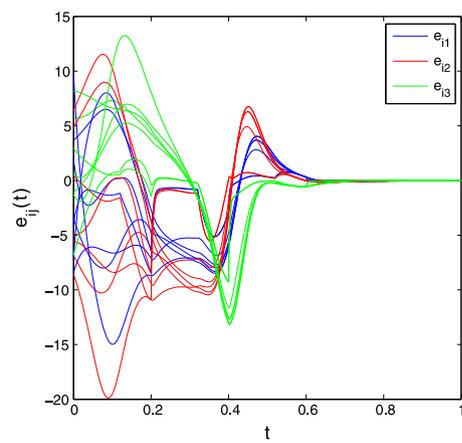
Example 2 Choosing the node dynamics as the following well-known time-delayed Chua oscillator:

$$\dot{x}(t) = \mathcal{M}x(t) + g_1\big(x(t)\big) + g_2\big(x(t-\tau)\big),$$

where

$$x(t) = \big(x_1(t), x_2(t), x_3(t)\big)^T, \quad \tau = 0.02,$$

$$\mathcal{M} = \begin{bmatrix} -a(1 + m_2) & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & -d \end{bmatrix},$$

$$g_1\big(x(t)\big) = \begin{bmatrix} -0.5a(m_1 - m_2)(|x_1(t) + 1| - |x_1(t) - 1|) \\ 0 \\ 0 \end{bmatrix},$$

$$g_2\big(x(t - \tau)\big) = \begin{bmatrix} 0 \\ 0 \\ -b\chi_0 \sin(\upsilon x_1(t - \tau))^T \end{bmatrix},$$

$a = 10$, $b = 19$, $d = 0.1636$, $m_1 = -1.4325$, $m_2 = -0.7831$, $\upsilon = 0.5$, $\chi_0 = 0.2$. Choose the positive constant $L = 12$ such that Assumption 2.1 holds. For simplicity, in the numerical simulations, assuming $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $T_1 = 0.2$, $T_2 = 0.1$, $\eta_1 = 0.9$, $\eta_2 = 0.8$, the time-varying $\tau(t) = 1 - 0.5e^{-t}$, then $\dot{\tau}(t) = 0.5e^{-t} \in (0, 0.5] \leq \frac{1}{2} \doteq \mu < 1$. A complex network consisting of 19 nodes with three communities is shown in Figure 2 ($A = B$). The community network is constructed by integrating three BA networks consisting of 50 nodes with $m_0 = m = 3$. For any pair of communities, four edges are chosen to connect them randomly (Figure 6). The feedback control gains as $k_i = 20$ for $i \in \bar{V}_{\varphi i}$. By simple calculations, $\theta = \frac{1}{15}$, $\rho = 0.867$, for Theorem 3.1, one can choose $\alpha = 35$, $\beta = 5$, and $c < -\frac{\kappa}{\lambda_{\max}(\hat{A}_l)}$ such that condition (13)-(15) hold. Therefore, the CLGOS can be achieved for any initial values. Figures 7-9 show the orbits of linear generalized outer synchronization errors.

5 Conclusions

In this paper, we investigated the problems of CLGOS in community networks via pinning control with two different switch periods. Using Lyapunov stability theory, linear matrix inequality (LMI), sufficient CLGOS criteria for community networks are

Figure 6 A community network is constructed by integrating three BA networks consisting of 50 nodes with $m_0 = m = 8$. For any pair of communities, four edges are chosen to connect them randomly.

Figure 7 The orbits of linear generalized outer synchronization errors in the first community.

Figure 8 The orbits of linear generalized outer synchronization errors in the second community.

Figure 9 The orbits of linear generalized outer synchronization errors in the third community.
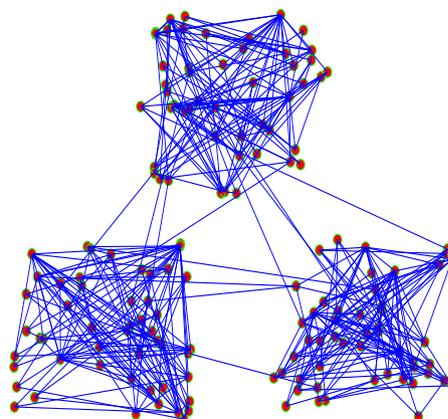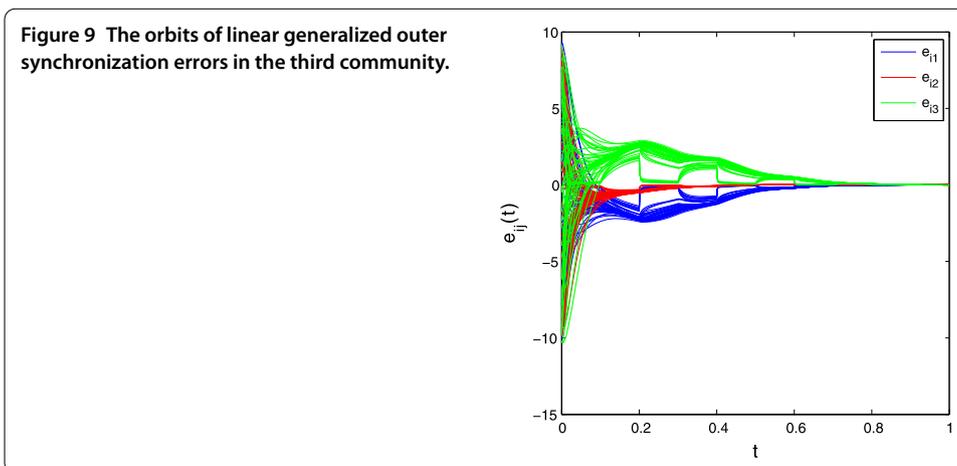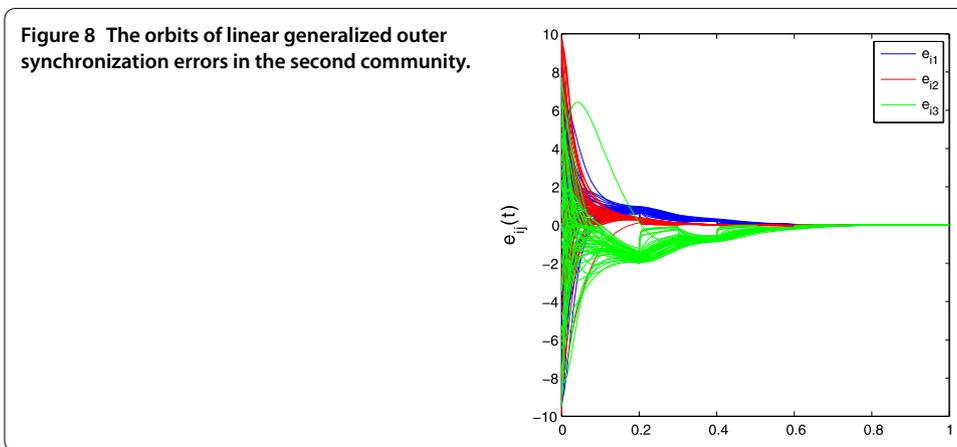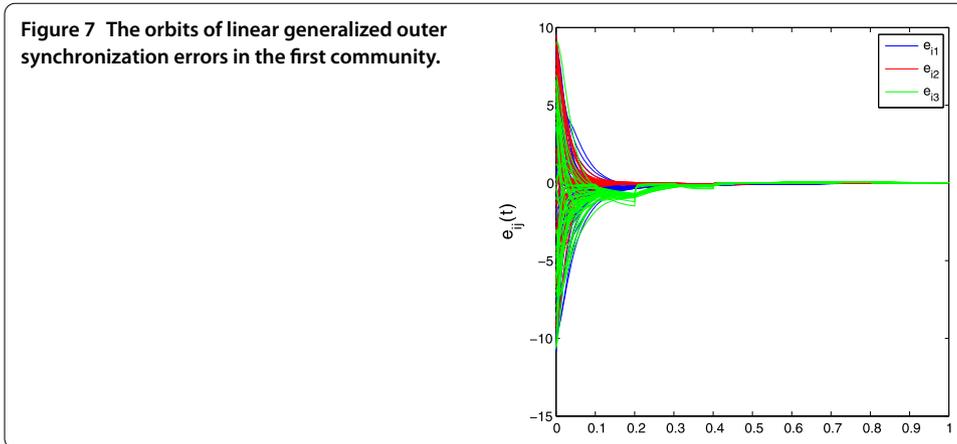
derived. Both community networks with identical nodes and non-identical nodes are investigated. Therefore, our proposed control schemes are better applicable technically. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed method.

Authors' contributions
YHL carried out the main part of this article, corrected and revised the manuscript, HL, SMZ, QSZ brought forward some suggestions on this article. All authors have read and approved the final manuscript.

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