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Extinction for a discrete competition system with feedback controls

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Abstract
We consider a nonautonomous discrete competition system with nonlinear interinhibition terms and feedback controls. By constructing a suitable Lyapunov function, we obtain some criteria about the extinction of one of the two species and the corresponding feedback controls varieties. Our conclusions not only supplement but also improve some existing ones. Numerical simulations are used to illustrate our analytic analysis. We show that feedback control variables play an important role in the extinction property of the system.

Keywords: extinction; discrete; competition; feedback controls

1 Introduction

Recently, much attention has been paid to the competition systems. For example, Wang *et al.* [1] considered the following two-species competition system with nonlinear interinhibition terms:

$$\begin{cases} \dot{x_1}(t) = x_1(t)\{r_1(t) - a_1(t)x_1(t) - \frac{c_2(t)x_2(t)}{1+x_2(t)}\}, \\ \dot{x_2}(t) = x_2(t)\{r_2(t) - a_2(t)x_2(t) - \frac{c_1(t)x_1(t)}{1+x_1(t)}\}, \end{cases} \tag{1.1}$$

where $x_1(t)$, $x_2(t)$ are the population densities of two competing species, $a_1(t)$, $a_2(t)$ are the intraspecific competition rate of the first and second species, $c_1(t)$, $c_2(t)$ represent the interspecific competing rates and $r_1(t)$, $r_2(t)$ are the intrinsic growth rates of species. Wang *et al.* [1] showed the existence and global asymptotic stability of positive almost periodic solutions of model (1.1). For the ecological sense of model (1.1), we refer to [2] and the references therein.

Considering that the discrete-time models governed by difference equations are more appropriate than the continuous ones when the populations have a short life expectancy and nonoverlapping generations, Qin *et al.* [3] introduced the following discrete analogue of system (1.1):

$$\begin{cases} x_1(n+1) = x_1(n)\exp\{r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)}\}, \\ x_2(n+1) = x_2(n)\exp\{r_2(n) - a_2(n)x_2(n) - \frac{c_1(n)x_1(n)}{1+x_1(n)}\}. \end{cases} \tag{1.2}$$

A good understanding of the permanence, existence, and global stability of positive periodic solutions was obtained in [3]. As for the almost periodic case, Wang and Liu [4] further studied the existence, uniqueness, and uniformly asymptotic stability of a positive almost periodic solution of system (1.2). Extinction of species and the stability property of another species were considered in [5]. Yue [6] investigated system (1.2) with one toxin producing species. Sufficient conditions that guarantee the extinction of one of the components and the global attractivity of the other one were obtained in [6].

Noting that ecosystems in the real world are often distributed by unpredictable forces that can result in changes in biological parameters, Wang *et al.* [7] proposed the following model, system (1.2) with feedback controls:

$$\begin{cases} x_1(n+1) = x_1(n)\exp\{r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)} - e_1(n)u_1(n)\}, \\ x_2(n+1) = x_2(n)\exp\{r_2(n) - a_2(n)x_2(n) - \frac{c_1(n)x_1(n)}{1+x_1(n)} - e_2(n)u_2(n)\}, \\ \Delta u_1(n) = -b_1(n)u_1(n) + d_1(n)x_1(n), \Delta u_2(n) = -b_2(n)u_2(n) + d_2(n)x_2(n). \end{cases} \tag{1.3}$$

Wang *et al.* [7] established a criterion for the existence and uniformly asymptotic stability of unique positive almost periodic solution of system (1.3) with almost periodic parameters. Yu [8] further considered the influence of feedback control variables on the persistent property of the system. On the other hand, as we all know, the extinction property is also an important topic in the study of mathematical biology; however, until now there are still no scholars investigating this property of system (1.3). Indeed, in this paper, we apply the analysis technique of Chen *et al.* [9], Xu *et al.* [10], and Zhang *et al.* [11] to obtain a set of sufficient conditions that guarantee one of the two species and the corresponding feedback controls varieties will be driven to extinction. For more works in this direction, we refer to [12–21] and the references therein.

For any bounded sequence $\{g(n)\}$, we denote $g^L = \inf_{n\in Z}\{g(n)\}$, $g^M = \sup_{n\in Z}\{g(n)\}$. For convenience, we introduce the following assumptions:

(H_1) $\{r_i(n)\}$, $i = 1, 2$, are bounded sequences defined on Z, and $\{a_i(n)\}$, $\{c_i(n)\}$, $\{d_i(n)\}$, and $\{e_i(n)\}$, $i = 1, 2$, are bounded nonnegative sequences defined on Z.

(H_2) Sequences $\{b_i(n)\}$ satisfy $0 < b_i^L \le b_i^M < 1$ for all $n \in Z$.

(H_3) There exists a positive integer ω such that, for each $i = 1, 2$,

$$\liminf_{n\to\infty} \sum_{s=n}^{n+\omega-1} r_i(s) > 0.$$

(H_4) There exists a positive integer ρ such that, for each $i = 1, 2$,

$$\limsup_{n\to\infty} \prod_{s=n}^{n+\rho-1} \bigl(1 - b_i(s)\bigr) < 1.$$

As regards the biological background, we focus our discussion on the positive solutions of system (1.3). So, we consider (1.3) together with the following initial conditions:

$$x_i(0) > 0, \qquad u_i(0) > 0, \quad i = 1, 2. \tag{1.4}$$

It is obvious that the solutions of (1.3)-(1.4) are well defined and satisfy

$$x_i(n) > 0, \qquad u_i(n) > 0, \quad i = 1, 2, \text{for } n \in Z. \tag{1.5}$$

The rest of this paper is organized as follows. In the next section, we study the extinction of one species and the corresponding feedback control varieties of system (1.3). Some examples together with their numerical simulations are carried out to show the feasibility of our results in Section 3. We end this paper with a brief discussion.

2 Extinction

In this section, we'll establish sufficient conditions on the extinction of one of the two species and the corresponding feedback controls varieties of system (1.3). Wang *et al.* [7] showed that the positive solutions of system (1.3) were bounded eventually:

Lemma 2.1 (see [7]) *Any positive solution* $(x_1(n), x_2(n), u_1(n), u_2(n))^T$ *of system* (1.3) *satisfies*

$$\limsup_{n\to\infty} x_i(n) \le B_i, \qquad \limsup_{n\to\infty} u_i(n) \le D_i, \tag{2.1}$$

where $B_i = \frac{\exp(r_i^M - 1)}{a_i^L}$ *and* $D_i = \frac{B_i d_i^M}{b_i^L}$ *for* $i = 1, 2$.

We now come to study the extinction of species x_2 and the feedback controls varieties u_2 of system (1.3).

Theorem 2.1 *In addition to* (H_1)-(H_4), *further suppose that*:

$$(H_5) \quad \limsup_{n\to\infty} \frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} < \liminf_{n\to\infty} \frac{a_2(n)}{c_2(n)}$$

and

$$(H_6) \quad \limsup_{n\to\infty} \frac{e_1(n)}{b_1(n)} < \liminf_{n\to\infty} \left(\frac{c_1(n)}{(1+B_1)d_1(n)} \liminf_{n\to\infty} \frac{\sum_{s=n}^{n+\omega-1} r_1(s)}{\sum_{s=n}^{n+\omega-1} r_2(s)} - \frac{a_1(n)}{d_1(n)} \right),$$

where B_1 *is defined in Lemma* 2.1. *Then* x_2 *and* u_2 *will be driven to extinction, that is, for any positive solution* $(x_1(n), x_2(n), u_1(n), u_2(n))^T$ *of system* (1.3), $\lim_{n\to\infty} x_2(n) = 0$ *and* $\lim_{n\to\infty} u_2(n) = 0$.

Proof According to Lemma 2.1, for any $\varepsilon > 0$ small enough, there exists $n_1 > 0$ large enough such that, for $n \ge n_1$,

$$x_1(n) \le B_1 + \varepsilon, \qquad u_1(n) \le D, \qquad u_2(n) \le D, \tag{2.2}$$

where $D = \max\{D_1 + \varepsilon, D_2 + \varepsilon\}$. Thus, it follows from (H_3) that there exist positive constants η_0 and $n_2 \ge n_1$ such that

$$\sum_{s=n}^{n+\omega-1} r_i(s) \ge \eta_0 \quad \text{for all } n \ge n_2.$$

By (H_1), (H_2), and (H_5) we can obtain that

$$\liminf_{n\to\infty} \frac{e_2(n)}{b_2(n)} > \limsup_{n\to\infty} \left(\frac{c_2(n)}{d_2(n)} \limsup_{n\to\infty} \frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} - \frac{a_2(n)}{d_2(n)} \right). \tag{2.3}$$

For the same ε, according to (H_5)-(H_6) and (2.3), we can choose positive constants $\alpha, \beta, \gamma, \delta$, and $n_3 \geq n_2$ such that

$$\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} < \frac{\alpha}{\beta} - \varepsilon < \frac{\alpha}{\beta} < \frac{a_2(n)}{c_2(n)},$$
$$\frac{e_1(n)}{b_1(n)} < \frac{\gamma}{\alpha} < \frac{\beta c_1(n) - \alpha(1 + B_1 + \varepsilon)a_1(n)}{\alpha(1 + B_1 + \varepsilon)d_1(n)}$$

and

$$\frac{e_2(n)}{b_2(n)} > \frac{\delta}{\beta} > \frac{\alpha c_2(n) - \beta a_2(n)}{\beta d_2(n)}$$

for all $n \geq n_3$. Hence, we have:

$$\sum_{s=n}^{n+\omega-1} \bigl(\beta r_2(s) - \alpha r_1(s)\bigr) < -\varepsilon\beta\eta_0, \tag{2.4}$$

$$\alpha e_1(n) - \gamma b_1(n) < 0, \tag{2.5}$$

$$\alpha a_1(n) - \frac{\beta c_1(n)}{1 + B_1 + \varepsilon} + \gamma d_1(n) < 0, \tag{2.6}$$

$$\delta b_2(n) - \beta e_2(n) < 0, \tag{2.7}$$

$$\alpha c_2(n) - \beta a_2(n) - \delta d_2(n) < 0. \tag{2.8}$$

Consider the Lyapunov function

$$V(n) = x_1^{-\alpha}(n)x_2^{\beta}(n)\exp\bigl\{\gamma u_1(n) - \delta u_2(n)\bigr\}. \tag{2.9}$$

By calculating we get

$$\begin{aligned}
\frac{V(n+1)}{V(n)} = \exp\Biggl\{&\beta r_2(n) - \alpha r_1(n) + \bigl(\alpha e_1(n) - \gamma b_1(n)\bigr)u_1(n) \\
&+ \bigl(\delta b_2(n) - \beta e_2(n)\bigr)u_2(n) + \left(\alpha a_1(n) - \frac{\beta c_1(n)}{1 + x_1(n)} + \gamma d_1(n)\right)x_1(n) \\
&+ \left(\frac{\alpha c_2(n)}{1 + x_2(n)} - \beta a_2(n) - \delta d_2(n)\right)x_2(n)\Biggr\} \\
\leq \exp\Biggl\{&\beta r_2(n) - \alpha r_1(n) + \bigl(\alpha e_1(n) - \gamma b_1(n)\bigr)u_1(n) \\
&+ \bigl(\delta b_2(n) - \beta e_2(n)\bigr)u_2(n) + \left(\alpha a_1(n) - \frac{\beta c_1(n)}{1 + B_1 + \varepsilon} + \gamma d_1(n)\right)x_1(n) \\
&+ \bigl(\alpha c_2(n) - \beta a_2(n) - \delta d_2(n)\bigr)x_2(n)\Biggr\}.
\end{aligned}$$

It follows that from (2.5)-(2.8) that

$$V(n+1) \leq V(n)\exp\bigl\{\beta r_2(n) - \alpha r_1(n)\bigr\} \quad \text{for all } n \geq n_3. \tag{2.10}$$

For any $n \geq n_3$, we choose an integer $m \geq 0$ such that $n \in [n_3 + m\omega, n_3 + (m+1)\omega)$. Integrating (2.10) from n_3 to $n-1$ leads to

$$\begin{aligned} V(n) &\leq V(n_3)\exp\left\{\sum_{s=n_3}^{n-1}\bigl(\beta r_2(s) - \alpha r_1(s)\bigr)\right\} \\ &\leq V(n_3)\exp\left\{\sum_{s=n_3}^{n_3+m\omega-1} + \sum_{s=n_3+m\omega-1}^{n-1}\right\}\bigl(\beta r_2(s) - \alpha r_1(s)\bigr) \\ &\leq V(n_3)\exp\{-\varepsilon\beta\eta_0 m + M_1\} \\ &\leq V(n_3)\exp\left\{-\varepsilon\beta\eta_0\left(\frac{n-n_3}{\omega} - 1\right) + M_1\right\} \\ &\leq V(n_3)\exp\left\{-\frac{\varepsilon\beta\eta_0 n}{\omega} + M_1^*\right\}, \end{aligned} \tag{2.11}$$

where $M_1^* = \frac{\varepsilon\beta\eta_0 n_3}{\omega} + \varepsilon\beta\eta_0 + M_1$ and $M_1 = \sup_{n\in Z}|\beta r_2(n) - \alpha r_1(n)|\omega$. Relations (2.2), (2.9), and (2.11) imply that that, for $n \geq n_3$,

$$x_2(n) < \left[x_1^{-\alpha}(n_3)x_2^{\beta}(n_3)\exp\bigl\{(\gamma+\delta)D\bigr\}(B_1+\varepsilon)^{\alpha}\exp\bigl\{M_1^*\bigr\}\right]^{\frac{1}{\beta}}\exp\left\{-\frac{\varepsilon\eta_0 n}{\omega}\right\}. \tag{2.12}$$

Hence, $x_2(n) \to 0$ exponentially as $n \to \infty$. Similarly to corresponding proof of Theorem 3.1 in Chen *et al.* [9], we can easily see that $u_2(n) \to 0$ as $n \to \infty$. This ends the proof of Theorem 2.1. □

Now, let us investigate the extinction property of species x_1 and the feedback controls varieties u_1 in system (1.3), which is also an interesting problem, and we obtain the following result.

Theorem 2.2 *Let* $(x_1(n), x_2(n), u_1(n), u_2(n))^T$ *be any positive solution of system* (1.3). *Suppose that* (H_1)-(H_4) *and the following inequalities hold*:

$$(H_7) \quad \liminf_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} > \limsup_{n\to\infty}\frac{c_1(n)}{a_1(n)},$$

$$(H_8) \quad \limsup_{n\to\infty}\frac{e_2(n)}{b_2(n)} < \liminf_{n\to\infty}\left(\frac{c_2(n)}{(1+B_2)d_2(n)}\liminf_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} - \frac{a_2(n)}{d_2(n)}\right),$$

where B_2 *is defined in Lemma* 2.1. *Then* $\lim_{n\to\infty} x_1(n) = 0$ *and* $\lim_{n\to\infty} u_1(n) = 0$.

Proof According to Lemma 2.1, for any $\varepsilon > 0$ small enough, there exists a positive constant $n_4 > n_3$ such that, for $n \geq n_4$,

$$x_2(n) \leq B_2 + \varepsilon. \tag{2.13}$$

By (H_1), (H_2), and (H_7) we obtain that

$$\liminf_{n\to\infty}\frac{e_1(n)}{b_1(n)} > \limsup_{n\to\infty}\left(\frac{c_1(n)}{d_1(n)}\limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_1(s)}{\sum_{s=n}^{n+\omega-1} r_2(s)} - \frac{a_1(n)}{d_1(n)}\right). \tag{2.14}$$

For the same ε, according to (H_7)-(H_8) and (2.14), we can choose positive constants $\alpha, \beta, \gamma, \delta$, and $n_5 \geq n_4$ such that:

$$\begin{aligned}
&\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} > \frac{\alpha}{\beta} + \varepsilon > \frac{\alpha}{\beta} > \frac{a_2(n)}{c_2(n)}, \\
&\frac{e_2(n)}{b_2(n)} < \frac{\delta}{\beta} < \frac{\alpha c_2(n) - \beta(1 + B_2 + \varepsilon)a_2(n)}{\beta(1 + B_2 + \varepsilon)d_2(n)}
\end{aligned}$$

and

$$\frac{e_1(n)}{b_1(n)} > \frac{\gamma}{\alpha} > \frac{\beta c_1(n) - \alpha a_1(n)}{\alpha d_1(n)}$$

for all $n \geq n_5$. Hence, we have:

$$\sum_{s=n}^{n+\omega-1} \bigl(\alpha r_1(s) - \beta r_2(s)\bigr) < -\varepsilon\beta\eta_0, \tag{2.15}$$

$$\beta e_2(n) - \delta b_2(n) < 0, \tag{2.16}$$

$$-\frac{\alpha c_2(n)}{1 + B_2 + \varepsilon} + \beta a_2(n) + \delta d_2(n) < 0, \tag{2.17}$$

$$\gamma b_1(n) - \alpha e_1(n) < 0, \tag{2.18}$$

$$-\alpha a_1(n) + \beta c_1(n) - \gamma d_1(n) < 0. \tag{2.19}$$

Consider the Lyapunov function

$$V(n) = x_1^{\alpha}(n) x_2^{-\beta}(n) \exp\bigl\{\delta u_2(n) - \gamma u_1(n)\bigr\}. \tag{2.20}$$

By calculating and inequalities (2.16)-(2.19) we obtain that

$$\begin{aligned}
\frac{V(n+1)}{V(n)} &= \exp\Biggl\{\alpha r_1(n) - \beta r_2(n) + \bigl(\gamma b_1(n) - \alpha e_1(n)\bigr)u_1(n) \\
&\quad + \bigl(\beta e_2(n) - \delta b_2(n)\bigr)u_2(n) + \left(-\alpha a_1(n) + \frac{\beta c_1(n)}{1 + x_1(n)} - \gamma d_1(n)\right)x_1(n) \\
&\quad + \left(-\frac{\alpha c_2(n)}{1 + x_2(n)} + \beta a_2(n) + \delta d_2(n)\right)x_2(n)\Biggr\} \\
&\leq \exp\Biggl\{\alpha r_1(n) - \beta r_2(n) + \bigl(\gamma b_1(n) - \alpha e_1(n)\bigr)u_1(n) \\
&\quad + \bigl(\beta e_2(n) - \delta b_2(n)\bigr)u_2(n) + \bigl(-\alpha a_1(n) + \beta c_1(n) - \gamma d_1(n)\bigr)x_1(n) \\
&\quad + \left(-\frac{\alpha c_2(n)}{1 + B_2 + \varepsilon} + \beta a_2(n) + \delta d_2(n)\right)x_2(n)\Biggr\} \\
&\leq \exp\bigl\{\alpha r_1(n) - \beta r_2(n)\bigr\}.
\end{aligned} \tag{2.21}$$

From (2.21), similarly to the analysis of of Theorem 2.1, we can get the conclusion that $x_1(n) \to 0$ and $u_1(n) \to 0$ as $n \to \infty$. This ends the proof of Theorem 2.1. □

When $e_i(n) = b_i(n) = d_i(n) = 0$ $(i = 1,2)$, (1.3) becomes (1.2), as discussed in [5]. Similarly to the proofs of Theorems 2.1 and 2.2, we can obtain the following:

Corrolary 2.1 *In addition to* (H_1)-(H_3), *further suppose that*

$$(H_9)\quad \limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} < \min\left\{\liminf_{n\to\infty}\frac{a_2(n)}{c_2(n)}, \liminf_{n\to\infty}\frac{c_1(n)}{(1+B_1)a_1(n)}\right\},$$

where B_i $(i = 1,2)$ *are defined in Lemma* 2.1. *Then the species* x_2 *will be driven to extinction, that is, for any positive solution* $(x_1(n), x_2(n))^T$ *of system* (1.2), $\lim_{n\to\infty} x_2(n) = 0$.

Corrolary 2.2 *Let* $(x_1(n), x_2(n))^T$ *be any positive solution of system* (1.2). *Suppose that*

$$(H_{10})\quad \liminf_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} > \max\left\{\limsup_{n\to\infty}\frac{c_1(n)}{a_1(n)}, \limsup_{n\to\infty}\frac{(1+B_2)a_2(n)}{c_2(n)}\right\},$$

where B_2 *is defined in Lemma* 2.1. *Then* $\lim_{n\to\infty} x_1(n) = 0$.

Remark 2.1 Comparing with Assumptions (H_1) and (H_2) given in [5], we can see that our assumptions in Corollaries 2.1 and 2.2 are weaker.

3 Example and numeric simulation

In this section, we give the following two examples to illustrate our main results.

Example 3.1 Consider the following system:

$$\begin{cases} x_1(n+1) = x_1(n)\exp\{1.4 - 1.75x_1(n) - \frac{(1+0.3\sin(n))x_2(n)}{1+x_2(n)} - 0.9u_1(n)\}, \\ x_2(n+1) = x_2(n)\exp\{0.7 - 2.6x_2(n) - \frac{(6+2\cos(n))x_1(n)}{1+x_1(n)} - 1.5u_2(n)\}, \\ \Delta u_1(n) = -(0.8+0.1\sin(n))u_1(n) + 0.5x_1(n), \\ \Delta u_2(n) = -(0.7+0.2\cos(n))u_2(n) + 0.4x_2(n). \end{cases} \tag{3.1}$$

In this case, we have that (H_1)-(H_4) hold and $B_1 = \frac{\exp(r_1^M-1)}{a_1^L} \approx 0.8525$, and hence

$$\begin{aligned} &\limsup_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} = 0.5 < \liminf_{n\to\infty}\frac{a_2(n)}{c_2(n)} \approx 2, \\ &\limsup_{n\to\infty}\frac{e_1(n)}{b_1(n)} \approx 1.2857, \\ &\liminf_{n\to\infty}\left(\frac{c_1(n)}{(1+B_1)d_1(n)}\liminf_{n\to\infty}\frac{\sum_{s=n}^{n+\omega-1} r_1(s)}{\sum_{s=n}^{n+\omega-1} r_2(s)} - \frac{a_1(n)}{d_1(n)}\right) \approx 5.1370. \end{aligned}$$

So all conditions in Theorem 2.1 ares satisfied, and x_2 and u_2 in system (3.1) are extinct. Our numerical simulation supports this result (see Figure 1).

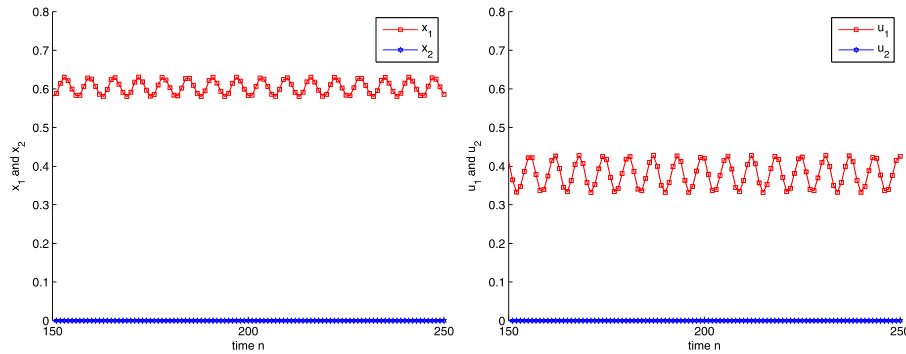

Figure 1 **Dynamic behavior of system (3.1) with the initial conditions $(x_1(0), x_2(0), u_1(0), u_2(0)) = (0.1, 0.3, 0.2, 0.04)^T$ and $(0.2, 0.1, 0.6, 0.5)^T$.**

Example 3.2 Consider the following system:

$$\begin{cases} x_1(n+1) = x_1(n)\exp\{0.4 - 1.75x_1(n) - \frac{(3.4+0.4\sin(n))x_2(n)}{1+x_2(n)} - 0.9u_1(n)\}, \\ x_2(n+1) = x_2(n)\exp\{1.6 - 1.3x_2(n) - \frac{(3+\cos(n))x_1(n)}{1+x_1(n)} - 1.5u_2(n)\}, \\ \Delta u_1(n) = -(0.8 + 0.1\sin(n))u_1(n) + 0.5x_1(n), \\ \Delta u_2(n) = -(0.7 + 0.2\cos(n))u_2(n) + 0.4x_2(n). \end{cases} \tag{3.2}$$

In this case, we have that (H_1)-(H_4) hold and $B_2 = \frac{\exp(r_2^M - 1)}{a_2^L} \approx 1.4016$, and hence

$$\liminf_{n\to\infty} \frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} = 4 > \limsup_{n\to\infty} \frac{c_1(n)}{a_1(n)} \approx 2.2857,$$

$$\limsup_{n\to\infty} \frac{e_2(n)}{b_2(n)} \approx 3,$$

$$\liminf_{n\to\infty}\left(\frac{c_2(n)}{(1+B_2)d_2(n)} \liminf_{n\to\infty} \frac{\sum_{s=n}^{n+\omega-1} r_2(s)}{\sum_{s=n}^{n+\omega-1} r_1(s)} - \frac{a_2(n)}{d_2(n)}\right) \approx 9.2417.$$

So all conditions in Theorem 2.2 are satisfied, and x_1 and u_1 in system (3.2) are extinct. Numerical simulation also confirms our result (see Figure 2).

4 Discussion

In this paper, we consider a two-species nonautonomous discrete competition system with nonlinear interinhibition terms and feedback controls, that is, (1.3) which was discussed in [7, 8]. However, until now, there are still no scholars investigating the extinction property of system (1.3), which is also an important topic in mathematical biology. By developing the analysis technique of Chen *et al.* [9], Xu *et al.* [10], and Zhang *et al.* [11] we obtain sufficient conditions that guarantee the extinction of one of the components and the corresponding feedback controls varieties. When $e_i(n) = b_i(n) = d_i(n) = 0$ $(i = 1, 2)$, (1.3) becomes (1.2), as discussed in [3–5]. As direct results of Theorems 2.1 and 2.2, Corollaries 2.1 and 2.2 improve and supplement those of [5, 7, 8]. Moreover, by comparing Theorem 2.1 with Corollary 2.1, and Theorem 2.2 with Corollary 2.2 we also found that, for such a kind of

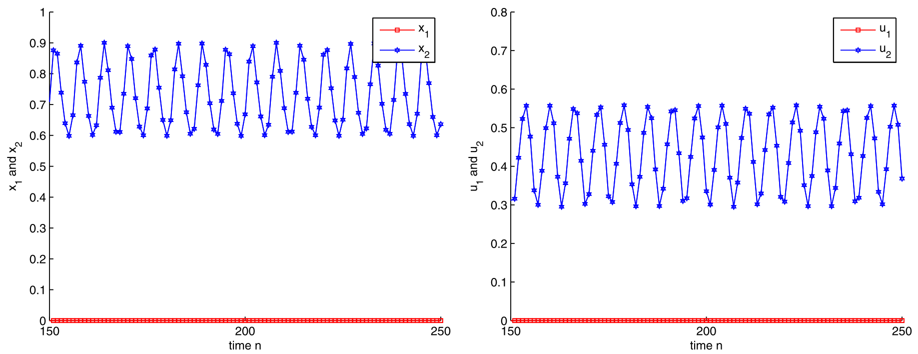

Figure 2 Dynamic behavior of system (3.1) with the initial conditions $(x_1(0), x_2(0), u_1(0), u_2(0)) = (0.1, 0.3, 0.2, 0.04)^T$ and $(0.2, 0.1, 0.6, 0.5)^T$.

systems, feedback control variables play an important role in the extinction property of the system.

Competing interests
The author declares that there is no conflict of interests regarding the publication of this paper.

Author's contributions
The author wrote the manuscript carefully, read, and approved the final manuscript.

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