Advances in Difference Equations
a SpringerOpen Journal

RESEARCH

Open Access

Permanence for a modified Leslie-Gower predator-prey model with Beddington-DeAngelis functional response and feedback controls

Qin Yue[*]

[*]Correspondence:
xhper1978@sina.com
College of Finance and
Mathematics, West Anhui University,
Liuan, Anhui 237000, P.R. China

Abstract

A modified Leslie-Gower predator-prey system with Beddington-DeAngelis functional response and feedback controls is studied. By applying the differential inequality theory, sufficient conditions which guarantee the permanence of the system are obtained. Our results improve the main results of Zhang *et al.* (Abstr. Appl. Anal. 2014:252579, 2014). One example is presented to verify our main results.

Keywords: permanence; Leslie-Gower model; Beddington-DeAngelis functional response; feedback controls

1 Introduction

Let $f(t)$ be a continuous bounded function on $\mathbb{R}$, and we set

$$f^l = \inf_{t \in \mathbb{R}} f(t), \qquad f^u = \sup_{t \in \mathbb{R}} f(t).$$

Leslie [1, 2] introduced the following two species Leslie-Gower predator-prey model:

$$\begin{cases} \dot{x}(t) = (r_1 - b_1 x(t))x(t) - p(x(t))y(t), \\ \dot{y}(t) = (r_2 - a_2 \frac{y(t)}{x(t)})y(t), \end{cases} \tag{1.1}$$

where $x(t)$, $y(t)$ stand for the population (the density) of the prey and the predator at time t, respectively. The parameters r_1 and r_2 are the intrinsic growth rates of the prey and the predator, respectively. b_1 measures the strength of competition among individuals of species x. The value $\frac{r_1}{b_1}$ is the carrying capacity of the prey in the absence of predation. The predator consumes the prey according to the functional response $p(x)$ and grows logistically with growth rate r_2 and carrying capacity $\frac{r_2 x}{a_2}$ proportional to the population size of the prey (or prey abundance). The parameter a_2 is a measure of the food quantity that the prey provides and converted to predator birth. The term y/x is the Leslie-Gower term which measures the loss in the predator population due to rarity (*per capita* y/x) of its favorite food. Leslie model is a predator-prey model where the carrying capacity of the

predator is proportional to the number of prey, stressing the fact that there are upper limits to the rates of increase in both prey x and predator y, which are not recognized in the Lotka-Volterra model.

The main shortcoming of (1.1) is that in the case of severe scarcity, y can switch over to other populations but its growth will be limited by the fact that its most favorite food x is not available in abundance. In order to overcome this shortcoming, recently, Aziz-Alaoui and Daher Okiye [3] suggested to add a positive constant d to the denominator and proposed the following predator-prey model with modified Leslie-Gower and Holling-type II schemes:

$$\begin{cases} \dot{x}(t) = (r_1 - b_1 x(t) - \frac{a_1 y(t)}{x(t)+k_1})x(t), \\ \dot{y}(t) = (r_2 - \frac{a_2 y(t)}{x(t)+k_2})y(t), \end{cases} \tag{1.2}$$

where r_1, b_1, r_2, a_2 have the same meaning as in models (1.1). a_1 is the maximum value of the *per capita* reduction rate of x due to y, k_1 (respectively, k_2) measures the extent to which the environment provides protection to prey x (respectively, to the predator y). They obtained the boundedness and global stability of positive equilibrium of system (1.2). Yu [4] studied the structure, linearized stability and the global asymptotic stability of equilibria of (1.2). Zhu and Wang [5] obtained sufficient conditions for the existence and global attractivity of positive periodic solutions of system (1.2) with periodic coefficients. Yu and Chen [6] further considered the permanence and existence of a unique globally attractive positive almost periodic solution of system (1.2) with almost periodic coefficients and mutual interference. Considering that Beddington-DeAngelis functional response preformed better than Holling-type II functional response, Yu [7] incorporated the Beddington-DeAngelis functional response into system (1.2) and considered the following model which is the generalization of model (1.2):

$$\begin{cases} \dot{x}(t) = (r_1 - b_1 x(t) - \frac{a_1 y(t)}{\alpha + \beta x(t) + \gamma y(t)})x(t), \\ \dot{y}(t) = (r_2 - \frac{a_2 y(t)}{x(t)+k_2})y(t). \end{cases} \tag{1.3}$$

Sufficient conditions on the global asymptotic stability of a positive equilibrium were obtained by Yu [7]. Pal and Mandal [8] considered the Hopf bifurcation of system (1.3) with strong Allee effect. Zhang [9] studied the permanence and existence of an almost periodic solution of system (1.3) with almost periodic coefficients.

Based on Zhang [9], Zhang *et al.* [10] incorporated the feedback control into model (1.3) with almost periodic coefficients and considered the following model:

$$\begin{cases} \dot{x}(t) = x(t)(a_1(t) - b(t)x(t) - \frac{c(t)y(t)}{\alpha(t)+\beta(t)x(t)+\gamma(t)y(t)} - e_1(t)u(t)), \\ \dot{y}(t) = y(t)(a_2(t) - \frac{r(t)y(t)}{x(t)+k(t)} - e_2(t)v(t)), \\ \dot{u}(t) = -d_1(t)u(t) + p_1(t)x(t-\tau), \\ \dot{v}(t) = -d_2(t)v(t) + p_2(t)y(t-\tau), \end{cases} \tag{1.4}$$

where $b(t), c(t), r(t), k(t), \alpha(t), \beta(t), \gamma(t), a_i(t), d_i(t), p_i(t)$ and $e_i(t)$ ($i = 1, 2$) are all continuous, almost periodic functions and satisfy

$$\min_{i=1,2}\{b^l, c^l, r^l, k^l, \alpha^l, \beta^l, \gamma^l, a_i^l, d_i^l, p_i^l, e_i^l\} > 0,$$

$$\max_{i=1,2}\{b^u, c^u, r^u, k^u, \alpha^u, \beta^u, \gamma^u, a_i^u, d_i^u, p_i^u, e_i^u\} < +\infty.$$

Set

$$M_1 = \frac{a_1^u}{b^l}, \qquad M_2 = \frac{a_2^u(M_1 + k^u)}{r^l}, \qquad L_1 = \frac{p_1^u M_1}{d_1^l}, \qquad L_2 = \frac{p_2^u M_2}{d_2^l}. \tag{1.5}$$

Suppose that system (1.4) holds together with the following initial conditions:

$$
\begin{aligned}
x(s) &= \phi(s) \geq 0, \quad s \in [-\tau, 0], \qquad \phi(0) > 0, \\
y(s) &= \varphi(s) \geq 0, \quad s \in [-\tau, 0], \qquad \varphi(0) > 0, \\
u(0) &> 0, \qquad v(0) > 0.
\end{aligned} \tag{1.6}
$$

By using the comparison theorem of differential equation, Zhang *et al.* [10] obtained the following result.

Theorem A ([10]) *Assume that*

$$(\mathrm{H}_1) \quad a_1^l - \frac{c^u}{\gamma^l} - e_1^u L_1 > 0 \quad \text{and} \quad a_2^l - e_2^u L_2 > 0$$

hold, then system (1.4) *with initial conditions* (1.6) *is permanent, i.e., any positive solution* $(x(t), y(t), u(t), v(t))^T$ *of system* (1.4) *with initial conditions* (1.6) *satisfies*

$$m_1 \leq \liminf_{t \to +\infty} x(t) \leq \limsup_{t \to +\infty} x(t) \leq M_1, \qquad m_2 \leq \liminf_{t \to +\infty} y(t) \leq \limsup_{t \to +\infty} y(t) \leq M_2,$$

$$l_1 \leq \liminf_{t \to +\infty} u(t) \leq \limsup_{t \to +\infty} y(t) \leq L_1, \qquad l_2 \leq \liminf_{t \to +\infty} v(t) \leq \limsup_{t \to +\infty} y(t) \leq L_2,$$

where m_i, l_i, M_i *and* L_i $(i = 1, 2)$ *are positive constants.*

Theorem A shows that feedback control variables play important roles in the persistent property of system (1.4). But the question is whether or not the feedback control variables have influence on the permanence of the system. Many papers (see [11–13] and the references cited therein) have showed that feedback control variables have no influence on the permanent property of continuous system with feedback control. Thus, in this paper, we will apply the analysis technique of Chen *et al.* [11] to establish sufficient conditions, which is independent of feedback control variables, to ensure the permanence of the system. In fact, we obtain the following main result.

Theorem B *Assume that*

$$(\mathrm{H}_2) \quad a_1^l - \frac{c^u}{\gamma^l} > 0$$

holds, then system (1.4) *with initial conditions* (1.6) *is permanent.*

Comparing with Theorem A, it is easy to see that (H_2) in Theorem B is weaker than (H_1) in Theorem A, and feedback control variables have no influence on the permanent property of system (1.4), so our results improve the main results in Zhang *et al.* [10].

The organization of this paper is as follows. In Section 2, we introduce several lemmas, and the permanence of system (1.4) is then studied in this section. In Section 3, a suitable example together with its numerical simulations is given to illustrate the feasibility of the main results.

2 Permanence

Now let us state several lemmas which will be useful in proving the main results of this section.

Lemma 2.1 ([14]) *If $a > 0$, $b > 0$ and $\dot{x} \geq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have*

$$\liminf_{t \to +\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0$, $b > 0$ and $\dot{x} \leq x(b - ax)$, when $t \geq 0$ and $x(0) > 0$, we have

$$\limsup_{t \to +\infty} x(t) \leq \frac{b}{a}.$$

Lemma 2.2 ([11]) *Assume that $a > 0$, $b(t) > 0$ is a bounded continuous function and $x(0) > 0$. Further suppose that:*
  (i)

$$\dot{x}(t) \leq -ax(t) + b(t),$$

  then for all $t \geq s$,

$$x(t) \leq x(t - s) \exp\{-as\} + \int_{t-s}^{t} b(\tau) \exp\{a(\tau - t)\} \, d\tau.$$

  Especially, if $b(t)$ is bounded above with respect to M, then

$$\limsup_{t \to +\infty} x(t) \leq \frac{M}{a}.$$

  (ii)

$$\dot{x}(t) \geq -ax(t) + b(t),$$

  then for all $t \geq s$,

$$x(t) \geq x(t - s) \exp\{-as\} + \int_{t-s}^{t} b(\tau) \exp\{a(\tau - t)\} \, d\tau.$$

  Especially, if $b(t)$ is bounded above with respect to m, then

$$\liminf_{t \to +\infty} x(t) \geq \frac{m}{a}.$$

The following lemma is a direct conclusion of [10].

Lemma 2.3 *For any positive solution* $(x(t), y(t), u(t), v(t))^T$ *of system* (1.4) *with initial conditions* (1.6), *there exist four positive constants* M_i *and* L_i $(i = 1, 2)$ *such that*

$$
\begin{aligned}
&\limsup_{t \to +\infty} x(t) \le M_1, \qquad \limsup_{t \to +\infty} y(t) \le M_2, \\
&\limsup_{t \to +\infty} u(t) \le L_1, \qquad \limsup_{t \to +\infty} v(t) \le L_2,
\end{aligned}
\tag{2.1}
$$

where M_i *and* L_i $(i = 1, 2)$ *are defined in* (1.5).

Lemma 2.4 *Suppose that* (H_2) *holds, then there exist two positive constants* m_1 *and* l_1 *such that any positive solution* $(x(t), y(t), u(t), v(t))^T$ *of system* (1.4) *with initial conditions* (1.6) *satisfies*

$$
\liminf_{t \to +\infty} x(t) \ge m_1, \qquad \liminf_{t \to +\infty} u(t) \ge l_1,
$$

where m_1 *and* l_1 *are defined in the proof.*

Proof According to Lemma 2.3, there exists enough large $T_1 > 0$ such that for $t \ge T_1$,

$$
x(t) \le 2M_1, \qquad u(t) \le 2L_1.
\tag{2.2}
$$

Thus, it follows from (2.2) and the first equation of system (1.4) that

$$
\begin{aligned}
\dot{x}(t) &= x(t)\left(a_1(t) - b(t)x(t) - \frac{c(t)y(t)}{\alpha(t) + \beta(t)x(t) + \gamma(t)y(t)} - e_1(t)u(t) \right) \\
&\ge x(t)\left(a_1^l - 2b^u M_1 - \frac{c^u}{\gamma^l} - 2e_1^u L_1 \right) \\
&\triangleq Qx(t),
\end{aligned}
\tag{2.3}
$$

where $Q = a_1^l - 2b^u M_1 - \frac{c^u}{\gamma^l} - 2e_1^u L_1 < a_1^l - 2b^u M_1 < a_1^u - 2b^l \frac{a_1^u}{b^l} = -a_1^u < 0$.

Integrating both sides of (2.3) from η $(\eta \le t)$ to t leads to

$$
\frac{x(t)}{x(\eta)} \ge \exp\{Q(t - \eta)\}
$$

or

$$
x(\eta) \le x(t)\exp\{-Q(t - \eta)\}.
\tag{2.4}
$$

Particularly, take $\eta = t - \tau$, one can get

$$
x(t - \tau) \le x(t)\exp\{-Q\tau\}.
\tag{2.5}
$$

Substituting (2.5) into the third equation of system (1.4) leads to

$$
\dot{u}(t) \le -d_1^l u(t) + p_1^u x(t)\exp\{-Q\tau\}.
\tag{2.6}
$$

Applying Lemma 2.2(i) to the above differential inequality, for $0 \le s \le t$, one has

$$
\begin{aligned}
u(t) \quad &\le \quad u(t-s)\exp\{-d_1^l s\} + \int_{t-s}^{t} p_1^u x(\eta)\exp\{-Q\tau\}\exp\{d_1^l(\eta - t)\}\,d\eta \\
&\overset{\text{from (2.4)}}{\le} \quad u(t-s)\exp\{-d_1^l s\} \\
&\qquad + \int_{t-s}^{t} p_1^u x(t)\exp\{-Q(t-\eta)\}\exp\{-Q\tau\}\exp\{d_1^l(\eta - t)\}\,d\eta \\
&\le \quad u(t-s)\exp\{-d_1^l s\} + p_1^u x(t)\frac{1}{Q}\big(1 - \exp\{-Qs\}\big)\exp\{-Q\tau\},
\end{aligned}
\tag{2.7}
$$

where we have used the fact that $\max_{\eta \in [t-s,t]}\exp\{d_1^l(\eta - t)\} = \exp\{0\} = 1$.

Note that there exists K such that $2e_1^u L_1 \exp\{-d_1^l K\} < \frac{\beta}{2}$, as $s \ge K$, where $\beta = a_1^l - \frac{c^u}{\gamma^l} > 0$ according to (H$_2$). In fact, we can choose $K > \frac{1}{d_1^l}\ln\frac{4e_1^u L_1}{\beta}$. And so, fix K, combined with (2.7), we can obtain

$$
\begin{aligned}
u(t) &\le u(t-K)\exp\{-d_1^l K\} + p_1^u x(t)\frac{1}{Q}\big(1 - \exp\{-QK\}\big)\exp\{-Q\tau\} \\
&\le 2L_1 \exp\{-d_1^l K\} + p_1^u x(t)\frac{1}{Q}\big(1 - \exp\{-QK\}\big)\exp\{-Q\tau\} \\
&\le 2L_1 \exp\{-d_1^l K\} + Dx(t)
\end{aligned}
\tag{2.8}
$$

for all $t > T_1 + K$, where $D = p_1^u \frac{1}{Q}(1 - \exp\{-QK\})\exp\{-Q\tau\} > 0$.

Substituting (2.8) into the first equation of system (1.4), for all $t > T_1 + K$, one has

$$
\begin{aligned}
\dot{x}(t) &\ge x(t)\left(a_1^l - b^u x(t) - \frac{c^u}{\gamma^l} - 2e_1^u L_1 \exp\{-d_1^l K\} - e_1^u Dx(t) \right) \\
&\ge x(t)\left(\frac{\beta}{2} - \big(b^u + e_1^u D\big)x(t) \right).
\end{aligned}
\tag{2.9}
$$

By Lemma 2.1, we have

$$
\liminf_{t\to+\infty} x(t) \ge \frac{\beta}{2(b^u + e_1^u D)} \triangleq m_1.
\tag{2.10}
$$

Thus, there exists $T_2 > T_1 + K$ such that for all $t > T_2$,

$$
x(t) \ge \frac{m_1}{2}.
\tag{2.11}
$$

Inequality (2.11) together with the third equation of (1.4) leads to

$$
\dot{u}(t) \ge -d_1^u u(t) + p_1^l \frac{m_1}{2} \quad \text{for all } t > T_2 + \tau.
$$

By applying Lemma 2.2(ii) to the above differential inequality, we have

$$
\liminf_{t\to+\infty} u(t) \ge \frac{p_1^l m_1}{2d_1^u} \triangleq l_1.
\tag{2.12}
$$

Obviously, m_1 and l_1 are independent of the solution of system (1.4). Inequalities (2.10) and (2.12) show that the conclusion of Lemma 2.4 holds. The proof is completed. □

Lemma 2.5 *For any positive solution* $(x(t), y(t), u(t), v(t))^T$ *of system* (1.4) *with initial conditions* (1.6), *there exist two positive constants* m_2 *and* l_2 *satisfying*

$$\liminf_{t \to +\infty} y(t) \geq m_2, \qquad \liminf_{t \to +\infty} v(t) \geq l_2,$$

where m_2 *and* l_2 *are defined in the proof.*

Proof The proof of Lemma 2.5 is similar to the proof of Lemma 2.4. However, for the sake of completeness, we give the complete proof here.

According to Lemma 2.3, there exists enough large $T_3 > 0$ such that for $t \geq T_3$,

$$y(t) \leq 2M_2, \qquad v(t) \leq 2L_2. \tag{2.13}$$

Thus, it follows from (2.13) and the second equation of system (1.4) that

$$\begin{aligned}
\dot{y}(t) &= y(t)\left(a_2(t) - \frac{r(t)y(t)}{x(t) + k(t)} - e_2(t)v(t) \right) \\
&\geq y(t)\left(a_2^l - \frac{2r^u M_2}{k^l} - 2e_2^u L_2 \right) \\
&\triangleq Py(t),
\end{aligned} \tag{2.14}$$

where $P = a_2^l - \frac{2r^u M_2}{k^l} - 2e_2^u L_2 < 0$.

Integrating both sides of (2.14) from η ($\eta \leq t$) to t, leads to

$$\frac{y(t)}{y(\eta)} \geq \exp\{P(t - \eta)\}$$

or

$$y(\eta) \leq y(t)\exp\{-P(t - \eta)\}. \tag{2.15}$$

Particularly, take $\eta = t - \tau$, one can get

$$y(t - \tau) \leq y(t)\exp\{-P\tau\}. \tag{2.16}$$

Substituting (2.16) into the fourth equation of system (1.4) leads to

$$\dot{v}(t) \leq -d_2^l v(t) + p_2^u y(t)\exp\{-P\tau\}. \tag{2.17}$$

Applying Lemma 2.2(i) to the above differential inequality, for $0 \leq s \leq t$, one has

$$\begin{aligned}
v(t) &\leq v(t - s)\exp\{-d_2^l s\} + \int_{t-s}^{t} p_2^u y(\eta)\exp\{-P\tau\}\exp\{d_2^l(\eta - t)\}\, d\eta \\
&\overset{\text{from (2.15)}}{\leq} v(t - s)\exp\{-d_2^l s\}
\end{aligned}$$

$$+ \int_{t-s}^{t} p_2^u y(t) \exp\{-P(t-\eta)\} \exp\{-P\tau\} \exp\{d_1^l(\eta - t)\} \, d\eta$$

$$\leq \quad v(t-s) \exp\{-d_2^l s\} + p_2^u y(t) \frac{1}{P} \big(1 - \exp\{-Ps\}\big) \exp\{-P\tau\}, \tag{2.18}$$

where we have used the fact that $\max_{\eta \in [t-s,t]} \exp\{d_2^l(\eta - t)\} = \exp\{0\} = 1$.

Note that there exists K_1 such that $2e_2^u L_2 \exp\{-d_2^l K_1\} < \frac{a_2^l}{2}$ as $s \geq K_1$. In fact, we can choose $K_1 > \frac{1}{d_2^l} \ln \frac{4e_2^u L_2}{a_2^l}$. And so, fix K_1, combined with (2.18), we can obtain

$$v(t) \leq v(t - K_1) \exp\{-d_2^l K_1\} + p_2^u y(t) \frac{1}{P} \big(1 - \exp\{-PK_1\}\big) \exp\{-P\tau\}$$

$$\leq 2L_2 \exp\{-d_2^l K_1\} + p_2^u y(t) \frac{1}{P} \big(1 - \exp\{-PK_1\}\big) \exp\{-P\tau\}$$

$$\leq 2L_2 \exp\{-d_2^l K_1\} + By(t) \tag{2.19}$$

for all $t > T_1 + K_1$, where $B = p_2^u \frac{1}{P}(1 - \exp\{-PK_1\}) \exp\{-P\tau\}$.

Substituting (2.19) into the second equation of system (1.4), for all $t > T_3 + K_1$, one has

$$\dot{y}(t) \geq y(t) \left(a_2^l - \frac{r^u y(t)}{k^l} - e_2^u \big(2L_2 \exp\{-d_2^l K_1\} + By(t)\big) \right)$$

$$= y(t) \left(a_2^l - 2e_2^u L_2 \exp\{-d_2^l K_1\} - \left(\frac{r^u}{k^l} + e_2^u B \right) y(t) \right)$$

$$\geq x(t) \left(\frac{a_2^l}{2} - \left(\frac{r^u}{k^l} + e_2^u B \right) y(t) \right). \tag{2.20}$$

By Lemma 2.1, we have

$$\liminf_{t \to +\infty} x(t) \geq \frac{a_2^l k^l}{2(r^u + e_2^u k^l B)} \triangleq m_2. \tag{2.21}$$

Thus, there exists $T_4 > T_3 + K_1$ such that for all $t > T_4$,

$$y(t) \geq \frac{m_2}{2}. \tag{2.22}$$

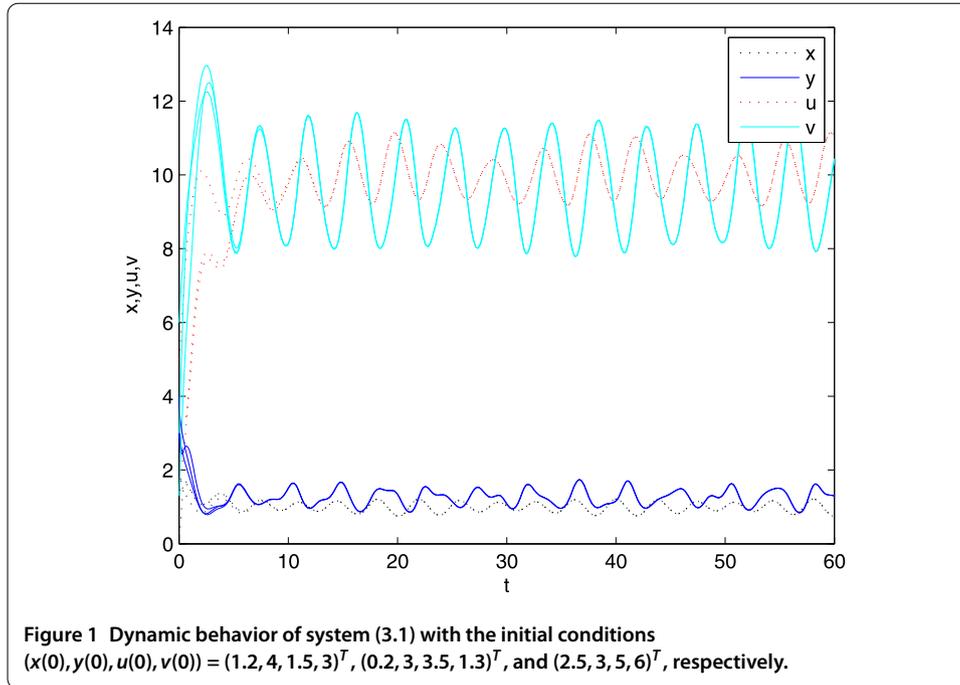Inequality (2.22) together with the fourth equation of (1.4) leads to

$$\dot{v}(t) \geq -d_2^u v(t) + p_2^l \frac{m_2}{2} \quad \text{for all } t > T_4 + \tau.$$

By applying Lemma 2.2(ii) to the above differential inequality, we have

$$\liminf_{t \to +\infty} v(t) \geq \frac{p_2^l m_2}{2 d_2^u} \triangleq l_2. \tag{2.23}$$

Obviously, m_2 and l_2 are independent of the solution of system (1.4). Inequalities (2.21) and (2.23) show that the conclusion of Lemma 2.5 holds. The proof is completed. □

Lemmas 2.3-2.5 show that the conclusion of Theorem B holds.

Figure 1 Dynamic behavior of system (3.1) with the initial conditions $(x(0), y(0), u(0), v(0)) = (1.2, 4, 1.5, 3)^T$, $(0.2, 3, 3.5, 1.3)^T$, **and** $(2.5, 3, 5, 6)^T$, **respectively.**

3 Examples and numeric simulations

Consider the following example:

$$
\begin{cases}
\dot{x}(t) = x(t)(11 + \cos\sqrt{3}t - 6x(t) - \frac{(0.8 + 0.7\sin\sqrt{5}t)y(t)}{20 + (12 + \sin\sqrt{7}t)x(t) + 5y(t)} - 0.5u(t)), \\
\dot{y}(t) = y(t)(5 + 0.3\sin\sqrt{7}t - \frac{(5 + 0.2\cos\sqrt{11}t)y(t)}{x(t)+2} - 0.3v(t)), \\
\dot{u}(t) = -0.2u(t) + (2 + \sin\sqrt{2}t)x(t - 0.5), \\
\dot{v}(t) = -(0.4 + 0.2\cos\sqrt{2}t)v(t) + 3y(t - 0.5),
\end{cases}
\tag{3.1}
$$

in this case, we have

$$
a_1^l - \frac{c^u}{\gamma^l} = 9.7 > 0.
\tag{3.2}
$$

Equation (3.2) shows that (H$_2$) holds, so system (3.1) is permanent according to Theorem B. Our numerical simulation supports our result (see Figure 1). However,

$$
a_1^l - \frac{c^u}{\gamma^l} - e_1^u L_1 = -5.3 < 0 \quad \text{and} \quad a_2^l - e_2^u L_2 = -15.175 < 0,
\tag{3.3}
$$

that is to say, (H$_1$) does not hold and we cannot obtain the result of the permanence from Theorem A. Thus our results improve the main results in Zhang *et al.* [10].

References
1. Leslie, PH: Some further notes on the use of matrices in population mathematics. Biometrika **35**, 213-245 (1948)
2. Leslie, PH: A stochastic model for studying the properties of certain biological systems by numerical methods. Biometrika **45**, 16-31 (1958)
3. Aziz-Alaoui, MA, Daher Okiye, M: Boundedness and global stability for a predator-prey model with modified Leslie-Gower and Holling-type II schemes. Appl. Math. Lett. **16**, 1069-1075 (2003)
4. Yu, S: Global asymptotic stability of a predator-prey model with modified Leslie-Gower and Holling-type II schemes. Discrete Dyn. Nat. Soc. **2012**, Article ID 208167 (2012)
5. Zhu, Y, Wang, K: Existence and global attractivity of positive periodic solutions for a predator-prey model with modified Leslie-Gower Holling-type II schemes. J. Math. Anal. Appl. **384**, 400-408 (2011)
6. Yu, S, Chen, F: Almost periodic solution of a modified Leslie-Gower predator-prey model with Holling-type II schemes and mutual interference. Int. J. Biomath. **7**, 1450028 (2014)
7. Yu, S: Global stability of a modified Leslie-Gower model with Beddington-DeAngelis functional response. Adv. Differ. Equ. **2014**, 84 (2014)
8. Pal, P, Mandal, P: Bifurcation analysis of a modified Leslie-Gower predator-prey model with Beddington-DeAngelis functional response and strong Allee effect. Math. Comput. Simul. **97**, 123-146 (2014)
9. Zhang, Z: Almost periodic solution of a modified Leslie-Gower predator-prey model with Beddington-DeAngelis functional response. J. Appl. Math. **2013**, Article ID 834047 (2013)
10. Zhang, K, Li, J, Yu, A: Almost periodic solution of a modified Leslie-Gower predator-prey model with Beddington-DeAngelis functional response and feedback controls. Abstr. Appl. Anal. **2014**, Article ID 252579 (2014)
11. Chen, F, Yang, J, Chen, L: Note on the persistent property of a feedback control system with delays. Nonlinear Anal., Real World Appl. **11**, 1061-1066 (2010)
12. Chen, F, Yang, J, Chen, L, Xie, X: On a mutualism model with feedback controls. Appl. Math. Comput. **214**, 581-587 (2009)
13. Wu, H, Yu, S: Permanence, extinction, and almost periodic solution of a Nicholson's blowflies model with feedback control and time delay. Discrete Dyn. Nat. Soc. **2013**, Article ID 798961 (2013)
14. Chen, F, Li, Z, Huang, Y: Note on the permanence of a competitive system with infinite delay and feedback controls. Nonlinear Anal., Real World Appl. **8**, 680-687 (2007)