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The critical exponent for fast diffusion equation with nonlocal source

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Abstract

This paper considers the Cauchy problem for fast diffusion equation with nonlocal source $u_t = \Delta u^m + (\int_{\mathbb{R}^n} u^q(x,t)\,dx)^{\frac{p-1}{q}} u^{r+1}$, which was raised in [Galaktionov et al. in Nonlinear Anal. 34:1005–1027, 1998]. We give the critical Fujita exponent $p_c = m + \frac{2q-n(1-m)-nqr}{n(q-1)}$, namely, any solution of the problem blows up in finite time whenever $1 < p \le p_c$, and there are both global and non-global solutions if $p > p_c$.

MSC: 35B33; 35K65

Keywords: Critical exponents; Fast diffusion; Nonlocal; Global solutions; Blow-up

1 Introduction

In this paper, we study the following Cauchy problem of fast diffusion parabolic equation with a nonlinear nonlocal source:

$$\begin{cases} u_t = \Delta u^m + (\int_{\mathbb{R}^n} u^q(x,t)\,dx)^{\frac{p-1}{q}} u^{r+1}, & (x,t) \in \mathbb{R}^n \times (0,T), \\ u(x,0) = u_0(x), & x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

where the spatial dimension $n \ge 1$, the coefficients m, p, q, r satisfy $\max\{0, 1 - \frac{2}{n} + r\} < m < 1$, $p > 1$, $q \ge 1$, $0 \le r < \frac{2}{n}$, and the initial data $u_0(x)$ is a nontrivial nonnegative continuous function.

The quasilinear parabolic equations involving a nonlocal term originate in the phenomena of diffusion about concentration of some Newtonian fluids or the density of some biological species and heat transfer in a special medium with nonlocal source (see [2, 3] and the references therein). In the past three decades, various nonlocal mathematical models were established to describe many physical phenomena (see [1, 4–9] and references therein). At the same time, many important results have appeared on the blow-up problem for a nonlinear parabolic equation with nonlocal source (see [2, 6, 8–11] and references therein), and for nonlocal nonlinear diffusion equations [12, 13]. However, most of efforts have been devoted in bounded domains, there were few researches for the Cauchy problems (see [1, 14, 15]).

It is well known that the classical Cauchy problem

$$u_t = \Delta u + u^p \quad \text{in } \mathbb{R}^n \times (0,T) \tag{1.2}$$

possesses the critical exponent $1 + \frac{2}{n}$ [16–19], that is to say, any nontrivial solution blows up in finite time if $1 < p \le 1 + \frac{2}{n}$, whereas global and non-global solutions coexist if $p > 1 + \frac{2}{n}$, depending on the size of initial data. From then on, the Fujita phenomenon has been observed for many nonlinear PDEs (see surveys [20, 21] and references therein).

The study for the Cauchy problem of nonlocal nonlinear parabolic equation was proposed by Galaktionov et al. [1], in which it was proved that the Cauchy problem (1.1) with $m = 1$ has a critical Fujita exponent, and Wang et al. [15] obtained similar results by other methods. Recently, Zhou [14] considered the global and non-global existence of solutions for (1.1) with $m > 1$.

The present paper investigates a fast diffusion parabolic equation (1.1) ($\max\{0, 1 - \frac{2}{n}\} < m < 1$) with a nonlocal source, and establishes the critical Fujita exponent $p_c = m + \frac{2q-n(1-m)-nqr}{n(q-1)}$. Comparing with the known result for the parallel problem with a local source

$$u_t = \Delta u^m + u^p \quad \text{in } \mathbb{R}^n \times (0, T),$$

the critical Fujita exponent was obtained in [22, 23] and shown to be $p_c = 1 + \frac{2m}{n}$.

In the rest of the paper, we always let u be a solution to (1.1), and $p_c = m + \frac{2q-n(1-m)-nqr}{n(q-1)}$. The main results are stated in the following theorems.

Theorem 1.1 *For $1 < p \le p_c$, there are no global nontrivial solutions to* (1.1).

Theorem 1.2 *For $p > p_c$, there are both global and non-global solutions to* (1.1).

This paper is organized as follows. Section 2 concerns the non-global solution to prove Theorem 1.1. Section 3 deals with the global existence to prove Theorem 1.2. And Sect. 4 shows in what ways the parameter q of the nonlocal source affects the behavior of solutions in the fast diffusion problem (1.1).

2 Non-global solutions

This section mainly applies the test function method (refer to [15, 22]) to prove that any solution of (1.1) must blow up in finite time for $1 < p \le p_c$. Introducing the test function

$$\varphi_k(x) = \left(\frac{k}{\pi}\right)^{\frac{n}{2}} e^{-k|x|^2} \tag{2.1}$$

for some $k > 0$, we can simply verify that

$$\int_{\mathbb{R}^n} \varphi_k(x)\,dx = 1, \qquad \left\|\varphi_k(x)\right\|_{L^\infty} = \left(\frac{k}{\pi}\right)^{\frac{n}{2}}, \qquad \Delta\varphi_k(x) \ge -2kn\varphi_k(x).$$

Define

$$F(t) = \int_{\mathbb{R}^n} u(x,t)\varphi_k(x)\,dx.$$

It is sufficient to show that $F(t)$ blows up in finite time as $1 < p \le p_c$ to deal with Theorem 1.1.

Proof of Theorem 1.1 Firstly, we consider the case of $1 < p < p_c$. Multiplying equation (1.1) by $\varphi_k(x)$ and integrating by parts in $\mathbb{R}^n$, we get

$$
\begin{aligned}
F'(t) &= \int_{\mathbb{R}^n} u_t \varphi_k \, dx \\
&= \int_{\mathbb{R}^n} \Delta u^m \varphi_k \, dx + \left(\int_{\mathbb{R}^n} u^q \, dx \right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} u^{r+1} \varphi_k \, dx \\
&\geq -2kn \int_{\mathbb{R}^n} u^m \varphi_k \, dx + \|\varphi_k\|_{L^\infty}^{-\frac{p-1}{q}} \left(\int_{\mathbb{R}^n} u^q \varphi_k \, dx \right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} u^{r+1} \varphi_k \, dx.
\end{aligned}
$$

Using Jensen's inequality for $m < 1$, $q > 1$, and $r > 0$,

$$
\begin{aligned}
F'(t) &\geq -2knF^m(t) + \left(\frac{k}{\pi} \right)^{-\frac{n(p-1)}{2q}} F^{p+r}(t) \\
&= F^{p+r}(t) \left(\left(\frac{k}{\pi} \right)^{-\frac{n(p-1)}{2q}} - 2knF^{-(p+r-m)}(t) \right).
\end{aligned}
$$

Assuming

$$
F(t) > \left(\pi^{\frac{n(p-1)}{2q}} (4n)^{-1} \right)^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p-1)}{2q(p+r-m)}}, \tag{2.2}
$$

we obtain

$$
\left(\frac{k}{\pi} \right)^{-\frac{n(p-1)}{2q}} > 4knF^{-(p+r-m)}(t),
$$

and

$$
F'(t) \geq \frac{1}{2} \left(\frac{k}{\pi} \right)^{-\frac{n(p-1)}{2q}} F^{p+r}(t). \tag{2.3}
$$

This implies

$$
F(t) \geq \left(F^{-(p+r-1)}(0) - \frac{p+r-1}{2} \left(\frac{k}{\pi} \right)^{-\frac{n(p-1)}{2q}} t \right)^{-\frac{1}{p+r-1}}.
$$

Obviously, $F(t)$ blows up for any nonnegative initial data as $t \to T = \frac{2F^{-(p+r-1)}(0)}{p+r-1} (\frac{k}{\pi})^{\frac{n(p-1)}{2q}}$. In the following, we show that

$$
F(0) > \left(\pi^{\frac{n(p-1)}{2q}} (4n)^{-1} \right)^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p-1)}{2q(p+r-m)}} \tag{2.4}
$$

is a sufficient condition to prove condition (2.2). If not, there exists some τ, such that

$$
F(\tau) = \left(\pi^{\frac{n(p-1)}{2q}} (4n)^{-1} \right)^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p-1)}{2q(p+r-m)}},
$$

and

$$F(t) > \left(\pi^{\frac{n(p-1)}{2q}}(4n)^{-1}\right)^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p-1)}{2q(p+r-m)}}, \quad t \in [0,\tau).$$

This implies $F'(\tau_0) < 0$ for some $\tau_0 \in (0,\tau)$, which contradicts $F'(t) \geq 0$, $t \in (0,\tau)$. Thereby, to prove that a solution of (1.1) blows up in finite time, we only show (2.4) is true for any nonnegative nontrivial initial data $u_0(x)$. Since $\frac{n}{2} < \frac{2q+n(p-1)}{2q(p+r-m)}$ which was derived by $1 < p < p_c$, there exists a $k > 0$ small enough, such that

$$F(0) = \left(\frac{k}{\pi}\right)^{\frac{n}{2}} \int_{\mathbb{R}^n} \mathrm{e}^{-k|x|^2} u_0(x)\,dx > \left(\pi^{\frac{n(p-1)}{2q}}(4n)^{-1}\right)^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p-1)}{2q(p+r-m)}}.$$

Next, we consider the case of $p = p_c$. Supposing a solution of (1.1) is global for any $t \geq 0$, it holds that

$$F(t) = \left(\frac{k}{\pi}\right)^{\frac{n}{2}} \int_{\mathbb{R}^n} \mathrm{e}^{-k|x|^2} u(x,t)\,dx \leq \left(\pi^{\frac{n(p-1)}{2q}}(4n)^{-1}\right)^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p-1)}{2q(p+r-m)}}. \tag{2.5}$$

That is, if (2.5) is not true, namely $F(t_1) > (\pi^{\frac{n(p-1)}{2q}}(4n)^{-1})^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p-1)}{2q(p+r-m)}}$ for some $t_1 > 0$, then the solution $u(x,t)$ must blow up in finite time by the above proof. The condition $p = p_c$ means $\frac{n}{2} = \frac{2q+n(p-1)}{2q(p+r-m)}$, and (2.5) can be rewritten as

$$\int_{\mathbb{R}^n} \mathrm{e}^{-k|x|^2} u(x,t)\,dx \leq \pi^{\frac{n}{2}} \left(\pi^{\frac{n(p-1)}{2q}}(4n)^{-1}\right)^{-\frac{1}{p+r-m}} \quad \text{for } t > 0. \tag{2.6}$$

Without loss of generality, assuming $u_0(x)$ has compact support in $\mathbb{R}^n$, we get that $u(x,t) \in L(\mathbb{R}^n)$ for any fixed $t > 0$ (see [24]). By Lebesgue Dominated Convergence Theorem, as $k \to 0$ in (2.6),

$$\int_{\mathbb{R}^n} u(x,t)\,dx \leq \pi^{\frac{n}{2}} \left(\pi^{\frac{n(p-1)}{2q}}(4n)^{-1}\right)^{-\frac{1}{p+r-m}}. \tag{2.7}$$

Integrating equation (1.1) on $\mathbb{R}^n \times [0,t]$, we have

$$\int_{\mathbb{R}^n} u(x,t)\,dx - \int_{\mathbb{R}^n} u_0(x)\,dx = \int_0^t \left(\int_{\mathbb{R}^n} u^q\,dx\right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} u^{r+1}\,dx\,dt.$$

And then

$$\int_0^t \left(\int_{\mathbb{R}^n} u^q\,dx\right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} u^{r+1}\,dx\,dt \leq \int_{\mathbb{R}^n} u(x,t)\,dt \leq \pi^{\frac{n}{2}} \left(\pi^{\frac{n(p-1)}{2q}}(4n)^{-1}\right)^{-\frac{1}{p+r-m}}$$

as $u_0(x,t) \geq 0$. This implies that

$$\int_0^\infty \left(\int_{\mathbb{R}^n} u^q\,dx\right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} u^{r+1}\,dx\,dt < +\infty. \tag{2.8}$$

On the other hand, from [25] we know that there exists $\delta > 0$ such that the solution of (1.1) satisfies

$$u(x,\tau) > \delta\bigl(1 + B|x|^2\bigr)^{-\frac{1}{1-m}}$$

for $B = \frac{(1-m)\alpha\delta^{1-m}}{2mn}$ and some $\tau > 0$. Setting

$$\underline{u}(x,t) = \delta(1+t)^{-\alpha}\bigl(1 + B|x|^2(1+t)^{-\frac{2\alpha}{n}}\bigr)^{-\frac{1}{1-m}}$$

with $\alpha = \frac{n}{2-n(1-m)}$, it is simple to verify

$$\underline{u}(x,t) \le u(x,t+\tau) \quad \text{for } x \in \mathbb{R}^n, t > 0.$$

And we have

$$\begin{aligned}
&\int_0^\infty \left(\int_{\mathbb{R}^n} u^q(x,t)\,dx\right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} u^{r+1}(x,t)\,dx\,dt \\
&\quad \ge \int_0^\infty \left(\int_{\mathbb{R}^n} u^q(x,t+\tau)\,dx\right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} u^{r+1}(x,t+\tau)\,dx\,dt \\
&\quad \ge \int_0^\infty \left(\int_{\mathbb{R}^n} \underline{u}^q(x,t)\,dx\right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n} \underline{u}^{r+1}(x,t)\,dx\,dt \\
&\quad = B^{-\frac{p+q-1}{2q}}\delta^{p+r}\left(\int_{\mathbb{R}^n}\bigl(1+|\xi|^2\bigr)^{-\frac{q}{1-m}}\,d\xi\right)^{\frac{p-1}{q}} \int_{\mathbb{R}^n}\bigl(1+|\xi|^2\bigr)^{-\frac{r+1}{1-m}}\,d\xi \int_0^\infty (1+t)^{-1}\,dt \\
&\quad = +\infty,
\end{aligned}$$

since $-\alpha(p+r-1) + \frac{\alpha(p-1)}{q} = -1$ for $p = p_c$ and $\xi = \sqrt{B}x(1+t)^{-\frac{\alpha}{n}}$. This contradicts (2.8), and so our assumption that the solution of (1.1) globally exist for $t > 0$ is not true, which proves Theorem 1.1 with $p = p_c$. □

3 Coexistence of global and non-global solutions

This section mainly deals with the global solution for the case of $p > p_c$ to derive Theorem 1.2.

Proof of Theorem 1.2 Firstly, we show that the solution of (1.1) must blow up in finite time for large initial data $u_0(x)$. The proof of Theorem 1.1 means that $u(x,t)$ does not exist globally, provided u_0 satisfies

$$\left(\frac{k}{\pi}\right)^{\frac{n}{2}} \int_{\mathbb{R}^n} e^{-k|x|^2} u_0(x)\,dx > \bigl(\pi^{\frac{n(p-1)}{2q}}(4n)^{-1}\bigr)^{-\frac{1}{p+r-m}} k^{\frac{2q+n(p+1)}{2q(p+r-m)}}. \tag{3.1}$$

For any fixed $k = k_0 > 0$, we can choose large $u_0(x)$ to fulfil condition (3.1).

Next, we prove that the solution of (1.1) exists globally for any small initial data $u_0(x)$. Let

$$\bar{u} = (t+1)^{-\beta}\bigl(D_1 + D_2|x|^2(t+1)^{-\beta(1-m)-1}\bigr)^{-\frac{1}{1-m}},$$

where $\beta = \frac{n(p-1)+2q}{2q(p+r-1)-n(1-m)(p-1)}$, and $D_1, D_2 > 0$ are to be determined. We demonstrate that $\bar{u}$ is a global supersolution of (1.1) for suitable D_1 and D_2. Setting

$$Z = D_1 + D_2|x|^2(t+1)^{-\beta(1-m)-1} =: D_1 + D_2 z,$$

with $z = |x|^2(t+1)^{-\beta(1-m)-1}$, we have

$$\begin{aligned}
&\bar{u}_t - \Delta\bar{u}^m - \left(\int_{\mathbb{R}^n} \bar{u}^q\,dx\right)^{\frac{p-1}{q}} \bar{u}^{r+1} \\
&\quad = (t+1)^{-\beta-1} Z^{-\frac{1}{1-m}-1}\Bigg[-\beta Z + \frac{D_2(\beta-\beta m+1)}{1-m} z + \frac{2mD_2 n}{1-m} Z - \frac{4mD_2^2}{(1-m)^2} z \\
&\qquad - (t+1)^{-\beta r+1}\left(\int_{\mathbb{R}^n} (t+1)^{-\beta q}\big(D_1 + D_2|y|^2(t+1)^{-1-\beta(1-m)}\big)^{-\frac{q}{1-m}}\,dy\right)^{\frac{p-1}{q}} Z^{-\frac{r}{1-m}+1}\Bigg] \\
&\quad =: (t+1)^{-\beta-1} Z^{-\frac{1}{1-m}-1} G(Z).
\end{aligned} \tag{3.2}$$

For $\max\{0, 1-\frac{2}{n}+r\} < m < 1$, $q \geq 1$, $r \geq 0$ implying $\frac{2q}{1-m} \geq \frac{2}{1-m} > n$, there exists a constant $C > 0$ such that

$$\begin{aligned}
&\int_{\mathbb{R}^n} (t+1)^{-\beta q}\big(D_1 + D_2|y|^2(t+1)^{-1-\beta(1-m)}\big)^{-\frac{q}{1-m}}\,dy \\
&\quad = \int_{\mathbb{R}^n} (t+1)^{-\beta q + \frac{n+n\beta(1-m)}{2}}\big(D_1 + D_2|w|^2\big)^{-\frac{q}{1-m}}\,dw \\
&\quad \leq C(t+1)^{-\beta q + \frac{n+n\beta(1-m)}{2}}.
\end{aligned}$$

Substituting the above inequity into the expression of $G(Z)$ in (3.2), and using $D_2 z = Z - D_1$, $\beta = \frac{n(p-1)+2q}{2q(p+r-1)-n(1-m)(p-1)}$, we have

$$\begin{aligned}
G(Z) &\geq -\beta Z + \frac{D_2(\beta-\beta m+1)}{1-m} z + \frac{2mD_2 n}{1-m} Z - \frac{4mD_2^2}{(1-m)^2} z \\
&\quad - C(t+1)^{-\beta(p+r-1)+\frac{n+n\beta(1-m)}{2q}(p-1)+1} Z^{-\frac{r}{1-m}+1} \\
&= \left(-\beta + \frac{\beta-\beta m+1}{1-m} + \frac{2mD_2 n}{1-m} - \frac{4mD_2}{(1-m)^2}\right) Z \\
&\quad - \left(\frac{\beta-\beta m+1}{1-m} - \frac{4mD_2}{(1-m)^2}\right) D_1 - CZ^{-\frac{r}{1-m}+1} \\
&=: F(Z).
\end{aligned} \tag{3.3}$$

To describe $F(Z) \geq 0$ for some D_1 and D_2, we have to show (i) $F(D_1) \geq 0$ and (ii) $F'(Z) \geq 0$ for $Z \geq D_1$.

(i) $F(D_1) = (-\beta + \frac{2mDn}{1-m})D_1 - CD_1^{-\frac{r}{1-m}+1} \geq 0$ is equivalent to

$$D_1^{-\frac{r}{1-m}} \leq \frac{1}{C}\left(-\beta + \frac{2mD_2 n}{1-m}\right), \tag{3.4}$$

$$D_2 > \frac{\beta(1-m)}{2mn}. \tag{3.5}$$

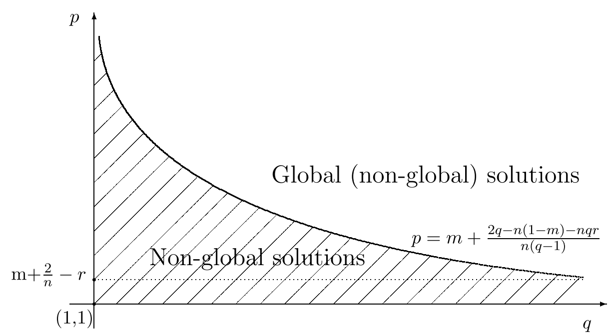

Figure 1 Critical Fujita exponent curve in q–p plane

(ii) By simple computation, $F'(Z) = -\beta + \frac{\beta-\beta m+1}{1-m} + \frac{2mD_2n}{1-m} - \frac{4mD_2}{(1-m)^2} - C(1-\frac{r}{1-m})Z^{-\frac{r}{1-m}}$. If $1-\frac{r}{1-m} \leq 0$, condition (ii) is ensured by

$$-\beta + \frac{\beta-\beta m+1}{1-m} + \frac{2mD_2n}{1-m} - \frac{4mD_2}{(1-m)^2} > 0. \tag{3.6}$$

If $1-\frac{r}{1-m} > 0$, condition (ii) is ensured by (3.6) and

$$D_1^{-\frac{r}{1-m}} \leq \frac{1-m}{C(1-m-r)}\left(-\beta + \frac{\beta-\beta m+1}{1-m} + \frac{2mD_2n}{1-m} - \frac{4mD_2}{(1-m)^2}\right). \tag{3.7}$$

Inequalities (3.5) and (3.6) require

$$\frac{\beta(1-m)}{2mn} < D_2 < \frac{1-m}{2m(2-n(1-m))}. \tag{3.8}$$

Due to $\beta = \frac{n(p-1)+2q}{2q(p+r-1)-n(1-m)(p-1)}$ and $p > p_c$, we can choose some $D_2 > 0$ that fulfils (3.8). For such D_2, choose $D_1 > 0$ large enough to satisfy (3.4) and (3.7).

In conclusion, $\bar{u}$ is a global supersolution to problem (1.1) with small initial data $u_0(x) \leq \bar{u}(x,0) = (D_1 + D_2|x|^2)^{-\frac{1}{1-m}}$. □

4 Conclusion

This paper shows that the model (1.1) possesses critical Fujita exponent $p_c = m + \frac{2q-n(1-m)-nqr}{n(q-1)}$ in Theorems 1.1 and 1.2, and we find that the coefficient q of the nonlocal term affects the critical Fujita exponent. It's easy to see that p_c is decreasing in q with $\lim_{q\to\infty} p_c = m + \frac{2}{n} - r$ and $\lim_{q\to 1} p_c = \infty$. That is to say, the scope $1 < p \leq p_c$ for the blow-up of any nontrivial solutions will be enlarged as q is decreasing, and any nontrivial solution of (1.1) will blow up when $p > 1$ and $q = 1$. Refer to Fig. 1.

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Authors' contributions
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