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Ostrowski-type inequalities pertaining to Atangana–Baleanu fractional operators and applications containing special functions

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Abstract
The objective of this article is to incorporate the concept of the Ostrowski inequality with the Atangana–Baleanu fractional integral operator. A novel integral identity for twice-differentiable functions is established after a rigorous investigation of several basic definitions and existing ideas related to inequalities and fractional calculus. Following that, numerous Ostrowski-type inequalities are provided based on this identity, which uses Mittag–Leffler as its kernel structure. Some specific applications, such as q-digamma functions and modified Bessel functions, are also investigated. Choosing $s = 1$, we also analyze new results for convex functions as special cases. Our findings corroborate some well-documented inequalities.

Keywords: Ostrowski inequality; Convex functions; Atangana–Baleanu fractional operator; q-digamma functions; Modified Bessel functions

1 Introduction

The hypothesis of convex functions gives us amazing standards and methods to concentrate on a wide class of issues in both pure and applied sciences. Several paragons of sciences reliably endeavor to use and benefit the original musings for the delight and beautification of the convexity hypothesis. This hypothesis assumes an important and pivotal part in applied mathematics, particularly in nonlinear programming, financial mathematics, mathematical statistics, optimization theory, and functional analysis. The theory of convexity plays a vital role in the exploration of mathematical inequalities. There exists a strong relationship between the theory of inequality, fractional integrals, and convex functions due to the behavior of their definitions and properties.

Definition 1.1 ([1]) A function $\mathcal{Q}\colon \mathcal{J} \subseteq \mathcal{R} \to \mathcal{R}$ is said to be convex if

$$\mathcal{Q}\bigl(\Phi \mathfrak{p} + (1-\Phi)\mathfrak{q}\bigr) \leq \Phi \mathcal{Q}(\mathfrak{p}) + (1-\Phi)\mathcal{Q}(\mathfrak{q}), \tag{1.1}$$

holds true for all $[\mathfrak{p}, \mathfrak{q}] \in \mathcal{J}$ and $\Phi \in [0,1]$. We say that $\mathcal{Q}$ is concave if $(-\mathcal{Q})$ is convex.

Convex functions are used to create inequalities such as the Hermite–Hadamard (H – H) inequality, the Ostrowski inequality, and Simpson's inequality. The H – H double inequality is one of the most extensively researched results involving convex functions. This conclusion provides us with the necessary and sufficient conditions for a function to be convex. The H – H inequality has been considered as one of the most useful results in mathematical analysis. It is also known as the H – H inequality's classical equation.

The Hermite–Hadamard inequality (see [2]) asserts that, if a mapping $\mathcal{Q} : \mathcal{J} \subset \mathcal{R} \to \mathcal{R}$ is convex in $\mathcal{J}$ for $\mathfrak{p}, \mathfrak{q} \in \mathcal{J}$ and $\mathfrak{q} > \mathfrak{p}$, then

$$\mathcal{Q}\left(\frac{\mathfrak{p}+\mathfrak{q}}{2}\right) \leq \frac{1}{\mathfrak{q}-\mathfrak{p}} \int_{\mathfrak{p}}^{\mathfrak{q}} \mathcal{Q}(\Phi)\, d\Phi \leq \frac{\mathcal{Q}(\mathfrak{p}) + \mathcal{Q}(\mathfrak{q})}{2}. \tag{1.2}$$

In 1938, Ostrowski [3] investigated the following interesting integral inequality as:

Let $\mathcal{Q}$: $\mathcal{J} \subseteq \mathcal{R} \to \mathcal{R}$ be a differentiable mapping on $\mathcal{J}^\circ$, such that $\mathcal{Q} \in \mathcal{L}[\mathfrak{p}, \mathfrak{q}]$, where $\mathfrak{p}, \mathfrak{q} \in \mathcal{J}$ with $\mathfrak{q} > \mathfrak{p}$. If $|\mathcal{Q}'(z)| \leq K$, for all $\omega \in [\mathfrak{p}, \mathfrak{q}]$, then

$$\left|\mathcal{Q}(\omega) - \frac{1}{\mathfrak{q}-\mathfrak{p}} \int_{\mathfrak{p}}^{\mathfrak{q}} \mathcal{Q}(u)\, du\right| \leq K(\mathfrak{q}-\mathfrak{p})\left[\frac{1}{4} + \frac{(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2})^2}{(\mathfrak{q}-\mathfrak{p})^2}\right], \tag{1.3}$$

holds true.

This result in the literature is studied extensively and is famously known as the Ostrowski inequality. This inequality gives an upper bound of $\frac{1}{\mathfrak{q}-\mathfrak{p}} \int_{\mathfrak{p}}^{\mathfrak{q}} \mathcal{Q}(u)\, du$ by $\mathcal{Q}(u)$.

Definition 1.2 ([4, 5]) A function $\mathcal{Q} : [0, +\infty) \to \mathcal{R}$ is called s-convex in the second sense, if

$$\mathcal{Q}\big(\Phi\mathfrak{p} + (1-\Phi)\mathfrak{q}\big) \leq \Phi^s \mathcal{Q}(\mathfrak{p}) + (1-\Phi)^s \mathcal{Q}(\mathfrak{q}), \tag{1.4}$$

holds true $\forall\ \mathfrak{p}, \mathfrak{q} \in [0, +\infty)$, $s \in (0, 1]$ and $\Phi \in [0, 1]$.

For $\mathcal{R}(a), \mathcal{R}(b) > 0$, the Beta function is expressed as:

$$\beta(a,b) = \int_0^1 \nu^{a-1}(1-\nu)^{b-1}\, d\nu.$$

Dragomir *et al.* [4], established the following integral inequality under the assumption of an s-convex function as:

$$2^{s-1}\mathcal{Q}\left(\frac{\mathfrak{p}+\mathfrak{q}}{2}\right) \leq \frac{1}{\mathfrak{q}-\mathfrak{p}} \int_{\mathfrak{p}}^{\mathfrak{q}} \mathcal{Q}(\Phi)\, d\Phi \leq \frac{\mathcal{Q}(\mathfrak{p}) + \mathcal{Q}(\mathfrak{q})}{s+1}. \tag{1.5}$$

Dragomir and Rassias [6], investigated the Ostrowski-type inequality for convex functions as:

$$\left|\mathcal{Q}(\omega) - \frac{1}{\mathfrak{q}-\mathfrak{p}} \int_{\mathfrak{p}}^{\mathfrak{q}} \mathcal{Q}(u)\, du\right| \leq \frac{K}{(\mathfrak{q}-\mathfrak{p})}\left[\frac{(\omega-\mathfrak{p})^2 + (\mathfrak{q}-\omega)^2}{2}\right]. \tag{1.6}$$

Alomari *et al.* [7], investigated the Ostrowski-type inequality for an s-convex function in the second sense as:

$$\left|\mathcal{Q}(\omega)-\frac{1}{\mathfrak{q}-\mathfrak{p}}\int_{\mathfrak{p}}^{\mathfrak{q}}\mathcal{Q}(u)\,du\right|\leq\frac{K}{(\mathfrak{q}-\mathfrak{p})}\left[\frac{(\omega-\mathfrak{p})^2+(\mathfrak{q}-\omega)^2}{(s+1)}\right]. \tag{1.7}$$

Many mathematicians generalized the Ostrowski inequality in different directions. In particular, several scientific articles have been published in this regard taking various forms of convexities into account. For example, Alomari *et al.* [7] used the notion of s-convexity and İşcan *et al.* [8] used the notion of an harmonically s-convex function. Set [9] introduced the fractional version of the Ostrowski-type inequality employing Riemann–Liouville fractional operators. Liu [10] used the equality proved by Set to establish new refinements of the Ostrowski-type inequality for an MT-convex function. Tunç [11], studied the Ostrowski-type inequality for an h-convex function. Ozdemir *et al.* [12], obtained a new version of the Ostrowski-type inequality for an (α, m)-convex function. Agarwal *et al.*[13], investigated a more generalized Ostrowski-type inequality via a Raina fractional integral operator. Sarikaya *et al.* [14], employed local fractional integrals to obtain new generalizations of the Ostrowski-type inequality. Gürbuz *et al.* [15], used a Katugampola fractional operator for a generalized version of Ostrowski inequality. Ahmad *et al.* [16], established some novel generalization of the Ostrowski inequality via an Atangana–Baleanu fractional operator for differentiable convex functions. To acquire detailed information about recent advancements of the Ostrowski-type inequality, we direct the readers to the following references (see [17–20]).

Fractional calculus forms an important area of research in the fields of pure and applied sciences. In particular, in mathematical analysis, it is used to solve the uniqueness of various fractional differential equations and boundary value problems. It also helps in solving many real-world problems. The main motivation of fractional calculus is to propose new notions of fractional derivatives and integrals and study their properties, applications, and advantages over other fractional operators. With regard to this interest, several new variants of fractional models such as Riemann–Liouville [21], k-Riemann–Liouville [22], Katugampola [23], Caputo–Fabrizio [24], Atangana–Baleanu [25], etc. have been introduced in some of the recent articles. They all have distinct conditions and properties, which make them not identical to each other. The main focus of this article to study the correlation between mathematical inequality and fractional operators. The improvements of fractional operators are backed by presenting different types of inequalities such as H – H type [26, 27], Minkowski type [28, 29], Grüss type [30, 31], Pólya–Szegö type [32], and Chebyshev type [33] employing these operators. Lately, many mathematicians have incorporated the concepts of new notions of fractional integrals and well-known inequalities. To know more about the recent developments about the theory of fractional integral inequalities, we suggest interested readers follow the articles [34–38].

Before discussing our main results, let us focus on some basic definitions and related results for fractional integral inequalities.

Definition 1.3 ([21, 34]) Let $\mathcal{Q}\in\mathcal{L}[\mathfrak{p},\mathfrak{q}]$ be the set of all Lebesgue measurable functions on $[\mathfrak{p},\mathfrak{q}]$. Then, for the order $\varsigma>0$, the left and right Riemann–Liouville (R–L) fractional integrals are defined as follows:

$$\mathcal{I}_{\mathfrak{p}^+}^{\varsigma}\mathcal{Q}(t)=\frac{1}{\Gamma(\varsigma)}\int_{\mathfrak{p}}^{t}(t-y)^{\varsigma-1}\mathcal{Q}(y)\,dy,\quad (t>\mathfrak{p})$$

and

$$\mathcal{I}_{\mathfrak{q}^-}^{\varsigma}\mathcal{Q}(t)=\frac{1}{\Gamma(\varsigma)}\int_{t}^{\mathfrak{q}}(y-t)^{\varsigma-1}\mathcal{Q}(y)\,dy\quad (t<\mathfrak{q}),$$

respectively, where $\Gamma(\varsigma)=\int_0^\infty y^{\varsigma-1}e^{-y}\,dy$ is the Euler gamma function.

Set *et al.* [9] proved the following equality and established several fractional Ostrowski-type inequalities.

Lemma 1.1 *Suppose* $\mathcal{Q}:\mathcal{J}=[\mathfrak{p},\mathfrak{q}]\to\mathcal{R}$ *is a twice-differentiable mapping on* $(\mathfrak{p},\mathfrak{q})$ *with* $\mathfrak{p}<\mathfrak{q}$. *If* $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$, *then for all* $\omega\in[\mathfrak{p},\mathfrak{q}]$ *and* $\varsigma>0$, *the following equality for AB-fractional integrals*

$$\begin{aligned}&\frac{(\omega-\mathfrak{p})^{\varsigma}-(\mathfrak{q}-\omega)^{\varsigma}}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)-\frac{\Gamma(\varsigma+1)}{\mathfrak{q}-\mathfrak{p}}\bigl\{\mathcal{I}_{\omega^+}^{\varsigma}\mathcal{Q}(\mathfrak{q})+\mathcal{I}_{\omega^-}^{\varsigma}\mathcal{Q}(\mathfrak{p})\bigr\}\\&\quad=\frac{(\omega-\mathfrak{p})^{\varsigma+1}}{\mathfrak{q}-\mathfrak{p}}\int_0^1\Phi^{\varsigma}\mathcal{Q}'\bigl(\Phi\omega+(1-\Phi)\mathfrak{p}\bigr)\,d\Phi\\&\qquad-\frac{(\mathfrak{q}-\omega)^{\varsigma+1}}{\mathfrak{q}-\mathfrak{p}}\int_0^1\Phi^{\varsigma}\mathcal{Q}'\bigl(\Phi\omega+(1-\Phi)\mathfrak{q}\bigr)\,d\Phi,\end{aligned}\tag{1.8}$$

holds true for $\Phi\in[0,1]$.

Definition 1.4 ([24]) Let $\mathcal{Q}\in H^1(\mathfrak{p},\mathfrak{q})$, $\mathfrak{q}>\mathfrak{p}$, $\varsigma\in[0,1]$, then the definition of the new Caputo fractional derivative is:

$${}^{\mathrm{CF}}D^{\varsigma}\mathcal{Q}(t)=\frac{\mathrm{B}(\varsigma)}{1-\varsigma}\int_{\mathfrak{p}}^{t}\mathcal{Q}'(y)\exp\biggl[-\frac{\varsigma}{(1-\varsigma)}(t-y)\biggr]\,dy,$$

where $\mathrm{B}(\varsigma)$ is a normalization function.

Definition 1.5 ([39]) Let $\mathcal{Q}\in H^1(\mathfrak{p},\mathfrak{q})$, $\mathfrak{q}>\mathfrak{p}$, $\varsigma\in[0,1]$, then the left and right Caputo–Fabrizio fractional integrals are defined as:

$$\bigl({}_{\mathfrak{p}}^{\mathrm{CF}}\mathcal{I}^{\varsigma}\mathcal{Q}\bigr)(t)=\frac{1-\varsigma}{\mathrm{B}(\varsigma)}\mathcal{Q}(t)+\frac{\varsigma}{\mathrm{B}(\varsigma)}\int_{\mathfrak{p}}^{t}\mathcal{Q}(y)\,dy,$$

and

$$\bigl({}^{\mathrm{CF}}\mathcal{I}_{\mathfrak{q}}^{\varsigma}\mathcal{Q}\bigr)(t)=\frac{1-\varsigma}{\mathrm{B}(\varsigma)}\mathcal{Q}(t)+\frac{\varsigma}{\mathrm{B}(\varsigma)}\int_{t}^{\mathfrak{q}}\mathcal{Q}(y)\,dy,$$

where $\mathrm{B}(\varsigma)$ is a normalization function.

The Atangana–Baleanu fractional operator containing the Mittag–Leffler function in the kernel was introduced by Atangana and Baleanu in [25], which solves the problem of retrieving the original function. It is seen that the Mittag–Leffler function is more appropriate than the power law in many physical phenomena. Due to its effectiveness, many researchers have shown a keen interest in utilizing this operator. Alina *et al.* [40] applied the Atangana–Baleanu fractional integral operator to multiplier transformations and obtained a new operator. Refai [41] presented the weighted fractional operators associated with the Atangana–Baleanu fractional operators. Very recently, Refai and Baleanu [42] in their short article extended the fractional integral in relation to the Mittag–Leffler kernel, which admits an integrable singular kernel at the origin. They introduced some modified ABC fractional operators and also solved related differential equations. Many researchers [43–47] have studied the fractional integral, which Atangana and Baleanu [25] generalized. The corresponding derivative operator in the Caputo and Liouville–Reimann senses is

Definition 1.6 ([25]) Let $\mathfrak{q} > \mathfrak{p}$, $\varsigma \in [0,1]$ and $\mathcal{Q} \in H^1(\mathfrak{p},\mathfrak{q})$. The new fractional derivative is given as:

$$ {}^{ABC}_{\ \mathfrak{p}}D^{\varsigma}_{t}\left[\mathcal{Q}(t)\right] = \frac{\mathrm{B}(\varsigma)}{1-\varsigma}\int_{\mathfrak{p}}^{t}\mathcal{Q}'(y)E_{\varsigma}\left[-\varsigma\frac{(t-y)^{\varsigma}}{(1-\varsigma)}\right]dy. $$

However, in the same paper they gave the corresponding Atangana–Baleanu (A–B) fractional integral operators as:

Definition 1.7 ([25]) The fractional integral operator with nonlocal kernel of a function $\mathcal{Q} \in H^1(\mathfrak{p},\mathfrak{q})$ is defined as:

$$ {}^{AB}_{\ \mathfrak{p}}\mathcal{I}^{\varsigma}_{t}\left\{\mathcal{Q}(t)\right\} = \frac{1-\varsigma}{\mathrm{B}(\varsigma)}\mathcal{Q}(t) + \frac{\varsigma}{\mathrm{B}(\varsigma)\Gamma(\varsigma)}\int_{\mathfrak{p}}^{t}\mathcal{Q}(y)(t-y)^{\varsigma-1}\,dy, $$

where $\mathfrak{q} > \mathfrak{p}$, $\varsigma \in [0,1]$.

In [48], the right-hand side of the AB-fractional integral operator was given as;

$$ {}^{AB}_{\ \mathfrak{q}}\mathcal{I}^{\varsigma}_{t}\left\{\mathcal{Q}(t)\right\} = \frac{1-\varsigma}{\mathrm{B}(\varsigma)}\mathcal{Q}(t) + \frac{\varsigma}{\mathrm{B}(\varsigma)\Gamma(\varsigma)}\int_{t}^{\mathfrak{q}}\mathcal{Q}(y)(y-t)^{\varsigma-1}\,dy. $$

Here, $\Gamma(\varsigma)$ is the Gamma function. The positivity of the normalization function $\mathrm{B}(\varsigma)$ implies that the fractional AB-integral of a positive function is positive. It is worth noting the case that when the order $\varsigma \to 1$, it yields the classical integral and the case when $\varsigma \to 0$, it provides the initial function.

Motivated by the above results and the literature, the main motivation of this article is to use the Atangana–Baleanu fractional integrals to prove some novel inequalities for twice-differentiable s-convex functions and some interesting applications related to modified Bessel functions and q-digamma functions. The rest of the paper is structured as follows: In Sect. 2, we establish a new identity and then apply it to derive new fractional Ostrowski-type inequalities for s-convex functions. Further, with the help of the improved Hölder's inequality, results for functions with a bounded second derivative are presented in Sect. 3. In order to illustrate the efficiency of our main results, some applications to modified Bessel functions and q-digamma functions are obtained in Sect. 4. Finally, in Sect. 5 a brief conclusion and future plans are discussed.

2 Main results

In this section, first we prove an Atangana–Baleanu fractional identity for twice-differentiable functions. Then, employing this and some fundamental integral inequalities, we present our main results.

Lemma 2.1 *Suppose $\mathcal{Q}:\mathcal{J}=[\mathfrak{p},\mathfrak{q}]\to\mathcal{R}$ is a twice-differentiable mapping on $(\mathfrak{p},\mathfrak{q})$ with $\mathfrak{p}<\mathfrak{q}$. If $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$, then for all $\omega\in[\mathfrak{p},\mathfrak{q}]$ and $\varsigma>0$, the following equality for AB-fractional integrals*

$$\begin{aligned}
&\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega) \\
&\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega) \\
&\quad=\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\,d\Phi \\
&\qquad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\,d\Phi,
\end{aligned}\tag{2.1}$$

holds true for $\Phi\in[0,1]$.

Proof Let us suppose that

$$\begin{aligned}
&\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\,d\Phi \\
&\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\,d\Phi \\
&I=\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{I}_1+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{I}_2,
\end{aligned}\tag{2.2}$$

where

$$\begin{aligned}
\mathcal{I}_1&=\int_0^1\Phi^{\varsigma+1}\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\,d\Phi \\
&=\frac{\Phi^{\varsigma+1}\mathcal{Q}'(\Phi\omega+(1-\Phi)\mathfrak{p})}{\omega-\mathfrak{p}}\bigg|_0^1-\int_0^1\frac{(\varsigma+1)\Phi^{\varsigma}\mathcal{Q}'(\Phi\omega+(1-\Phi)\mathfrak{p})}{\omega-\mathfrak{p}}\,d\Phi \\
&=\frac{\mathcal{Q}'(\omega)}{\omega-\mathfrak{p}}-\frac{\varsigma+1}{\omega-\mathfrak{p}}\int_0^1\Phi^{\varsigma}\mathcal{Q}'\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\,d\Phi \\
&=\frac{\mathcal{Q}'(\omega)}{\omega-\mathfrak{p}}-\frac{\varsigma+1}{(\omega-\mathfrak{p})^2}\mathcal{Q}(\omega)+\frac{\varsigma(\varsigma+1)}{(\omega-\mathfrak{p})^2}\int_0^1\Phi^{\varsigma-1}\mathcal{Q}\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\,d\Phi \\
&=\frac{\mathcal{Q}'(\omega)}{\omega-\mathfrak{p}}-\frac{\varsigma+1}{(\omega-\mathfrak{p})^2}\mathcal{Q}(\omega)+\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma+2)}{\varsigma(\omega-\mathfrak{p})^{\varsigma+2}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})-\frac{1-\varsigma}{\mathrm{B}(\varsigma)}\mathcal{Q}(\omega)\right\}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\mathcal{I}_2 &= \int_0^1 \Phi^{\varsigma+1}\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\,d\Phi \\
&= \frac{\Phi^{\varsigma+1}\mathcal{Q}'(\Phi\omega+(1-\Phi)\mathfrak{q})}{\omega-\mathfrak{q}}\bigg|_0^1 - \int_0^1 \frac{(\varsigma+1)\Phi^{\varsigma}\mathcal{Q}'(\Phi\omega+(1-\Phi)\mathfrak{q})}{\omega-\mathfrak{q}}\,d\Phi \\
&= -\frac{\mathcal{Q}'(\omega)}{\mathfrak{q}-\omega} - \frac{\varsigma+1}{\omega-\mathfrak{q}}\int_0^1 \Phi^{\varsigma}\mathcal{Q}'\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\,d\Phi \\
&= -\frac{\mathcal{Q}'(\omega)}{\mathfrak{q}-\omega} - \frac{\varsigma+1}{(\mathfrak{q}-\omega)^2}\mathcal{Q}(\omega) + \frac{\varsigma(\varsigma+1)}{(\mathfrak{q}-\omega)^2}\int_0^1 \Phi^{\varsigma-1}\mathcal{Q}\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\,d\Phi \\
&= -\frac{\mathcal{Q}'(\omega)}{\mathfrak{q}-\omega} - \frac{\varsigma+1}{(\mathfrak{q}-\omega)^2}\mathcal{Q}(\omega) + \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma+2)}{\varsigma(\mathfrak{q}-\omega)^{\varsigma+2}}\bigg\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q}) - \frac{1-\varsigma}{\mathrm{B}(\varsigma)}\mathcal{Q}(\omega)\bigg\},
\end{aligned}$$

using $\mathcal{I}_1$ and $\mathcal{I}_2$ with (2.2), we obtain (2.3). □

Theorem 2.1 *Suppose* $\mathcal{Q}:\mathcal{J}\subset[0,\infty)\to\mathcal{R}$ *is a twice-differentiable mapping on* $(\mathfrak{p},\mathfrak{q})$ *with* $\mathfrak{p}<\mathfrak{q}$ *such that* $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$. *If* $|\mathcal{Q}''|$ *is an s-convex function on* $[\mathfrak{p},\mathfrak{q}]$ *for some fixed* $s\in(0,1]$, *then for all* $\varsigma>0$, *the following AB-fractional integral inequality*

$$\begin{aligned}
&\bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega) - \frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega) \\
&\quad - \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\big\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\big\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\bigg| \\
&\quad\le \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\bigg\{\frac{|\mathcal{Q}''(\omega)|}{(\varsigma+s+2)} + \big|\mathcal{Q}''(\mathfrak{p})\big|\beta(\varsigma+2,s+1)\bigg\} \\
&\qquad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\bigg\{\frac{|\mathcal{Q}''(\omega)|}{(\varsigma+s+2)} + \big|\mathcal{Q}''(\mathfrak{q})\big|\beta(\varsigma+2,s+1)\bigg\},
\end{aligned}\tag{2.3}$$

holds true for $\Phi\in[0,1]$.

Proof From Lemma 2.1 and since $|\mathcal{Q}''|$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$, we obtain

$$\begin{aligned}
&\bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega) - \frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega) \\
&\quad - \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\big\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\big\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\bigg| \\
&\quad\le \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\big|\,d\Phi \\
&\qquad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\big|d\Phi \\
&\quad\le \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\big\{\Phi^{s}\big|\mathcal{Q}''(\omega)\big| + (1-\Phi)^{s}\big|\mathcal{Q}''(\mathfrak{p})\big|\big\}\,d\Phi \\
&\qquad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\big\{\Phi^{s}\big|\mathcal{Q}''(\omega)\big| + (1-\Phi)^{s}\big|\Phi''(\mathfrak{q})\big|\big\}\,d\Phi \\
&\quad\le \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\bigg\{\frac{|\mathcal{Q}''(\omega)|}{(\varsigma+s+2)} + \big|\mathcal{Q}''(\mathfrak{p})\big|\beta(\varsigma+2,s+1)\bigg\}
\end{aligned}$$

$$+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\mathcal{Q}''(\omega)|}{(\varsigma+s+2)}+\left|\mathcal{Q}''(\mathfrak{q})\right|\beta(\varsigma+2,s+1)\right\}.$$

Therefore, the proof is completed. □

Corollary 2.1 *If we set $s=1$ in Theorem 2.1, then we have the following new Ostrowski-type inequality for a convex function*:

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\left.\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\quad\le\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\mathcal{Q}''(\omega)|}{\varsigma+3}+\frac{|\mathcal{Q}''(\mathfrak{p})|}{(\varsigma+2)(\varsigma+3)}\right\}\\&\qquad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\mathcal{Q}''(\omega)|}{\varsigma+3}+\frac{|\mathcal{Q}''(\mathfrak{q})|}{(\varsigma+2)(\varsigma+3)}\right\}.\end{aligned}$$

Remark 2.1 If we set $\varsigma=1$ in Theorem 2.1, then (Theorem 4 of [49]) is recovered;

$$\begin{aligned}&\left|\frac{1}{\mathfrak{q}-\mathfrak{p}}\int_{\mathfrak{p}}^{\mathfrak{q}}\mathcal{Q}(u)\,du-\mathcal{Q}(\omega)+\left(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\mathcal{Q}'(\omega)\right|\\&\quad\le\frac{(\omega-\mathfrak{p})^{3}}{2(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\mathcal{Q}''(\omega)|}{s+3}+\frac{2|\mathcal{Q}''(\mathfrak{p})|}{(s+1)(s+2)(s+3)}\right\}\\&\qquad+\frac{(\mathfrak{q}-\omega)^{3}}{2(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\mathcal{Q}''(\omega)|}{s+3}+\frac{2|\mathcal{Q}''(\mathfrak{q})|}{(s+1)(s+2)(s+3)}\right\}.\end{aligned}$$

Corollary 2.2 *By using Corollary 2.1 with $|\mathcal{Q}''|\le\mathcal{M}$, we obtain the following inequality*

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\left.\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\quad\le\mathcal{M}\left(\frac{1}{(\varsigma+1)(\varsigma+2)(\mathfrak{q}-\mathfrak{p})}\right)\left[(\omega-\mathfrak{p})^{\varsigma+2}+(\mathfrak{q}-\omega)^{\varsigma+2}\right].\end{aligned}$$

Theorem 2.2 *Suppose $\mathcal{Q}:\mathcal{J}\subset[0,\infty)\to\mathcal{R}$ is a twice-differentiable mapping on $(\mathfrak{p},\mathfrak{q})$ with $\mathfrak{p}<\mathfrak{q}$ such that $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$. If $|\mathcal{Q}''|^{q}$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$ for some fixed $s\in(0,1]$, $q>1$, then for all $\varsigma>0$, the following A-B fractional integral inequality*

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\left.\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\quad\le\left(\frac{1}{(\varsigma+1)p+1}\right)^{\frac{1}{p}}\left[\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^{q}+|\mathcal{Q}''(\mathfrak{p})|^{q}}{s+1}\right)^{\frac{1}{q}}\right.\end{aligned}\qquad(2.4)$$

$$+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{q})|^q}{s+1}\right)^{\frac{1}{q}}\Bigg],$$

holds true for $\Phi\in[0,1]$, *where* $q^{-1}+p^{-1}=1$.

Proof Suppose that $q>1$. From Lemma 2.1, by using the well-known Hölder integral inequality and the s-convexity of $|\mathcal{Q}''|^q$, we obtain

$$\begin{aligned}&\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\\&\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg|\\&\quad\le\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\big|\,d\Phi\\&\quad\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\big|\,d\Phi\\&\quad\le\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\int_0^1\Phi^{(\varsigma+1)p}\,d\Phi\right)^{\frac{1}{p}}\left(\int_0^1\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\big|^q\,d\Phi\right)^{\frac{1}{q}}\\&\quad\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\int_0^1\Phi^{(\varsigma+1)p}\,d\Phi\right)^{\frac{1}{p}}\left(\int_0^1\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\big|^q\,d\Phi\right)^{\frac{1}{q}}.\end{aligned}\tag{2.5}$$

Since, $|\mathcal{Q}''|^q$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$, we obtain

$$\begin{aligned}&\int_0^1\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\big|^q\,d\Phi\\&\quad\le\int_0^1\left\{\Phi^s\big|\mathcal{Q}''(\omega)\big|^q+(1-\Phi)^s\big|\mathcal{Q}''(\mathfrak{p})\big|^q\right\}d\Phi\\&\quad=\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{p})|^q}{s+1}.\end{aligned}\tag{2.6}$$

Also,

$$\begin{aligned}&\int_0^1\big|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\big|^q\,d\Phi\\&\quad\le\int_0^1\left\{\Phi^s\big|\mathcal{Q}''(\omega)\big|^q+(1-\Phi)^s\big|\mathcal{Q}''(\mathfrak{q})\big|^q\right\}d\Phi\\&\quad=\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{q})|^q}{s+1}.\end{aligned}\tag{2.7}$$

By using (2.6) and (2.7) with (2.5), we obtain

$$\begin{aligned}&\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}-(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\\&\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg|\\&\quad\le\left(\frac{1}{(\varsigma+1)p+1}\right)^{\frac{1}{p}}\end{aligned}$$

$$\times\left[\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{p})|^q}{s+1}\right)^{\frac{1}{q}}+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{q})|^q}{s+1}\right)^{\frac{1}{q}}\right],$$

which completes the proof. □

Corollary 2.3 *If we set $s=1$ in Theorem 2.2, then we have the following Ostrowski-type inequality for a convex function*:

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\quad\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\quad\le\left(\frac{1}{(\varsigma+1)p+1}\right)^{\frac{1}{p}}\\&\quad\times\left[\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{p})|^q}{2}\right)^{\frac{1}{q}}+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{q})|^q}{2}\right)^{\frac{1}{q}}\right].\end{aligned}\tag{2.8}$$

Remark 2.2 If we set $\varsigma=1$ in Theorem 2.2, then we obtain (Theorem 5, [49]).

$$\begin{aligned}&\left|\frac{1}{\mathfrak{q}-\mathfrak{p}}\int_{\mathfrak{p}}^{\mathfrak{q}}\mathcal{Q}(u)\,du-\mathcal{Q}(\omega)+\left(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\mathcal{Q}'(\omega)\right|\\&\quad\le\left(\frac{1}{2p+1}\right)^{\frac{1}{p}}\\&\quad\times\left[\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{p})|^q}{s+1}\right)^{\frac{1}{q}}+\frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{q})|^q}{s+1}\right)^{\frac{1}{q}}\right].\end{aligned}$$

Corollary 2.4 *Using Corollary 2.3 with $|\mathcal{Q}''|\le\mathcal{M}$, we obtain*

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\quad\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\quad\le\mathcal{M}\left(\frac{1}{(\varsigma+1)p+1}\right)^{\frac{1}{p}}\left[\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\right].\end{aligned}$$

Theorem 2.3 *Suppose $\mathcal{Q}:\mathcal{J}\subset[0,\infty)\to\mathcal{R}$ is a twice-differentiable mapping on $(\mathfrak{p},\mathfrak{q})$ with $\mathfrak{p}<\mathfrak{q}$ such that $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$. If $|\mathcal{Q}''|^q$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$ for some fixed $s\in(0,1]$, $q\ge1$, then for all $\varsigma>0$, the following inequality for A-B fractional integrals*

$$\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\tag{2.9}$$

$$
\begin{aligned}
&- \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\bigg| \\
&\leq \left(\frac{1}{\varsigma+2}\right)^{1-\frac{1}{q}}\left[\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+s+2)} + \beta(\varsigma+2,s+1)\left|\mathcal{Q}''(\mathfrak{p})\right|^q\right)^{\frac{1}{q}}\right. \\
&\left.+ \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+s+2)} + \beta(\varsigma+2,s+1)\left|\mathcal{Q}''(\mathfrak{q})\right|^q\right)^{\frac{1}{q}}\right],
\end{aligned}
$$

holds true for $\Phi \in [0,1]$.

Proof Suppose that $q \geq 1$. From Lemma 2.1, by using the power-mean integral inequality and the s-convexity of $|\mathcal{Q}''|^q$, we obtain

$$
\begin{aligned}
&\bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega) - \frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega) \\
&- \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\bigg| \\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right|d\Phi \\
&+ \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right|d\Phi \\
&\leq \frac{(\omega-a)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\int_0^1 \Phi^{\varsigma+1}\,d\Phi\right)^{1-\frac{1}{q}}\left(\int_0^1\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right|^q d\Phi\right)^{\frac{1}{q}} \qquad (2.10) \\
&+ \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\int_0^1 \Phi^{\varsigma+1}\,d\Phi\right)^{1-\frac{1}{q}}\left(\int_0^1\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right|^q d\Phi\right)^{\frac{1}{q}}.
\end{aligned}
$$

Since $|\mathcal{Q}''|^q$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$, we obtain

$$
\begin{aligned}
&\int_0^1 \Phi^{\varsigma+1}\left|\Phi''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right|^q dt \\
&\leq \int_0^1 \Phi^{\varsigma+1}\left\{\Phi^s\left|\mathcal{Q}''(\omega)\right|^q + (1-\Phi)^s\left|\mathcal{Q}''(\mathfrak{p})\right|^q\right\}d\Phi \qquad (2.11) \\
&= \frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+s+2)} + \beta(\varsigma+2,s+1)\left|\mathcal{Q}''(\mathfrak{p})\right|^q
\end{aligned}
$$

and

$$
\begin{aligned}
&\int_0^1 \Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right|^q dt \\
&\leq \int_0^1 \Phi^{\varsigma+1}\left\{\Phi^s\left|\mathcal{Q}''(\omega)\right|^q + (1-\Phi)^s\left|\mathcal{Q}''(\mathfrak{q})\right|^q\right\}d\Phi \qquad (2.12) \\
&= \frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+s+2)} + \beta(\varsigma+2,s+1)\left|\mathcal{Q}''(\mathfrak{q})\right|^q.
\end{aligned}
$$

By using (2.11) and (2.12) with (2.10), we obtain

$$
\bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega) - \frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)
$$

$$
\begin{aligned}
&-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}_{\omega}^{AB}\mathcal{I}_{\mathfrak{p}}^{\varsigma}\mathcal{Q}(\mathfrak{p})+{}_{\omega}^{AB}\mathcal{I}_{\mathfrak{q}}^{\varsigma}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\bigg| \\
&\leq\left(\frac{1}{\varsigma+2}\right)^{1-\frac{1}{q}}\left[\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(\varsigma+s+2)}+\beta(\varsigma+2,s+1)\left|\mathcal{Q}''(\mathfrak{p})\right|^{q}\right)^{\frac{1}{q}}\right. \\
&\left.+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(\varsigma+s+2)}+\beta(\varsigma+2,s+1)\left|\mathcal{Q}''(\mathfrak{q})\right|^{q}\right)^{\frac{1}{q}}\right],
\end{aligned}
$$

which completes the proof. □

Corollary 2.5 *If we set $s=1$ in Theorem 2.3, then we have the following Ostrowski-type inequality for a convex function*:

$$
\begin{aligned}
&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right. \qquad (2.13)\\
&\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}_{\omega}^{AB}\mathcal{I}_{\mathfrak{p}}^{\varsigma}\mathcal{Q}(\mathfrak{p})+{}_{\omega}^{AB}\mathcal{I}_{\mathfrak{q}}^{\varsigma}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right| \\
&\leq\left(\frac{1}{\varsigma+2}\right)^{1-\frac{1}{q}}\left[\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{\varsigma+3}+\frac{|\mathcal{Q}''(\mathfrak{p})|^{q}}{(\varsigma+2)(\varsigma+3)}\right)^{\frac{1}{q}}\right. \\
&\left.+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{\varsigma+3}+\frac{|\mathcal{Q}''(\mathfrak{q})|^{q}}{(\varsigma+2)(\varsigma+3)}\right)^{\frac{1}{q}}\right].
\end{aligned}
$$

Remark 2.3 If we set $\varsigma=1$, in Theorem 2.3, then we recover (Theorem 6, [49])

$$
\begin{aligned}
&\left|\frac{1}{\mathfrak{q}-\mathfrak{p}}\int_{\mathfrak{p}}^{\mathfrak{q}}\mathcal{Q}(u)\,du-\mathcal{Q}(\omega)+\left(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\mathcal{Q}'(\omega)\right| \\
&\leq\left(\frac{1}{3}\right)^{1-\frac{1}{q}}\left[\frac{(\omega-\mathfrak{p})^{3}}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{s+3}+\frac{2|\mathcal{Q}''(\mathfrak{p})|^{q}}{(s+1)(s+2)(s+3)}\right)^{\frac{1}{q}}\right. \\
&\left.+\frac{(\mathfrak{q}-\omega)^{3}}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(s+3)}+\frac{2|\mathcal{Q}''(\mathfrak{q})|^{q}}{(s+1)(s+2)(s+3)}\right)^{\frac{1}{q}}\right].
\end{aligned}
$$

Corollary 2.6 *Under the same assumptions of Corollary 2.5 with $|\mathcal{Q}''|\leq\mathcal{M}$, we obtain the following inequality*

$$
\begin{aligned}
&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right. \\
&\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}_{\omega}^{AB}\mathcal{I}_{\mathfrak{p}}^{\varsigma}\mathcal{Q}(\mathfrak{p})+{}_{\omega}^{AB}\mathcal{I}_{\mathfrak{q}}^{\varsigma}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right| \\
&\leq\mathcal{M}\left(\frac{1}{(\varsigma+1)(\varsigma+2)(\mathfrak{q}-\mathfrak{p})}\right)\left[(\omega-\mathfrak{p})^{\varsigma+2}+(\mathfrak{q}-\omega)^{\varsigma+2}\right].
\end{aligned}
$$

Theorem 2.4 *Suppose $\mathcal{Q}:\mathcal{J}\subset[0,\infty)\to\mathcal{R}$ is a twice-differentiable mapping on $(\mathfrak{p},\mathfrak{q})$ with $\mathfrak{p}<\mathfrak{q}$ such that $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$. If $|\mathcal{Q}''|^q$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$ for some fixed $s\in(0,1]$, $q>1$, then for all $\varsigma>0$, following the inequality for AB-fractional integrals*

$$
\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right. \qquad (2.14)
$$

$$
\begin{aligned}
&- \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg| \\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{((\varsigma+1)p+1)p} + \frac{|\mathcal{Q}''(\omega)|^{q}+|\mathcal{Q}''(\mathfrak{p})|^{q}}{(s+1)q}\right\} \\
&\quad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{((\varsigma+1)p+1)p} + \frac{|\mathcal{Q}''(\omega)|^{q}+|\mathcal{Q}''(\mathfrak{q})|^{q}}{(s+1)q}\right\},
\end{aligned}
$$

holds true for $\Phi \in [0,1]$.

Proof From Lemma 2.1, we obtain

$$
\begin{aligned}
&\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega) - \frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega) \\
&\quad - \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg| \\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right| d\Phi \\
&\quad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1 \Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right| d\Phi.
\end{aligned}
$$

By using Young's inequality as

$$
\begin{aligned}
&UV < \frac{1}{p}U^{p} + \frac{1}{q}V^{q}, \\
&\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega) - \frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega) \\
&\quad - \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg| \\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{p}\int_0^1 \Phi^{(\varsigma+1)p}\,d\Phi + \frac{1}{q}\int_0^1 \left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right|^{q} d\Phi\right\} \\
&\quad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{p}\int_0^1 \Phi^{(\varsigma+1)p}\,d\Phi + \frac{1}{q}\int_0^1 \left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right|^{q} d\Phi\right\} \\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{p}\int_0^1 \Phi^{(\varsigma+1)p}\,d\Phi + \frac{1}{q}\int_0^1 \left\{\Phi^{s}\left|\mathcal{Q}''(\omega)\right|^{q} + (1-\Phi)^{s}\left|\mathcal{Q}''(\mathfrak{p})\right|^{q}\right\}\right\} \\
&\quad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{p}\int_0^1 \Phi^{(\varsigma+1)p}\,d\Phi + \frac{1}{q}\int_0^1 \left\{\Phi^{s}\left|\mathcal{Q}''(\omega)\right|^{q} + (1-\Phi)^{s}\left|\mathcal{Q}''(\mathfrak{q})\right|^{q}\right\}\right\} \\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{((\varsigma+1)p+1)p} + \frac{|\mathcal{Q}''(\omega)|^{q}+|\mathcal{Q}''(\mathfrak{p})|^{q}}{(s+1)q}\right\} \\
&\quad + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left\{\frac{1}{((\varsigma+1)p+1)p} + \frac{|\mathcal{Q}''(\omega)|^{q}+|\mathcal{Q}''(\mathfrak{q})|^{q}}{(s+1)q}\right\},
\end{aligned}
$$

which completes the proof. □

Corollary 2.7 *If we set $s = 1$ in Theorem 2.4, then we have the following Ostrowski-type inequality for a convex function*:

$$\begin{aligned}&\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\\&\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\big\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\big\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg|\\&\quad\le\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\bigg\{\frac{1}{((\varsigma+1)p+1)p}+\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{p})|^q}{2q}\bigg\}\\&\quad\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\bigg\{\frac{1}{((\varsigma+1)p+1)p}+\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{q})|^q}{2q}\bigg\}.\end{aligned}\tag{2.15}$$

Corollary 2.8 *If we set $\varsigma = 1$ in Theorem 2.4, we obtain*

$$\begin{aligned}&\bigg|\frac{1}{\mathfrak{q}-\mathfrak{p}}\int_{\mathfrak{p}}^{\mathfrak{q}}\mathcal{Q}(u)\,du-\mathcal{Q}(\omega)+\Big(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\Big)\mathcal{Q}'(\omega)\bigg|\\&\quad\le\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\bigg[\frac{1}{(2p+1)p}+\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{p})|^q}{2q}\bigg]\\&\quad\quad+\frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\bigg[\frac{1}{(2p+1)p}+\frac{|\mathcal{Q}''(\omega)|^q+|\mathcal{Q}''(\mathfrak{q})|^q}{(s+1)q}\bigg].\end{aligned}$$

3 Further inequalities via an improved Hölder's inequality

Theorem 3.1 *Suppose $\mathcal{Q}:\mathcal{J}\subset[0,\infty)\to\mathcal{R}$ is a twice-differentiable mapping on $(\mathfrak{p},\mathfrak{q})$ with $\mathfrak{p}<\mathfrak{q}$ such that $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$. If $|\mathcal{Q}''|^q$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$ for some fixed $s\in(0,1]$, $q>1$, then for all $\varsigma>0$, the following A-B fractional integral inequality*

$$\begin{aligned}&\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\\&\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\big\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\big\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg|\\&\quad\le\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\Bigg[\bigg(\frac{1}{(\varsigma p+p+1)(\varsigma p+p+2)}\bigg)^{\frac{1}{p}}\bigg(\frac{|\mathcal{Q}''(\omega)|^q}{(s+1)(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{p})|^q}{s+2}\bigg)^{\frac{1}{q}}\\&\quad\quad+\bigg(\frac{1}{(\varsigma+1)p+2}\bigg)^{\frac{1}{p}}\bigg(\frac{|\mathcal{Q}''(\omega)|^q}{(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{p})|^q}{(s+1)(s+2)}\bigg)^{\frac{1}{q}}\Bigg]\\&\quad\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\Bigg[\bigg(\frac{1}{(\varsigma p+p+1)(\varsigma p+p+2)}\bigg)^{\frac{1}{p}}\bigg(\frac{|\mathcal{Q}''(\omega)|^q}{(s+1)(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{q})|^q}{s+2}\bigg)^{\frac{1}{q}}\\&\quad\quad+\bigg(\frac{1}{(\varsigma+1)p+2}\bigg)^{\frac{1}{p}}\bigg(\frac{|\mathcal{Q}''(\omega)|^q}{(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{q})|^q}{(s+1)(s+2)}\bigg)^{\frac{1}{q}}\Bigg],\end{aligned}\tag{3.1}$$

holds true for $\Phi\in[0,1]$, where $q^{-1}+p^{-1}=1$.

Proof From Lemma 2.1, by using the Hölder–Işcan integral inequality (see in [50]) and the s-convexity of $|\mathcal{Q}''|^q$, we obtain

$$\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)$$

$$
\begin{aligned}
& - \frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}} \left\{ {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}} \mathcal{Q}(\mathfrak{p}) + {}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}} \mathcal{Q}(\mathfrak{q}) \right\} - \frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}} \mathcal{Q}(\omega) \Bigg| \\
\leq {} & \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \int_0^1 \Phi^{\varsigma+1} \left| \mathcal{Q}''\big(\Phi\omega + (1-\Phi)\mathfrak{p}\big) \right| d\Phi \\
& + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \int_0^1 \Phi^{\varsigma+1} \left| \mathcal{Q}''\big(\Phi\omega + (1-\Phi)\mathfrak{q}\big) \right| d\Phi \\
\leq {} & \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \Bigg[\left(\int_0^1 (1-\Phi)\Phi^{(\varsigma+1)p} \, d\Phi \right)^{\frac{1}{p}} \\
& \times \left(\int_0^1 (1-\Phi) \left| \mathcal{Q}''\big(\Phi\omega + (1-\Phi)\mathfrak{p}\big) \right|^q d\Phi \right)^{\frac{1}{q}} \\
& + \left(\int_0^1 \Phi^{(\varsigma+1)p+1} \, d\Phi \right)^{\frac{1}{p}} \left(\int_0^1 \Phi \left| \mathcal{Q}''\big(\Phi\omega + (1-\Phi)\mathfrak{p}\big) \right|^q d\Phi \right)^{\frac{1}{q}} \Bigg] \\
& + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \Bigg[\left(\int_0^1 (1-\Phi)\Phi^{(\varsigma+1)p} \, d\Phi \right)^{\frac{1}{p}} \\
& \times \left(\int_0^1 (1-\Phi) \left| \mathcal{Q}''\big(\Phi\omega + (1-\Phi)\mathfrak{q}\big) \right|^q d\Phi \right)^{\frac{1}{q}} \\
& + \left(\int_0^1 \Phi^{(\varsigma+1)p+1} \, d\Phi \right)^{\frac{1}{p}} \left(\int_0^1 \Phi \left| \mathcal{Q}''\big(\Phi\omega + (1-\Phi)\mathfrak{q}\big) \right|^q d\Phi \right)^{\frac{1}{q}} \Bigg] \\
\leq {} & \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \Bigg[\left(\int_0^1 (1-\Phi)\Phi^{(\varsigma+1)p} \, d\Phi \right)^{\frac{1}{p}} \\
& \times \left(\int_0^1 (1-\Phi) \left\{ \Phi^s \left| \mathcal{Q}''(\omega) \right|^q + (1-\Phi)^s \left| \mathcal{Q}''(\mathfrak{p}) \right|^q \right\} d\Phi \right)^{\frac{1}{q}} \\
& + \left(\int_0^1 \Phi^{(\varsigma+1)p+1} \, d\Phi \right)^{\frac{1}{p}} \left(\int_0^1 \Phi \left\{ \Phi^s \left| \mathcal{Q}''(\omega) \right|^q + (1-\Phi)^s \left| \mathcal{Q}''(\mathfrak{p}) \right|^q \right\} d\Phi \right)^{\frac{1}{q}} \Bigg] \\
& + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \Bigg[\left(\int_0^1 (1-\Phi)\Phi^{(\varsigma+1)p} \, d\Phi \right)^{\frac{1}{p}} \\
& \times \left(\int_0^1 (1-\Phi) \left\{ \Phi^s \left| \mathcal{Q}''(\omega) \right|^q + (1-\Phi)^s \left| \mathcal{Q}''(\mathfrak{q}) \right|^q \right\} d\Phi \right)^{\frac{1}{q}} \\
& + \left(\int_0^1 \Phi^{(\varsigma+1)p+1} \, d\Phi \right)^{\frac{1}{p}} \left(\int_0^1 \Phi \left\{ \Phi^s \left| \mathcal{Q}''(\omega) \right|^q + (1-\Phi)^s \left| \mathcal{Q}''(\mathfrak{q}) \right|^q \right\} d\Phi \right)^{\frac{1}{q}} \Bigg]. \\
\leq {} & \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \Bigg[\left(\frac{1}{(\varsigma p + p + 1)(\varsigma p + p + 2)} \right)^{\frac{1}{p}} \left(\frac{|\mathcal{Q}''(\omega)|^q}{(s+1)(s+2)} + \frac{|\mathcal{Q}''(\mathfrak{p})|^q}{s+2} \right)^{\frac{1}{q}} \\
& + \left(\frac{1}{(\varsigma+1)p+2} \right)^{\frac{1}{p}} \left(\frac{|\mathcal{Q}''(\omega)|^q}{(s+2)} + \frac{|\mathcal{Q}''(\mathfrak{p})|^q}{(s+1)(s+2)} \right)^{\frac{1}{q}} \Bigg] \\
& + \frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})} \Bigg[\left(\frac{1}{(\varsigma p + p + 1)(\varsigma p + p + 2)} \right)^{\frac{1}{p}} \left(\frac{|\mathcal{Q}''(\omega)|^q}{(s+1)(s+2)} + \frac{|\mathcal{Q}''(\mathfrak{q})|^q}{s+2} \right)^{\frac{1}{q}} \\
& + \left(\frac{1}{(\varsigma+1)p+2} \right)^{\frac{1}{p}} \left(\frac{|\mathcal{Q}''(\omega)|^q}{(s+2)} + \frac{|\mathcal{Q}''(\mathfrak{q})|^q}{(s+1)(s+2)} \right)^{\frac{1}{q}} \Bigg].
\end{aligned}
$$

This completes the proof. □

Corollary 3.1 *If we set $s = 1$ in Theorem 3.1, then we have the following Ostrowski-type inequality for a convex function*:

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\quad\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\quad\le\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(\varsigma p+p+1)(\varsigma p+p+2)}\right)^{\frac{1}{p}}\left(\frac{1}{6}\left|\mathcal{Q}''(\omega)\right|^{q}+\frac{1}{3}\left|\mathcal{Q}''(\mathfrak{p})\right|^{q}\right)^{\frac{1}{q}}\right.\\&\quad\left.+\left(\frac{1}{(\varsigma+1)p+2}\right)^{\frac{1}{p}}\left(\frac{1}{3}\left|\mathcal{Q}''(\omega)\right|^{q}+\frac{1}{6}\left|\mathcal{Q}''(\mathfrak{p})\right|^{q}\right)^{\frac{1}{q}}\right]\\&\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(\varsigma p+p+1)(\varsigma p+p+2)}\right)^{\frac{1}{p}}\left(\frac{1}{6}\left|\mathcal{Q}''(\omega)\right|^{q}+\frac{1}{3}\left|\mathcal{Q}''(\mathfrak{q})\right|^{q}\right)^{\frac{1}{q}}\right.\\&\quad\left.+\left(\frac{1}{(\varsigma+1)p+2}\right)^{\frac{1}{p}}\left(\frac{1}{3}\left|\mathcal{Q}''(\omega)\right|^{q}+\frac{1}{6}\left|\mathcal{Q}''(\mathfrak{q})\right|^{q}\right)^{\frac{1}{q}}\right].\end{aligned}$$

Corollary 3.2 *If we set $\varsigma = 1$ in Theorem 3.1, we obtain*

$$\begin{aligned}&\left|\frac{1}{\mathfrak{q}-\mathfrak{p}}\int_{\mathfrak{p}}^{\mathfrak{q}}\mathcal{Q}(u)\,du-\mathcal{Q}(\omega)+\left(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\mathcal{Q}'(\omega)\right|\\&\quad\le\frac{(\omega-\mathfrak{p})^{3}}{2(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(2p+1)(2p+2)}\right)^{\frac{1}{p}}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(s+1)(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{p})|^{q}}{s+2}\right)^{\frac{1}{q}}\right.\\&\quad\left.+\left(\frac{1}{2p+2}\right)^{\frac{1}{p}}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{p})|^{q}}{(s+1)(s+2)}\right)^{\frac{1}{q}}\right]\\&\quad+\frac{(\mathfrak{q}-\omega)^{3}}{2(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(2p+1)(2p+2)}\right)^{\frac{1}{p}}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(s+1)(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{q})|^{q}}{s+2}\right)^{\frac{1}{q}}\right.\\&\quad\left.+\left(\frac{1}{2p+2}\right)^{\frac{1}{p}}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(s+2)}+\frac{|\mathcal{Q}''(\mathfrak{q})|^{q}}{(s+1)(s+2)}\right)^{\frac{1}{q}}\right].\end{aligned}$$

Corollary 3.3 *Using the same assumptions in Corollary 3.1 with $|\mathcal{Q}''| \le \mathcal{M}$, we obtain*

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\quad\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\quad\le\frac{\mathcal{M}}{2^{\frac{1}{q}}(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(\varsigma p+p+1)(\varsigma p+p+2)}\right)^{\frac{1}{p}}+\left(\frac{1}{(\varsigma+1)p+2}\right)^{\frac{1}{p}}\right]\\&\quad\times\left[(\omega-\mathfrak{p})^{\varsigma+2}+(\mathfrak{q}-\omega)^{\varsigma+2}\right].\end{aligned}$$

Theorem 3.2 *Suppose $\mathcal{Q}:\mathcal{J}\subset[0,\infty)\to\mathcal{R}$ is a twice-differentiable mapping on $(\mathfrak{p},\mathfrak{q})$ with $\mathfrak{p}<\mathfrak{q}$ such that $\mathcal{Q}''\in\mathcal{L}_1[\mathfrak{p},\mathfrak{q}]$. If $|\mathcal{Q}''|^q$ is an s-convex function on $[\mathfrak{p},\mathfrak{q}]$ for some fixed*

$s \in (0,1]$, $q \geq 1$, then for all $\varsigma > 0$, the inequality for A-B fractional integrals

$$\begin{aligned} &\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right. \\ &\qquad\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right| \\ &\quad\leq\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(\varsigma+2)(\varsigma+3)}\right)^{1-\frac{1}{q}}\right. \\ &\qquad\times\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(\varsigma+s+2)(\varsigma+s+3)}+\left|\mathcal{Q}''(\mathfrak{p})\right|^{q}\beta(s+2,\varsigma+2)\right)^{\frac{1}{q}} \\ &\qquad\left.+\left(\frac{1}{\varsigma+3}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{\varsigma+s+3}+\left|\mathcal{Q}''(\mathfrak{p})\right|^{q}\beta(\varsigma+3,s+1)\right)^{\frac{1}{q}}\right] \\ &\qquad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(\varsigma+2)(\varsigma+3)}\right)^{1-\frac{1}{q}}\right. \\ &\qquad\times\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(\varsigma+s+2)(\varsigma+s+3)}+\left|\mathcal{Q}''(\mathfrak{q})\right|^{q}\beta(s+2,\varsigma+2)\right)^{\frac{1}{q}} \\ &\qquad\left.+\left(\frac{1}{\varsigma+3}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^{q}}{(\varsigma+s+3)}+\left|\mathcal{Q}''(\mathfrak{q})\right|^{q}\beta(\varsigma+3,s+1)\right)^{\frac{1}{q}}\right], \end{aligned} \tag{3.2}$$

holds true for $\Phi \in [0,1]$.

Proof From Lemma 2.1, the improved power-mean integral inequality (see in [50]), and the s-convexity of $|\mathcal{Q}''|^{q}$, we obtain

$$\begin{aligned} &\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right. \\ &\qquad\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right| \\ &\quad\leq\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right|d\Phi \\ &\qquad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\int_0^1\Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right|d\Phi \\ &\quad\leq\frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\int_0^1(1-\Phi)\Phi^{\varsigma+1}\,d\Phi\right)^{1-\frac{1}{q}}\right. \\ &\qquad\times\left(\int_0^1(1-\Phi)\Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right|^{q}d\Phi\right)^{\frac{1}{q}} \\ &\qquad\left.+\left(\int_0^1\Phi^{\varsigma+2}\,d\Phi\right)^{1-\frac{1}{q}}\left(\int_0^1\Phi^{\varsigma+2}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{p}\big)\right|^{q}d\Phi\right)^{\frac{1}{q}}\right] \\ &\qquad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\int_0^1(1-\Phi)\Phi^{\varsigma+1}\,d\Phi\right)^{1-\frac{1}{q}}\right. \\ &\qquad\times\left(\int_0^1(1-\Phi)\Phi^{\varsigma+1}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right|^{q}d\Phi\right)^{\frac{1}{q}} \end{aligned}$$

$$
\begin{aligned}
&+\left(\int_0^1 \Phi^{\varsigma+2}\,d\Phi\right)^{1-\frac{1}{q}}\left(\int_0^1 \Phi^{\varsigma+2}\left|\mathcal{Q}''\big(\Phi\omega+(1-\Phi)\mathfrak{q}\big)\right|^q d\Phi\right)^{\frac{1}{q}}\Bigg]\\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\Bigg[\left(\int_0^1 (1-\Phi)\Phi^{\varsigma+1}\,d\Phi\right)^{1-\frac{1}{q}}\\
&\quad\times\left(\int_0^1 (1-\Phi)\Phi^{\varsigma+1}\left\{\Phi^s\left|\mathcal{Q}''(\omega)\right|^q+(1-\Phi)^s\left|\mathcal{Q}''(\mathfrak{p})\right|^q\right\}d\Phi\right)^{\frac{1}{q}}\\
&\quad+\left(\int_0^1 \Phi^{\varsigma+2}\,d\Phi\right)^{1-\frac{1}{q}}\left(\int_0^1 \Phi^{\varsigma+2}\left\{\Phi^s\left|\mathcal{Q}''(\omega)\right|^q+(1-\Phi)^s\left|\mathcal{Q}''(\mathfrak{p})\right|^q\right\}d\Phi\right)^{\frac{1}{q}}\Bigg]\\
&\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\Bigg[\left(\int_0^1 (1-\Phi)\Phi^{\varsigma+1}\,d\Phi\right)^{1-\frac{1}{q}}\\
&\quad\times\left(\int_0^1 (1-\Phi)\Phi^{\varsigma+1}\left\{\Phi^s\left|\mathcal{Q}''(\omega)\right|^q+(1-\Phi)^s\left|\mathcal{Q}''(\mathfrak{q})\right|^q\right\}d\Phi\right)^{\frac{1}{q}}\\
&\quad+\left(\int_0^1 \Phi^{\varsigma+2}\,d\Phi\right)^{1-\frac{1}{q}}\left(\int_0^1 \Phi^{\varsigma+2}\left\{\Phi^s\left|\mathcal{Q}''(\omega)\right|^q+(1-\Phi)^s\left|\mathcal{Q}''(\mathfrak{q})\right|^q\right\}d\Phi\right)^{\frac{1}{q}}\Bigg]\\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\Bigg[\left(\frac{1}{(\varsigma+2)(\varsigma+3)}\right)^{1-\frac{1}{q}}\\
&\quad\times\left(\frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+s+2)(\varsigma+s+3)}+\left|\mathcal{Q}''(\mathfrak{p})\right|^q\beta(s+2,\varsigma+2)\right)^{\frac{1}{q}}\\
&\quad+\left(\frac{1}{\varsigma+3}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{\varsigma+s+3}+\left|\mathcal{Q}''(\mathfrak{p})\right|^q\beta(\varsigma+3,s+1)\right)^{\frac{1}{q}}\Bigg]\\
&\quad+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\Bigg[\left(\frac{1}{(\varsigma+2)(\varsigma+3)}\right)^{1-\frac{1}{q}}\\
&\quad\times\left(\frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+s+2)(\varsigma+s+3)}+\left|\mathcal{Q}''(\mathfrak{q})\right|^q\beta(s+2,\varsigma+2)\right)^{\frac{1}{q}}\\
&\quad+\left(\frac{1}{\varsigma+3}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+s+3)}+\left|\mathcal{Q}''(\mathfrak{q})\right|^q\beta(\varsigma+3,s+1)\right)^{\frac{1}{q}}\Bigg].
\end{aligned}
$$

This completes the proof. □

Corollary 3.4 *If we set $s=1$ in Theorem 3.2, then we have the following Ostrowski-type inequality for a convex function*:

$$
\begin{aligned}
&\Bigg|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\\
&\quad-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\Bigg|\\
&\leq \frac{(\omega-\mathfrak{p})^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\Bigg[\left(\frac{1}{(\varsigma+2)(\varsigma+3)}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+3)(\varsigma+4)}+\frac{2|\mathcal{Q}''(\mathfrak{p})|^q}{(\varsigma+2)(\varsigma+3)(\varsigma+4)}\right)^{\frac{1}{q}}\\
&\quad+\left(\frac{1}{\varsigma+3}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{\varsigma+4}+\frac{|\mathcal{Q}''(\mathfrak{p})|^q}{(\varsigma+3)(\varsigma+4)}\right)^{\frac{1}{q}}\Bigg]
\end{aligned}
$$

$$+\frac{(\mathfrak{q}-\omega)^{\varsigma+2}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(\varsigma+2)(\varsigma+3)}\right)^{1-\frac{1}{q}}\times\left(\frac{|\mathcal{Q}''(\omega)|^q}{(\varsigma+3)(\varsigma+4)}+\frac{2|\mathcal{Q}''(\mathfrak{q})|^q}{(\varsigma+2)(\varsigma+3)(\varsigma+4)}\right)^{\frac{1}{q}}+\left(\frac{1}{\varsigma+3}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{\varsigma+4}+\frac{|\mathcal{Q}''(\mathfrak{q})|^q}{(\varsigma+3)(\varsigma+4)}\right)^{\frac{1}{q}}\right].$$

Corollary 3.5 *If we set* $\varsigma=1$ *in Theorem* 3.2, *we obtain*

$$\begin{aligned}&\left|\frac{1}{\mathfrak{q}-\mathfrak{p}}\int_{\mathfrak{p}}^{\mathfrak{q}}\mathcal{Q}(u)\,du-\mathcal{Q}(\omega)+\left(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\mathcal{Q}'(\omega)\right|\\&\leq\frac{(\omega-\mathfrak{p})^3}{(2)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{12}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{(s+3)(s+4)}+\frac{2|\mathcal{Q}''(\mathfrak{p})|^q}{(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right.\\&\left.+\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{s+4}+\frac{6|\mathcal{Q}''(\mathfrak{p})|^q}{(s+1)(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right]\\&+\frac{(\mathfrak{q}-\omega)^3}{(2)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{12}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{(s+3)(s+4)}+\frac{2|\mathcal{Q}''(\mathfrak{q})|^q}{(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right.\\&\left.+\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left(\frac{|\mathcal{Q}''(\omega)|^q}{(s+4)}+\frac{6|\mathcal{Q}''(\mathfrak{q})|^q}{(s+1)(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right].\end{aligned}$$

Corollary 3.6 *Using the same assumption of Corollary* 3.4 *with* $|\mathcal{Q}''|\leq\mathcal{M}$, *we obtain*

$$\begin{aligned}&\left|\frac{(\omega-\mathfrak{p})^{\varsigma+1}-(\mathfrak{q}-\omega)^{\varsigma+1}}{(\varsigma+1)(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}'(\omega)-\frac{(\omega-\mathfrak{p})^{\varsigma}+(\mathfrak{q}-\omega)^{\varsigma}}{(\mathfrak{q}-\mathfrak{p})}\mathcal{Q}(\omega)\right.\\&\left.-\frac{\mathrm{B}(\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\left\{{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{p}}\mathcal{Q}(\mathfrak{p})+{}^{AB}_{\omega}\mathcal{I}^{\varsigma}_{\mathfrak{q}}\mathcal{Q}(\mathfrak{q})\right\}-\frac{2(1-\varsigma)\Gamma(\varsigma)}{\mathfrak{q}-\mathfrak{p}}\mathcal{Q}(\omega)\right|\\&\leq\frac{\mathcal{M}}{(\varsigma+1)(\varsigma+2)(\mathfrak{q}-\mathfrak{p})}\left[(\omega-\mathfrak{p})^{\varsigma+2}+(\mathfrak{q}-\omega)^{\varsigma+2}\right].\end{aligned}$$

4 Applications

4.1 *q*-digamma function

The q-digamma(psi) function ϱ_ρ, is the ρ-analog of the digamma function ϱ (see [51]) given as:

$$\begin{aligned}\varrho_\rho(\gamma)&=-\ln(1-\rho)+\ln\rho\sum_{k=0}^{\infty}\frac{\rho^{k+\gamma}}{1-\rho^{k+\gamma}}\\&=-\ln(1-\rho)+\ln\rho\sum_{k=1}^{\infty}\frac{\rho^{k\gamma}}{1-\rho^{k\gamma}}.\end{aligned}$$

For $\rho>1$ and $\gamma>0$, ρ-digamma function ϱ_ρ can be given as:

$$\varrho_\rho(\gamma)=-\ln(\rho-1)+\ln\rho\left[\gamma-\frac{1}{2}-\sum_{k=0}^{\infty}\frac{\rho^{-(k+\gamma)}}{1-\rho^{-(k+\gamma)}}\right]$$

$$= -\ln(\rho-1) + \ln\rho\left[\gamma - \frac{1}{2} - \sum_{k=1}^{\infty}\frac{\rho^{-k\gamma}}{1-\rho^{-k\gamma}}\right].$$

From the definition of q-digamma functions, it is seen that $\mathcal{Q}(x) = \varrho'_{\rho}(x)$ is completely monotonic on $(0,\infty)$. If we set $\mathcal{Q}(x) = \varrho'_{\rho}(x)$, then $\mathcal{Q}''(x) = \varrho'''_{\rho}(x)$ is also completely monotonic on $(0,\infty)$.

Proposition 4.1 *Assuming all the above conditions and applying Remark 2.1, we have*

$$\begin{aligned}
&\left|\frac{\varrho_{\rho}(\mathfrak{q}) - \varrho_{\rho}(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \varrho'_{\rho}(\omega) + \left(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\varrho''_{\rho}(\omega)\right| \\
&\quad\le \frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\varrho'''_{\rho}(\omega)|}{s+3} + \frac{2|\varrho'''_{\rho}(\mathfrak{p})|}{(s+1)(s+2)(s+3)}\right\} \\
&\qquad + \frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\varrho'''_{\rho}(\omega)|}{s+3} + \frac{2|\varrho'''_{\rho}(\mathfrak{q})|}{(s+1)(s+2)(s+3)}\right\}.
\end{aligned}$$

Proposition 4.2 *Considering all the conditions of Proposition 4.1 and applying Remark 2.2, we have*

$$\begin{aligned}
&\left|\frac{\varrho_{\rho}(\mathfrak{q}) - \varrho_{\rho}(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \varrho'_{\rho}(\omega) + \left(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\varrho''_{\rho}(\omega)\right| \\
&\quad\le \left(\frac{1}{2p+1}\right)^{\frac{1}{p}} \\
&\qquad\times\left[\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\varrho'''_{\rho}(\omega)|^q + |\varrho'''_{\rho}(\mathfrak{p})|^q}{s+1}\right)^{\frac{1}{q}} + \frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\varrho'''_{\rho}(\omega)|^q + |\varrho'''_{\rho}(\mathfrak{q})|^q}{s+1}\right)^{\frac{1}{q}}\right].
\end{aligned}$$

Proposition 4.3 *Considering all the conditions of Proposition 4.1 and applying Remark 2.3, we have*

$$\begin{aligned}
&\left|\frac{\varrho_{\rho}(\mathfrak{q}) - \varrho_{\rho}(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \varrho'_{\rho}(\omega) + \left(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\varrho''_{\rho}(\omega)\right| \\
&\quad\le \left(\frac{1}{3}\right)^{1-\frac{1}{q}}\left[\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\varrho'''_{\rho}(\omega)|^q}{s+3} + \frac{2|\varrho'''_{\rho}(\mathfrak{p})|^q}{(s+1)(s+2)(s+3)}\right)^{\frac{1}{q}}\right. \\
&\qquad\left. + \frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left(\frac{|\varrho'''_{\rho}(\omega)|^q}{(s+3)} + \frac{2|\varrho'''_{\rho}(\mathfrak{q})|^q}{(s+1)(s+2)(s+3)}\right)^{\frac{1}{q}}\right].
\end{aligned}$$

Proposition 4.4 *Considering all the conditions of Proposition 4.1 and applying Corollary 2.8 we have*

$$\begin{aligned}
&\left|\frac{\varrho_{\rho}(\mathfrak{q}) - \varrho_{\rho}(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \varrho'_{\rho}(\omega) + \left(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\varrho''_{\rho}(\omega)\right| \\
&\quad\le \frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\frac{1}{(2p+1)p} + \frac{|\varrho'''_{\rho}(\omega)|^q + |\varrho'''_{\rho}(\mathfrak{p})|^q}{2q}\right] \\
&\qquad + \frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\frac{1}{(2p+1)p} + \frac{|\varrho'''_{\rho}(\omega)|^q + |\varrho'''_{\rho}(\mathfrak{q})|^q}{(s+1)q}\right].
\end{aligned}$$

Proposition 4.5 *Considering all the conditions of Proposition 4.1 and applying Corollary 3.2 we have*

$$\begin{aligned}
&\left|\frac{\varrho_\rho(\mathfrak{q})-\varrho_\rho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}}-\varrho_\rho'(\omega)+\left(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\varrho_\rho''(\omega)\right| \\
&\quad\le \frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(p+p+1)(p+p+2)}\right)^{\frac{1}{p}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{(s+1)(s+2)}+\frac{|\varrho_\rho'''(\mathfrak{p})|^q}{s+2}\right)^{\frac{1}{q}}\right. \\
&\quad\quad\left.+\left(\frac{1}{2p+2}\right)^{\frac{1}{p}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{(s+2)}+\frac{|\varrho_\rho'''(\mathfrak{p})|^q}{(s+1)(s+2)}\right)^{\frac{1}{q}}\right] \\
&\quad\quad+\frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(p+p+1)(p+p+2)}\right)^{\frac{1}{p}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{(s+1)(s+2)}+\frac{|\varrho_\rho'''(\mathfrak{q})|^q}{s+2}\right)^{\frac{1}{q}}\right. \\
&\quad\quad\left.+\left(\frac{1}{2p+2}\right)^{\frac{1}{p}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{(s+2)}+\frac{|\varrho_\rho'''(\mathfrak{q})|^q}{(s+1)(s+2)}\right)^{\frac{1}{q}}\right].
\end{aligned}$$

Proposition 4.6 *Considering all the conditions of Proposition 4.1 and applying Corollary 2.8 we have*

$$\begin{aligned}
&\left|\frac{\varrho_\rho(\mathfrak{q})-\varrho_\rho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}}-\varrho_\rho'(\omega)+\left(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2}\right)\varrho_\rho''(\omega)\right| \\
&\quad\le \frac{(\omega-\mathfrak{p})^3}{(2)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{12}\right)^{1-\frac{1}{q}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{(s+3)(s+4)}+\frac{2|\varrho_\rho'''(\mathfrak{p})|^q}{(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right. \\
&\quad\quad\left.+\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{s+4}+\frac{6|\varrho_\rho'''(\mathfrak{p})|^q}{(s+1)(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right] \\
&\quad\quad+\frac{(\mathfrak{q}-\omega)^3}{(2)(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{12}\right)^{1-\frac{1}{q}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{(s+3)(s+4)}+\frac{2|\varrho_\rho'''(\mathfrak{q})|^q}{(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right. \\
&\quad\quad\left.+\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left(\frac{|\varrho_\rho'''(\omega)|^q}{(s+4)}+\frac{6|\varrho_\rho'''(\mathfrak{q})|^q}{(s+1)(s+2)(s+3)(s+4)}\right)^{\frac{1}{q}}\right].
\end{aligned}$$

4.2 Modified Bessel functions

Let the function $\mathcal{K}_\varrho:\mathcal{R}\to[1,\infty)$ be defined [51] as

$$\mathcal{K}_\varrho(u)=2^\varrho\Gamma(\varrho+1)u^{-\delta}\mathrm{I}_\varrho(u),\quad u\in\mathcal{R}.$$

Here, we consider the modified Bessel function of the first kind given by

$$\mathrm{I}_\varrho(u)=\sum_{n=0}^{\infty}\frac{(\frac{u}{2})^{\varrho+2n}}{n!\Gamma(\varrho+n+1)}.$$

The first-, second-, and third-order derivatives are given as

$$\mathcal{K}_\varrho'(u)=\frac{u}{2(\varrho+1)}\mathcal{K}_{\varrho+1}(u),$$

$$\mathcal{K}_\varrho''(u)=\frac{1}{4(\varrho+1)}\left[\frac{u^2}{(\varrho+2)}\mathcal{K}_{\varrho+2}(u)+2\mathcal{K}_{\varrho+1}(u)\right],$$

$$\mathcal{K}_\varrho'''(u) = \frac{u}{4(\varrho+1)(\varrho+2)}\left[\frac{u^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(u) + 3\mathcal{K}_{\varrho+2}(u)\right].$$

If we use, $\Phi(u) = \mathcal{K}_\varrho'(u)$ and the above functions, we have

Proposition 4.7 *Considering all the above conditions and applying Remark 2.1 we have*

$$\begin{aligned}
&\left|\frac{\mathcal{K}_\varrho(\mathfrak{q}) - \mathcal{K}_\varrho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \frac{\omega}{2(\varrho+1)}\mathcal{K}_{\varrho+1}(\omega) + \frac{(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2})}{4(\varrho+1)}\left[\frac{\omega^2}{(\varrho+2)}\mathcal{K}_{\varrho+2}(\omega) + 2\mathcal{K}_{\varrho+1}(\omega)\right]\right| \\
&\leq \frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\omega[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|}{4(\varrho+1)(\varrho+2)(s+3)}\right. \\
&\quad \left. + \frac{|\mathfrak{p}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|}{2(\varrho+1)(\varrho+2)(s+1)(s+2)(s+3)}\right\} \\
&\quad + \frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left\{\frac{|\omega[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|}{4(\varrho+1)(\varrho+2)(s+3)}\right. \\
&\quad \left. + \frac{|\mathfrak{q}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|}{2(\varrho+1)(\varrho+2)(s+1)(s+2)(s+3)}\right\}.
\end{aligned}$$

Proposition 4.8 *Considering all the conditions of Proposition 4.7 and applying Remark 2.2 we have*

$$\begin{aligned}
&\left|\frac{\mathcal{K}_\varrho(\mathfrak{q}) - \mathcal{K}_\varrho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \frac{\omega}{2(\varrho+1)}\mathcal{K}_{\varrho+1}(\omega) + \frac{(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2})}{4(\varrho+1)}\left[\frac{\omega^2}{(\varrho+2)}\mathcal{K}_{\varrho+2}(\omega) + 2\mathcal{K}_{\varrho+1}(\omega)\right]\right| \\
&\leq \left(\frac{1}{2p+1}\right)^{\frac{1}{p}}\left[\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\right. \\
&\quad \times \left(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q + |\frac{\mathfrak{p}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|^q}{s+1}\right)^{\frac{1}{q}} \\
&\quad + \frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})} \\
&\quad \left. \times \left(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q + |\frac{\mathfrak{q}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|^q}{s+1}\right)^{\frac{1}{q}}\right].
\end{aligned}$$

Proposition 4.9 *Considering all the conditions of Proposition 4.7 and applying Corollary 2.3, we have*

$$\begin{aligned}
&\left|\frac{\mathcal{K}_\varrho(\mathfrak{q}) - \mathcal{K}_\varrho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \frac{\omega}{2(\varrho+1)}\mathcal{K}_{\varrho+1}(\omega) + \frac{(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2})}{4(\varrho+1)}\left[\frac{\omega^2}{(\varrho+2)}\mathcal{K}_{\varrho+2}(\omega) + 2\mathcal{K}_{\varrho+1}(\omega)\right]\right| \\
&\leq \left(\frac{1}{3}\right)^{1-\frac{1}{q}}\left[\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\right. \\
&\quad \times \left(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q}{s+3}\right. \\
&\quad \left. + \frac{2|\frac{\mathfrak{p}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|^q}{(s+1)(s+2)(s+3)}\right)^{\frac{1}{q}}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\\
&\times\left(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega)+3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+3)}\right.\\
&\left.+\frac{2|\frac{\mathfrak{q}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q})+3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|^q}{(s+1)(s+2)(s+3)}\right)^{\frac{1}{q}}\Bigg].
\end{aligned}
$$

Proposition 4.10 *Considering all the conditions of Proposition 4.7 and applying Corollary 2.8, we have*

$$
\begin{aligned}
&\left|\frac{\mathcal{K}_\varrho(\mathfrak{q})-\mathcal{K}_\varrho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}}-\frac{\omega}{2(\varrho+1)}\mathcal{K}_{\varrho+1}(\omega)+\frac{(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2})}{4(\varrho+1)}\left[\frac{\omega^2}{(\varrho+2)}\mathcal{K}_{\varrho+2}(\omega)+2\mathcal{K}_{\varrho+1}(\omega)\right]\right|\\
&\leq\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\frac{1}{(2p+1)p}\right.\\
&\left.+\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega)+3\mathcal{K}_{\varrho+2}(\omega)]|^q+|\frac{\mathfrak{p}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p})+3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|^q}{2q}\right]\\
&+\frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\frac{1}{(2p+1)p}\right.\\
&\left.+\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega)+3\mathcal{K}_{\varrho+2}(\omega)]|^q+|\frac{\mathfrak{q}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q})+3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|^q}{(s+1)q}\right].
\end{aligned}
$$

Proposition 4.11 *Considering all the conditions of Proposition 4.7 and applying Corollary 3.2, we have*

$$
\begin{aligned}
&\left|\frac{\mathcal{K}_\varrho(\mathfrak{q})-\mathcal{K}_\varrho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}}-\frac{\omega}{2(\varrho+1)}\mathcal{K}_{\varrho+1}(\omega)+\frac{(\omega-\frac{\mathfrak{p}+\mathfrak{q}}{2})}{4(\varrho+1)}\left[\frac{\omega^2}{(\varrho+2)}\mathcal{K}_{\varrho+2}(\omega)+2\mathcal{K}_{\varrho+1}(\omega)\right]\right|\\
&\leq\frac{(\omega-\mathfrak{p})^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(2p+1)(2p+2)}\right)^{\frac{1}{p}}\right.\\
&\times\left(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega)+3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+1)(s+2)}\right.\\
&\left.+\frac{|\frac{\mathfrak{p}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p})+3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|^q}{s+2}\right)^{\frac{1}{q}}\\
&+\left(\frac{1}{2p+2}\right)^{\frac{1}{p}}\\
&\times\left(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega)+3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+2)}\right.\\
&\left.\left.+\frac{|\frac{\mathfrak{p}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p})+3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|^q}{(s+1)(s+2)}\right)^{\frac{1}{q}}\right]\\
&+\frac{(\mathfrak{q}-\omega)^3}{2(\mathfrak{q}-\mathfrak{p})}\left[\left(\frac{1}{(2p+1)(2p+2)}\right)^{\frac{1}{p}}\right.\\
&\times\left(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega)+3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+1)(s+2)}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{|\frac{\mathfrak{q}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|^q}{s+2}\Bigg)^{\frac{1}{q}} \\
&+ \left(\frac{1}{2p+2}\right)^{\frac{1}{p}} \\
&\times \Bigg(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+2)} \\
&+ \frac{|\frac{\mathfrak{q}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|^q}{(s+1)(s+2)}\Bigg)^{\frac{1}{q}}\Bigg].
\end{aligned}
$$

Proposition 4.12 *Considering all the conditions of Proposition* 4.7 *and applying Corollary* 3.5, *we have*

$$
\begin{aligned}
&\left|\frac{\mathcal{K}_\varrho(\mathfrak{q}) - \mathcal{K}_\varrho(\mathfrak{p})}{\mathfrak{q}-\mathfrak{p}} - \frac{\omega}{2(\varrho+1)}\mathcal{K}_{\varrho+1}(\omega) + \frac{(\omega - \frac{\mathfrak{p}+\mathfrak{q}}{2})}{4(\varrho+1)}\left[\frac{\omega^2}{(\varrho+2)}\mathcal{K}_{\varrho+2}(\omega) + 2\mathcal{K}_{\varrho+1}(\omega)\right]\right| \\
&\quad\le \frac{(\omega-\mathfrak{p})^3}{(2)(\mathfrak{q}-\mathfrak{p})}\Bigg[\left(\frac{1}{12}\right)^{1-\frac{1}{q}} \\
&\quad\times \Bigg(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+3)(s+4)} \\
&\quad+ \frac{2|\frac{\mathfrak{p}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|^q}{(s+2)(s+3)(s+4)}\Bigg)^{\frac{1}{q}} \\
&\quad+ \left(\frac{1}{4}\right)^{1-\frac{1}{q}} \\
&\quad\times \Bigg(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q}{s+4} \\
&\quad+ \frac{6|\frac{\mathfrak{p}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{p}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{p}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{p})]|^q}{(s+1)(s+2)(s+3)(s+4)}\Bigg)^{\frac{1}{q}}\Bigg] \\
&\quad+ \frac{(\mathfrak{q}-\omega)^3}{(2)(\mathfrak{q}-\mathfrak{p})}\Bigg[\left(\frac{1}{12}\right)^{1-\frac{1}{q}} \\
&\quad\times \Bigg(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+3)(s+4)} \\
&\quad+ \frac{2|\frac{\mathfrak{q}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|^q}{(s+2)(s+3)(s+4)}\Bigg)^{\frac{1}{q}} \\
&\quad+ \left(\frac{1}{4}\right)^{1-\frac{1}{q}} \\
&\quad\times \Bigg(\frac{|\frac{\omega}{4(\varrho+1)(\varrho+2)}[\frac{\omega^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\omega) + 3\mathcal{K}_{\varrho+2}(\omega)]|^q}{(s+4)} \\
&\quad+ \frac{6|\frac{\mathfrak{q}}{4(\varrho+1)(\varrho+2)}[\frac{\mathfrak{q}^2}{(\varrho+3)}\mathcal{K}_{\varrho+3}(\mathfrak{q}) + 3\mathcal{K}_{\varrho+2}(\mathfrak{q})]|^q}{(s+1)(s+2)(s+3)(s+4)}\Bigg)^{\frac{1}{q}}\Bigg].
\end{aligned}
$$

5 Conclusion

In recent times most of the work on inequality is based on revealing new bounds of some well-known inequalities using fractional calculus. In this direction, we have investigated the correlation between the theory of inequality and fractional calculus. We have considered the Ostrowski-type inequality in the setting of Atangana–Baleanu fractional calculus. Fractional integral operators play a major role in the advancement of the theory of mathematical inequalities. For this reason, first, we established a new equality for differentiable functions, and using this we have proved our main results. The main objective is to employ Atangana–Baleanu fractional integrals and an s-convex function to provide new bounds of the Ostrowski-type inequality. Several special cases of the main results are rediscovered as well. To be more specific in our main results, if we put $s = 1$, we obtain new Ostrowski-type inequalities for the convex function. Hence, in this paper, we show results for both s-convex and convex functions. In the future, we will use the novel concepts and the modified fractional operators introduced by Refai and Baleanu [42].

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Declarations

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