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Some extensions for Geragthy type contractive mappings

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Abstract
In this paper, we establish some fixed point theorems on some extensions of Geragthy contractive type mappings in the context of b-metric-like spaces.

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1 Introduction and preliminaries

One of the interesting extensions of the notion of a metric space is the dislocated space, introduced by Hitzler [1]. This notion was rediscovered by Amini-Harandi [2] and given the name of a metric-like space.

Definition 1.1 On a nonempty set X we define a function $\sigma : X \times X \to [0,\infty)$ such that for all $x,y,z \in X$:

($\sigma 1$) if $\sigma(x,y) = 0$ then $x = y$;
($\sigma 2$) $\sigma(x,y) = \sigma(y,x)$;
($\sigma 3$) $\sigma(x,y) \le \sigma(x,z) + \sigma(z,y)$;

and the pair (X,σ) is called a dislocated (metric-like) space.

Throughout this paper, we suppose that $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ where $\mathbb{N}$ denotes the set of all positive integers. Further, the symbols $\mathbb{R}^+$ and $\mathbb{R}_0^+$ denotes the set of positive reals and the set of non-negative reals. First, we recall some basic concepts and notations.

The concept of a b-metric was introduced by Czerwik [3] as a generalization of the metric (see also Bakhtin [4] and Bourbaki [5]) to extend the celebrated Banach contraction mapping principle. Following this initial paper of Czerwik [3], a number of researchers in nonlinear analysis investigated the topology of the paper and proved several fixed point theorems in the context of complete b-metric spaces (see *e.g.* [6–10] and references therein).

Definition 1.2 [3] Let X be a nonempty set and $s \ge 1$ be a given real number. A mapping $d: X \times X \to [0,\infty)$ is said to be a b-metric if for all $x,y,z \in X$ the following conditions are satisfied:

(bM_1) $d(x,y) = 0$ if and only if $x = y$;
(bM_2) $d(x,y) = d(y,x)$;

(bM_3) $d(x,z) \leq s[d(x,y) + d(y,z)]$.

In this case, the pair (X,d) is called a b-metric space (with constant s).

In what follows, we recall the notion of b-metric-like space which is an interesting generalization of both b-metric space and metric-like space.

Definition 1.3 [11] Let X be a nonempty set and $s \geq 1$ be a given real number. A mapping $d\colon X \times X \to [0,\infty)$ is said to be b-metric-like if for all $x,y,z \in X$ the following conditions are satisfied:

(bML_1) if $d(x,y) = 0$ then $x = y$;
(bML_2) $d(x,y) = d(y,x)$;
(bML_3) $d(x,z) \leq s[d(x,y) + d(y,z)]$.

In this case, the pair (X,d) is called a b-metric-like space (with constant s).

Example 1.4 Let $X = C([0,T])$ be the set of all real continuous functions on the closed interval $[0,T]$. Let $d : X \times X \to \mathbb{R}_0^+$ be defined

$$d(f,g) = \max\bigl(\bigl|f(t) - g(t)\bigr|\bigr)^p + a,$$

for all $f,g \in X$, $a \in \mathbb{R}_0^+$, and $p > 1$. It is easy to see that (X,d) is a complete b-metric-like space with $s = 2^{p-1}$. For more examples, see *e.g.* [11].

Remark 1.5 Let (X,d) be a b-metric-like space with constant $s \geq 1$. Then it is clear that $d^s(x,y) = |2d(x,y) - d(x,x) - d(y,y)|$ satisfies the following:
(S1) $d^s(x,x) = 0$ for all $x \in X$.

Definition 1.6 [11] Let (X,d) be a b-metric-like space. Then:
(1) a sequence $\{x_n\}$ in X is called convergent to $x \in X$ if and only if $\lim_{n\to\infty} d(x_n,x) = d(x,x)$;
(2) a sequence $\{x_n\}$ in X is called Cauchy sequence if and only if $\lim_{n,m\to\infty} d(x_n,x_m)$ exists and finite;
(3) (X,d) is complete if and only if every Cauchy sequence $\{x_n\}$ in X converges to $x \in X$ so that

$$\lim_{n\to\infty} d(x_n,x) = d(x,x) = \lim_{m,n\to\infty} d(x_n,x_m).$$

Proposition 1.7 [11] *Let (X,d) be a b-metric-like space with constant s and let $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} d(x_n,x) = 0$. Then*:
(1) *x is unique.*
(2) *$\frac{1}{s}d(x,y) \leq \lim_{n\to\infty} d(x_n,y) \leq sd(x,y)$ for all $y \in X$.*

Lemma 1.8 [11] *Let (X,d) be a b-metric-like space with constant s and $\{x_n\}$ a sequence in X such that*

$$d(x_{n+1},x_{n+2}) \leq kd(x_n,x_{n+1}), \quad n = 0,1,\ldots,$$

where $0 \leq k$ and $sk < 1$. Then $\{x_n\}$ is a Cauchy sequence in X and $\lim_{n,m\to\infty} d(x_n,x_m) = 0$.

Lemma 1.9 [12] *Let (X,d) be a b-metric-like space with constant s and assume that $\{x_n\}$ and $\{y_n\}$ are sequences in X converging to x and y, respectively. Then*

$$\begin{aligned}\frac{1}{s^2}d(x,y)-\frac{1}{s}d(x,x)-d(y,y)&\leq\liminf_{n\to\infty}d(x_n,y_n)\leq\limsup_{n\to\infty}d(x_n,y_n)\\&\leq sd(x,x)+s^2d(y,y)+s^2d(x,y).\end{aligned}$$

In particular, if $d(x,y)=0$ then $\lim_{n\to\infty}d(x_n,y_n)=0$.

Moreover, for each $z\in X$ we have

$$\frac{1}{s}d(x,z)-d(x,x)\leq\liminf_{n\to\infty}d(x_n,z)\leq\limsup_{n\to\infty}d(x_n,z)\leq sd(x,z)+sd(x,x). \tag{1}$$

In particular, if $d(x,x)=0$, then

$$\frac{1}{s}d(x,z)\leq\liminf_{n\to\infty}d(x_n,z)\leq\limsup_{n\to\infty}d(x_n,z)\leq sd(x,z).$$

Notice that, in general, a b-metric-like mapping does not need to be continuous.

The notion of α-admissible and triangular α-admissible mappings were introduced by Samet *et al.* [13] and Karapınar *et al.* [14], respectively.

Definition 1.10 Let $T:X\to X$ be a mapping and $\alpha:X\times X\to[0,\infty)$ be a function. We say that T is an α-admissible mapping if

$$x,y\in X,\quad\alpha(x,y)\geq1\quad\Rightarrow\quad\alpha(Tx,Ty)\geq1.$$

Moreover, a self-mapping T is called triangular α-admissible if T is α-admissible and

$$x,y,z\in X,\quad\alpha(x,z)\geq1\text{ and }\alpha(z,y)\geq1\quad\Rightarrow\quad\alpha(x,y)\geq1.$$

For more details on α-admissible and triangular α-admissible mappings, see *e.g.* [13–17].

Very recently, Popescu [18] refined the notion of triangular α-orbital admissible as follows.

Definition 1.11 [18] Let $T:X\to X$ be a mapping and $\alpha:X\times X\to[0,\infty)$ be a function. We say that T is α-orbital admissible if

$$\alpha(x,Tx)\geq1\quad\Rightarrow\quad\alpha\bigl(Tx,T^2x\bigr)\geq1.$$

Furthermore, T is called triangular α-orbital admissible if T is α-orbital admissible and

$$\alpha(x,y)\geq1\text{ and }\alpha(y,Ty)\geq1\quad\Rightarrow\quad\alpha(x,Ty)\geq1.$$

As mentioned in [18] each α-admissible (respectively, triangular α-admissible) mapping is an α-orbital admissible (respectively, triangular α-orbital admissible) mapping. In the following example we shall show that the converse is not true.

Example 1.12 Let $X = \{a,b,c,d,e,f,g,h\}$. We define a self-mapping $T : X \to X$ such that $Tx = x$, for $x = a, d$ and

$$Tx = y \quad \text{for } (x,y) \in \bigl\{(b,c),(c,b),(e,f),(f,e),(g,h),(h,g)\bigr\}.$$

Moreover, we define $\alpha : X \times X \to \mathbb{R}_0^+$, such that

$$\alpha(x,y) = \begin{cases} 1 & \text{if } (x,y) \in \{(a,b),(a,c),(b,b),(c,c),(b,c),(c,b),(b,d),(c,d),(d,e)\}, \\ 0 & \text{otherwise.} \end{cases}$$

Note that T is α-orbital admissible, since $\alpha(b,Tb) = \alpha(b,c) = 1$ and $\alpha(c,Tc) = \alpha(c,b) = 1$. On the other hand, we have $\alpha(d,e) = 1$, but $\alpha(Td,Te) = \alpha(d,f) = 0$. Hence, T is not α-admissible.

Lemma 1.13 [18] *Let $T : X \to X$ be a triangular α-orbital admissible mapping. Assume that there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for each $n \in \mathbb{N}_0$. Then we have $\alpha(x_n,x_m) \geq 1$ for all $m,n \in \mathbb{N}$ with $n < m$.*

Lemma 1.14 *Let $T : X \to X$ be a triangular α-orbital admissible mapping. Assume that there exists $x_0 \in X$ such that $\alpha(Tx_0,x_0) \geq 1$. Define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for each $n \in \mathbb{N}_0$. Then we have $\alpha(x_m,x_n) \geq 1$ for all $m,n \in \mathbb{N}$ with $n < m$.*

We characterize the notion of α-regular in the setting of a b-metric-like space.

Definition 1.15 (*cf.* [18]) Let (X,d) be a b-metric-like space, X is said to be α-regular, if for every sequence $\{x_n\}$ in X such that $\alpha(x_n,x_{n+1}) \geq 1$ (respectively, $\alpha(x_{n+1},x_n) \geq 1$) for all n and $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k},x) \geq 1$ (respectively, $\alpha(x,x_{n_k}) \geq 1$) for all k.

In this paper, we shall prove the existence and uniqueness of a fixed point for certain operators in the setting of b-metric-like spaces. The presented results improve, extend, and unify a number of existing results in the literature.

2 Main result for *b*-metric-like spaces

In this section, we shall state and prove our main results. First, we recall the following classes of auxiliary functions. Let Ψ be the set of all increasing and continuous functions $\psi : [0,\infty) \to [0,\infty)$ with $\psi^{-1}(\{0\}) = \{0\}$. Let $\mathcal{F}_s$ be the family of all functions $\beta : [0,\infty) \to [0,\frac{1}{s})$ which satisfy the condition

$$\lim_{n\to\infty} \beta(t_n) = \frac{1}{s} \quad \text{implies} \quad \lim_{n\to\infty} t_n = 0, \tag{2}$$

for some $s \geq 1$.

Definition 2.1 Let (X,d) be a b-metric-like space with constant $s \geq 1$, and $T : X \to X$ be a map. We say that T is a generalized almost α-ψ-ϕ-Geraghty contractive type mapping if there exist a function $\alpha : X \times X \to [0,\infty)$, $\psi, \phi \in \Psi$, $\beta \in \mathcal{F}_s$, and some $L \geq 0$ such that

$$\alpha(x,y)\psi\bigl(s^2 d(Tx,Ty)\bigr) \leq \beta\bigl(\psi\bigl(M(x,y)\bigr)\bigr)\psi\bigl(M(x,y)\bigr) + L\phi\bigl(N(x,y)\bigr), \tag{3}$$

for all $x, y \in X$, where

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{4s}\right\} \quad \text{and} \tag{4}$$

$$N(x,y) = \min\left\{d^s(x,Ty), d^s(y,Tx), d(x,Tx), d(y,Ty)\right\}. \tag{5}$$

Remark 2.2 Since the functions belonging to $\mathcal{F}_s$ are strictly smaller than $\frac{1}{s}$, for some $s \geq 1$, the expression $\beta(\psi(M(x,y)))$ in (3) can be estimated from above as follows:

$$\beta\left(\psi\left(M(x,y)\right)\right) < \frac{1}{s} \quad \text{for any } x, y \in X.$$

Theorem 2.3 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ and $T : X \to X$ be a generalized almost α-ψ-ϕ-Geraghty contractive type mapping. We suppose also that*

(i) *T is triangular α-orbital admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *T is continuous.*

Then T has a fixed point, $u \in X$ with $d(u,u) = 0$.

Proof By (ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Define a sequence $\{x_n\} \subset X$ by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}_0$. As T is triangular α-orbital admissible, by Lemma 1.13 we have $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}_0$. Throughout the proof, we suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}_0$. Indeed, if there exists n_0 such that $x_{n_0} = x_{n_0+1}$, then x_{n_0} becomes the fixed point of T, which completes the proof.

Since T is a generalized almost α-ψ-ϕ-Geraghty contractive type mapping we have

$$\begin{aligned} \psi\left(s^2 d(x_{n+1}, x_{n+2})\right) &\leq \alpha(x_n, x_{n+1})\psi\left(s^2 d(Tx_n, Tx_{n+1})\right) \\ &\leq \beta\left(\psi\left(M(x_n, x_{n+1})\right)\right)\psi\left(M(x_n, x_{n+1})\right) + L\phi\left(N(x_n, x_{n+1})\right). \end{aligned} \tag{6}$$

Thus, we have

$$\psi\left(s^2 d(x_{n+1}, x_{n+2})\right) < \frac{1}{s}\psi\left(M(x_n, x_{n+1})\right) + L\phi\left(N(x_n, x_{n+1})\right), \tag{7}$$

where $N(x_n, x_{n+1}) = \min\{d^s(x_n, x_{n+2}), d^s(x_{n+1}, x_{n+1}), d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\} = 0$, and

$$M(x_n, x_{n+1}) = \max\left\{d(x_n, x_{n+1}), d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2}), \frac{d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1})}{4s}\right\}.$$

Note that

$$\begin{aligned} \frac{d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1})}{4s} &\leq \frac{s[d(x_n, x_{n+1}) + 3d(x_{n+1}, x_{n+2})]}{4s} \\ &= \frac{[d(x_n, x_{n+1}) + 3d(x_{n+1}, x_{n+2})]}{4} \\ &\leq \max\left\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\right\}. \end{aligned}$$

Consequently, we have

$$M(x_n, x_{n+1}) = \max\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\}.$$

If $M(x_n, x_{n+1}) = d(x_{n+1}, x_{n+2})$, then from (7) we have

$$\psi(s^2 d(x_{n+1}, x_{n+2})) < \frac{1}{s}\psi(d(x_{n+1}, x_{n+2})) \leq \psi(d(x_{n+1}, x_{n+2})).$$

Since ψ is increasing, we derive that $s^2 d(x_{n+1}, x_{n+2}) < d(x_{n+1}, x_{n+2})$, which is a contradiction as $s \geq 1$. Thus, $M(x_n, x_{n+1}) = d(x_n, x_{n+1})$. Again by (7), we find

$$\psi(s^2 d(x_{n+1}, x_{n+2})) < \frac{1}{s}\psi(d(x_n, x_{n+1})) \leq \psi(d(x_n, x_{n+1})).$$

Hence, we get

$$s^2 d(x_{n+1}, x_{n+2}) \leq d(x_n, x_{n+1}) \quad \text{equivalently} \quad d(x_{n+1}, x_{n+2}) \leq \frac{1}{s^2} d(x_n, x_{n+1}). \tag{8}$$

Case (i): $s > 1$. Since $\frac{1}{s^2} > 0$ and $s\frac{1}{s^2} = \frac{1}{s} < 1$, by Lemma 1.8, the sequence $\{x_n\}$ is Cauchy and

$$\lim_{n,m\to\infty} d(x_n, x_m) = 0. \tag{9}$$

Case (ii): $s = 1$. From (8), we have $d(x_{n+1}, x_{n+2}) \leq d(x_n, x_{n+1})$ for all n. Thus, we conclude that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = r, \tag{10}$$

for some $r \geq 0$. We shall prove that $r = 0$. Suppose, on the contrary, that $r > 0$. Note that, for $s = 1$, the inequality (6) turns into

$$\psi(d(x_{n+1}, x_{n+2})) < \beta(\psi(M(x_n, x_{n+1})))\psi(M(x_n, x_{n+1})) + L\phi(N(x_n, x_{n+1})), \tag{11}$$

where $N(x_n, x_{n+1}) = 0$ and $M(x_n, x_{n+1}) = d(x_n, x_{n+1})$ as evaluated above. Thus, (11) yields

$$\frac{\psi(d(x_{n+1}, x_{n+2}))}{\psi(d(x_n, x_{n+1}))} \leq \beta(\psi(d(x_n, x_{n+1}))) < 1. \tag{12}$$

By taking the limit as $n \to \infty$ in (12) and regarding the continuity of ψ, we get

$$\lim_{n\to\infty} \beta(\psi(d(x_n, x_{n+1}))) = 1.$$

Hence, we have

$$\lim_{n\to\infty} \psi(d(x_n, x_{n+1})) = 0 \quad \text{and so} \quad \lim_{n\to\infty} d(x_n, x_{n+1}) = 0.$$

Consequently $r = 0$. In what follows, we shall prove that $\{x_n\}$ is a Cauchy sequence. Indeed we will prove that $\lim_{m,n\to\infty} d(x_n, x_m) = 0$. Suppose, on the contrary, that there exist $\varepsilon > 0$ and corresponding subsequences $\{n_k\}$ and $\{m_k\}$ of $\mathbb{N}$ satisfying $n_k > m_k > k$ for which

$$d(x_{m_k}, x_{n_k}) \geq \varepsilon, \tag{13}$$

where n_k, m_k are chosen as the smallest integers satisfying (13), that is,

$$d(x_{m_k}, x_{n_k-1}) < \varepsilon. \tag{14}$$

By (13), (14), and the triangle inequality, we easily derive that

$$\varepsilon \leq d(x_{m_k}, x_{n_k}) \leq d(x_{m_k}, x_{n_k-1}) + d(x_{n_k-1}, x_{n_k}) < \varepsilon + d(x_{n_k-1}, x_{n_k}). \tag{15}$$

Using (15) and the squeeze theorem we get

$$\lim_{k\to\infty} d(x_{m_k}, x_{n_k}) = \varepsilon. \tag{16}$$

In a similar way, we can prove that $\lim_{k\to\infty} d(x_{m_k}, x_{n_k+1}) = 0$, $\lim_{k\to\infty} d(x_{n_k}, x_{m_k+1}) = 0$.

Regarding that T is a generalized almost α-ψ-ϕ-Geraghty contractive type mapping, we have

$$\begin{aligned} \psi\bigl(d(x_{m_k+1}, x_{n_k+1})\bigr) &\leq \alpha(x_{m_k}, x_{n_k})\psi\bigl(d(Tx_{m_k}, Tx_{n_k})\bigr) \\ &\leq \beta\bigl(\psi\bigl(M(x_{m_k}, x_{n_k})\bigr)\bigr)\psi\bigl(M(x_{m_k}, x_{n_k})\bigr) + L\phi\bigl(N(x_{m_k}, x_{n_k})\bigr), \end{aligned} \tag{17}$$

for all $x, y \in X$, where

$$\begin{aligned} M(x_{m_k}, x_{n_k}) = \max\biggl\{ & d(x_{m_k}, x_{n_k}), d(x_{m_k}, x_{m_k+1}), d(x_{n_k}, x_{n_k+1}), \\ & \frac{d(x_{m_k}, x_{n_k+1}) + d(x_{n_k}, x_{m_k+1})}{4} \biggr\}, \end{aligned} \tag{18}$$

and

$$N(x_{m_k}, x_{n_k}) = \min\bigl\{d^s(x_{m_k}, x_{n_k+1}), d^s(x_{n_k}, x_{m_k+1}), d(x_{m_k}, x_{m_k+1}), d(x_{n_k}, x_{n_k+1})\bigr\}.$$

By taking the limit as $k \to \infty$ in (17) and taking (18), (10) into account, we get

$$\psi(\varepsilon) \leq \lim_{k\to\infty} \beta\bigl(\psi\bigl(M(x_{m_k}, x_{n_k})\bigr)\bigr)\psi(\varepsilon). \tag{19}$$

Since β is a Geraghty function, we derive that $\psi(M(x_{m_k}, x_{n_k})) \to 0$. Consequently, we have $d(x_{m_k}, x_{n_k}) \to 0$, which is a contradiction. Hence, we conclude that $\lim_{m,n\to\infty} d(x_n, x_m) = 0$, and the sequence $\{x_n\}$ is Cauchy for any $s \geq 1$.

By completeness of (X, d), there exists $u \in X$ such that

$$\lim_{n\to\infty} d(x_n, u) = \lim_{n,m\to\infty} d(x_n, x_m) = d(u, u) = 0.$$

Since T is continuous,

$$Tu = T\Big(\lim_{n\to\infty} x_n\Big) = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} x_{n+1} = u,$$

and u is a fixed point for T. □

In what follows, we replace the condition of continuity of the operator by the condition of α-regularity of the space.

Theorem 2.4 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ and $T : X \to X$ be a generalized almost α-ψ-ϕ-Geraghty contractive type mapping. We suppose also that*:

(i) *T is triangular α-orbital admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *X is α-regular and d is continuous.*

Then T has a fixed point, $u \in X$ with $d(u,u) = 0$.

Proof Following the lines of the proof of Theorem 2.3, we conclude that there exists $u \in X$ such that

$$\lim_{n\to\infty} d(x_n,u) = \lim_{n,m\to\infty} d(x_n,x_m) = d(u,u) = 0.$$

Since X is α-regular, $\alpha(x_n,x_{n+1}) \geq 1$ for all n. Due to the fact that $\lim_{n\to\infty} x_n = u$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k},u) \geq 1$ for all k. To prove that u is a fixed point for T, suppose on the contrary that $d(u,Tu) > 0$.

Now, by using the properties of ψ and as T is a generalized almost α-ψ-ϕ-Geraghty contractive type mapping we have

$$\begin{aligned}\psi\big(d(x_{n_k+1},Tu)\big) &\leq \alpha(x_{n_k},u)\psi\big(s^2 d(Tx_{n_k},Tu)\big)\\ &\leq \beta\big(\psi\big(M(x_{n_k},u)\big)\big)\psi\big(M(x_{n_k},u)\big) + L\phi\big(N(x_{n_k},u)\big).\end{aligned}$$

Thus we have

$$\psi\big(d(x_{n_{k+1}},Tu)\big) < \frac{1}{s}\psi\big(M(x_{n_k},u)\big) + L\phi\big(N(x_{n_k},u)\big), \tag{20}$$

where $N(x_{n_k},u) = \min\{d^s(x_{n_k},Tu), d^s(u,x_{n_k+1}), d(x_{n_k},x_{n_k+1}), d(u,Tu)\}$, and note that $\lim_{k\to\infty} N(x_{n_k},u) = 0$. Moreover,

$$\begin{aligned}M(x_{n_k},u) &= \max\left\{d(x_{n_k},u), d(x_{n_k},x_{n_k+1}), d(u,Tu), \frac{d(x_{n_k},Tu) + d(u,x_{n_k+1})}{4s}\right\}\\ &\leq \max\Bigg\{d(x_{n_k},u), d(x_{n_k},x_{n_k+1}), d(u,Tu),\\ &\qquad \frac{d(x_{n_k},u) + d(u,Tu) + d(u,x_{n_k}) + d(x_{n_k},x_{n_k+1})}{4}\Bigg\}.\end{aligned}$$

Hence

$$\lim_{k\to\infty} M(x_{n_k},u) \le \max\left\{0, d(u,Tu), \frac{d(u,Tu)}{4}\right\} = d(u,Tu),$$

and by the definition of $M(x_{n_k},u)$ we have $\lim_{k\to\infty} M(x_{n_k},u) = d(u,Tu)$.

By the continuity of ψ and the b-metric-like d, taking the limit as k goes to ∞ on both sides of (20) we have

$$\psi\bigl(d(u,Tu)\bigr) \le \frac{1}{s}\psi\bigl(d(u,Tu)\bigr).$$

Thus $1 = \frac{\psi(d(u,Tu))}{\psi(d(u,Tu))} \le \frac{1}{s}$, which is a contradiction in the case $s > 1$. Hence $d(u,Tu) = 0$; therefore $Tu = u$. In the case $s = 1$ we take the limit as k goes to ∞ on both sides of

$$\psi\bigl(d(x_{n_k+1},Tu)\bigr) \le \beta\bigl(\psi\bigl(M(x_{n_k},u)\bigr)\bigr)\psi\bigl(M(x_{n_k},u)\bigr)$$

and get $\lim_{k\to\infty} \beta(\psi(M(x_{n_k},u))) = 1$ and as $\beta \in \mathcal{F}_1$ so we have $\lim_{k\to\infty} \psi(M(x_{n_k},u)) = 0$. Thus we have $d(u,Tu) = 0$; therefore $Tu = u$. □

For the uniqueness of a fixed point of a generalized α-ψ-ϕ contractive mapping, we will consider the following hypothesis.

(H) For all $x,y \in \operatorname{Fix}(T)$, either $\alpha(x,y) \ge 1$ or $\alpha(y,x) \ge 1$.

Here, $\operatorname{Fix}(T)$ denotes the set of fixed points of T.

Theorem 2.5 *Adding condition* (H) *to the hypotheses of Theorem* 2.3 (*or Theorem* 2.4), *we obtain the uniqueness of the fixed point of* T.

Proof Suppose that x^* and y^* are two fixed points of T. Then it is obvious that $M(x^*,y^*) = d(x^*,y^*)$ and $N(x^*,y^*) = 0$. So, we have

$$\begin{aligned}\psi\bigl(d(x^*,y^*)\bigr) &\le \psi\bigl(s^2 d(Tx^*,Ty^*)\bigr)\\ &\le \alpha(x^*,y^*)\psi\bigl(s^2 d(Tx^*,Ty^*)\bigr)\\ &\le \beta\bigl(\psi\bigl(d(x^*,y^*)\bigr)\bigr)\psi\bigl(d(x^*,y^*)\bigr)\\ &< \frac{1}{s}\psi\bigl(d(x^*,y^*)\bigr),\end{aligned}$$

which is a contradiction. □

Definition 2.6 Let (X,d) be a b-metric-like space with constant $s \ge 1$, $T: X \to X$ be a map, we say that T is a generalized rational α-ψ-ϕ-Geraghty contractive mapping of type (I) if there exist a function $\alpha: X\times X \to [0,\infty)$, $\psi,\phi \in \Psi$, $\beta \in \mathcal{F}_s$, and some $L \ge 0$ such that

$$\alpha(x,y)\psi\bigl(s^2 d(Tx,Ty)\bigr) \le \beta\bigl(\psi\bigl(K(x,y)\bigr)\bigr)\psi\bigl(K(x,y)\bigr) + L\phi\bigl(N(x,y)\bigr), \tag{21}$$

for all $x,y \in X$, where $N(x,y)$ is defined as in (5) and

$$K(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{1+d(x,y)}, \frac{d(x,Tx)d(y,Ty)}{1+d(Tx,Ty)}\right\}. \tag{22}$$

Definition 2.7 Let (X,d) be a b-metric-like space with constant $s \geq 1$, $T : X \to X$ be a map, we say that T is a generalized rational α-ψ-ϕ-Geraghty contractive of type (II) mapping if there exist a function $\alpha : X \times X \to [0,\infty)$, $\psi, \phi \in \Psi$, $\beta \in \mathcal{F}_s$ and some $L \geq 0$ such that

$$\alpha(x,y)\psi\bigl(s^2 d(Tx,Ty)\bigr) \leq \beta\bigl(\psi\bigl(Q(x,y)\bigr)\bigr)\psi\bigl(Q(x,y)\bigr) + L\phi\bigl(N(x,y)\bigr), \tag{23}$$

for all $x, y \in X$, where $N(x,y)$ is defined as in (5) and

$$Q(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty) + d^s(x,Ty)d^s(y,Tx)}{1 + s(d(x,Tx) + d(y,Ty))}\right\}. \tag{24}$$

Theorem 2.8 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ and $T : X \to X$ be a generalized rational α-ψ-ϕ-Geraghty contractive mapping of type* (I) *such that*:

(i) *T is triangular α-orbital admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;
(iii) *T is continuous.*

Then T has a fixed point, $u \in X$ with $d(u,u) = 0$.

Proof We shall use the same techniques as in the proof of Theorem 2.3. First of all, we shall construct a sequence $\{x_n\} \subset X$ where $x_{n+1} = Tx_n$ for which $\alpha(x_n, x_{n+1}) \geq 1$ and $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}_0$.

Since T is generalized rational α-ψ-ϕ-Geraghty contractive of type (I) we have

$$\begin{aligned}\psi\bigl(s^2 d(x_{n+1},x_{n+2})\bigr) &\leq \alpha(x_n,x_{n+1})\psi\bigl(s^2 d(Tx_n,Tx_{n+1})\bigr) \\ &\leq \beta\bigl(\psi\bigl(K(x_n,x_{n+1})\bigr)\bigr)\psi\bigl(K(x_n,x_{n+1})\bigr) + L\phi\bigl(N(x_n,x_{n+1})\bigr).\end{aligned} \tag{25}$$

Since $N(x_n,x_{n+1}) = 0$, the above inequality implies that

$$\psi\bigl(s^2 d(x_{n+1},x_{n+2})\bigr) \leq \beta\bigl(\psi\bigl(K(x_n,x_{n+1})\bigr)\bigr)\psi\bigl(K(x_n,x_{n+1})\bigr), \tag{26}$$

where

$$K(x_n,x_{n+1}) = \max\left\{d(x_n,x_{n+1}), \frac{d(x_n,x_{n+1})d(x_{n+1},x_{n+2})}{1 + d(x_n,x_{n+1})}, \frac{d(x_n,x_{n+1})d(x_{n+1},x_{n+2})}{1 + d(x_{n+1},x_{n+2})}\right\}.$$

On the other hand, we have

$$\frac{d(x_n,x_{n+1})d(x_{n+1},x_{n+2})}{1 + d(x_{n+1},x_{n+2})} \leq \frac{d(x_n,x_{n+1})d(x_{n+1},x_{n+2})}{d(x_{n+1},x_{n+2})} = d(x_n,x_{n+1})$$

and

$$\frac{d(x_n,x_{n+1})d(x_{n+1},x_{n+2})}{1 + d(x_n,x_{n+1})} \leq \frac{d(x_n,x_{n+1})d(x_{n+1},x_{n+2})}{d(x_n,x_{n+1})} = d(x_{n+1},x_{n+2}).$$

Consequently, we get $K(x_n,x_{n+1}) \leq \max\{d(x_n,x_{n+1}), d(x_{n+1},x_{n+2})\}$.

If $\max\{d(x_n,x_{n+1}), d(x_{n+1},x_{n+2})\} = d(x_{n+1},x_{n+2})$, then from (26) together with Remark 2.2, we have

$$\psi\bigl(s^2 d(x_{n+1},x_{n+2})\bigr) < \frac{1}{s}\psi\bigl(d(x_{n+1},x_{n+2})\bigr).$$

This is a contradiction since ψ is increasing. Thus, we have $\max\{d(x_n,x_{n+1}),d(x_{n+1},x_{n+2})\} = d(x_n,x_{n+1})$ and by the definition of $K(x_n,x_{n+1})$ we shall have $K(x_n,x_{n+1}) = d(x_n,x_{n+1})$. Consequently, the inequality (26) turns into

$$\psi\bigl(d(x_{n+1},x_{n+2})\bigr) \le \psi\bigl(s^2 d(x_{n+1},x_{n+2})\bigr) \le \beta\bigl(\psi\bigl(K(x_n,x_{n+1})\bigr)\bigr)\psi\bigl(d(x_n,x_{n+1})\bigr). \tag{27}$$

By Remark 2.2, we get

$$\psi\bigl(s^2 d(x_{n+1},x_{n+2})\bigr) < \psi\bigl(d(x_n,x_{n+1})\bigr) \text{ and hence, } d(x_{n+1},x_{n+2}) < \frac{1}{s^2}d(x_n,x_{n+1}). \tag{28}$$

Case (i): $s > 1$. Since $\frac{1}{s^2} > 0$ and $s\frac{1}{s^2} = \frac{1}{s} < 1$, by Lemma 1.8, the sequence $\{x_n\}$ is Cauchy and

$$\lim_{n,m\to\infty} d(x_n,x_m) = 0. \tag{29}$$

Case (ii): $s = 1$. Since $\{d(x_n,x_{n+1})\}$ is a decreasing sequence, there exists $r \ge 0$ such that

$$\lim_{n\to\infty} d(x_n,x_{n+1}) = r, \tag{30}$$

for some $r \ge 0$. We shall prove that $r = 0$. Suppose, on the contrary, that $r > 0$. By letting $n \to \infty$ in (27) we find

$$\psi(r) \le \lim_{n\to\infty} \beta\bigl(\psi\bigl(K(x_n,x_{n+1})\bigr)\bigr)\psi(r).$$

It yields $1 = \lim_{n\to\infty} \beta(\psi(K(x_n,x_{n+1})))$. Since $\beta \in \mathcal{F}_1$, we get $\psi(K(x_n,x_{n+1})) \to 0$, which implies that $d(x_n,x_{n+1}) \to 0$, that is, $r = 0$.

In what follows, we shall prove that $\{x_n\}$ is a Cauchy sequence. Indeed we will prove that $\lim_{m,n\to\infty} d(x_n,x_m) = 0$. Suppose, on the contrary, that there exist $\varepsilon > 0$ and corresponding subsequences $\{n_k\}$ and $\{m_k\}$ of $\mathbb{N}$ satisfying $n_k > m_k > k$ for which

$$d(x_{m_k},x_{n_k}) \ge \varepsilon, \tag{31}$$

where n_k, m_k are chosen as the smallest integers satisfying (31), that is,

$$d(x_{m_k},x_{n_k-1}) < \varepsilon. \tag{32}$$

By (31), (32), and the triangle inequality, we easily derive that

$$\begin{aligned}\varepsilon \le d(x_{m_k},x_{n_k}) &\le d(x_{m_k},x_{n_k-1}) + d(x_{n_k-1},x_{n_k})\\ &< \varepsilon + d(x_{n_k-1},x_{n_k}).\end{aligned} \tag{33}$$

Using (33) and the squeeze theorem we get

$$\lim_{k\to\infty} d(x_{m_k},x_{n_k}) = \varepsilon. \tag{34}$$

Regarding that T is generalized rational α-ψ-ϕ-Geraghty contractive mapping of type (I), we have

$$\begin{aligned}\psi\bigl(d(x_{m_k+1},x_{n_k+1})\bigr) &\le \alpha(x_{m_k},x_{n_k})\psi\bigl(d(Tx_{m_k},Tx_{n_k})\bigr)\\ &\le \beta\bigl(\psi\bigl(K(x_{m_k},x_{n_k})\bigr)\bigr)\psi\bigl(K(x_{m_k},x_{n_k})\bigr)+L\phi\bigl(N(x_{m_k},x_{n_k})\bigr),\end{aligned} \tag{35}$$

for all $x,y\in X$, where

$$\begin{aligned}K(x_{m_k},x_{n_k}) = \max\biggl\{&d(x_{m_k},x_{n_k}),\frac{d(x_{m_k},x_{m_k+1})d(x_{n_k},x_{n_k+1})}{1+d(x_{m_k},x_{n_k})},\\ &\frac{d(x_{m_k},x_{m_k+1})d(x_{n_k},x_{n_k+1})}{1+d(x_{m_k+1},x_{n_k+1})}\biggr\}\end{aligned} \tag{36}$$

and

$$N(x_{m_k},x_{n_k}) = \min\bigl\{d^s(x_{m_k},x_{n_k+1}),d^s(x_{n_k},x_{m_k+1}),d(x_{m_k},x_{m_k+1}),d(x_{n_k},x_{n_k+1})\bigr\}.$$

It is clear that

$$\lim_{k\to\infty}K(x_{m_k},x_{n_k})=\varepsilon \quad\text{and}\quad \lim_{k\to\infty}N(x_{m_k},x_{n_k})=0.$$

By taking the limit as $k\to\infty$ in (35) and taking (36), (30) into account, we get

$$\begin{aligned}\psi(\varepsilon) &\le \lim_{k\to\infty}\beta\bigl(\psi\bigl(K(x_{m_k},x_{n_k})\bigr)\bigr)\lim_{k\to\infty}\psi\bigl(K(x_{m_k},x_{n_k})\bigr)\\ &\le \lim_{k\to\infty}\beta\bigl(\psi\bigl(K(x_{m_k},x_{n_k})\bigr)\bigr)\psi(\varepsilon).\end{aligned} \tag{37}$$

Since β is a Geraghty function, we derive that $\psi(K(x_{m_k},x_{n_k}))\to 0$. Consequently, we have $d(x_{m_k},x_{n_k})\to 0$, which is a contradiction. Hence $\lim_{m,n\to\infty}d(x_n,x_m)=0$, and the sequence $\{x_n\}$ is Cauchy for any $s\ge 1$.

By completeness of (X,d), there exists $u\in X$ such that

$$\lim_{n\to\infty}d(x_n,u)=d(u,u)=\lim_{n,m\to\infty}d(x_n,x_m)=0.$$

Now, if T is continuous, then

$$Tu=T\Bigl(\lim_{n\to\infty}x_n\Bigr)=\lim_{n\to\infty}Tx_n=\lim_{n\to\infty}x_{n+1}=u,$$

and u is a fixed point for T. □

Theorem 2.9 *Let (X,d) be a complete b-metric-like space with constant $s\ge 1$ and $T:X\to X$ be a generalized rational α-ψ-ϕ-Geraghty contractive of mapping type* (I) *such that*:

(i) *T is triangular α-orbital admissible*;

(ii) *there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$*;

(iii) *X is α-regular and d is continuous.*

Then T has a fixed point, $u\in X$ with $d(u,u)=0$.

Proof Verbatim of the proof of Theorem 2.8, we conclude that the iterative sequence $\{x_n\}$ is Cauchy and converges to $u \in X$. Since X is α-regular, then, as in the proof of Theorem 2.4, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that

$$\psi\left(d(x_{n_k+1}, Tu)\right) < \frac{1}{s}\psi\left(K(x_{n_k}, u)\right) + L\phi\left(N(x_{n_k}, u)\right), \tag{38}$$

where $K(x_{n_k}, u) = \max\{d(x_{n_k}, u), \frac{d(x_{n_k}, x_{n_k+1})d(u,Tu)}{1+d(x_{n_k},u)}, \frac{d(x_{n_k}, x_{n_k+1})d(u,Tu)}{1+d(x_{n_k+1},Tu)}\}$.

Hence $\lim_{k\to\infty} K(x_{n_k}, u) = 0$ and as in the proof of Theorem 2.4 $\lim_{k\to\infty} N(x_{n_k}, u) = 0$.

Thus taking the limit as $k \to \infty$ on both sides of (38) and keeping in mind that ψ and d are continuous we have $\psi(d(u, Tu)) \leq 0$. Hence $d(u, Tu) = 0$; therefore $Tu = u$. □

Theorem 2.10 *Adding condition* (H) *to the hypotheses of Theorem* 2.8 (*or Theorem* 2.9), *we obtain uniqueness of the fixed point of* T.

Proof As in the proof of Theorem 2.5, we suppose that x^* and y^* are two fixed points of T. Then, clearly, we have $K(x^*, y^*) = d(x^*, y^*)$ and $N(x^*, y^*) = 0$. So, we have

$$\begin{aligned}
\psi\left(d\left(x^*, y^*\right)\right) &\leq \psi\left(s^2 d\left(Tx^*, Ty^*\right)\right) \\
&\leq \alpha\left(x^*, y^*\right)\psi\left(s^2 d\left(Tx^*, Ty^*\right)\right) \\
&\leq \beta\left(\psi\left(d\left(x^*, y^*\right)\right)\right)\psi\left(d\left(x^*, y^*\right)\right) \\
&< \frac{1}{s}\psi\left(d\left(x^*, y^*\right)\right),
\end{aligned}$$

which is a contradiction. □

Theorem 2.11 *Let* (X, d) *be a complete b-metric-like space with constant* $s \geq 1$ *and* $T : X \to X$ *be a generalized rational* α-ψ-ϕ-*Geraghty contractive mapping of type* (II) *such that*:

(i) T *is triangular* α-*orbital admissible*;

(ii) *there exists* $x_0 \in X$ *such that* $\alpha(x_0, Tx_0) \geq 1$;

(iii) T *is continuous*.

Then T *has a fixed point*, $u \in X$ *with* $d(u, u) = 0$.

Proof Verbatim of the lines in the proof of Theorem 2.3, we construct a sequence $\{x_n\} \subset X$ where $x_{n+1} = Tx_n$ for which $\alpha(x_n, x_{n+1}) \geq 1$ and $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}_0$. Moreover, by using the fact that T is a generalized rational α-ψ-ϕ-Geraghty contractive mapping of type (II) and the property of ψ we have

$$\begin{aligned}
\psi\left(s^2 d(x_{n+1}, x_{n+2})\right) &\leq \alpha(x_n, x_{n+1})\psi\left(s^2 d(Tx_n, Tx_{n+1})\right) \\
&\leq \beta\left(\psi\left(Q(x_n, x_{n+1})\right)\right)\psi\left(Q(x_n, x_{n+1})\right) + L\phi\left(N(x_n, x_{n+1})\right).
\end{aligned} \tag{39}$$

By using the same arguments as in the proof of Theorem 2.3, we derive that

$$\psi\left(s^2 d(x_{n+1}, x_{n+2})\right) < \frac{1}{s}\psi\left(Q(x_n, x_{n+1})\right), \tag{40}$$

where

$$Q(x_n,x_{n+1}) = \max\left\{d(x_n,x_{n+1}), \frac{d(x_n,x_{n+1})d(x_{n+1},x_{n+2}) + d^s(x_n,x_{n+2})d^s(x_{n+1},x_{n+1})}{1+s[d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})]}\right\}.$$

Since $d(x_n,x_{n+2}) \leq s[d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})]$, we have

$$\begin{aligned} d(x_n,x_{n+1}) &\geq \frac{d(x_n,x_{n+1})d(x_n,x_{n+2})}{1+d(x_n,x_{n+2})} \\ &\geq \frac{d(x_n,x_{n+1})d(x_n,x_{n+2})}{1+s[d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})]}. \end{aligned}$$

Hence, we get $Q(x_n,x_{n+1}) = d(x_n,x_{n+1})$.

By using (40) we get $\psi(s^2 d(x_{n+1},x_{n+2})) \leq \psi(d(x_n,x_{n+1}))$. Since ψ is increasing we have $d(x_{n+1},x_{n+2}) < \frac{1}{s^2}d(x_n,x_{n+1})$. If $s > 1$ then, as in the proof of Theorem 2.3, by using Lemma 1.8, we conclude that $\{x_n\}$ is a Cauchy sequence and $\lim_{n,m\to\infty} d(x_n,x_m) = 0$. If $s = 1$, by verbatim of the proof of Theorem 2.3, we deduce that $\{x_n\}$ is a Cauchy sequence.

Since (X,d) is complete, there exists $u \in X$ such that $0 = \lim_{n,m\to\infty} d(x_n,x_m) = \lim_{n\to\infty} d(x_n,u) = d(u,u)$. Now, since T is continuous, $Tu = T(\lim_{n\to\infty} x_n) = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} x_{n+1} = u$ and u is a fixed point for T. □

Theorem 2.12 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ and $T: X \to X$ be a generalized rational α-ψ-ϕ-Geraghty contractive mapping of type* (II) *such that*:

(i) *T is triangular α-orbital admissible*;
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$*;
(iii) *X is α-regular and d is continuous.*

Then T has a fixed point, $u \in X$ with $d(u,u) = 0$.

Proof By following the proof of Theorem 2.11 line by line, we see that $\{x_n\}$ converges to $u \in X$. Due to the fact that X is α-regular and by following the lines of the proof of Theorem 2.4 there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that

$$\psi\big(d(x_{n_k+1},Tu)\big) < \frac{1}{s}\psi\big(Q(x_{n_k},u)\big) + L\phi\big(N(x_{n_k},u)\big), \tag{41}$$

where

$$Q(x_{n_k},u) = \max\left\{d(x_{n_k},u), \frac{d(x_{n_k},x_{n_k+1})d(u,Tu) + d^s(x_{n_k},Tu)d^s(u,x_{n_k+1})}{1+s[d(x_{n_k},x_{n_k+1})+d(u,Tu)]}\right\}.$$

Note that $\lim_{k\to\infty} Q(x_{n_k},u) = 0$ and $\lim_{k\to\infty} N(x_{n_k},u) = 0$. Thus taking the limit as $n \to \infty$ on both sides of (41) and keeping in mind that ψ and d are continuous we have $\psi(d(u,Tu)) = 0$, and so $d(u,Tu) = 0$. Thus $u = Tu$. □

Theorem 2.13 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ and $T: X \to X$ be a mapping. Suppose that there exist a function $\alpha: X \times X \to [0,\infty)$, $\psi \in \Psi$, $\beta \in \mathcal{F}_s$ such that*

$$\alpha(x,y)\psi\big(s^2 d(Tx,Ty)\big) \leq \beta\big(\psi\big(d(x,y)\big)\big)\psi\big(d(x,y)\big), \tag{42}$$

for all $x, y \in X$. Suppose also that:
(i) *T is triangular α-orbital admissible;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
(iii) *T is continuous.*
Then T has a fixed point, $u \in X$ with $d(u, u) = 0$.

Proof By (ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Define a sequence $\{x_n\} \subset X$ by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}_0$. As T is triangular α-orbital admissible, by Lemma 1.13 we have $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}_0$. Notice that if there exists a natural number n_0 such that $x_{n_0} = x_{n_0+1}$, then the proof is complete. To avoid this trivial case, from now on, we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}_0$.

Since T satisfies (42) we have

$$\begin{aligned}\psi\bigl(s^2 d(x_{n+1}, x_{n+2})\bigr) &\leq \alpha(x_n, x_{n+1})\psi\bigl(s^2 d(Tx_n, Tx_{n+1})\bigr)\\ &\leq \beta\bigl(\psi\bigl(d(x_n, x_{n+1})\bigr)\bigr)\psi\bigl(d(x_n, x_{n+1})\bigr).\end{aligned}$$

Thus, we have

$$\psi\bigl(s^2 d(x_{n+1}, x_{n+2})\bigr) < \frac{1}{s}\psi\bigl(d(x_n, x_{n+1})\bigr) < \psi\bigl(d(x_n, x_{n+1})\bigr). \tag{43}$$

Since ψ is increasing, we have $d(x_{n+1}, x_{n+2}) < \frac{1}{s^2} d(x_n, x_{n+1})$.

Case (i): If $s > 1$, then, since $\frac{1}{s^2} > 0$ and $s\frac{1}{s^2} = \frac{1}{s} < 1$, by Lemma 1.8, $\{x_n\}$ is a Cauchy sequence and $\lim_{n,m\to\infty} d(x_n, x_m) = 0$.

Case (ii): If $s = 1$, then as $d(x_{n+1}, x_{n+2}) < d(x_n, x_{n+1})$, there exists $r \geq 0$ such that $\lim_{n\to\infty} d(x_n, x_{n+1}) = r$. If $r > 0$ then taking the limit as $n \to \infty$ on both sides of

$$\psi\bigl(d(x_{n+1}, x_{n+2})\bigr) \leq \beta\bigl(\psi\bigl(d(x_n, x_{n+1})\bigr)\bigr)\psi\bigl(d(x_n, x_{n+1})\bigr)$$

we get $\lim_{n\to\infty} \beta(\psi(d(x_n, x_{n+1}))) = 1$ and as β is a Geraghty function, we derive that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{44}$$

That is, $r = 0$. In what follows we shall prove that $\{x_n\}$ is a Cauchy sequence. Indeed we will prove that $\lim_{m,n\to\infty} d(x_n, x_m) = 0$. Suppose, on the contrary, that there exist $\varepsilon > 0$ and corresponding subsequences $\{n_k\}$ and $\{m_k\}$ of $\mathbb{N}$ satisfying $n_k > m_k > k$ for which

$$d(x_{m_k}, x_{n_k}) \geq \varepsilon, \tag{45}$$

where n_k, m_k are chosen as the smallest integers satisfying (45), that is,

$$d(x_{m_k}, x_{n_k-1}) < \varepsilon. \tag{46}$$

By (45), (46), and the triangle inequality, we easily derive that

$$\varepsilon \leq d(x_{m_k}, x_{n_k}) \leq d(x_{m_k}, x_{n_k-1}) + d(x_{n_k-1}, x_{n_k}) < \varepsilon + d(x_{n_k-1}, x_{n_k}). \tag{47}$$

Using (47) and the squeeze theorem we get

$$\lim_{k\to\infty} d(x_{m_k},x_{n_k}) = \varepsilon. \tag{48}$$

As T satisfies (42), we have

$$\begin{aligned}\psi\bigl(d(x_{m_k+1},x_{n_k+1})\bigr) &\le \alpha(x_{m_k},x_{n_k})\psi\bigl(d(Tx_{m_k},Tx_{n_k})\bigr)\\ &\le \beta\bigl(\psi\bigl(d(x_{m_k},x_{n_k})\bigr)\bigr)\psi\bigl(d(x_{m_k},x_{n_k})\bigr).\end{aligned} \tag{49}$$

Taking the limit as $k\to\infty$ for (49) we get $\lim_{k\to\infty}\beta(\psi(d(x_{m_k},x_{n_k}))) = 1$. Thus

$$\lim_{k\to\infty} d(x_{m_k},x_{n_k}) = 0.$$

Therefore, $\{x_n\}$ is a Cauchy sequence for $s\ge 1$. By completeness of (X,d), there exists $u\in X$ such that $0 = \lim_{n,m\to\infty} d(x_n,x_m) = \lim_{n\to\infty} d(x_n,u) = d(u,u)$.

Since T is continuous, $Tu = T(\lim_{n\to\infty} x_n) = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} x_{n+1} = u$ and u is a fixed point for T. □

Theorem 2.14 *Let (X,d) be a complete b-metric-like space with constant $s\ge 1$ and $T: X\to X$ be a mapping. Suppose that there exist a function $\alpha: X\times X\to[0,\infty)$, $\psi\in\Psi$, and $\beta\in\mathcal{F}_s$ such that*

$$\alpha(x,y)\psi\bigl(s^2 d(Tx,Ty)\bigr) \le \beta\bigl(\psi\bigl(d(x,y)\bigr)\bigr)\psi\bigl(d(x,y)\bigr), \tag{50}$$

for all $x,y\in X$. Suppose also that:

(i) *T is triangular α-orbital admissible;*
(ii) *there exists $x_0\in X$ such that $\alpha(x_0,Tx_0)\ge 1$;*
(iii) *X is α-regular and d is continuous.*

Then T has a fixed point, $u\in X$ with $d(u,u)=0$.

Theorem 2.15 *Adding condition* (H) *to the hypotheses of Theorem* 2.13 (*or Theorem* 2.14), *we obtain the uniqueness of the fixed point of T.*

Remark 2.16 Notice that we get several corollaries by replacing the auxiliary functions ψ and β in a proper way. In particular, by taking $\psi(t)=t$ we find the extended version of several existing results.

3 Expected consequences

In this section, we shall consider some immediate consequences of our main results.

The following result is obtained by letting $L=0$ in Theorem 2.3 or 2.4.

Corollary 3.1 *Let (X,d) be a complete b-metric-like space with constant $s\ge 1$ and $T: X\to X$ be a mapping. Suppose that there exist $\alpha: X\times X\to[0,\infty)$, $\psi\in\Psi$, $\beta\in\mathcal{F}_s$ such that*

$$\alpha(x,y)\psi\bigl(s^2 d(Tx,Ty)\bigr) \le \beta\bigl(\psi\bigl(M(x,y)\bigr)\bigr)\psi\bigl(M(x,y)\bigr), \tag{51}$$

for all $x, y \in X$, *where*

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{4s}\right\}. \tag{52}$$

Suppose also that:

(i) *T is triangular α-orbital admissible*;

(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;

(iii) *T is continuous or* (iii)′ *X is α-regular and d is continuous*.

Then T has a fixed point.

Adding condition (H) to the hypothesis of Corollary 3.1, we guarantee the uniqueness of the fixed point.

Again by letting $L = 0$ in Theorem 2.5 and Theorem 2.10 we get two more corollaries as Corollary 3.1. We skip the details regarding the volume of the paper.

Corollary 3.2 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$, $T : X \to X$ be a map and $\alpha : X \times X \to [0,\infty)$ be a function. Suppose that T satisfies at least one of the following conditions*:

(a) $\alpha(x,y)d(Tx,Ty) \leq \frac{1}{2s^3}M(x,y)$;

(b) $\alpha(x,y)d(Tx,Ty) \leq \frac{1}{2s^3}K(x,y)$;

where $M(x,y)$, $K(x,y)$ are defined as in (4), (22). *Suppose also that*:

(i) *T is triangular α-orbital admissible*;

(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;

(iii) *T is continuous or* (iii)′ *X is α-regular and d is continuous*.

Then T has a unique fixed point $u \in X$ with $d(u,u) = 0$.

Proof It is sufficient to take $L = 0$, $\psi(t) = t$, and $\beta(t) = \frac{1}{2s}$ in Theorem 2.5 and Theorem 2.10 (and thus, Theorem 2.3 or Theorem 2.4, Theorem 2.8 or Theorem 2.9, respectively). □

Adding condition (H) to the hypothesis of Corollary 3.2, we guarantee the uniqueness of the fixed point.

Corollary 3.3 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$, $T : X \to X$ be a map, and $\alpha : X \times X \to [0,\infty)$ be a function. Suppose that T satisfies at least one of the following conditions*:

(c) $\alpha(x,y)d(Tx,Ty) \leq \frac{1}{2s^3}Q(x,y)$,

where $M(x,y)$, $K(x,y)$, $Q(x,y)$ are defined as in (24). *Suppose also that*:

(i) *T is triangular α-orbital admissible*;

(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$*;

(iii) *T is continuous or* (iii)′ *X is α-regular and d is continuous*.

Then T has a fixed point $u \in X$ with $d(u,u) = 0$.

Proof It is sufficient to take $L = 0$, $\psi(t) = t$, and $\beta(t) = \frac{1}{2s}$ in Theorem 2.11 or Theorem 2.12, respectively. □

3.1 For standard *b*-metric-like spaces

If we set $\alpha(x,y) = 1$ for all $x, y \in X$ in Theorem 2.5, then we derive the following results.

Corollary 3.4 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ such that d is continuous and $T : X \to X$ be a mapping. Suppose that there exist $\psi, \phi \in \Psi$, $\beta \in \mathcal{F}_s$, and some $L \geq 0$ such that*

$$\psi\bigl(s^2 d(Tx,Ty)\bigr) \leq \beta\bigl(\psi\bigl(M(x,y)\bigr)\bigr)\psi\bigl(M(x,y)\bigr) + L\phi\bigl(N(x,y)\bigr), \tag{53}$$

for all $x, y \in X$, where

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{4s}\right\} \quad \text{and} \tag{54}$$

$$N(x,y) = \min\bigl\{d^s(x,Ty), d^s(y,Tx), d(x,Tx), d(y,Ty)\bigr\}. \tag{55}$$

Then T has a unique fixed point $u \in X$ with $d(u,u) = 0$.

If we set $\alpha(x,y) = 1$ for all $x, y \in X$ in Theorem 2.10, then we derive the following results.

Corollary 3.5 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ such that d is continuous and $T : X \to X$ be a mapping. Suppose that there exist $\psi, \phi \in \Psi$, $\beta \in \mathcal{F}_s$, and some $L \geq 0$ such that*

$$\psi\bigl(s^2 d(Tx,Ty)\bigr) \leq \beta\bigl(\psi\bigl(K(x,y)\bigr)\bigr)\psi\bigl(K(x,y)\bigr) + L\phi\bigl(N(x,y)\bigr), \tag{56}$$

for all $x, y \in X$, where

$$K(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{1+d(x,y)}, \frac{d(x,Tx)d(y,Ty)}{1+d(Tx,Ty)}\right\} \quad \text{and} \tag{57}$$

$$N(x,y) = \min\bigl\{d^s(x,Ty), d^s(y,Tx), d(x,Tx), d(y,Ty)\bigr\}. \tag{58}$$

Then T has a unique fixed point $u \in X$ with $d(u,u) = 0$.

If we set $\alpha(x,y) = 1$ for all $x, y \in X$ in Theorem 2.11, then we derive the following results.

Corollary 3.6 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ such that d is continuous and $T : X \to X$ be a mapping. Suppose that there exist $\psi, \phi \in \Psi$, $\beta \in \mathcal{F}_s$, and some $L \geq 0$ such that*

$$\psi\bigl(s^2 d(Tx,Ty)\bigr) \leq \beta\bigl(\psi\bigl(Q(x,y)\bigr)\bigr)\psi\bigl(Q(x,y)\bigr) + L\phi\bigl(N(x,y)\bigr), \tag{59}$$

for all $x, y \in X$, where

$$Q(x,y) = \max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty)+d^s(x,Ty)d^s(y,Tx)}{1+s(d(x,Tx)+d(y,Ty))}\right\} \quad \text{and} \tag{60}$$

$$N(x,y) = \min\bigl\{d^s(x,Ty), d^s(y,Tx), d(x,Tx), d(y,Ty)\bigr\}. \tag{61}$$

Then T has a fixed point $u \in X$ with $d(u,u) = 0$.

If take $L = 0$ in Corollaries 3.4-3.6, we get three more consequences. Regarding the volume of the paper, we skip the details.

Corollary 3.7 *Let (X,d) be a complete b-metric-like space with constant $s \geq 1$ such that d is continuous and $T : X \to X$ be mapping. Suppose that there exist $\psi \in \Psi$ and $\beta \in \mathcal{F}_s$ such that*

$$\psi\big(s^2 d(Tx,Ty)\big) \leq \beta\big(\psi\big(d(x,y)\big)\big)\psi\big(d(x,y)\big), \tag{62}$$

for all $x,y \in X$. Then T has a unique fixed point $u \in X$ with $d(u,u) = 0$.

Proof It follows from Theorem 2.15 by $\alpha(x,y) = 1$ for all $x,y \in X$. □

3.2 For *b*-metric-like spaces endowed with a partial order

In this section, from our main results, we shall derive easily various fixed point results on a *b*-metric-like space endowed with a partial order. We, first, recall some notions.

Definition 3.8 Let $(X,\preceq)$ be a partially ordered set and $T : X \to X$ be a given mapping. We say that T is nondecreasing with respect to $\preceq$ if

$$x,y \in X, \quad x \preceq y \quad \Longrightarrow \quad Tx \preceq Ty.$$

Definition 3.9 Let $(X,\preceq)$ be a partially ordered set. A sequence $\{x_n\} \subset X$ is said to be nondecreasing (respectively, nonincreasing) with respect to $\preceq$ if $x_n \preceq x_{n+1}$ (respectively, $x_{n+1} \preceq x_n$ for all n).

Definition 3.10 Let $(X,\preceq)$ be a partially ordered set and d be a b-metric-like on X. We say that $(X,\preceq,d)$ is regular if for every nondecreasing (respectively, nonincreasing) sequence $\{x_n\} \subset X$ such that $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \preceq x$ (respectively, $x_{n_k} \succeq x$) for all k.

We have the following result.

Corollary 3.11 *Let $(X,\preceq)$ be a partially ordered set (which does not contain an infinite totally unordered subset) and d be a b-metric-like on X with constant $s \geq 1$ such that (X,d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exist $\psi \in \Psi$, $\beta \in \mathcal{F}_s$ such that*

$$\psi\big(s^2 d(Tx,Ty)\big) \leq \beta\big(\psi\big(M(x,y)\big)\big)\psi\big(M(x,y)\big), \tag{63}$$

for all $x,y \in X$ with $x \succeq y$ or $y \succeq x$ where $M(x,y)$ is defined as in (4). Suppose also that the following conditions hold:

(i) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*
(ii) *T is continuous or* (ii)′ *$(X,\preceq,d)$ is regular and d is continuous.*

Then T has a fixed point $u \in X$ with $d(u,u) = 0$.

Proof Define the mapping $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} 1 & \text{if } x \preceq y \text{ or } x \succeq y, \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, T satisfies (51), that is,

$$\alpha(x,y)\psi\bigl(s^2 d(Tx,Ty)\bigr) \le \beta\bigl(\psi\bigl(M(x,y)\bigr)\bigr)\psi\bigl(M(x,y)\bigr),$$

for all $x, y \in X$. From condition (i), we have $\alpha(x_0, Tx_0) \ge 1$. Moreover, for all $x, y \in X$, from the monotone property of T, we have

$$\alpha(x,y) \ge 1 \quad \Longrightarrow \quad x \succeq y \text{ or } x \preceq y \quad \Longrightarrow \quad Tx \succeq Ty \text{ or } Tx \preceq Ty \quad \Longrightarrow \quad \alpha(Tx,Ty) \ge 1.$$

Hence, the self-mapping T is α-admissible. Similarly, we can prove that T is triangular α-admissible and so triangular α-orbital admissible. Now, if T is continuous, the existence of a fixed point follows from Corollary 3.1. Suppose now that $(X, \preceq, d)$ is regular. Let $\{x_n\}$ be a sequence in X such that $\alpha(x_n, x_{n+1}) \ge 1$ for all n and $x_n \to x \in X$ as $n \to \infty$. From the regularity hypothesis and as X does not contain an infinite totally unordered subset, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \preceq x$ or $x \preceq x_{n_k}$ for all k. This implies from the definition of α that $\alpha(x_{n_k}, x) \ge 1$ for all k. In this case, the existence of a fixed point follows again from Corollary 3.1. □

In an analogous way, we derive the following results from Theorem 2.8 and Theorem 2.11, respectively.

Corollary 3.12 *Let $(X, \preceq)$ be a partially ordered set (which does not contain an infinite totally unordered subset) and d be a b-metric-like on X with constant $s \ge 1$ such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exist $\psi \in \Psi$, $\beta \in \mathcal{F}_s$ such that*

$$\psi\bigl(s^2 d(Tx,Ty)\bigr) \le \beta\bigl(\psi\bigl(K(x,y)\bigr)\bigr)\psi\bigl(K(x,y)\bigr), \tag{64}$$

for all $x, y \in X$ with $x \succeq y$ or $y \succeq x$ where $M(x,y)$ is defined as in (22). Suppose also that the following conditions hold:

(i) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*

(ii) *T is continuous or* (ii)′ *$(X, \preceq, d)$ is regular and d is continuous.*

Then T has a fixed point $u \in X$ with $d(u,u) = 0$.

Corollary 3.13 *Let $(X, \preceq)$ be a partially ordered set (which does not contain an infinite totally unordered subset) and d be a b-metric-like on X with constant $s \ge 1$ such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exist $\psi \in \Psi$, $\beta \in \mathcal{F}_s$ such that*

$$\psi\bigl(s^2 d(Tx,Ty)\bigr) \le \beta\bigl(\psi\bigl(Q(x,y)\bigr)\bigr)\psi\bigl(Q(x,y)\bigr), \tag{65}$$

for all $x, y \in X$ with $x \succeq y$ or $y \succeq x$ where $M(x,y)$ is defined as in (24). Suppose also that the following conditions hold:

(i) *there exists* $x_0 \in X$ *such that* $x_0 \preceq Tx_0$;
(ii) T *is continuous or* (ii)′ $(X, \preceq, d)$ *is regular and* d *is continuous.*
Then T *has a fixed point* $u \in X$ *with* $d(u,u) = 0$.

Corollary 3.14 *Let* $(X, \preceq)$ *be a partially ordered set* (*which does not contain an infinite totally unordered subset*) *and* d *be b-metric-like on* X *with constant* $s \geq 1$ *such that* (X,d) *is complete. Let* $T : X \to X$ *be a nondecreasing mapping with respect to* $\preceq$. *Suppose that there exist* $\psi \in \Psi$, $\beta \in \mathcal{F}_s$ *such that*

$$\psi\bigl(s^2 d(Tx,Ty)\bigr) \leq \beta\bigl(\psi\bigl(d(x,y)\bigr)\bigr)\psi\bigl(d(x,y)\bigr), \tag{66}$$

for all $x, y \in X$ *with* $x \succeq y$ *or* $y \succeq x$. *Suppose also that the following conditions hold*:
(i) *there exists* $x_0 \in X$ *such that* $x_0 \preceq Tx_0$;
(ii) T *is continuous or* (ii)′ $(X, \preceq, d)$ *is regular and* d *is continuous.*
Then T *has a fixed point* $u \in X$ *with* $d(u,u) = 0$.

Example 3.15 Let $X = [0,\infty)$ define $d : X \times X \to [0,\infty)$ by $d(x,y) = |x-y|^2$. Then (X,d) is complete b-metric (so b-metric-like) space with constant $s = 2$. Define $T : X \to X$ and $\alpha(x,y) : X \times X \to [0,\infty)$ as follows:

$$Tx = \begin{cases} \frac{x}{4} & \text{if } x \in [0,1], \\ \frac{x^2}{x+1} & \text{if } x \in (1,\infty), \end{cases}$$

and

$$\alpha(x,y) = \begin{cases} 1 & \text{if } x,y \in [0,1], \\ 0 & \text{otherwise.} \end{cases}$$

It is clear that T is triangular α-orbital admissible and we have $\alpha(0,T0) \geq 1$. Moreover, X is α-regular and d is continuous.

Let $\psi(t) = t$, $\beta(t) = \frac{1}{4}$ then clearly $\psi \in \Psi$ and $\beta \in \mathcal{F}_2$. Moreover, T satisfies (50) for the following reason: if $x, y \in [0,1]$, then

$$\alpha(x,y)\psi\bigl(2^2 d(Tx,Ty)\bigr) = 2^2\left(\frac{x}{4} - \frac{y}{4}\right)^2 = \frac{(x-y)^2}{4} = \beta\bigl(\psi\bigl(d(x,y)\bigr)\bigr)\psi(d(x,y).$$

Otherwise,

$$\alpha(x,y)\psi\bigl(2^2 d(Tx,Ty)\bigr) = 0 \leq \beta\bigl(\psi\bigl(d(x,y)\bigr)\bigr)\psi(d(x,y).$$

Therefore, by Theorem 2.14, T has a fixed point $x = 0$.

Example 3.16 Let $X = \{0,1,2\}$ define $d : X \times X \to [0,\infty)$ by $d(x,y) = (\max\{x,y\})^{\frac{3}{2}}$. Then (X,d) is complete b-metric-like space with constant $s = 2^{\frac{3}{2}-1} = 2^{\frac{1}{2}}$ such that d is continuous. Define $T : X \to X$ by $T = \{(0,0),(1,0),(2,0)\}$.

Let $\psi(t) = t$, $\beta(t) = \frac{1}{2^{\frac{1}{2}}}e^{-t}$ or $\beta(t) = \frac{1}{2^{\frac{1}{2}}+t}$, then clearly $\psi \in \Psi$ and $\beta \in \mathcal{F}_{2^{\frac{1}{2}}}$. Note that $K(0,1) = 1$, $K(0,2) = K(1,2) = 2^{\frac{3}{2}}$, and clearly T satisfies (56) with $L = 0$ Therefore, by Corollary 3.5, T has a fixed point $x = 0$.

4 Conclusion

It is clear that we can list several more results by replacing the b-metric-like space, with some other abstract space, such as a b-metric space, a metric space, a metric-like space, a partial metric space, and so on.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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