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RESEARCH

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A novel joint multi-target detection and tracking approach based on Bayes joint decision and estimation

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Abstract

This paper proposes a novel joint decision and estimation (JDE) solution for the multi-target detection and tracking (MDT) problem. MDT aims to jointly detect the number of targets and estimate their states, which is essentially a JDE problem since detection and tracking are highly coupled. Thus, a joint solution which can utilize the coupling is preferable. However, the existing JDE approach has either poor performance or excessive design parameters without considering the MDT problem realities, i.e., the losses that different decisions may lead to. Therefore, we propose a compact conditional JDE (CCJDE)-based MDT method with less design parameters but superior performance. Specifically, we propose a CCJDE-based MDT risk which unifies the detection and tracking risks in a compact way. Then, we derive the joint detection and tracking solution accounting for their couplings, where the joint probabilistic data association filter is adopted due to its advantageous performance and the adaptability to the JDE framework. Then, an efficient CCJDE-MDT algorithm is developed. Besides, some parameter designing guidelines are presented by considering the MDT realities. Simulation results verify the effectiveness of the proposed CCJDE-MDT method, which outperforms the traditional decision-then-estimation in joint performance and also beats the existing recursive joint decision and estimation(RJDE) method in many cases.

1 Introduction

Multi-target detection and tracking (MDT) is a challenging problem and has wide applications, especially in military and civilian surveillance [1–7]. In an MDT problem, both the number of targets and their states are unknown and vary with time due to targets appearing and disappearing. Moreover, the indetermination of measurements with miss detections and false alarms due to clutters makes the MDT problem much more difficult. How to detect the number of targets and track the multiple targets from a sequence of noisy and cluttered observation sets jointly becomes a difficult issue and is critical in both academic and engineering research.

MDT has been studied extensively and abundant results are available. Most traditional multiple target tracking formulations involve explicit associations between measurements and targets. Among these methods, multiple hypothesis tracking (MHT) [8] and its variations concern the propagation of association hypothesis in time. The joint probabilistic data association (JPDA) [1, 9] method uses observations weighted by their association probabilities. These methods, however, either assume a known number of targets or determine the number of targets first and then estimate their states based on the determined number. In some other applications, tracking-before-detection (TBD) [10–13] and detection-before-tracking (DBT) are adopted. Specifically, [12, 13] proposed and assessed new TBD strategies in the context of space-time adaptive processing. Except for the association-based methods, the random finite set (RFS) approach has attracted much attention in recent years and fruitful research results have been achieved [14–24]. RFS methods assumed the states and measurements of multiple targets as random finite sets, and their advantages in handling multiple targets tracking have been verified. However, this paper focuses on the traditional data association-based multiple target tracking, and thus, RFS methods are not discussed.

Actually, MDT is a joint decision and estimation (JDE) problem with dual goals: deciding on the number of targets and estimating their states, furthermore, they are interdependent. On the one hand, a correct decision on the number can help state estimation since accurate state estimate relies on correct judge of targets number; On the other hand, accurate state estimation can also provide information which benefits making a correct decision on the target number. In essence, this is a so-called JDE problem [25] involving coupled decision and estimation, and good solutions require solving them jointly.

Traditionally, solutions for JDE problems contain [26]: (1) separate decision and estimation, which does not consider their couplings at all [27, 28]. (2) Decision then estimation (DTE), in which decision is made first disregarding estimation and then do estimation as if the decision were surely correct [29, 30]. (3) Estimation then decision (ETD), in which estimation is made first and then decision is made based on it [12, 13, 31, 32]. (4) Density-based method. This is beyond the scope of this paper, which is for point inference.

However, these solutions all have their respective drawbacks [36]. For the first category, decision and estimation are handled separately using their own information and techniques, and the interdependence between them is completely ignored. This is information loss, and the final joint performance is limited due to this loss. For the second category, the disadvantage is that: estimation is made completely on the decision without considering any possible decision error, where there may probably a decision error; meanwhile, decision is made disregarding the quality of the estimation it would lead to. The detection-before-tracking strategy belongs to this category, and thus has the above disadvantages. That is, only the effect of detection on tracking is considered but the effect of tracking on detection is ignored. For the third category, i.e., estimation then decision, it does not work well if estimation depends significantly on decision. The tracking-then-detection strategy belongs to this category and thus has the disadvantage, i.e., it does not consider the effect of detection on tracking, which limits its joint performance. For the fourth category, it is not in the scope of this paper. In this paper, we focus on point estimation rather than density estimation, where the latter may suffer from high computational complexity.

For JDE problems, [25] proposed an integrated paradigm based on a new Bayes risk, which unifies the traditional Bayes decision and estimation risks. This is inherently superior to traditional methods for utilizing the couplings between decision and estimation [26, 34–39]. Generally, MDT is a typical JDE problem while the available JDE framework just committed to such problems; thus, it is promising to tackling the MDT problem within the JDE approach.

Although the JDE framework is advantageous in handling problems involving coupled decision and estimation, difficulties arise when it comes to MDT problems mainly because of its particularities and complexities. First, the estimation cost for multi-targets is not well defined, especially when the true target state and the estimated target state have different dimensions. Although reference [40] proposes a recursive JDE (RJDE)-based MDT method by introducing a new estimation cost, parameter designing is especially complicated and the MDT problem realities is also not considered during the designing process. Here, *MDT problem realties* mean the possible losses caused by different types of incorrect decisions (in the following part, it has the same meaning). Second, there are already three design parameters in RJDE while MDT brings extra parameters. Too many design parameters are challenging to be determined but can definitely increase the calculation complexity, which would aggravate the already complicated MDT problems. Third, when designing parameters, the existing RJDE method does not take the MDT problem realities into consideration, which is actually important information and thus it is better to be utilized.

In general, the existing MDT method within JDE framework has complex implementation but limited performance. To overcome these shortcomings, a new and applicable JDE approach for MDT problems is required, which needs to satisfy two requirements: guaranteed good joint performance and easy implementation. Within the JDE framework, the conditional JDE (CJDE) method has been verified to have superior joint performance and easy implementation by introducing the on-line data [36]. Based on this, a compact CJDE (CCJDE) method is also proposed with less design parameters [37], which is especially suitable for complex JDE problems. This is consistent with the above requirements for MDT problems.

Inspired by the above, we consider solving MDT problems by introducing the idea of CCJDE due to its performance advantages and easy implementation. To achieve this goal, we first propose a CCJDE-based MDT risk, which is a joint Bayes risk unifying the detection and tracking risks through only one parameter. By this unification, the interdependence between detection and tracking is taken into consideration.

Based on the proposed CCJDE-MDT risk, we derive a joint solution containing a detector and a tracker, which have analytical forms. To obtain this joint solution, several quantities are required such as hypothesis conditioned estimates and the probability of each hypothesis. To determine these quantities, this paper proposes to introducing the JPDA filter for two main reasons. First, JPDA is widely used in multiple target tracking area, which is simple and effective. Second and more importantly, JPDA suits MDT problems well, i.e., it can output all the quantities required in the JDE framework. Besides, the parameter designing is specially studied by considering the MDT realities; subsequently, some designing guidelines are provided for application.

The advantages of the proposed CCJDE-MDT method are verified through an illustrative MDT example under different scenarios. They show that both the proposed CCJDE-MDT and RJDE methods outperform the traditional DTE method in joint performance, and CCJDE-MDT outperforms RJDE in many cases. These demonstrate the advantages of CCJDE-MDT in two aspects: (1) as a joint method, CCJDE-MDT can fully utilize the coupling between detection and tracking in MDT problems and finally achieve a satisfactory joint performance; (2) compared with RJDE, CCJDE-MDT not only has easy implementation, but also has performance advantages by considering the MDT problem realities when designing the parameters.

More specifically, the contributions of this work are summarized as follows:

- We propose a CCJDE-MDT risk for handling the multi-target detection and tracking problems. This is a joint risk, which unifies the detection and tracking risks in a compact way; therefore, the coupling between target detection and tracking is incorporated.
- We derive an analytical CCJDE-MDT solution based on the above joint risk. Specifically, the joint solution containing a detector and a tracker is presented, which fully utilizes the coupling between detection and tracking, and also accounts for the particularities of MDT problems. During the derivation, the JPDA filter is introduced since it has fine tracking performance, simple calculation, and also suits the JDE framework.
- Parameter designing is specially and deeply studied. We propose that parameters should be designed by considering the realities of MDT problems, i.e., the losses each incorrect decision may bring to. Then, penalize the incorrect decision which may lead to larger losses heavier than those lead to smaller losses so that the joint Bayes risk could be better minimized. Following this idea, some guidelines are provided for practical parameter designing.
- Simulation results demonstrate the effectiveness and advantages of the proposed CCJDE-MDT approach. It beats the traditional DTE in joint performance and also outperforms RJDE in many cases. This verifies that the proposed CCJDE-MDT method not only has easy implementation but also has superior joint performance by utilizing the coupling between detection and tracking and also the MDT realities when designing parameters..

This paper is organized as follows: Section 2 formulates the MDT problem. Section 3 proposes a new CCJDE-based MDT method which contains a joint CCJDE-based MDT risk, a joint solution, and a joint algorithm. Also, parameter designing is deeply studied by considering the MDT reality. Section 4 presents an illustrative yet typical MDT example compared with DTE and RJDE methods. Section 5 concludes the paper.

2 Problem formulation

Suppose there are multiple targets moving in the field of interests, and the goal of multi-targets detection and tracking (MDT) is to detect the number of targets and track their states jointly using all available data. In MDT, detection and tracking are highly coupled: on the one hand, correct detection of targets number can help tracking since accurate state estimation relies on correct targets number; on the other hand, accurate tracking can help detection since it can provide critical information about the targets number.

Specifically, consider that there are N moving targets, and the nth ($n = 1, \ldots, N$) target is modeled by the following state-space equations:

$$\begin{aligned} x_{k+1}^n &= F^n x_k^n + w_k^n \\ y_k^n &= H^n x_k^n + v_k^n \end{aligned}$$

where x_k^n and y_k^n are the state vector and measurement vector of the nth target at time k, respectively. F^n and H^n are known transition matrices. The process noise w_k^n and the measurement noise v_k^n are mutually uncorrelated zero-mean white Gaussian noise sequences, and their covariances are Q_k^n and R_k^n, respectively.

Suppose that there are totally m_k^z measurements at time k. One target can only generate at most one measurement and one measurement only has one source, i.e., either from a target or a clutter. Usually, it is not easy to distinguish whether a measurement originated from a target or from a clutter. Denote z_k^r as the rth kinematic measurement at time k ($r = 1, 2, \ldots, m_k^z$), then

$$z_k^r = \begin{cases} y_k^n & \text{if } z_k^r \text{ is from target } n \\ z_k^{r,c} & \text{if } z_k^c \text{ is from clutter} \end{cases}$$

where each measurement $z_k^{r,c}$ is assumed to be uniformly distributed in the surveillance region.

Based on the above models, MDT aims to jointly determine the targets number N and their states $x^n (n = 1, \ldots, N)$, which are interdependent. Correct detection of targets number N facilitates the state estimation x^n while accurate estimation x^n also helps determine N. Essentially, MDT is a joint problem as is involves inference of interdependent discrete value (the targets number N) and continuous value (the targets states x^n) uncertainties. For such joint problems, exploration and appropriate utilization of the coupling information can help improve the joint performance of detection and tracking.

3 CCJDE-based joint multi-target detection and tracking

As analyzed above, MDT is a typically joint decision and estimation (JDE) problem, and handling detection and tracking jointly is promising to achieve a superior joint performance. In the following, after reviewing the existing JDE framework, we propose a novel MDT method by fully utilizing the coupling between detection and tracking.

3.1 Review of joint decision and estimation (JDE)

The basic idea of the JDE approach is to minimize the following generalized Bayes risk [25]

$$\bar{R} = \sum_{i,j} (\alpha_{ij} c_{ij} + \beta_{ij} E[C(x, \hat{x}) | D^i, H^j]) P\{D^i, H^j\} \tag{1}$$

where D^i stands for the ith decision while H^j stands for the jth hypothesis; c_{ij} is the cost of deciding on D^i while the truth is H^j; $P\{D^i, H^j\}$ is the joint probability of decision D^i and hypothesis H^j; $C(x, \hat{x})$ is the cost of estimating x by $\hat{x}$; $E[C(x, \hat{x}) | D^i, H^j]$ is the expected cost conditioned on the fact that D^i is decided but H^j is true; α_{ij} and β_{ij} are the weight coefficients of decision and estimation costs, respectively. Generally, the joint risk $\bar{R}$ is a generalization of the traditional Bayes risks for decision and estimation.

JDE is theoretically superior to the existing separate decision and estimation or two-stage methods by explicitly accounting for the interdependence between decision and estimation. Within the JDE framework, we develop a conditional JDE (CJDE) approach by introducing the on-line data, which inherits the theoretical superiorities of JDE but has simpler calculation. Specifically, the CJDE risk is as follows:

$$\bar{R}_C(z) = \sum_{i,j}(\alpha_{ij}c_{ij} + \beta_{ij}E[C(x,\hat{x})|D^i, H^j, z])P\{D^i, H^j, z\}$$

where CJDE risk differs from JDE risk by introducing the data z.

Note that in CJDE, there are parameters $\alpha_{ij}, c_{ij}, \beta_{ij}$ need to be designed, which is specifically difficult for complicated JDE problems. Therefore, we further proposed a compact CJDE (CCJDE) algorithm with less parameters. Specifically, let $\alpha_{ij} = 0$ in the CJDE risk, and assign β_{ij} dual function—tuning the relative weight of decision and estimation, and also tuning different types of errors. More details about CCJDE can be found in [37].

3.2 Motivation

As is analyzed in Introduction, MDT is essentially a JDE problem, and good solutions require solving detection and tracking sub-problems jointly. For such joint problems, the available JDE framework provides an effective joint solution, which has advantageous performance by accounting for the interdependence between decision and estimation.

However, when it comes to MDT problems, difficulties arise due to particularities and complexities caused by multi-targets. Specifically, the estimation cost $E[C(x,\hat{x})|D^i, H^j]$ for multi-targets is not defined originally, especially when the hypothesized target number and decided number is mis-matched. For example, what is the error of an estimator $(\hat{x}_1, \hat{x}_2)$ assuming two targets, each with a state dimension of n, while there is only a single target with an n-dimensional state x?

Although [40] presents a RJDE method with a new estimation cost, there are excessive design parameters. Too many design parameters are challenging to be determined but can definitely increase the calculation complexity, which would aggravate the already complicated MDT problems. Specifically, there are four parameters in RJDE: $\alpha_{ij}, \beta_{ij}, c_{ij}$ and η, where $\alpha_{ij}, \beta_{ij}, c_{ij}$ are explained in (1), and η is a constant denoting the estimation cost for the mis-matched case. It is obvious that parameter designing is already complicated in RJDE, not to mention that detection and tracking multiple targets (e.g., MDT problems) is much more complex than single target.

Furthermore, the MDT problem realities are not considered when designing parameters in RJDE, i.e., the losses that different types of incorrect detections may lead to. Actually, this MDT problem realities contain useful information and taking them into consideration may help improve the algorithm performance. The essential reason is that by doing this, the particularities of the MDT problems can be captured and the coupling between detection and tracking can be further explored, which may lead to better joint performance.

Based on the above, a simplified joint solution for MDT is promising, which satisfies three requirements: it accounts for the coupling between detection and tracking; it is easy for implementation; it can accounts for the MDT problem realities when designing parameters. Considering that CCJDE satisfies all the three requirements, we propose to solve the MDT problem by introducing the idea of CCJDE.

3.3 Joint CCJDE-based MDT Risk

Denote by X the true target state, which is a stacked vector comprised by the states of all targets. H^j denotes the hypothesis that there are j targets while D^i denotes that there are i targets being detected. The maximum total targets number is N. The target label is not considered in this paper, therefore, H^j contains all the combinations of hypothesized targets with total number j, and D^i contains all combinations of detected targets with total number i. Based on these denotations, we propose the following CCJDE risk for MDT:

$$R_C(z) = \sum_{i,j=1}^{N} \gamma_{ij} E[C(X,\hat{X})|D^i, H^j, Z])P\{D^i, H^j|Z\} \tag{2}$$

where γ_{ij} is the only parameter unifying the detection and tracking risks, $\hat{X}$ is the estimate of true target state X, and Z denotes all the available measurements. The expected estimation cost is defined as follows [40]:

$$\begin{aligned} \varepsilon_{ij} &\triangleq E\left[C\left(X,\hat{X}\right)|D^i, H^j, Z\right] \\ &= \begin{cases} \frac{\tau(i)}{i}\text{mse}\left(\hat{X}_k|D^i, H^j\right) & \text{if} \quad i=j \\ \eta & \text{if} \quad i \neq j \end{cases} \end{aligned} \tag{3}$$

where η is the estimation cost parameter for the case when hypothesized targets number and the detected targets number are mis-matched. It is intuitive that η should be a large constant so that the incorrect decision can be penalized. $\text{mse}(\hat{X}_k|D^i, H^j)$ denotes the estimation mean square error of $\hat{X}_k$ conditioned on hypothesis H^j and decision D^i.

$\tau(i)$ is a non-increasing positive function of i and $\tau(1) = 1$. $\frac{\tau(i)}{i}$ is the normalization factor, which is a ratio between the determined number of targets and an adjustment function. This is reasonable because the more targets being tracked, the greater estimation error usually being resulted in. For example, if an algorithm tracking 10 targets has the same average mse (i.e., normalized only by factor i) as an algorithm tracking only 1 target, it makes much sense to say the first algorithm does a better job than the second. Therefore, τ (i) is introduced to favor the algorithms tracking more targets.

Remark 1

The proposed CCJDE–MDT risk $R_C(z)$ is reasonable. Take γ_{ij} as decision cost and ε_{ij} as estimation cost, the joint risk $R_C(z) = \sum_{i,j} \gamma_{ij}\varepsilon_{ij}P\{D^i, H^j|Z\}$ can be considered as a product of decision cost and estimation cost. This differs from the original JDE risk $\bar{R}$, which is summation of decision cost and estimation cost. In general, both $\bar{R}$ and $R_C(z)$ unify the traditional decision and estimation but in different ways. $R_C(z)$ inherits the virtues of $\bar{R}$ by accounting the coupling between decision and estimation.

Remark 2

The proposed CCJDE-MDT risk $R_C(z)$ is a generalization of the traditional risks for detection and tracking, which is not only advantageous in performance but also simple in implementation, as follows:

First, the proposed CCJDE-MDT is advantageous in joint performance. That's because $R_C(z)$ is essentially a JDE risk which unifies the traditional detection and tracking risks into one framework in a *compact* way. By doing this, the interdependence between detection and tracking can be utilized, which is critical information in joint problems but usually ignored traditionally. This appropriate utilization helps the proposed approach achieving a good joint performance.

Second, the proposed CCJDE-MDT is simple in implementation. It is known that compared with single target, detection and tracking multiple targets is already complicated. However, in the original RJDE, extensive design parameters make it more complicated, i.e., there are parameters α_{ij}, β_{ij} and c_{ij}. In the proposed CCJDE-MDT risk, the design parameters are greatly reduced, i.e., only one parameter γ_{ij}.

Specifically, in the original RJDE, there are two steps in designing parameters. First, we need to first balance the contributions of decision and estimation by adjusting $\alpha_{ij}c_{ij}$ and $\beta_{ij}\varepsilon_{ij}$. This is important and not easy since c_{ij} and ε_{ij} usually have very different orders of magnitude. Therefore, we need to tune α_{ij} and β_{ij} appropriately to make $\alpha_{ij}c_{ij}$ and $\beta_{ij}\varepsilon_{ij}$ equivalent in terms of order of magnitude. By doing this, decision cost and estimation cost contribute equally into the joint risk. Second, we need to further design β_{ij} for different (i, j)s to penalize different incorrect decisions according to the cost they may bring to (the larger cost an incorrect decision may bring, the heavier penalization to such incorrect decision).

However, in our proposed CCJDE-MDT, only γ_{ij} needs to be designed appropriately. We just need to find a good trade-off between decision and estimation by tuning γ_{ij} for the sake of good joint performance. This simplification is very meaningful especially for the already complicated MDT problems.

Actually, the main advantage of using *product* of decision cost and estimation cost in the CCJDE risk rather than using their *summation* is **simplicity**. The former is simpler than the latter, as is analyzed above. This simplification is especially meaningful for complicated problems such as MDT. That's because such JDE problems are already complex, let alone adding complex design parameters.

Remark 3

In this joint risk $R_C(z)$, ε_{ij} is defined as a large constant η. The clarifications are as follows:

Since X and $\hat{X}$ have different dimensions if D^i does not match H^j, it is difficult to define in a unified way, as is analyzed in Motivation part. Intuitively, ε_{ij} should be related to i and j; however, it is hard to provide a strict mathematical definition for ε_{ij} when $i \neq j$.

Theoretically, $\varepsilon_{ij}(i \neq j)$ should be large so as to facilitate decision-making. Therefore, defining ε_{ij} by a large constant η is reasonable and effective way. Besides, this definition simplifies the joint risk and its solution (presented next). In general, this definition is reasonable, easy in implementation, and it provides an effective solution for the mis-matched case of decision and estimation.

Remark 4

The followings are more explanations about the target state X.

First, the capital X is a stacked vector comprised by the states of all targets. Therefore, X has different dimensions under different hypothesis H^j. Specifically, under hypothesis $H^j(j = 1, 2, \ldots, N)$, X is a stacked vector of the states of j targets.

Second, X under H^j and H^i $(i \neq j)$ cannot be directly subtracted from each other since they have different dimensions. This has already been stated above. Furthermore, this mis-match is one of the motivation of this paper. More specifically, suppose the hypothesized target number is one and the corresponding target state is $(\hat{x}_1)$, and the decided number is two and the corresponding state is $(\hat{x}_1, \hat{x}_2)$. Then, how should we define the state estimation error between $(\hat{x}_1)$ and $(\hat{x}_1, \hat{x}_2)$?

Third, due to the above, we define the estimation cost ε_{ij} by (3). That is, use a large constant η to denote the estimation cost for the mis-matched case. Although simple, it is reasonable that the estimation cost is large when the hypothesized target number and the decided target number are different.

3.4 Solution of CCJDE-MDT

The goal of MDT is to obtain the detection and tracking results, specifically, jointly infer the number of targets and their states. To achieve this goal, we need to minimize the above CCJDE-MDT risk. In the following, we focus on deriving the joint detector and tracker.

3.4.1 CCJDE-MDT detector and tracker

Let $Z^k = \{Z_1, Z_2, \cdots, Z_k\}$ denotes the set of all measurements up to time k. Here, $Z_k = \{z_k^1, z_k^2, \cdots, z_k^{m_k^z}\}$ is the set of measurements at time k, where z_k^i is the ith measurement and m_k^z is the number of total measurements at k. X_k and $\hat{X}_k$ denote the true state and the estimated state at time k, respectively. In the following, $i, j = 1, \ldots, N$, where N is the maximum number of targets.

Given any expected estimation cost $E[C(X_k, \hat{X}_k)|D^i, H^j, Z^k]$, the CCJDE-MDT detector D_k is

$$D_k = D_k^i, \text{ if } C_C^i(Z^k) \leqslant C_C^l(Z^k), \forall l \tag{4}$$

where the posterior cost is

$$C_C^i(Z^k) = \sum_j \gamma_{ij} \varepsilon_{ij}^k P\{H^j|Z^k\}$$

Given any decision D_k^i, with the quadratic estimation cost $C(X_k, \hat{X}_k) = \tilde{X}_k' \tilde{X}_k$, where $\tilde{X}_k = X_k - \hat{X}_k$, the CCJDE-MDT tracker for $R_C(z)$ is:

$$\check{X}_k^{(i)} = \sum_j \hat{X}_k^{(j)} \bar{P}_i\{H^j|Z^k\} \tag{5}$$

where the generalized posterior probability

$$\bar{P}_i\{H^j|Z^k\} = \gamma_{ij} P\{H^j|Z^k\} / \sum_l \gamma_{il} P\{H^l|Z^k\}$$

For clarification, $\hat{X}_k^{(j)}$ is the target state estimate under hypothesis H^j while $\check{X}_k^{(i)}$ is the target state estimate under decision D^i.

Specifically, to obtain the detector D_k, the key is to calculate the posterior cost $C_C^i(Z^k) = \sum_j \gamma_{ij}\varepsilon_{ij}^k P\{H^j|Z^k\} \quad (i = 1, \ldots, N)$. Note that $C_C^i(Z^k)$ is mainly determined by the expected estimation cost ε_{ij}^k and the posterior hypothesis probability $P\{H^j|Z^k\}(j = 1, \cdots, N)$. Furthermore, for ε_{ij}^k (defined in (3)), it can be seen that it is η and $\mathrm{mse}(\hat{X}_k|D^i, H^j)$ that affect ε_{ij}^k, where η is a constant parameter given in advance while the estimation mean square error $\mathrm{mse}(\hat{X}_k|D^i, H^j)$ requires to be calculated. Note that $\mathrm{mse}(\hat{X}_k|D^i, H^j)$ is required in ε_{ij}^k only when $i = j$, therefore, $\mathrm{mse}(\hat{X}_k|D^i, H^j) = mse(\hat{X}_k|H^j)$. Here, $\mathrm{mse}(\hat{X}_k|H^j)$ denotes the state estimation mean square error under hypothesis H^j. In the following, for the sake of simplicity, we use $\mathrm{mse}(\hat{X}_k|H^j)$ for denotation.

To obtain the tracker $\hat{X}_k = \check{X}_k^{(i)}$ (given in (5), which is a weighted sum of each hypothesis conditioned estimate $\hat{X}_k^{(j)}$, we need to calculate the hypothesis conditioned estimate $\hat{X}_k^{(j)}$ and the corresponding hypothesis probability $P\{H^j|Z^k\}\,(j = 1, \ldots, N)$. Note that since $\varepsilon_{ij}^k(i \neq j) = \eta$, which is a larger constant denoting the expected estimation error for the mis-matched case, it is actually $\hat{X}_k^{(j)}(i = j)$ contributes to $\check{X}_k^{(i)}$.

In summary, it is mainly the hypothesis probability $P\{H^j|Z^k\}$, the hypothesis conditioned estimate $\hat{X}_k^{(j)}$ and the corresponding estimation mean square error $\mathrm{mse}(\hat{X}_k|H^j)$ $(j = 1, \ldots, N)$ that affect the CCJDE-MDT detection and tracking results. In other words, $P\{H^j|Z^k\}, \hat{X}_k^{(j)}$, and $\mathrm{mse}(\hat{X}_k|H^j)$ are critical prerequisite quantities for obtaining the CCJDE-MDT solution. Therefore, we focus on determining $P\{H^j|Z^k\}, \hat{X}_k^{(j)}$, and $\mathrm{mse}(\hat{X}_k|H^j)$ in the following parts.

3.4.2 Detailed derivation of CCJDE-MDT detector and tracker

Given the hypothesis H^j, determining the target state $\hat{X}_k^{(j)}$ is actually a multi-target tracking problem with a known number of targets. This has been widely studied with abundant results. Among these, we propose to use the JPDA (joint probabilistic data association) algorithm for its effectiveness, simplicity, popularity, and also the adaptability to our problem.

The fundamental idea of JPDA filter is to compute the probabilities of all feasible measurement-to-target association events θ_k^i jointly. The track update is obtained by the probabilistic average over all θ_k^i. Details of JPDA are given in the Appendix.

Following the Bayesian formula and the JPDA filter, the probability $P\{H^j|Z^k\}$ at each time k can be updated by

$$P\{H^j|Z^k\} = \frac{1}{c} f(Z_k|H^j, Z^{k-1}) P\{H^j|Z^{k-1}\} \tag{6}$$

where

$$f(Z_k|H^j, Z^{k-1}) = \sum_l f\{Z_k|H^j, \theta_k^l, m_k^z, Z^{k-1}\}P\{\theta_k^l|H^j, m_k^z\}$$

is the likelihood of H^j; the summation is over all possible θ_k^l, which l denotes the lth measurement-to-target association event. c is a normalizing constant. $f\{Z_k|H^j, \theta_k^l, m_k^z, Z^{k-1}\}$ and $P\{\theta_k^l|H^j, m_k^z\}$ are obtained by JPDA filter.

Based on the above, all the hypothesis conditioned estimate $\hat{X}_k^{(j)}$ with the corresponding estimation mean square error matrix $P_k^{(j)}$ (by the JPDA filter), and the hypothesis posterior probability $P\{H^j|Z^k\}$ (by Eq. (6)) are obtained, which are all required in determining the CCJDE-MDT solution. Here, $P_k^{(j)}$ is the estimation mean square error matrix, and $\mathrm{tr}(P_k^{(j)}) = \mathrm{mse}(\hat{X}_k|H^j)$, where $\mathrm{tr}(\cdot)$ means the trace of a matrix.

Remark 5

The reasons for adopting the JPDA filter are as follows: First and most important, JPDA can output all quantities required in the CCJDE-MDT detector and tracker, i.e., the state estimation $\hat{X}_k^{(j)}$, the estimation mse $P_k^{(j)}$,and the hypothesis probability $P\{H^j|Z^k\}$. Second, JPDA has guaranteed estimation performance and is also easy for implementation, which have been illustrated in [1].

3.4.3 CCJDE-based MDT algorithm

Based on the above CCJDE-MDT detection and tracking results, we propose the following CCJDE-MDT algorithm.

Note that in step (b), with data Z_k available, the detailed state estimate $\hat{X}_k^{(j)}$ and the corresponding estimation MSE $P_k^{(j)}$ under hypothesis H^j can be determined by the JPDA filter, which is presented in Appendix A.

Remark 6

With this efficient algorithm, the number of targets and their states are inferred jointly without iterations. Moreover, the couplings between detection and tracking is fully accounted for.

Specifically, detection and tracking affect each other as follows: on the one hand, tracking affects detection through the expected estimation cost ε_{ij}^k, which can be output by tracking but is basically required in detection; on the other hand, detection affects tracking through the hypothesis probability $P\{H^j\}$, which can be output by detection but is critical for tracking.

Remark 7

In general, the proposed CCJDE-MDT solution is a **joint** *solution in the following sense.*

Firstly, detection and tracking are interdependent, as is analyzed in *Remark* 5. In other words, we can neither obtain detection without tracking nor obtain tracking without detection. This influence is mutual.

Second, the proposed CCJDE-MDT method completely differs from the traditional "detection-then-tracking" or "tracking-then-detection" methods. That's because in "detection-then-tracking", detection is done first and then tracking is done secondly. When handling detection, we do not consider anything about tracking since detection does not depend on tracking at all. Similarly, in "tracking-then-detection", tracking is done first without any consideration of detection, and then detection is done based on the tracking result. However, in our proposed method, we cannot determine detection/tracking without tracking/detection.

Third, Table 1 also clearly shows that in the proposed CCJDE-MDT algorithm, detection and tracking are obtained jointly rather than sequentially. From Table 1, the steps (a), (b), (c) are all preliminary calculations for the final joint solution. Based on these steps, the final joint solution containing a detector and a tracker can be output jointly. This significantly differs from the "detection-then-tracking" or "tracking-then-detection" strategies since in such "two-steps" strategies, there must be "detection step" then/before "tracking step" which are sequentially. However, in our proposed method, the CCJDE-MDT detector and tracker are determined jointly/simultaneously at the last step.

3.5 Analysis of parameters

As the only parameter in CCJDE-MDT, γ_{ij} tunes the relative weight of detection and tracking, which is a design parameter. Since γ_{ij} is problem-dependent, appropriate designing strategy with the MDT problem realities being considered is preferable. Here, MDT problem realities denote the losses that different types of incorrect decisions may bring to (it has been explained in Introduction). The following are guidelines for designing γ_{ij}.

Firstly, the incorrect decision should be penalized heavier than the correct one, i.e., $\gamma_{ij}\varepsilon_{ij}(i \neq j) > \gamma_{ij}\varepsilon_{ij}(i = j)$. This is natural and reasonable because it can avoid incorrect decision and also benefits the correct decision. Actually, this is a condition that must be satisfied so as to ensure the correctness of decision-making.

Table 1 CCJDE-MDT algorithm

(a) Initialization	Conditioned on each hypothesis H^j, initialize the state $\hat{X}_0^{(j)}$ and its MSE $P_0^{(j)}$
	Initialize the hypothesis probability $P\{H^j\} = 1/N$.
(b) Update	At time k, with data Z_k available:
	Update the state estimate $\hat{X}_k^{(j)}$ and its MSE $P_k^{(j)}$ under each hypothesis H^j;
	Update the posterior probability of each $P\{H^j\|Z^k\}$ according to (6)
(c) Further computation	Compute the expected estimation cost ε_{ij}^k according to (3);
	For each candidate D^i, compute the estimate $\check{X}_k^{(i)}$ according to (5)
(d) Output	The optimal CCJDE-MDT detection result is D_k^i: $C^i(Z^k) \leq C^l(Z^k), \forall l$;
	The corresponding CCJDE-MDT tracking result is $\check{X}_k^{(i)}$

Secondly, the penalization for different types of decision errors should be different according to the losses they may bring to. Take the CCJDE-MDT decision for illustration, which aims to minimize the Bayes decision risk $\bar{R}_D = \sum_{i,j} \gamma_{ij}\varepsilon_{ij}P\{H^j\}$. Assume that there are at most two targets $(i, j = 1, 2)$, misjudging one target into two targets (the corresponding parameter is γ_{21}) and misjudging two targets into one target (the corresponding parameter is γ_{12}) should have different penalty costs.

More specifically, suppose this is an MDT problem on the battlefield, where targets are enemies and we would like to destroy them after detection and tracking. In this case, misjudge one target into two targets is called *false alarm* while misjudge two targets into one is called a *miss*.

In this MDT problem, a miss causes greater losses than a false alarm. That's because, a miss would let an enemy target by, which is rather dangerous for us since the enemy may probably attack us. While the false alarm only wastes some materials. For example, we need missiles to destroy the targets, and suppose one missile is needed to destroy each target. Therefore, the more targets detected, the more missiles are needed for destroying. Based on these, for false alarm, it is detected that there are two targets (although the truth is only one target), and thus we launch two missiles for destroying. This is obviously a waste of missiles. In summary, a miss would lead to greater losses to us than a false alarm in this problem.

Therefore, it is reasonable to penalize *miss* heavier than *false alarm* so as to avoid miss. Note that within the Bayesian framework, the final goal is to minimize the Bayes risk, e.g., the joint Bayes risk $R_C(z)$ in this paper. Therefore, punishing such incorrect decisions that are likely to cause greater Bayes risk heavily is preferable because this is beneficial for reducing the final Bayes risk.

In general, taking the MDT problem realities into consideration when designing parameters can help improve the algorithm performance. The essential reason is that by doing this, more useful information about the problem can be explored and utilized. With more information, the algorithm performance is definitely improved.

4 Simulation and analysis

4.1 Basic assumptions

To illustrate our proposed CCJDE-MDT method, we apply it to a simple yet representative joint multi-target detection and tracking (MDT) problem. Our goal is to jointly detect the number of targets and track their states, and then take actions based on the result, e.g., destroy the targets. In this MDT problem, what we care about most is the joint performance since both correct target number judgement and accurate target state estimation are critical to us.

For simplicity, we have some basic assumptions [40]: (a) the number of targets $m_k^t \leqslant N$ is unknown but constant over time k. (b) A target can generate at most one measurement—no multipath; a measurement can have originated from at most one target—no unresolved measurements. (c) The number of false measurements is Poisson distributed. The false measurements are i.i.d and uniformly distributed in the surveillance region of a volume V. (d) All targets follow CV models with a linear measurement equation. To save space, the dynamic and measurement models are omitted.

4.2 Compared methods

The compared methods are as follows:

(a) The traditional decision then estimation (DTE) method. Specifically, in DTE, the optimal Bayes decision on the number of targets is made first following the Bayesian decision rule:

$$\frac{P\{H^1|Z^k\}}{P\{H^0|Z^k\}} \underset{D_0}{\overset{D^1}{\gtrless}} \frac{c_{10}-c_{00}}{c_{01}-c_{11}}$$

where $P\{H^i|Z^k\}$ is posterior probability of hypothesis H^i, c_{ij} is the cost of deciding on D^i while H^j is true. Then, based on this decision, the corresponding target state estimate are obtained.

(b) The RJDE method in [40].

(c) The proposed CCJDE-MDT method.

(d) Known number, in which the true target number is known. This is the ideal case and it sets a lower bound for the joint performance.

4.3 Performance evaluation

For a joint problem, the performance of algorithms require jointly evaluation [33]. In evaluating the performance of MDT algorithms, the goal is to measure the distance between two sets of tracks: the set of ground truth and the set of estimated tracks output by the algorithms. The OSPA (Optimal Sub-patten Assignment) metric proposed by [41] is widely used and also suitable for such problems.

OSPA measures the distance between two sets, which is defined as follows: Denote by $d^{(c)}(x,y)=\min(c,d(x,y))$ the distance between $x,y\in W$ cutoff at $c>0$, where W is a closed and bounded observation window and $W\in\mathbb{R}^N$. Π_k is the set of permutations on $\{1,2,\ldots,k\}$ for any $k\in\mathbb{N}=\{1,2,\ldots\}$. For $1\leqslant p<\infty, c>0$, and arbitrary finite subsets $X=\{x_1,\ldots,x_m\}$ and $Y=\{y_1,\ldots,y_n\}$ of W, where $m,n\in\mathbb{N}_0=\{0,1,2,\ldots\}$, define

$$\bar{d}_p^{(c)}(X,Y)\triangleq\left(\frac{1}{n}\left[\min_{\pi\in\Pi_n}\sum_{i=1}^{m}d^{(c)}(x_i,y_{\pi(i)})^p+c^p(n-m)\right]\right)^{1/p} \tag{7}$$

if $m\leqslant n$, and $\bar{d}_p^{(c)}(X,Y)=\bar{d}_p^{(c)}(Y,X)$ if $m>n$. The function $\bar{d}_p^{(c)}$ is called the OSPA metric of order p with cutoff c. In (7), p determines the sensitivity of $\bar{d}_p^{(c)}$ to outlier estimates, and c determines the relative weighting of how the metric penalizes the cardinality error as opposed to the localization error. Generally, OSPA considers both the cardinality error and the localization error, and its superiority is demonstrated in [41]. It can evaluate the tracking and detection performance jointly. Therefore, OSPA is adopted as the joint performance evaluation metric in this paper.

4.4 Simulation

4.4.1 Parameters setting

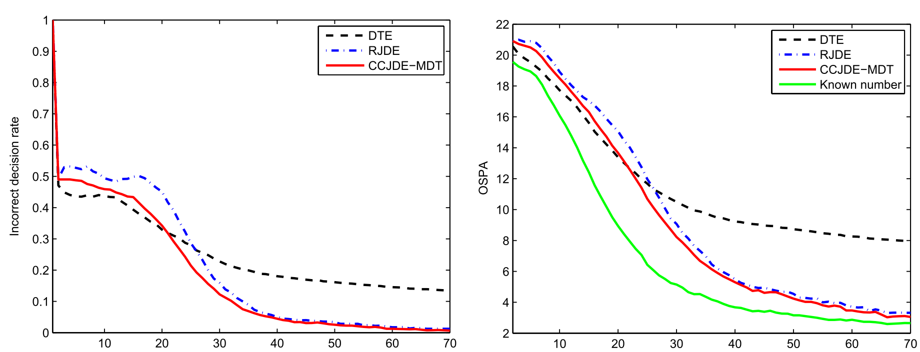
The maximum number of targets $N = 2$. The target number is uniformly and randomly distributed from 1 to N and remains constant in each run. The number of false measurement at each time step k is sampled from a Poisson distribution

$$P_f\{m\} = \frac{e^{-\lambda V}(-\lambda V)^m}{m!}$$

where λ is the clutter density and V is the volume of the surveillance region. All the false measurements are uniformly distributed within the region and independent of each other.

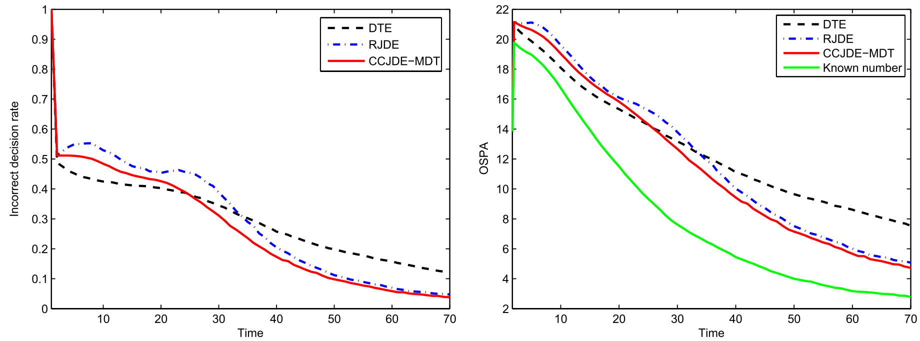
The covariances of the process noise and measurement noise are $Q = \text{diag}[1\text{m}^2, 0.01(m/s)^2, 1m^2, 0.01(m/s)^2]$ and $R = \text{diag}[100\text{m}^2, 100\text{m}^2]$, respectively. The initial position of two targets are $[-100m, 1m/s, -100m, 1m/s]$ and $[-150m, 1.5m/s, -150m, 1.5m/s]$, respectively. The parameters in RJDE are chosen as $\alpha_{ij} = 1$, $\beta_{ij} = 1(i \neq j), \beta_{ij} = 0(i = j), c_{ij} = 1(i \neq j), c_{ij} = 0(i = j)$. In CCJDE-MDT, the only parameter $\gamma_{ij} = [0, 1; 2, 1]'$. All results were obtained from 5000 Monte Carlo (MC) runs.

For performance evaluation, two evaluation metrics are used: the incorrect decision rate and the OSPA metric. Specifically, the incorrect decision rate is calculated as: $N_{\text{incorrect}}/N_{MC}$, where N_{MC} is the number of total Monte Carlo runs while $N_{\text{incorrect}}$ is the number of incorrect decisions. The incorrect decision rate actually reflects the percentage of the incorrect decisions in total decisions and is adopted as the decision performance metric in this paper. Obviously, the lower the $N_{\text{incorrect}}/N_{\text{MC}}$ is, the better the decision performance is. Besides, OSPA evaluates the joint performance, as is presented above. In the OSPA metric, we choose $p = 2$ and the cutoff value $c = 20$.



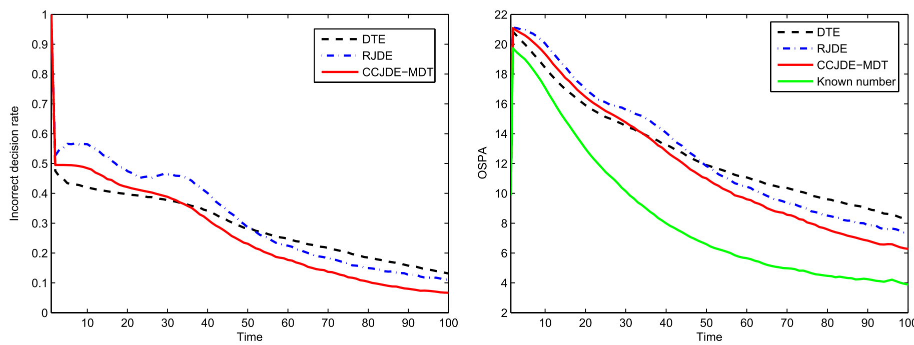
(a) Decision error rate (b) OSPA

Fig. 1 Simulation results of MDT, scenario 1. $\lambda = 5/m^3, P_d = 0.75$



(a) Decision error rate (b) OSPA

Fig. 2 Simulation results of MDT, scenario 2. $\lambda = 20/m^3, P_d = 0.75$



(a) Decision error rate (b) OSPA

Fig. 3 Simulation results of MDT, scenario 3. $\lambda = 20/m^3, P_d = 0.65$

4.4.2 Simulation results

Scenario 1

In this scenario, the clutter density $\lambda = 5/m^3$, and the detection probability $P_d = 0.75$. Here, $\lambda = 5/m^3$ means there are 5 clutters per cubic meter of volume. Simulation results are presented in Fig. 1. It can be seen that for the decision performance, both RJDE and the proposed CCJDE-MDT methods beat the traditional DTE method. This verifies that in JDE approach (both RJDE and CCJDE-MDT belong to JDE approach), tracking facilitates detection by utilizing their couplings.

For joint performance, CCJDE-MDT performs best. This verifies that CCJDE-MDT can take advantage of the couplings between detection and tracking, and finally achieves superior joint performance. Furthermore, compared with RJDE, CCJDE-MDT is simpler in implementation and superior in performance, where the reasons lie in fewer parameters and also the consideration of MDT problem realities.

Scenario 2

The clutter density $\lambda = 20/m^3$, which is heavier than that in scenario 1 while all other parameters are the same as in scenario 1. Simulation results are presented in Fig. 2. Similar to scenario 1, they show that CCJDE-MDT performs best in both decision and joint performance.

Scenario 3

The clutter density $\lambda = 20/\mathrm{m}^3$, the detection probability $P_d = 0.65$, and all other parameters are the same as in scenario 1. That is, the clutter density becomes heavier and the detection probability becomes lower compared with scenario 1. The simulation results are presented in Fig. 3.

Figure 3 shows that CCJDE-MDT significantly outperforms other methods in both decision and joint performance. Comparing with scenarios 1 and 2, although the performance of all three algorithms deteriorate, the advantages of CCJDE-MDT over other methods are more obvious. In other words, CCJDE-MDT has more significant advantages than other methods in scenario 3. Note that in this scenario, the time step is longer than that in other scenarios since in this "complex" scenario, detecting and tracking are difficult for all methods.

Scenario 4

The detection probability $P_d = 0.9$, which is higher than that in Scenario 1, and all other parameters are the same. The simulation results are presented in Fig. 4. They show that when the detection probability is high, both RJDE and CCJDE-MDT perform well, which are better than DTE. This still make sense as it demonstrates the effectiveness of CCJDE-MDT in "simple" scenario, i.e., the detection probability is high.

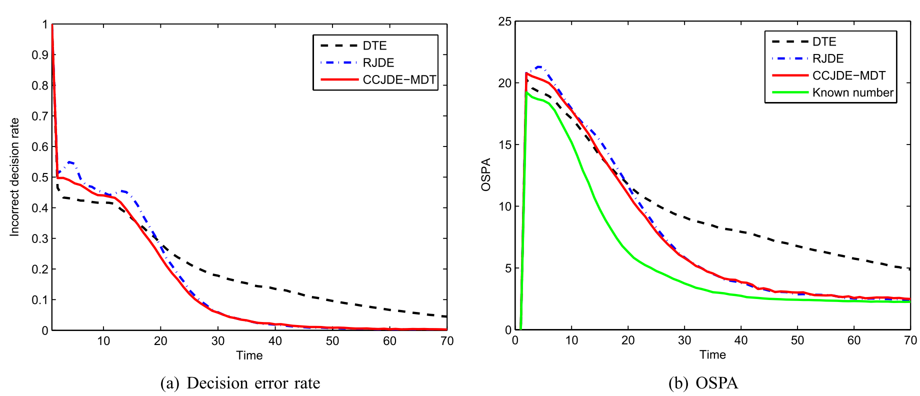


Fig. 4 Simulation results of MDT, scenario 4. $\lambda = 5/m^3, P_d = 0.9$

Remark 8

The above simulations fully demonstrate the effectiveness and advantages of the proposed CCJDE-MDT method. It outperforms the traditional DTE in both decision and joint performance (OSPA). The essential reason is that CCJDE-MDT can fully utilize the coupling between detection and tracking, and finally results in superior joint performance.

Furthermore, the advantages of CCJDE-MDT over RJDE are also demonstrated. Firstly, the complexity of CCJDE-MDT is greatly reduced compared with RJDE. This mainly results from less design parameters in CCJDE-MDT. Second, the performance of CCJDE-MDT is better than RJDE in many scenarios. This mainly results from the consideration of the MDT problem realities when designing parameters in CCJDE-MDT.

Specifically, in RJDE, β_{ij} is designed equally for any i and j, whose underlying assumption is that the penalization for different types of decision errors are the same. This design strategy does not consider the MDT problem realities. Actually, different types of incorrect decisions may cause different losses, and the penalization should be set according to the losses they may lead to, as is analyzed in Section III.E. In CCJDE-MDT, however, the only parameter γ_{ij} is designed skillfully by taking the reality into consideration. We choose $\gamma_{12} > \gamma_{21}$ in this MDT problem since *miss* usually brings heavier losses than *false alarm*. Finally, CCJDE-MDT outperforms RJDE in joint performance.

Remark 9

By comparing the simulation results in different scenarios, it can be seen that the more complex the scenario is, the more obvious the advantages of CCJDE-MDT are. Here, "complex" means the clutter density is high and the detection rate is low, which are not conductive to detection and tracking. In other words, in such "complex" scenarios, target detection and tracking are difficult.

Specifically, scenario 3 is a complex scenario, i.e., the clutter density is high and the detection probability is low. However, simulation results show that the performance advantages of CCJDE-MDT over other methods are more obvious in scenario 3 than in scenario 1/2/4, where in the latter, the clutter density is lower and detection rate is higher than in the former.

In summary, the advantages of the proposed CCJDE-MDT method are especially obvious in complex scenarios. This is of great significance because in practice, it is always such "complex" scenarios that requires more efforts and are also the focus of research.

Remark 10

In this paper, only simple simulation scenarios are presented for illustration since this paper focuses on verifying the effectiveness and superiorities of the newly proposed JDE-based MDT method. The focus of this paper is the utilization of the coupling between detection and tracking so as to improve the joint performance. What we most want to express through this paper is that the proposed method is superior in handling the interdependence between detection and tracking compared with the traditional methods.

Specifically, we did not consider the birth and death of the target, and the target number is assumed to be constant over time. Also, we adopted only $N = 2$ for illustration. These are all used for illustration. For more practical and complicated applications of our proposed method, we will investigate them in the future. Actually, for more targets ($N \geqslant 3$) and considering the birth and death, our proposed method will definitely work since they will not fundamentally change the problem but only increase the computational complexity.

Remark 11

The computational complexity of the proposed method is discussed as follows:

First, the proposed CCJDE-MDT method is point estimation-based method rather than density estimation-based method. Therefore, it has much lower computational complexity compared with the density estimation methods, such as random finite set (RFS) methods.

Second, the main computation comes from the JPDA filter. As is stated in the CCJDE-MDT method, many required quantities in our method can be output by the JPDA filtering. With these basic quantities, we only need to do Bayes optimization by minimizing the proposed CCJDE-MDT risk. The CCJDE-MDT detection and tracking results show that detection can be determined following the Bayes decision rule while tracking can be determined by the weighed combination of hypothesis conditioned state estimates. These are all basic operations which will only increase very few calculation. Therefore, the computational complexity of DTE, RJDE, and the proposed CCJDE-MDT is basically the same without essential difference.

5 Conclusions

This paper proposed a CCJDE-based multi-target detection and tracking (MDT) method for handling MDT problems, which involve interdependent detection and tracking. We first propose a joint CCJDE-based MDT risk, which unifies the detection and tracking risks for multi-targets through a compact way. Based on this risk, we derive the joint solution with an analytical form, where the coupling between detection and tracking is fully utilized. Specifically, JPDA filter is adopted due to its advantageous performance, easy implementation, and also the adaptability to the JDE framework. Also, a CCJDE-MDT algorithm is presented. Finally, parameter designing is systematically and deeply studied, and some important guidelines are provided for practical application.

Simulation results verify the effectiveness and advantages of the proposed CCJDE-MDT method. CCJDE-MDT outperforms the traditional DTE method in joint performance and is also better than RJDE in many cases. The essential reason is that CCJDE-MDT can fully utilize the coupling between detection and tracking, and also take the MDT problem realities into consideration. Finally, it performs best in joint performance. Applications to more complicated MDT problems will be investigated in the future.

Appendix

Denote by m_k^t the targets number in the surveillance region, and denote by n_k^t the number of true measurements at time k. m_k^z denotes all the measurements are received at time k, which contains both the measurements from targets and the measurements from clutters or false alarm measurements. Suppose m_k^t is unknown but constant over time k and its maximum number is N; one target can only generate at most one measurement; one measurement can only come from one target or one clutter. Denote by $Z^k = \{z_1, z_2, \cdots, z_k\}$ the set of all measurements up to time k, and $z_k = \{z_k^1, z_k^2, \cdots, z_k^{m_k^z}\}$ is the set of measurements at time k. Here, z_k^i is the ith measurement and m_k^z is the number of measurement at k.

For each θ_k^l, its posterior probability can be computed as:

$$\begin{aligned} P\{\theta_k^l|z_k, Z^{k-1}\} &= P\{\theta_k^l|z_k, m_k^z, Z^{k-1}\} \\ &= \frac{1}{c} f(z_k|\theta_k^l, m_k^z, Z^{k-1}) P\{\theta_k^l|m_k^z\} \end{aligned}$$

Assuming that each measurement is conditional independent, we have

$$f\{z_k|H^j, \theta_k^l, m_k^z, Z^{k-1}\} = \prod_{j=1}^{m_k^z} f(z_k^j|\theta_k^l, m_k^z, Z^{k-1})$$

and

$$f(z_k^j|\theta_k^l, m_k^z, Z^{k-1}) = \begin{cases} f_t^i(z_k^j), \text{ if } z_k^j \text{ is associated with target } i \\ f_f(z_k^l), \text{ if } z_k^j \text{ is not from any target} \end{cases}$$

where $f_t^i(\cdot)$ and $f_f(\cdot)$ are the probability density functions of the true and false measurements.

Given m_k^z and n_k^t, assume that each event θ_k^l has equal probability, $P\{\theta_k^l|m_k^z\}$ can be derived by

$$\begin{aligned} P\{\theta_k^l|m_k^z\} &= P\{\theta_k^l, n_k^t|m_k^z\} \\ &= P\{\theta_k^l|n_k^t, m_k^z\} P\{n_k^t|m_k^z\} \end{aligned}$$

where $P\{\theta_k^l|n_k^t, m_k^z\} = (m_k^z - n_k^t)!/m_k^z!, P\{n_k^t|m_k^z\} = (P_d P_G)^{n_k^t}(1 - P_d P_G)^{m_k^t - n_k^t} P_f\{m_k^z - n_k^t\}$, and $P_f\{\cdot\}$ is the probability mass function of the number of false measurements. P_d and P_G are the detection and gate probabilities, respectively.

The track update is obtained by the probabilistic average over all θ_k^i. Suppose that all $P\{\theta_k^l|Z_k\}$ are computed, the probability μ_k^{ij} (denotes the probability of associating z_k^j to target i) is obtained by summing up all the probabilities of θ_k^l that contains this association. Then, the track of the target i can be updated by

$$\begin{aligned} \hat{x}_k &= \hat{x}_{k|k-1} + K_k \overline{\overline{z}}_k \\ P_k &= P_k^0 \mu_k^{i0} + (1 - \mu_k^{i0}) P_k^{KF} + \widetilde{P}_k \end{aligned}$$

where $\mu_k^{i0} = 1 - \sum_{j=1}^{m_k^z} \mu_k^{ij}$ is the probability that no measurement is associated with the target, $\bar{\tilde{z}}_k = \sum_{j=1}^{m_k^z} \tilde{z}_k^j \mu_k^{ij}$ is the average measurement residual,

and

$$P_k^{KF} = P_{k|k-1} - K_k S_k^{-1} K_k'$$

$$\widetilde{P}_k = K_k [\sum_{j=1}^{m_k^z} \tilde{z}_k^j (\tilde{z}_k^j)' \mu_k^{ij} - (1 + \mu_k^{i0}) \bar{\tilde{z}}_k \bar{\tilde{z}}_k'] K_k'$$

$$P_k^0 = P_{k|k-1} + \frac{1-\beta}{1 - P_d P_G} K_k S_k^{-1} K_k'$$

K_k is the KF gain at time k, $\beta = \frac{\Gamma_{\gamma/2}(n_z/2+1)}{n_z/2\Gamma_{\gamma/2}(n_z/2)}$, and Γ is the incomplete Gamma function. The derivation for P_k^0 is the MSE matrix for the case that none of the measurements is associated with the target. More details about the JPDA filter can be found in [40].

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Author contributions
WC conceived the idea, proposed the novel approach, and wrote the majority of the manuscript. QL revised the manuscript and provided constructive suggestions. All authors read and approved the final manuscript.

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Declarations

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Not applicable.

Consent for publication
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Competing interests
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