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# A Novel Method for Type Synthesis of Parallel Mechanism Without Parasitic Motion Based on 2R1T Parallel Mechanism with Rotational Bifurcation

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## Abstract

The parasitic motion has been widely recognized as the major drawback of the parallel mechanism. Therefore a class of 2R1T PMs (parallel mechanism) without parasitic motion has been synthesized. However, these PMs can only rotate around two axes in sequential order. It decreases the performance of the balancing adjustment of the end-effector. In this paper, a family of 2R1T PMs without parasitic motion was reconstructed by using a novel method based on the remarkable properties of rotational bifurcation mechanisms, which can rotate in sequential order. Furthermore, some PMs rotating around two continuous axes in an arbitrary order are established by adding single joints. Taking the practicability of these structures into consideration, the workspace of 3-PRPS PM was analyzed as an example. Moreover, this study explores the practical application of the PMs without parasitic motion in developing balance mechanisms in rough-terrain fire-fighting robots. During the climbing process, the tank is adjusted to be parallel to the horizontal plane in real-time. It is proved that this kind of structure realizes continuous rotation around two rotation axes on the premise of no parasitic motion.

**Keywords:** Parallel mechanism, Rotational bifurcation, Parasitic motion, Type synthesis, Balance mechanism

## 1 Introduction

One of the most widely used types of lower mobility parallel mechanisms is the 2R1T parallel mechanism, where R refers to a rotational DOF and T means a translational DOF. Carretero et al. [1] found that the 3-PRS PM produced three types of motion in constrained DOFs: a rotation around the  $Z$ -axis, a translation along the  $X$ -axis, and a translation along the  $Y$ -axis during the motion, which was called parasitic motion. A major drawback of most 2R1T PMs is their inherent parasitic motion, which makes the position and direction of the end-effector less controllable. Consequently, 2R1T PMs without parasitic motion were proposed to eliminate the parasitic motion

which break up the rotation of the end-effector into a rotation around two orthogonal axes. Hence the rotation angles are controlled by only one or two actuators on each branch. On the other hand, the rotation center of the end-effector of such PMs only move axially in the direction of the translational DOF T, which means that the position of the rotation center does not change as a result of the rotation. Therefore, this type of PM is applied to the focusing system of astronomical telescopes and leveling system of onboard platforms, etc.

Many experts and scholars have devoted themselves to the study of 2R1T PMs without parasitic motion. These studies mainly focused on two parts: type analysis and application. For type analysis, a series of PMs without parasitic motion accompanied by five joints per branch [2] was synthesized using Lie group theory. And a 3-CRC symmetrical parallel mechanism was proposed by Chen

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et al. [3], which enabled one translation and two rotations without parasitic motion on the mid-plane. Zhao et al. [4] constructed a 3-UPU PM without parasitic motion, and the PM do non-parasitic 2R1T motion on the mid-plane as well. Besides, in the aspect of the application, the RPS/2-RPU PM [5] was used in the lockable spherical joint-based parallel kinematic machine. Gan et al. [6] investigated the geometrical characteristics of the 3-PUP mechanism in a special assembly and proposed an application in developing a rehabilitation device for the sprained subtalar joint. Yang et al. [7] designed a 3-PPS PM to perform continuous contact operations, such as deburring, grinding, and polishing. Chen et al. [8] presented a novel over-constrained 2RPU-RPS-UPS PMs without parasitic motion and analyzed the reason for non-parasitic motion. Although 2R1T PMs without parasitic motion have been extensively studied and designed, the end-effectors of most structures occurred the problem of compulsory sequential rotation under the circumstance of rotation containing two axes. Otherwise, these structures will generate infinitesimal rotations which indicated their bifurcate in another sequence.

To overcome the abovementioned drawback, the type synthesis of 2R1T PMs without parasitic motion is the priority in this study. Various methods have been proposed and applied to type synthesis. Most of these methods fell into two main categories: group theory [9, 10] and screw theory [11, 12]. Fan et al. [13] synthesized several 2R1T, 2R2T, and 2R PMs based on the integration of configuration evolution and Lie group theory. Wei and Dai [14] proposed a method for synthesizing metamorphic 2R1T PMs with Lie subgroup. Ye et al. [15] analyzed the motion of the UP-equivalent PMs, and synthesized the limb bonds based on Lie group theory. Xu et al. [16] discussed the ultimate constraint wrenches of 2R1T PMs and synthesized a series of novel overconstrained 2R1T PMs with the fewest kinematic joints. Using finite screw and topological theory, Yang et al. [17] presented a novel 3-DOF translational PM type synthesis method. Li et al. [18] proposed a method to synthesize serial limbs by adding single joints to the basic limbs and constructed single-loop PMs with 3T, 2R1T, and 1R2T motion. Combining the screw theory and space geometry, Xu et al. [19] synthesized several 2R PMs with two continuous axes. Kong and Gosselin [20, 21] constructed the UP-equivalent PMs and PPR-equivalent PMs by using a virtual-chain approach. Zhao and Zi [22] synthesized some types of PMs to construct a bionic human shoulder based on the structure of muscles and bones. Sun and Huo [23] discussed four cases of position and orientation of the rotation axis, showing the new method of type synthesis for a 2R1T PM with parasitic motion. Dos Santos et al. [24] added kinematic joints at the end of the branches

to limit the DOFs of the branches. Wang et al. [25] synthesized a parallel mechanism satisfying 2R1T motion by calculating the virtual work of the end-effector on the branch. Using the upper triangular matrix to express different branches, Zou et al. [26] proposed a new method for synthesizing 3T PM. Gao et al. [27, 28] proposed  $G_F$  set theory and developed a new method of type synthesis. Li et al. [29] proposed a new way to synthesize 2R1T and 2T1R 3-DOF redundant actuated PMs based on the Atlas method. However, there are few studies on the type synthesis of 2R1T PMs without parasitic motion. Li and Hervé [2] investigated the Lie groups and Lie subgroups of the 2R1T PMs without parasitic motion, and synthesized a series of types. Each branch of these structures contained at least 5 single joints, which are not the optimal structures of the branches. Therefore, it is necessary to propose a novel method for synthesizing 2R1T PMs without parasitic motion.

Considering the drawbacks mentioned above, in this paper, the 2R1T PMs without parasitic motion were synthesized through the characteristics of rotational bifurcation in Section 2. Then in Section 3, the approach to eliminate the compulsory sequential rotation is presented. In Section 4, the workspace of selected PMs are calculated analytically. Afterward, an application of a tank balance adjustment platform for a rough-terrain fire-fighting robot is shown in Section 5.

## 2 Type Synthesis of Novel 2R1T PMs Without Parasitic Motion

The optimal 2R1T mechanism consists of one decoupled translational DOF and two decoupled rotational DOFs, however, most of the 2R1T PMs are accompanied by displacement during rotating. 2R1T PMs without parasitic motion are equivalent to parallel mechanisms without parasitic translation according to the theory of Carretero et al. and Li et al. [1, 2], which means that the 2R1T PMs are decoupled along the direction of translational DOF T. It is worth noting that there is a feature of 2R1T PMs with rotational bifurcation which meets the requirement of no coupled translation, the end-effector will not translate along any axis during rotating, and the position of type rotation center change only along the direction of T DOF. Therefore, 2R1T PMs without parallel motion can be synthesized by using the characteristics of rotational bifurcation.

The main reason for the rotational bifurcation is the change of the correlation between constraint couples applied by branches to the end-effector. As shown in Figure 1, at the home position (the end-effector is parallel to the horizontal plane), each branch applies on the end-effector a constraint force parallel to the  $xoy$  plane and a constraint couple perpendicular to the plane formed by

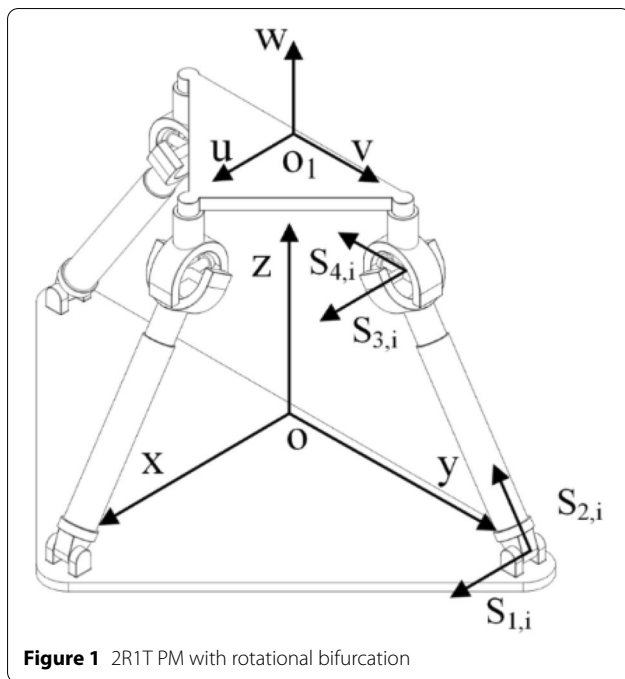


Figure 1 2R1T PM with rotational bifurcation

the two axes of the U joint. As the end-effector rotates, the normal line directions of the constraint couples change. According to Grassmann line geometry theory, the constraint couples' linearly independence number is 2, which means that the constraint couples restrict the rotation along w-direction as well as the rotation along the u-axis or v-axis. Consequently, the 2R1T PMs with rotational bifurcation may have the following two forms:

- (1) The directions of the constraint forces exerted by branches vary with the rotation of the end-effector, and the normal line directions of the constraint couples are parallel to the z-axis.

Since the constraint forces applied by branches to the end-effector vary with the rotation of the end-effector, the constraint forces of branches can be expressed as:

$$\begin{cases} \mathbf{S}_i^f = (\cos \beta \ 0 \ \sin \beta \ 0 \ 0 \ 0)^T, \\ \mathbf{S}_i^c = (0 \ \cos \alpha \ \sin \alpha \ 0 \ 0 \ 0)^T, \end{cases} \quad (1)$$

where  $\alpha$  indicates the angle of rotation of the moving stage around the  $u$ -axis;  $\beta$  indicates the angle of rotation of the moving stage around the  $v$ -axis.

When the end-effector is rotated, it is impossible to achieve decoupling because the constraint forces

Table 1 Constraint wrenches of 2R1T PMs with rotational bifurcation

Wrenches of 2R1T PMs with rotational bifurcation		
Case 1		Constraint forces are coplanar
Case 2		
Case 3		Constraint forces are planar convergences

restrict the translations in all directions, and the T DOF is eliminated. Consequently, the 2R1T PMs without parasitic motion can not be constructed in this condition.

- 2) The normal line directions of constraint couples imposed by branches vary with the rotation of the end-effector, and the directions of the constraint forces are parallel to the  $xoy$  plane. This condition is further split into three cases (Table 1).

**Case 1:** Constraint forces are coplanar, and the normal lines of the constraint couples vary with the rotation of the end-effector.

As shown in Table 1, the constraint forces exerted by the branches constitute a couple which normal line direction is parallel to the  $z$ -axis. Since the linear independence of the former couple and the constraint couples change during rotation, the rotational bifurcation occurred.

Assume that the origin of the base coordinate system is established at the projection of end-effector's rotation center on the fixed platform, and the origin of the local coordinate system is established at the rotation center of the end-effector. In order to describe the position of the end-effector of 2R1T PMs without parasitic motion, the position vector of the local coordinate system is:

$$P = [0 \ 0 \ \sigma]^T, \tag{2}$$

where  $\sigma$  is a variable that increases or decreases as the end-effector is translated along the decoupling direction. Then the twists of the end-effector in the base coordinate system can be expressed as follows:

$$S^m = \begin{cases} S_1 = (1 \ 0 \ 0 \ 0 \ \sigma \ 0)^T, \\ S_2 = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_3 = (0 \ 0 \ 0 \ 0 \ -\sin \alpha \ \cos \alpha)^T. \end{cases} \tag{3}$$

The sum aggregate of the wrenches imposed by the branches is:

$$S^r = \begin{cases} S_1^r = (1 \ 0 \ 0 \ 0 \ \sigma \ 0)^T, \\ S_2^r = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_3^r = (0 \ 0 \ 0 \ 0 \ -\sin \alpha \ \cos \alpha)^T. \end{cases} \tag{4}$$

According to Dai's theory [30], the wrenches transform from the base coordinate system to the local coordinate system of each branch through the transformation matrix  $N$ .

$$N = \begin{bmatrix} R & O \\ AR & R \end{bmatrix}, \tag{5}$$

where  $R$  is the rotation matrix from the base coordinate system to the local coordinate system, and  $A$  is the skew-symmetric matrix of the position vector in the base coordinate system concerning to the local coordinate system. So the wrenches in the local coordinate system can be derived as follows:

$$S_i^r = [N]S^r. \tag{6}$$

If an individual branch consists of three single joints, then the twists of each branch can be obtained from the wrenches of each branch by reciprocal product:

$$S_i^m = \begin{cases} S_{i,1} = (1 \ 0 \ 0 \ 0 \ \sigma \ -a)^T, \\ S_{i,2} = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_{i,3} = (0 \ 0 \ 0 \ 0 \ -\sin \alpha \ \cos \alpha)^T, \end{cases} \tag{7}$$

where  $a$  is the distance between the base coordinate system and the branch local coordinate system.

As the branches consist of joints in series, the twists of a branch can be obtained by adding up the twists of the joints, which means that:

$$S_i = \omega_1 S_{i1} + \omega_2 S_{i2} + \omega_3 S_{i3} + \dots, \tag{8}$$

where  $\omega_i$  is the amplitude of each joint, and  $S_{ij}$  is the axial direction of twist of the  $j$ th joint in the  $i$ th branch.

According to Eq. (8), each configuration is synthesized by linear combination. If the joint that connects to the

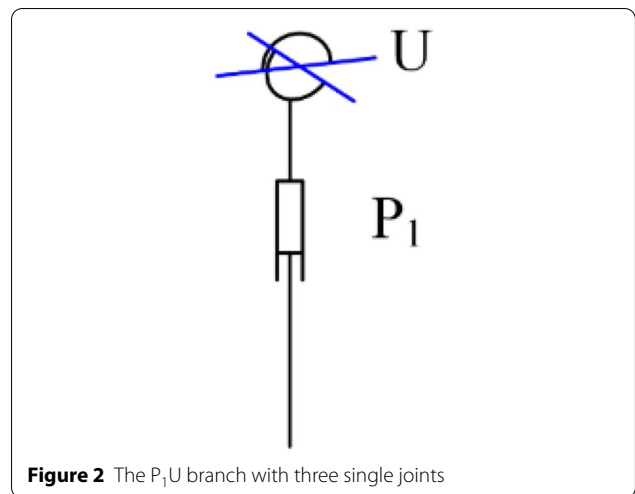


Figure 2 The P<sub>1</sub>U branch with three single joints

fixed platform has the same screw as  $S_{i,1}$  or  $S_{i,2}$  in Eq. (7), the position of the joint changes with  $\sigma$  because both  $S_{i,1}$  and  $S_{i,2}$  have variable  $\sigma$ , and  $\sigma$  cannot be eliminated by linear combination. Consequently, the structure of the three-single-joint branch can be established as shown in Figure 2.

Although the configuration of three single joints can be constructed, the branch only translates along the axial direction of  $P_1$ , and it cannot form 2R1T PMs with rotational bifurcation. So this group of wrenches is not suitable.

Suppose that each branch is composed of four single joints. In order to constitute a constraint torque whose normal line is parallel to the  $z$ -axis, the constraint forces of the branches should be coplanar and not intersect at a single point. Then according to Eq. (6), the wrenches imposed by each branch are

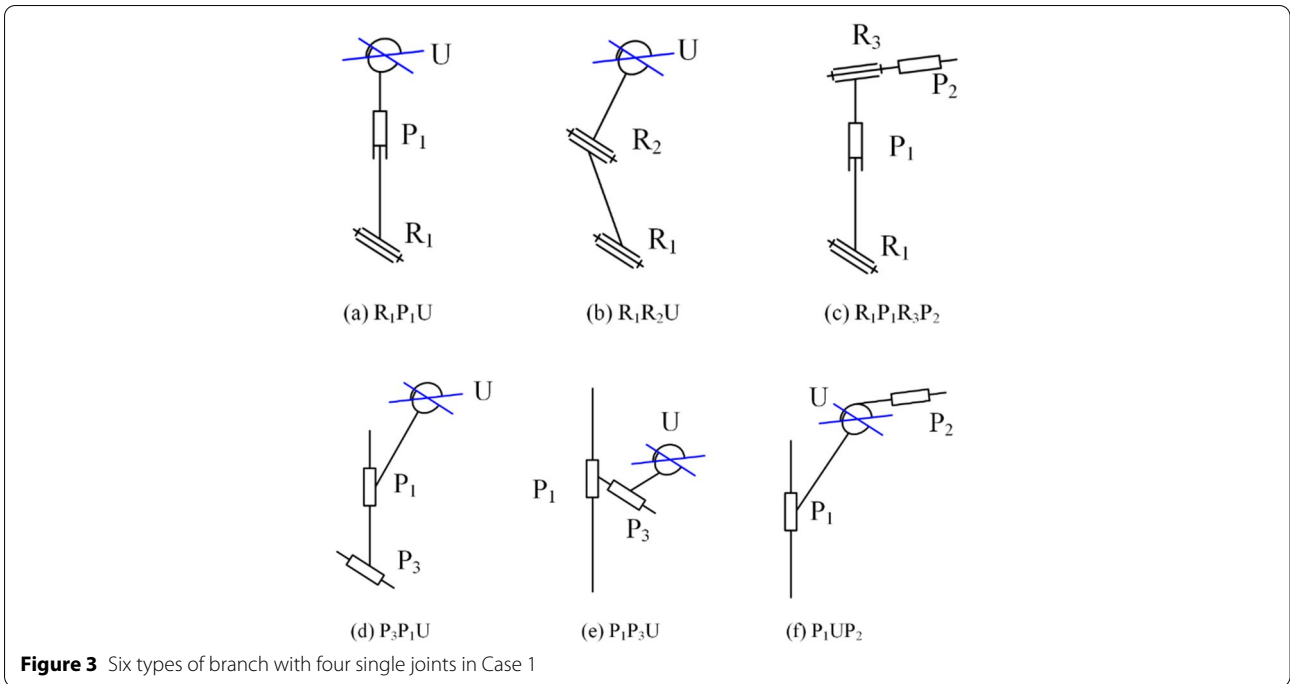
$$S_i^r = \begin{cases} S_{i,1}^r = (1 \ 0 \ 0 \ 0 \ \sigma \ -a)^T, \\ S_{i,2}^r = (0 \ 0 \ 0 \ 0 \ -\sin \alpha \ \cos \alpha)^T. \end{cases} \tag{9}$$

Moreover, the twists can be achieved by reciprocal product:

$$S_i^m = \begin{cases} S_{i,1} = (1 \ 0 \ 0 \ 0 \ \sigma \ -a)^T, \\ S_{i,2} = (0 \ \cos \alpha \ \sin \alpha \ -\sigma \ \cos \alpha + a \sin \alpha \ 0 \ 0)^T, \\ S_{i,3} = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T, \\ S_{i,4} = (0 \ 0 \ 0 \ 0 \ -\sin \alpha \ \cos \alpha)^T. \end{cases} \tag{10}$$

According to Eq. (8), the following configurations can be synthesized as Figure 3.

In the kinematic joints shown in Figure 3, the axis of  $R_1$  is perpendicular to the axis of  $R_3$ , and the axial direction of  $P_2$  is perpendicular to the axial direction of  $P_3$ .

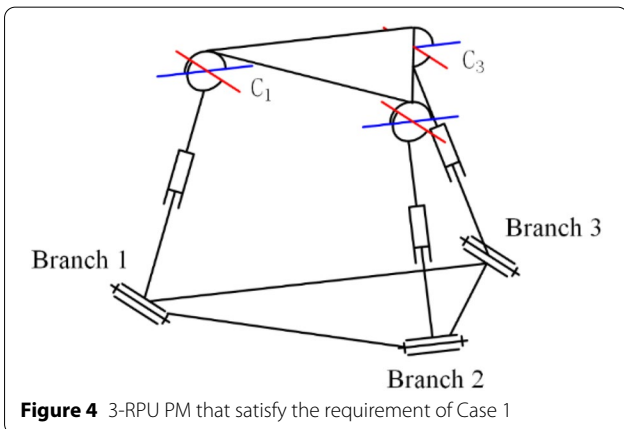


However, the axial direction of the first axis of the U-joint in a branch has to be changed when constructing a 3- R<sub>1</sub>P<sub>1</sub>U PM or 3- R<sub>1</sub>R<sub>2</sub>U PM with rotational bifurcation, as shown in Figure 4. The first axes of the U-joints are represented by red lines, and the axial directions of that in branches 1 and 3 are parallel to the revolute joints' axial direction which connect to the fixed platform, so that the normal lines of the constraint couples are perpendicular to the plane formed by the two rotation axes of the U-joint. To make the normal line of constraint couple parallel to that in branches 1 and 3, the direction of the first axis of the U joint in branch 2 is perpendicular to the revolute joint. C<sub>1</sub> and C<sub>3</sub> are the intersection points of the two rotation axes in branches 1 and

3, respectively. When the end-effector rotates around C<sub>1</sub>C<sub>3</sub>, the normal line direction of the constraint couples exerted by each branch varies with rotation.

**Case 2:** Constraint forces are coplanar. Normal lines of two constraint couples are parallel to each other, and another one has a different direction.

Case 2 is the same as Case 1, as shown in Table 1. In order for the end-effector to have bifurcated rotational DOFs, the normal lines of the two constraints couples are supposed to be parallel, and another normal line distinct from the former. When the end-effector rotates around any axis, there always be one or two normal lines that vary as the end-effector rotates, while the others are parallel to the z-axis. The wrenches and the twists for Case 2 are as follows:



$$S_i^r = \begin{cases} S_{i,1}^r = (1 \ 0 \ 0 \ 0 \ \sigma \ -a)^T, \\ S_{i,2}^r = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T, \end{cases} \quad (11)$$

$$S_i^m = \begin{cases} S_{i,1} = (1 \ 0 \ 0 \ 0 \ \sigma \ -a)^T, \\ S_{i,2} = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_{i,3} = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T, \\ S_{i,4} = (0 \ 0 \ 0 \ 0 \ 1 \ 0)^T. \end{cases} \quad (12)$$

The six structures obtained according to Eq. (8) are shown in Figure 5.

Case 2 illustrates a condition that one of the normal line directions of the constraint couples is parallel to



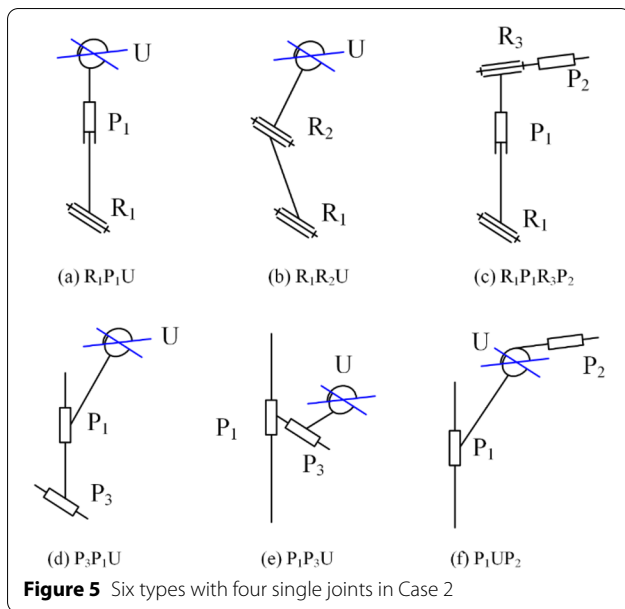


Figure 5 Six types with four single joints in Case 2

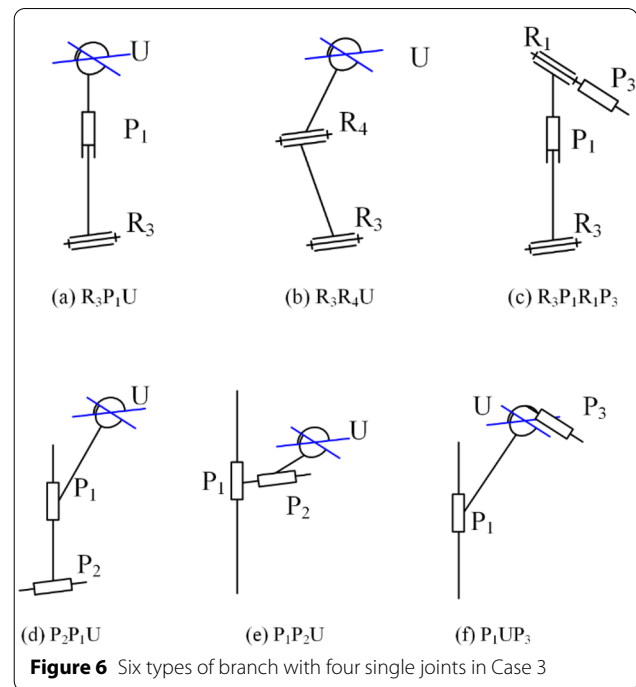


Figure 6 Six types of branch with four single joints in Case 3

the normal line direction of the torque formed by the constraint forces, which is a particular case of Case 1. These six structures mentioned in Case 2 are the same as those in Case 1 as well. However, in these structures, the first axis of the U-joint in each branch should be parallel to the direction of the constraint force. The PMs constructed by the configurations of Case 2 and Case 1 both have two decoupled rotational DOFs and a decoupled translational DOF at the initial position. However, when the end-effector rotates, the two rotational DOFs of the parallel mechanism bifurcate.

**Case 3:** Constraint forces are planar convergences. And normal lines of two constraint couples are parallel to each other, and another one has a different direction.

The difference between Case 2 and Case 3 is that the axial directions of the constraint forces intersect at a point. Respectively, the wrenches and twists required to be satisfied by the branches are:

$$S_i^r = \begin{cases} S_{i,1}^r = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_{i,2}^r = (0 \ 0 \ 0 \ 0 \ -\sin \alpha \ \cos \alpha)^T, \end{cases} \quad (13)$$

$$S_i^m = \begin{cases} S_{i,1}^m = (0 \ \cos \alpha \ \sin \alpha \ 0 \ 0 \ 0)^T, \\ S_{i,2}^m = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_{i,3}^m = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T, \\ S_{i,4}^m = (0 \ 0 \ 0 \ 1 \ 0 \ 0)^T. \end{cases} \quad (14)$$

According to Eq. (8), the suitable branches cannot be constructed through a linear combination of individual twists since  $S_{i,1}$  contains the variable  $\sigma$  and can't be eliminated.

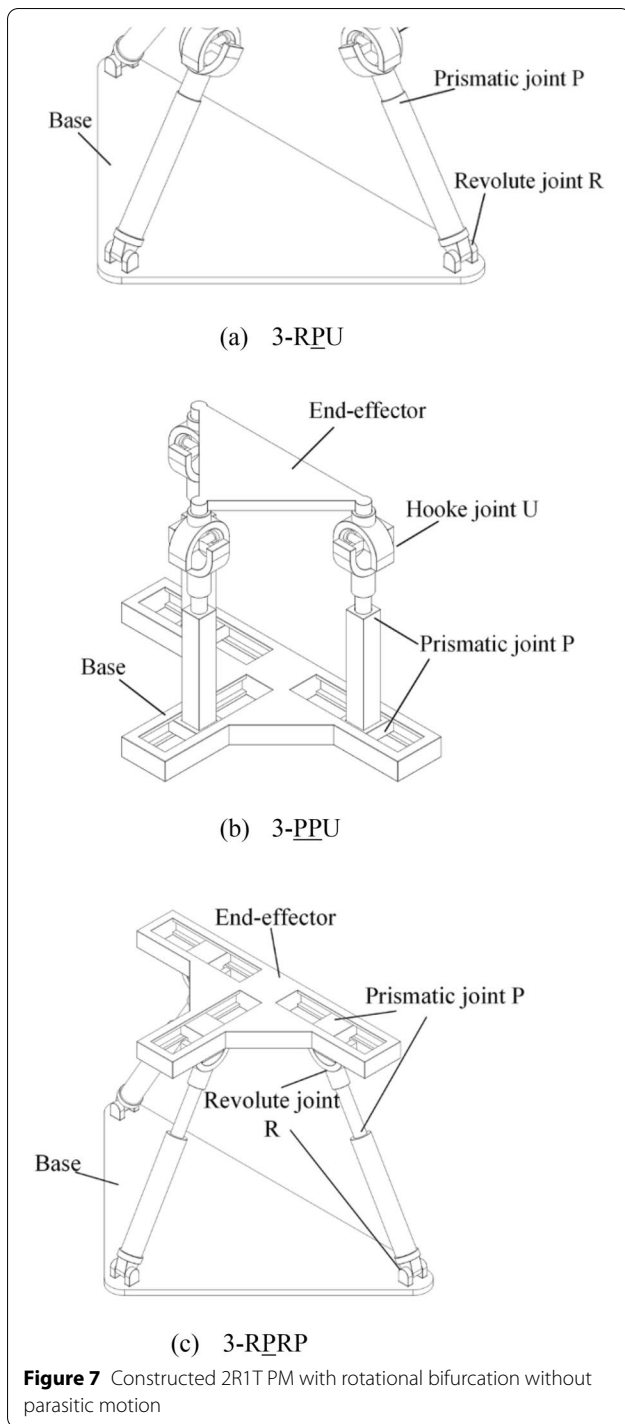
Considering that Case 3 may synthesize the types of 2R1T PMs with rotational bifurcation as well, a special case is discussed. Suppose that the normal line directions of the constraint couples are parallel to the z-axis in the home position, and at that moment the end-effector has the properties of the rotational bifurcation. Their wrenches and twists are as follows:

$$S_1^r = \begin{cases} S_{11}^r = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_{12}^r = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T, \end{cases} \quad (15)$$

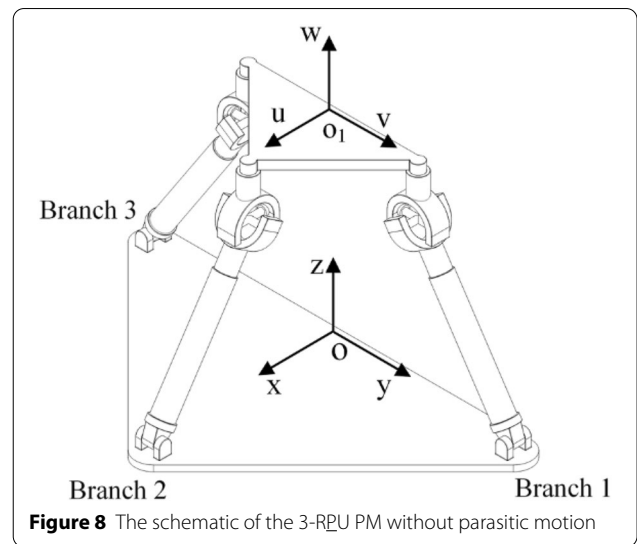
$$S_1^m = \begin{cases} S_1 = (1 \ 0 \ 0 \ 0 \ \sigma \ -a)^T, \\ S_2 = (0 \ 1 \ 0 \ -\sigma \ 0 \ 0)^T, \\ S_3 = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T, \\ S_4 = (0 \ 0 \ 0 \ 1 \ 0 \ 0)^T. \end{cases} \quad (16)$$

So that the structures are established as shown in Figure 6, the PMs formed by the branches cannot rotate along the selected axes, and the rotation center of the end-effector change with rotation. So Case 3 is inappropriate.

In Figures 3 and 5, structures (a) and (b) are equivalent tandem mechanisms to each other. Structures (e) and (f) are similar to (d) and can be seen as variants of (d). So the structures of three-branch 2R1T PMs with rotational bifurcation without parasitic motion are shown in Figure 7, in which all the branches have four single joints.



However, the structures described above are infinitesimal 2R1T mechanisms. It means that these structures can achieve flexible 2R1T movements only for infinitesimal rotation angles from the bifurcated configuration. When the end-effector is rotated, the plane formed by the two rotational axes is no longer parallel to the initial position,

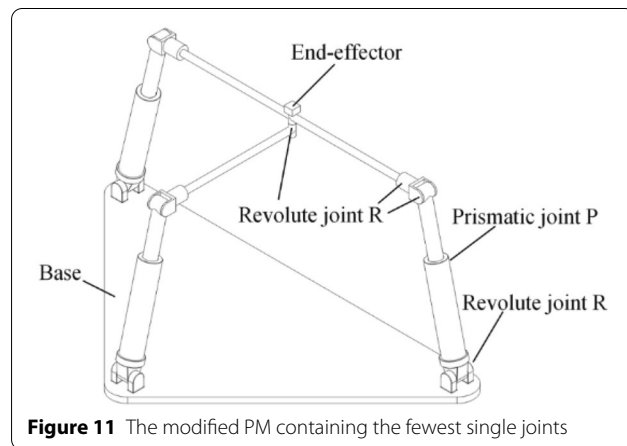
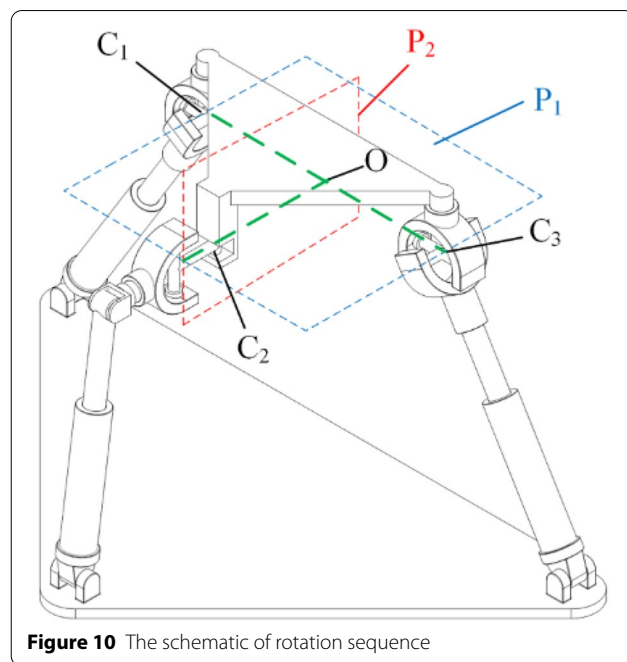
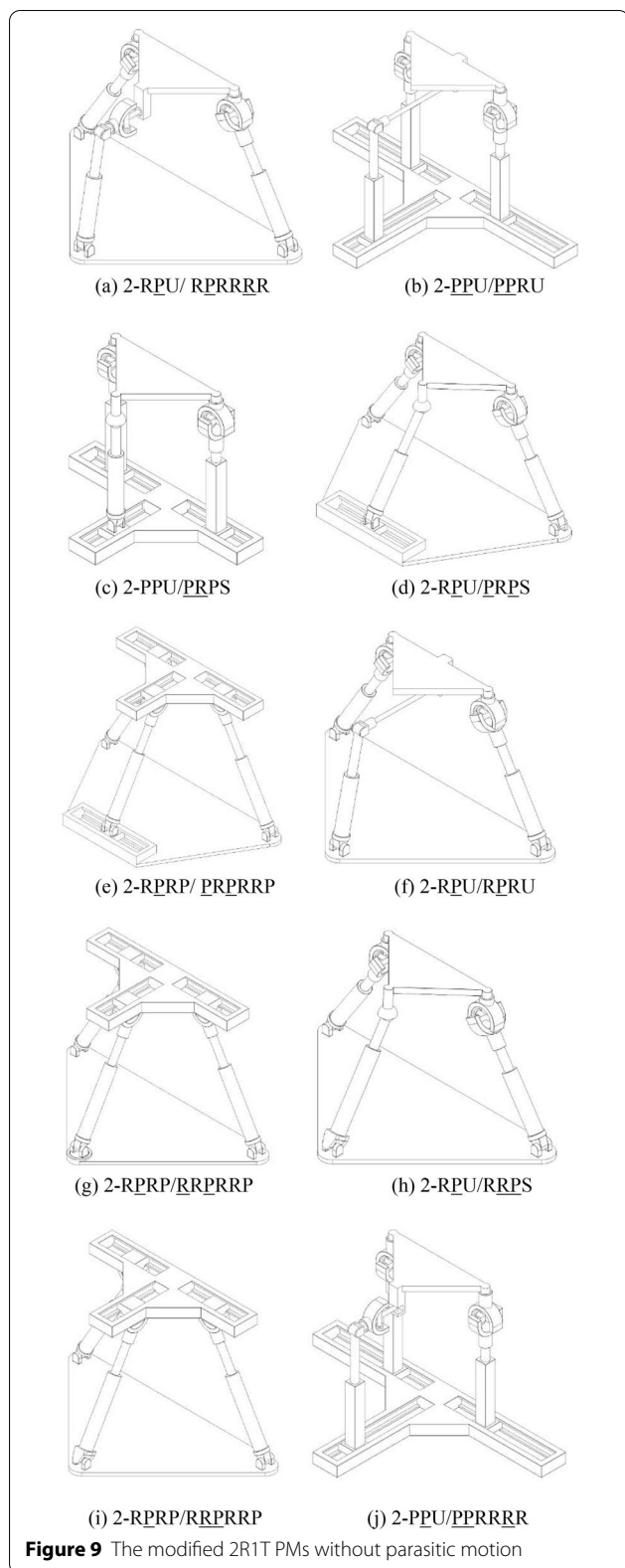


which results in a rotation being restricted, obviously, the 2R1T PMs are converted into 1R1T PMs, as shown in Figure 8. It happens because branch 1 and branch 3 can only move in the  $yoz$  plane, while branch 2 can only move in the  $xoz$  plane and lack a DOF in the plane's normal line direction. For such PMs, there is no way to achieve their goal when the rotational angle around the  $u$ -axis coexists with the rotational angle around the  $v$ -axis. This problem can be effectively solved by adding some single joints to the branches to give the parallel mechanism extra spatial freedom.

Figure 9 shows 10 structures that add single joints in only one branch, all of which are 2R1T PMs without parasitic motion.

### 3 Approach to Constructing PMs with an Arbitrary Rotation Sequence

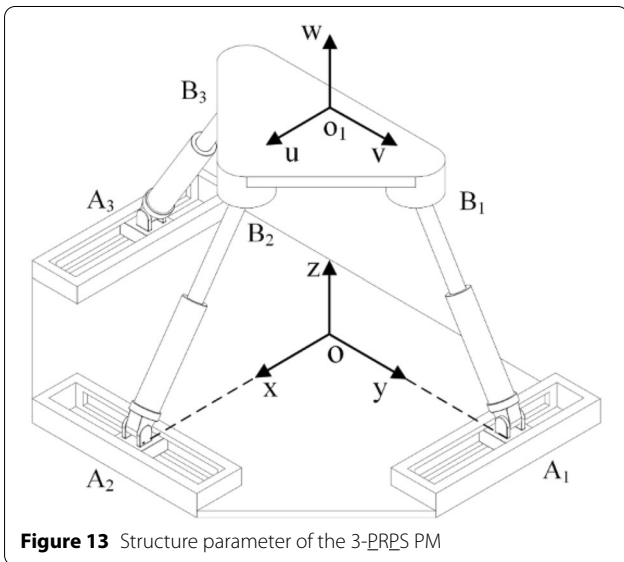
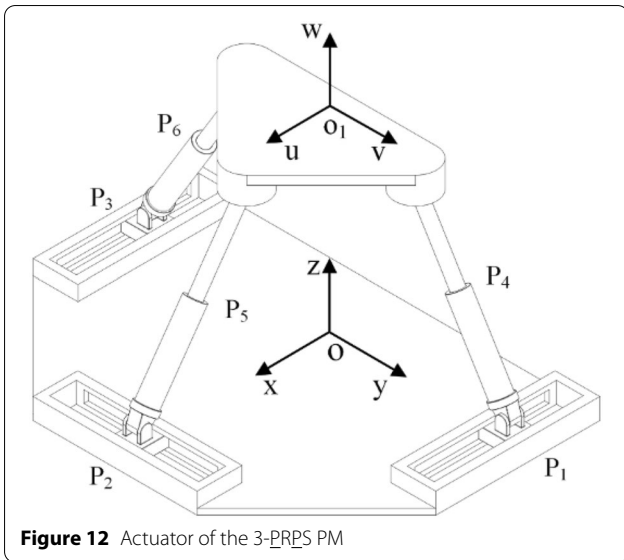
The 10 structures shown in Figure 9 must follow a particular rotation sequence to avoid infinitesimal rotations.  $2-RPU/RPRRRR$  structure of Figure 9a is used as an example, as shown in Figure 10. The plane  $P_1$  formed by the two rotation axes of the U-joints in the RPU branch is parallel to the end-effector. The plane  $P_2$  formed by the rotation axes of the last two R-joints in the  $RPRRRR$  branch is perpendicular to the end-effector plane. The line  $C_1C_3$  connecting the centers of the U-joints in two RPU branches forms the first revolute axis of the end-effector. And  $C_2O$ , which is the intersection line of  $P_2$  and  $P_1$ , is the second revolute axis of the end-effector. In the sequence of rotating around  $C_1C_3$  first and  $C_2O$  second, the end-effector satisfies the motion requirement of 2R1T. Whereas when the sequence is adjusted so that rotation around  $C_2O$  comes first and then around  $C_1C_3$ ,



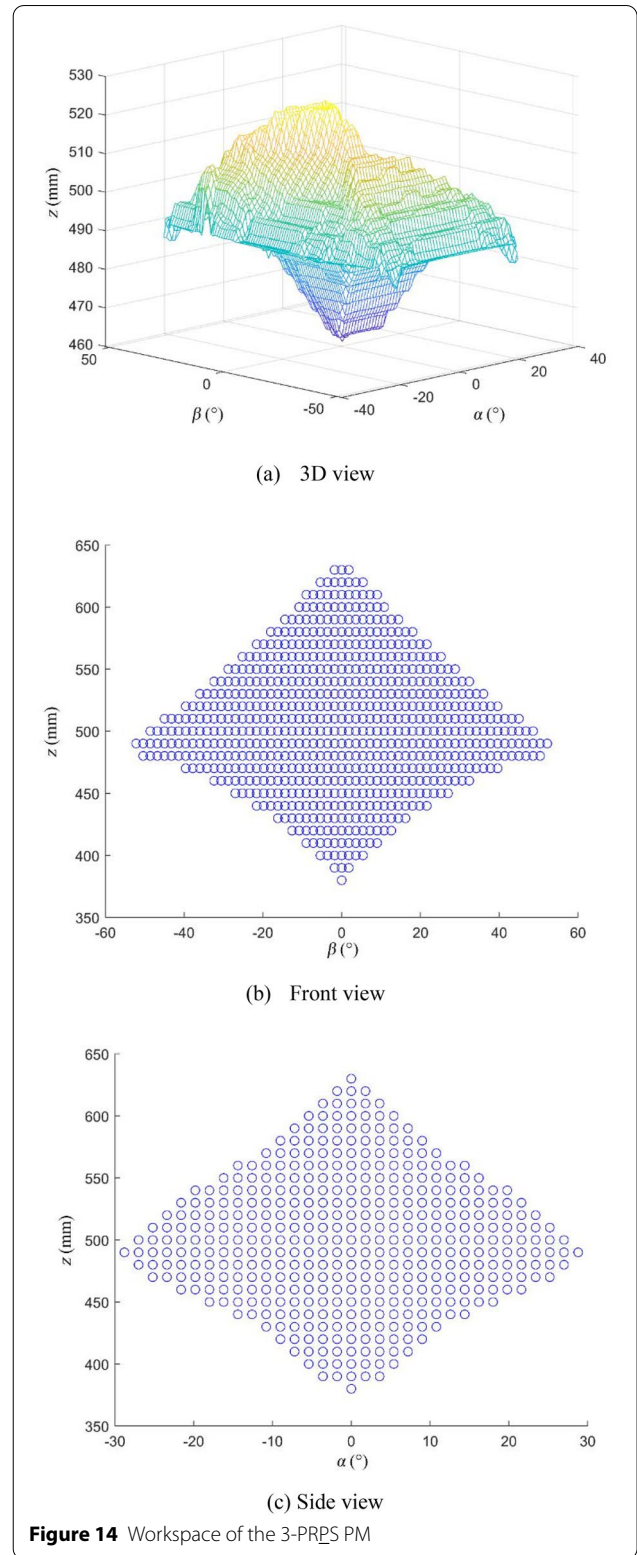
the rotational bifurcation appears, and the latter rotation is an infinitesimal one.

Adding single joints to all branches is indeed a feasible approach to establishing PMs without parasitic motion that rotate around two continuous axes in an arbitrary order. It is essential to note that the modified branches contain five or six single joints, and the parallel mechanisms constituted by these branches are 3R1T PM and 6-DOF PMs. In these structures, the modified PM shown in Figure 11 contains





the fewest single joints, which has three rotational DOFs and a translational DOF. However, the end-effector of this PM may occur constraint singularity because the connection between the branches and the end-effector are revolute joints installed in the rotation center. Therefore, only the modified 6-DOF PMs can simultaneously achieve no parasitic 2R1T motion and rotation in an arbitrary order. It requires rational allocation of actuators. The modified PM based on Figure 9d is shown in Figure 12 as an example. The actuator  $P_4$ ,  $P_5$  and  $P_6$  drive the end-effector to rotate. As the end effector rotates continuously around  $u$ -axis and



$v$ -axis, an extra rotation that rotates around  $w$ -axis will be generated. The extra rotational DOF is coupled with the other two rotational DOFs. The actuators  $P_1$ ,  $P_2$ , and  $P_3$  are used to compensate for this coupled rotation without actively affecting the motion of the end-effector.

#### 4 Workspace

In the previous section, the 10 2R1T PMs without parasitic motion were fully enumerated, and all of them can eliminate the rotational bifurcation caused by the rotation sequence by adding single joints. In order to find out the most suitable working condition for these structures, the analysis of the workspaces are priority. The modified PM shown in Figure 12 is used as an example in this section, and the structure parameter of it is illustrated in Figure 13. The main constraints and parameters of this structure are described below:

$$\begin{cases} o_1B_1 = o_1B_3 = 225 \text{ mm}; o_1B_2 = 300 \text{ mm}, \\ oA_1 = oA_3 = 350 \text{ mm}; oA_2 = 450 \text{ mm}, \\ 300 \text{ mm} \leq oo_1 \leq 800 \text{ mm}, \\ 400 \text{ mm} \leq A_iB_i \leq 650 \text{ mm}. \end{cases}$$

Because of the difference between  $oA_1$  and  $oA_2$ , the different rotation sequence will lead to different results. Consequently, the workspace is the intersection of the two results. The travel of the actuator in the sequence that rotates around the  $u$ -axis before rotating around the  $v$ -axis is derived below:

$$\begin{cases} A_2B_2 = \sqrt{(oA_2 - o_1B_2 \cos \alpha)^2 + (oo_1 + o_1B_2 \cos \beta \sin \alpha)^2}, \\ A_1B_1 = \sqrt{(-o_1B_1 \cos \beta + oA_1)^2 + (o_1B_1 \sin \beta + oo_1)^2}, \\ A_3B_3 = \sqrt{(o_1B_3 \cos \beta - oA_3)^2 + (-o_1B_3 \sin \beta + oo_1)^2}. \end{cases} \quad (18)$$

Consider another different rotation sequence:

$$\begin{cases} A_2B_2 = \sqrt{(oA_2 - o_1B_2 \cos \alpha)^2 + (oo_1 - o_1B_2 \sin \alpha)^2}, \\ A_1B_1 = \sqrt{(-o_1B_1 \cos \beta + oA_1)^2 + (-o_1B_1 \cos \alpha \sin \beta + oo_1)^2}, \\ A_3B_3 = \sqrt{(o_1B_3 \cos \beta - oA_3)^2 + (o_1B_3 \cos \alpha \sin \beta + oo_1)^2}. \end{cases} \quad (19)$$

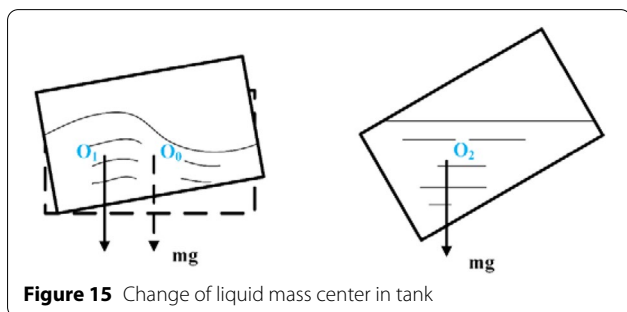


Figure 15 Change of liquid mass center in tank

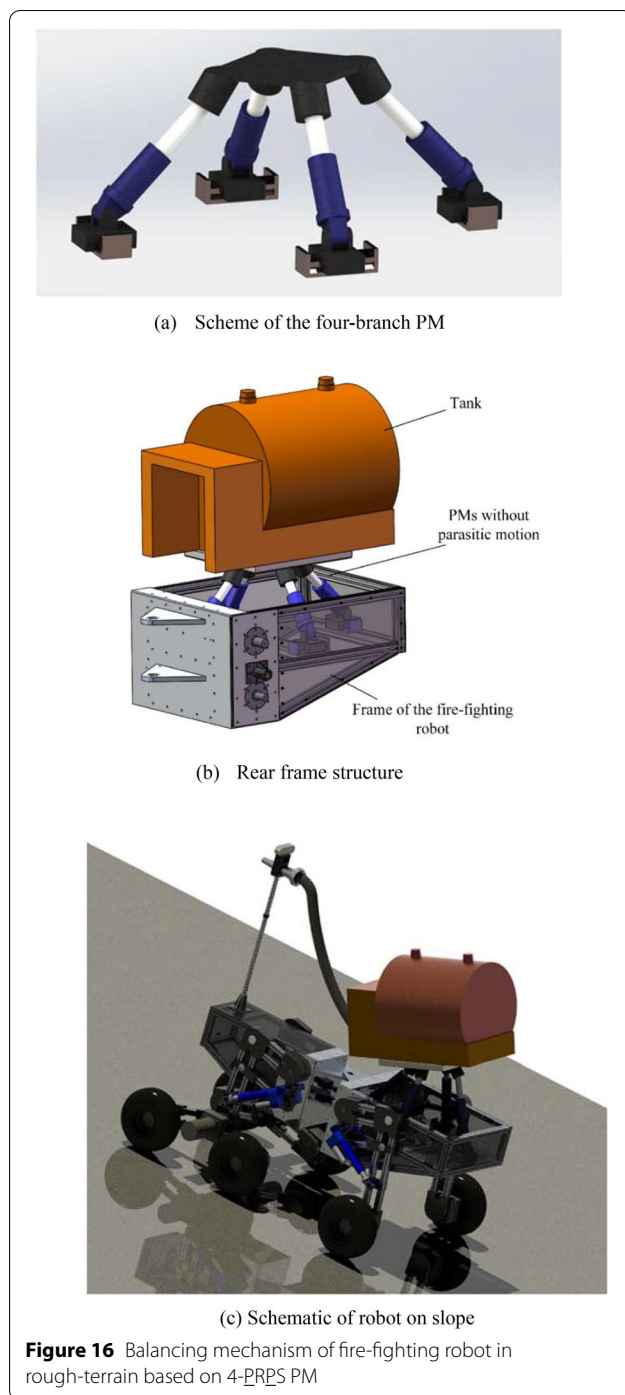
Its workspace is shown in Figure 14, in which Figure 14a–c show its 3D view and the front and side views respectively. The workspace is similar to two regular pyramids connected by the bottom, which has a cross-section approximating a square. Under the conditions of  $oo_1=490$  mm, the end-effector has maximum angles of rotation, which are  $\alpha=28.8^\circ$  and  $\beta=52.2^\circ$ . When the end-effector is rotated continuously around two axes, the operating angles are mainly in the range of  $-27.8^\circ < \alpha < 27.8^\circ$ ,  $-30.6^\circ < \beta < 30.6^\circ$ .

#### 5 Application of PMs Without Parasitic Motion in the Rough-Terrain Fire-Fighting Robots

Operating on rough terrain is a big test for the stability of fire-fighting robots that spray liquid fire retardants. Due to the complexity of the rough-terrain and the frequent highs and lows, the liquid in the tank oscillates with the frame, causing an impact on the tank walls, while the offset of the mass also makes the side acceleration of the frame increase, which is detrimental to the stability of the body. This effect is particularly pronounced in a half tank of liquid, as shown in Figure 15. As the angle of inclination increases, the mass center of the liquid in the tank changes from  $o_0$  to  $o_2$ .

Since the centroid position of the end-effector of the PMs without parasitic motion do not change with rotation, it can be used as the tank balancing adjustment mechanism to solve the above problems, and the barycenter of the tank will not change greatly, which is convenient for accurate control. As the axial directions of the R-axis which connect the fixed platform and branches are parallel or perpendicular, it is easy to extend from three to four branches as well. It has the same reachable angle and an increased support area. As Figure 16a shows, the four-branch PM has better symmetry as well as stiffness and stability.

As previously mentioned, the PM without parasitic motion has two continuous rotation axes in the workspace that are perpendicular to each other. Therefore, the end-effector allows horizontal adjustment by simply rotating around the two axes. Whether the robot is working uphill or on a slope, it is possible to keep the water tank parallel to the horizontal as long as the gradient is not more than operating angles, as shown in Figure 16b, c.



Compared to general PMs, the rotational DOFs of PMs without parasitic motion are decoupled from the translational DOFs, so that rotation is not accompanied by a change in the position of the rotation center, reducing the flow of liquid in the tank. On the other hand, as a balancing mechanism, it is easier to calculate the telescopic travel of the actuator when adjusting the angles, making it less difficult to control.

## 6 Conclusions

This study proposed a method to synthesize types of 2R1T PMs without parasitic motion based on screw theory, and improved the PMs to rotate in an arbitrary order. The conclusions are drawn as follows.

- (1) Non-parasitic 2R1T motion was categorized into three cases whose constraint wrenches were presented based on similarities between the 2R1T PMs with rotational bifurcation and the 2R1T PMs without parasitic motion.
- (2) We obtained the available branches constructing 2R1T PMs without parasitic motion algebraically by twists and wrenches of each branch. Further, we established ten types of 2R1T PMs without parasitic motion for acquiring modified structures that rotate in an arbitrary order.
- (3) The workspace of the 3-PRPS PM has been studied. The shape of the workspace is similar to two regular pyramids connected by the bottom. The operating angles are mainly in the range of  $-27.8^\circ < \alpha < 27.8^\circ$ ,  $-30.6^\circ < \beta < 30.6^\circ$ , which is possible to keep the water tank parallel to the horizontal on slopes less than  $30^\circ$ .
- (4) Applying PMs without parasitic motion, we established an application in the terrain-rough fire-fighting robots.

A comparative analysis of the performances of the whole structures which rotate in an arbitrary order, including workspace, stiffness, and singularity will be investigated in future work. We intend to manufacture the entity prototype of the fire-fighting robot and research the performance of the balancing adjustment as well.

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### Author Contributions

JQ conceived the basic idea, do the research, and wrote the manuscript. CY refined the method and provided theoretical guidance. ZS and LC contributed to the revision of this paper. All authors read and approved the final manuscript. All authors read and approved the final manuscript.

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### Competing Interests

The authors declare no competing financial interests.

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