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An Optimal Feed Interpolator Based on G^2 Continuous Bézier Curves for High-Speed Machining of Linear Tool Path

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Abstract

A numerical control (NC) tool path of digital CAD model is widely generated as a set of short line segments in machining. However, there are three shortcomings in the linear tool path, such as discontinuities of tangency and curvature, huge number of line segments, and short lengths of line segments. These disadvantages hinder the development of high speed machining. To smooth the linear tool path and improve machining efficiency of short line segments, this paper presents an optimal feed interpolator based on G^2 continuous Bézier curves for the linear tool path. First, the areas suitable for fitting are screened out based on the geometric characteristics of continuous short segments (CSSs). CSSs in every area are compressed and fitted into a G^2 Continuous Bézier curve by using the least square method. Then a series of cubic Bézier curves are generated. However, the junction between adjacent Bézier curves is only G^0 continuous. By adjusting the control points and inserting Bézier transition curves between adjacent Bézier curves, the G^2 continuous tool path is constructed. The fitting error is estimated by the second-order Taylor formula. Without iteration, the fitting algorithm can be implemented in real-time environment. Second, the optimal feed interpolator considering the comprehensive constraints (such as the chord error constraint, the maximum normal acceleration, servo capacity of each axis, etc.) is proposed. Simulation and experiment are conducted. The results shows that the proposed method can generate smooth path, decrease the amount of segments and reduce machining time for machining of linear tool path. The proposed research provides an effective method for high-speed machining of complex 2-D/3-D profiles described by short line segments.

Keywords: G^2 continuous path, Least square method, High-speed machining, Continuous short segments, Optimal feed interpolator, Data compression

1 Introduction

Linear tool path is the most widespread tool path representation form to approximate the complicated surface in machining; however, it introduces both the tangent and curvature discontinuities at the segment junctions. The inherent shortcomings are the most important sources of feed fluctuation. With the development of machining technology in modern industry, CNC machine tools ask for high interpolation technology, but linear tool path cannot satisfy the requirements of high-speed and

high-accuracy machining. Though look-ahead scheme has been integrated in current CNC systems to alleviate the frequent start and stop, the discontinuous still cause the fluctuations of feed speed and acceleration [1, 2]. Smoothing the linear tool path with parametric curves is an important way to reduce the feed fluctuation and improve the machining efficiency.

To achieve the smoothness of the linear tool path, two major approaches, i.e., curve fitting ones and transition ones, have been proposed in the previous studies to convert the linear segments to parametric splines. The fitting methods, including approximation algorithms and interpolation algorithms, are adopted by some advanced CNC systems. Local transition is mainly through the use of other curves to replace the original linear trajectory in

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the corner to achieve the purpose of smoothness [3–11]. Zhang et al. [3], proposed a transition algorithm for two continuous linear segments based on the cubic Hermite curve fitting approach. This algorithm guaranteed the tangent continuity of the new tool path. Bi et al. [4, 5], applied a cubic Bézier curve, whose middle two control points are identical, to realize continuous-curvature path-smoothing. An optimization problem needs to be solved to estimate the proper transition lengths. Zhao et al. [6] proposed a method for smoothing the linear tool path by using two transition cubic Bézier curves. One cubic Bézier curve smoothes the tool tip position path, and the other smoothes the orientation vector of tool axis. Zhao et al. [7], Sneha et al. [8] and Sun et al. [9] used a B-spline to round the corner. The approximation error at the rounded corner can be exactly controlled by the distribution of the control points. Shi et al. [10, 11], proposed a corner smoothing method for linear tool path based on quintic PH curve, which can restrict the approximation error exactly. Although the above transition methods can smooth the linear path, they increased the amount of data, shortened the lengths of the segments, and aggravated the computation burden of the system. Another solution is the global smoothing, which can fit a series of given points into a smooth parametric curve and can compress the data of the linear tool path. B-spline and NURBS, which have good smoothness characteristic and compression ratio, have received the widespread attention [12–19]. Yau and Kuo [12] proposed an offline NURBS fitting algorithm based on least square method. Wang and Yau [13] proposed a real-time look-ahead NURBS interpolation method. This method can fit consecutive short segments into NURBS online and can guarantee the C^2 continuity. Yeh and Su [14] designed an optimization search method to determine the NURBS order and the number of basis functions. Zhang et al. [15] adopts adaptive method to look for geometric feature points, and group a series of given points according to the feature points. This method improves the fitting efficiency. However, NURBS fitting methods have the following deficiencies: 1) heavy computational complexity of recursive algorithm, which is difficult to meet the real-time requirements of system; 2) inevitable errors introduced by employing a truncated Taylor series. In addition, some researchers have proposed the other parametric curves. Yau and Wang [16] proposed a real-time fast Bézier interpolation method. Its compression ratio is low, and tangency and curvature of the junction between adjacent curves are discontinuous. Wang et al. [17] presented an Akima curve fitting method, but it had large amount of calculation to obtain the slope vector of Akima curve. Tasi et al. [18] proposed a C^2 continuous Bézier fitting method, but this method increased the

amount of data. Zhang et al. [19] presented a five-order polynomial to ensure machining path of C^2 continuity, but it had large amount of calculation and poor real-time performance.

In this paper, an optimal feed interpolator based on G^2 Bézier transition algorithm is proposed to fit the linear tool path, which can satisfy the following requirements: 1) G^2 continuity; 2) data compression; 3) approximation control; 4) real-time performance. The remainder of this paper is organized as follows: Section 2 presents the G^2 continuous Bézier fitting algorithm based on least square method. Section 3 describes the proposed interpolator. Section 4 provides the simulation and experiment. Section 5 concludes the paper.

2 G^2 Continuity Bézier Fitting Method

In this section, a G^2 continuous Bézier fitting algorithm based on least square method is deduced. Firstly, several basic concepts, such as geometric continuity, cubic Bézier, consecutive short segment and breakpoint, are described. Then, a cubic Bézier curve is used to approximate the consecutive short segments based on Least Squares method, and the approximation error is calculated and restricted within a given tolerance. At last, the linear tool path is transformed to a G^2 continuous path.

2.1 Definitions of G^2 Continuity and Cubic Bézier

Continuity is basically a measure of the smoothness of a piecewise curve at its joining points. DeRose and Barsky [20] defined the order of smoothness by parametric and geometric continuity. Here geometric continuity is used to smooth the piecewise linear tool path. G^1 continuity is the first-order geometric continuity, which refers to continuity of the unit tangent, and G^2 continuity is the second-order geometric continuity, which refers to continuity of the curvature vectors at the joint.

In this research, a cubic Bézier curve, which is G^2 continuous, is adopted to fit consecutive short segments (CSSs) together. The cubic Bézier is given by Farin [21] as:

$$C(t) = \sum_0^3 \binom{3}{i} B_i t^i (1-t)^{3-i}, \quad 0 \leq t \leq 1, \quad (1)$$

where t is the curve parameter and $\{B_i\}$ are the control points.

Assuming that the two middle control points are the same, $B_1 = B_2 = B_{12}$, the cubic Bézier is simplified as:

$$C(t) = (1-t)^3 B_0 + 3t(1-t) B_{12} + t^3 B_3, \quad (0 \leq t \leq 1), \quad (2)$$

The first and second derivatives of the Bézier curve at $t=0$ and $t=1$ are calculated as follows:

$$C'(0) = 3(B_{12} - B_0), \quad C'(1) = 3(B_3 - B_{12}), \quad (3)$$

$$C''(0) = 6(B_0 - B_{12}), \quad C''(1) = 6(B_3 - B_{12}). \quad (4)$$

Then, the curvatures of the Bézier curve at $t=0$ and $t=1$ are obtained:

$$\kappa(t) = \frac{\|C'(t) \times C''(t)\|}{\|C'(t)\|^3}. \quad (5)$$

Substituting Eqs. (3) and (4) into Eq. (5), $\kappa(0) = \kappa(1) = 0$. The curvature is continuous and there is only one curvature extremum in the Bézier curve.

2.2 Definitions of Consecutive Short Segment and Breakpoint

In order to obtain good accuracy, the corner constraint and the bi-chord error constraint must be taken into consideration before least square curve approximation is used to fit linear segments. Yau and Wang [16] proposed an isosceles triangle method to compute the bi-chord errors, which are illustrated in Figure 1.

The corner of the two adjacent line segments $Q_{k-1}Q_k$ and Q_kQ_{k+1} is calculated as:

$$\theta_k = \arccos \left(\frac{\overrightarrow{Q_{k-1}Q_k} \cdot \overrightarrow{Q_kQ_{k+1}}}{\| \overrightarrow{Q_{k-1}Q_k} \| \cdot \| \overrightarrow{Q_kQ_{k+1}} \|} \right). \quad (6)$$

The bi-chord errors δ_1 and δ_2 are calculated as follows:

$$\delta_1 = R(1 - \cos \phi_1), \quad (7)$$

$$\delta_2 = R(1 - \cos \phi_2) = R(1 - \cos(\theta_k - \phi_1)), \quad (8)$$

where, $l_k = \|Q_{k-1}Q_k\|$, $l_{k+1} = \|Q_kQ_{k+1}\|$, $\phi_1 + \phi_2 = \theta_k$, $R = \frac{l_k}{2 \sin \phi_1}$, $\phi_1 = \arctan \left(\frac{l_k \sin \theta_k}{l_{k+1} + l_k \cos \theta_k} \right)$.

The criterions of the corner constraint and the bi-chord error constraint are set to θ_{\max} and ε_{\max} . If $\theta_k \leq \theta_{\max}$,

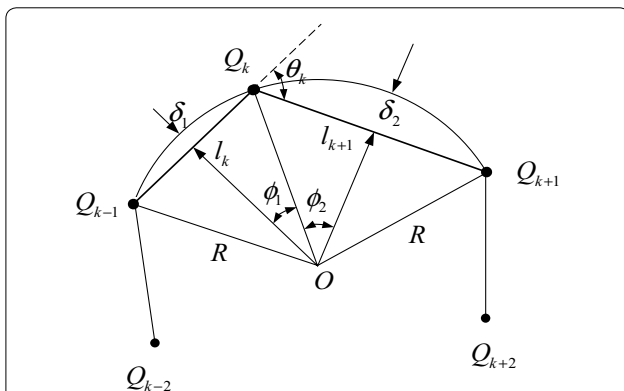


Figure 1 An isosceles triangle method to compute the bi-chord errors

and $\delta_1 \leq \varepsilon_{\max}$, and $\delta_2 \leq \varepsilon_{\max}$, the adjacent line segments $Q_{k-1}Q_k$ and Q_kQ_{k+1} are CSSs and can be fitted into Bézier; otherwise, the given point Q_k is set to be a breakpoint.

2.3 Bézier Curve Fitting Based on Least Squares (LS)

Method

To achieve the smoothness of the linear tool path, there are two major approaches, i.e., interpolation and approximation. In interpolation a curve which satisfies the given data precisely is constructed. The curve passes through all the given points. In approximation a curve which does not necessarily satisfy the given data precisely is constructed.

In this section, the LS fitting method is used to approximate the consecutive short segments. Compared with the interpolation curve, the curvature changes of the approximation curve are less. Moreover, it has a higher compression ratio.

As shown in Figure 2, $\{Q_i\}(i = 0, 1, \dots, m)$ are the given points generated by CAM software and meet the criterions of the corner constraint and the bi-chord error constraint. These points can be fitted into a cubic Bézier curve $C(t)$ by using LS method. $\{B_i\}(i = 0, 1, 2, 3)$ are the four corresponding control points. Assuming that $B_0 = Q_0$, $B_3 = Q_m$, $B_1 = B_2 = B_{12}$, only the point B_{12} needs to be determined.

Supposing that:

$$F(B_{12}) = \sum_{i=0}^m (C(t_i) - Q_i)^2,$$

where $C(t_i)(i = 0, 1, \dots, m)$ are the points on the curve corresponding to Q_i . Parameters $\{t_i\}$ are computed by chord length parameterization method [22]:

$$\begin{cases} t_0 = 0, \\ t_m = 1, \\ t_i = t_{i-1} + \frac{\|Q_i - Q_{i-1}\|}{L}, \quad i = 1, 2, \dots, m. \\ L = \sum_{i=1}^m \|Q_i - Q_{i-1}\|, \end{cases} \quad (9)$$

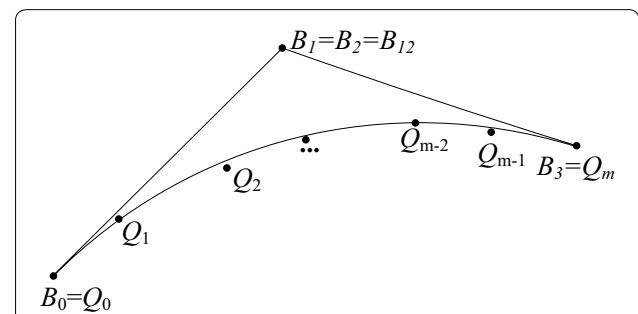


Figure 2 Get Bézier curve by the least square fitting method

According to the extreme conditions of continuous function, when $dF/dB_{12} = 0$, $F(B_{12})$ achieves the minimum:

$$\frac{dF}{dB_{12}} = \sum_{i=0}^m [6t_i(1-t_i)(C(t_i) - Q_i)] = 0. \quad (10)$$

Thus, B_{12} can be formulated as follows:

$$B_{12} = \frac{\sum_{i=0}^m (t_i(1-t_i)Q_i - t_i(1-t_i)^4B_0 - t_i^4(1-t_i)B_3)}{3 \sum_{i=0}^m t_i^2(1-t_i)^2}. \quad (11)$$

2.4 Approximation Error Control

Restricting the approximation error within a given tolerance is a key problem in tool path fitting process. Thus, it is of great importance to analyze and restrict the deviations between the original and fitted tool path. When $\varepsilon(Q_i)$ denotes the approximation error of $C(t)$ with respect to Q_i , the common computation methods are as follows.

- (1) $\varepsilon(Q_i)$ denotes the distance between Q_i and $C(t)$. $\varepsilon(Q_i)$ is calculated by iterative algorithm as there is no analytical solution. It is difficult to satisfy the real-time computing in the CNC system.
- (2) Define that $\|Q_i - C(t_i)\| = \varepsilon(Q_i)$, but $C(t_i)$ may not be the nearest point on $C(t)$ with respect to Q_i . This definition cannot meet the accurate requirement of CNC system.

In this paper, the method developed by Zhu et al. [23] is adopted to calculate $\varepsilon(Q_i)$.

As shown in Figure 3, Q_i is the given point, and $C(t_i)$ is the point on the curve corresponding to Q_i . t_i is calculated by Eq. (9). $D = Q_i - C(t_i)$. T is the unit tangent vector at position $C(t_i)$. C is the projection on the main plan of D at $C(t_i)$. $C(t'_i)$ is the nearest point on $C(t)$ with respect to Q_i . $\varepsilon(Q_i) = \|Q_i - C(t'_i)\|$.

According to Ref. [24], $\varepsilon(Q_i)$ is formulated as:

$$\varepsilon(Q_i) = D_C - \frac{\kappa(D_T)^2 C \cdot N}{2(1 - \kappa D_C C \cdot N)} + o^3(D_T), \quad (12)$$

where κ is the curvature at $C(t_i)$, D_T and D_C are the projection lengths of D on T and C respectively:

$$D_T = D \cdot T, \quad (13)$$

$$D_C = \frac{D \cdot C}{\|C\|}. \quad (14)$$

$C(t)$, κ , T , N and C can be computed by analytical formulations. The equations are all established with respect

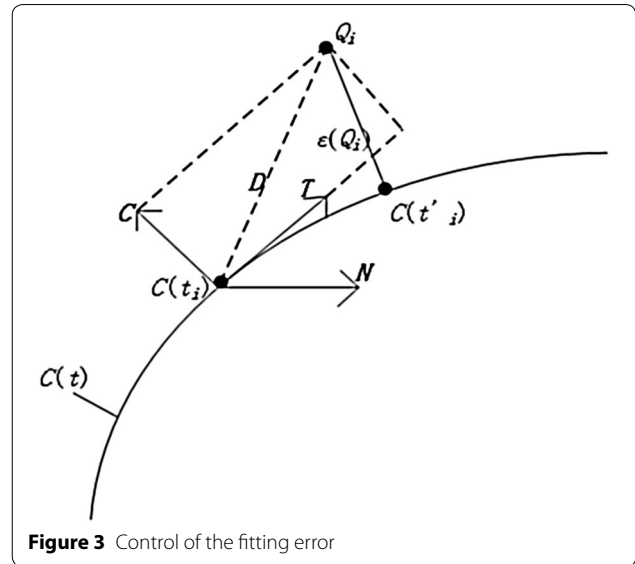


Figure 3 Control of the fitting error

to parameter $t \in [0, 1]$. κ is calculated by Eq. (5), and T is calculated by:

$$T = \frac{C'(t)}{\|C'(t)\|}. \quad (15)$$

Based on Frenet moving frame and Gramm–Schmidt orthogonalization, N and C are formulated:

$$N = \frac{C''(t) - (C''(t) \cdot T)T}{\|C''(t) - (C''(t) \cdot T)T\|}, \quad (16)$$

$$C = \frac{D - D_T T}{D_C}, \quad (17)$$

where $C'(t)$ and $C''(t)$ are the first and second order derivatives of $C(t)$.

Substituting Eqs. (13)–(17) into Eq. (12), the approximation errors $\{\varepsilon(Q_i)\}$ can be obtained. If $\{\varepsilon(Q_i)\}$ are all smaller than the given tolerance ε_{\max} , the fitting curve is obtained; otherwise, if $\varepsilon(Q_i) > \varepsilon_{\max}$, the given point Q_i is set to be a breakpoint, and the points between two adjacent breakpoints are fitted into a new Bézier curve again.

2.5 G² Continuous Path Construction

Through the above method, a series of cubic Bézier curves are generated. However, the junction between adjacent Bézier curves is only G^0 continuous. In order to construct a whole G^2 continuous tool path, the junction between adjacent Bézier curves should be smoothed.

As shown in Figure 4(a), assuming that $C_1(t)$ and $C_2(t)$ are two neighboring Bézier curves, and their control points are $\{P_i^1\}$ and $\{P_i^2\}$ respectively, P_3^1 is consistent

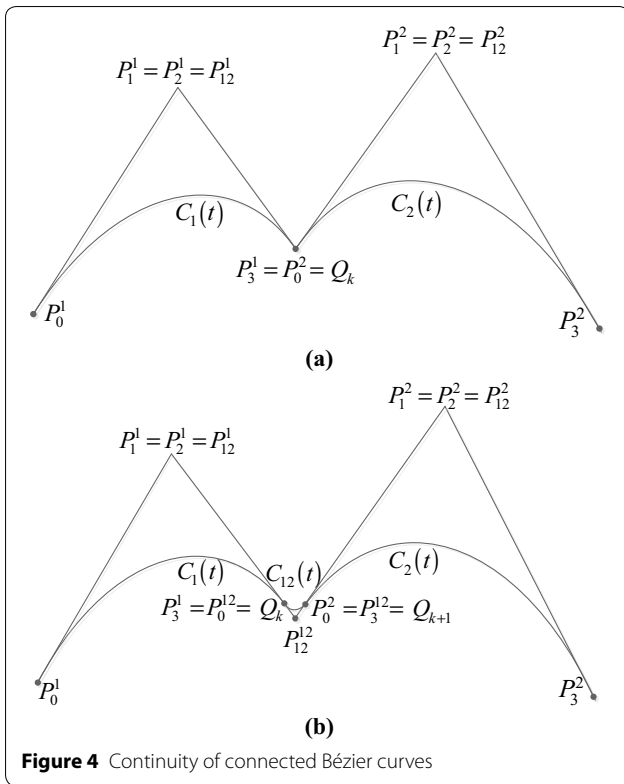


Figure 4 Continuity of connected Bézier curves

with P_0^2 at the point Q_k . $C_1(t)$ and $C_2(t)$ just touch at the join point.

As shown in Figure 4(b), P_3^1 is inconsistent with P_0^2 , $P_3^1 = Q_k$, $P_0^2 = Q_{k+1}$. A cubic Bézier transition curve $C_{12}(t)$ is constructed between the Q_k and Q_{k+1} . Its control points are $\{P_i^{12}\}$, $P_0^{12} = Q_k$, $P_3^{12} = Q_{k+1}$, $P_1^{12} = P_2^{12} = P_{12}^{12}$. P_{12}^{12} is the intersection of line $P_1^1P_3^1$ extension and line $P_{12}^2P_0^2$ extension. The proof for G^2 continuity between $C_1(t)$ and $C_{12}(t)$ is shown as follows.

(1) G^0 continuity:

$$C_1(1) = C_{12}(0) = Q_k.$$

(2) G^1 continuity:

$$\left. \begin{aligned} C_1'(1) &= 3(P_3^1 - P_{12}^{12}) \\ C_{12}'(0) &= 3(P_{12}^{12} - P_0^2) \end{aligned} \right\} \Rightarrow C_1'(1) // C_{12}'(0).$$

(3) G^2 continuity:

$$\left. \begin{aligned} C_1''(1) &= 6(P_3^1 - P_{12}^{12}) \\ C_{12}''(0) &= 6(P_{12}^{12} - P_0^2) \end{aligned} \right\} \Rightarrow \kappa^1(1) = \kappa^{12}(0) = 0.$$

Thus, $C_1(t)$ touches $C_{12}(t)$ with G^2 continuity. In the same way, $C_2(t)$ touches $C_{12}(t)$ with G^2 continuity. The whole tool path is G^2 continuous.

At the same time, the approximation of the transition curve $C_{12}(t)$ should be checked. According to Ref. [4], $C_{12}(0.5)$ is the farthest point from curve $C_{12}(t)$ to line

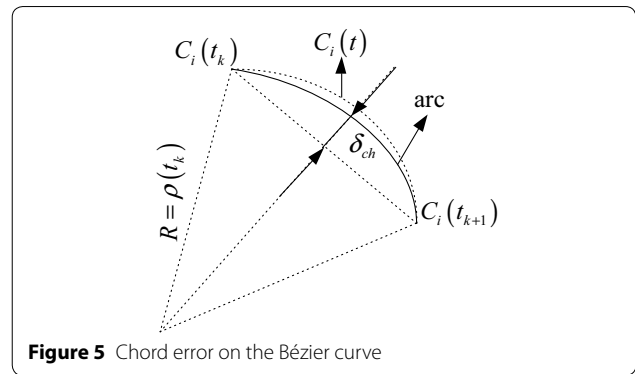


Figure 5 Chord error on the Bézier curve

Q_kQ_{k+1} . The distance from $C_{12}(0.5)$ to Q_kQ_{k+1} is the maximum deviation from $C_{12}(t)$ to Q_kQ_{k+1} . If the deviation error is bigger than ϵ_{max} , the transition curve $C_{12}(t)$ is cancelled and the line Q_kQ_{k+1} is used to connect Q_k and Q_{k+1} .

3 Optimal Feed Interpolation Algorithm

For interpolating the smoothed tool path, a real-time adaptive feedrate scheduling method considering the comprehensive constraints is proposed. Firstly, the comprehensive constraints (such as the chord error constraint, the maximum normal acceleration, servo capacity of each axis) are deduced to obtain the maximum feed and feasible acceleration range. On this basis, the optimal feed interpolator is introduced.

3.1 Comprehensive Constrains

3.1.1 Constraint of Chord Error

As shown in Figure 5, when a cutting tool moves linearly between contiguous interpolated points of the i th Bézier curve $C_i(t)$, contour error may occur. The chord error is the maximum distance between the curve segment and the chord. To confine the chord error within a given tolerance value ϵ_{max} , the feedrate is determined with an arc approximation method [25].

$$f_i(t) \leq f_{ch}(t) = \frac{2}{T_s} \sqrt{\rho_i^2(t) - (\rho_i(t) - \epsilon_{max})^2}, \quad (18)$$

where T_s is the interpolation period of CNC system, and $\rho_i(t)$ is called the radius of curvature.

3.1.2 Constraint of Maximum Normal Acceleration

The feedrate is also limited by the maximum normal acceleration A_{max} of the machine tool:

$$f_i(t) \leq f_{An}(t) = \sqrt{\rho_i(t)A_{max}}. \quad (19)$$

3.1.3 Constraint of Servo Capacity of Each Axis

One important error source in high-speed contouring is the error that results when the command input for each axis exceeds the acceleration/deceleration capacity of the servo system. To decrease this kind of error, the machine dynamics [26], including the constraints of velocity and acceleration represented by upper bounds for each axis, are taken into account.

Now suppose the feedrate of the tool tip is denoted as $f_i(t)$ and the displacement of the axis Λ ($\Lambda=X, Y, Z$) is denoted as $\theta_{\Lambda,i}(t)$. The feed of the axis Λ can be expressed as:

$$v_{\Lambda,i}(t) = \dot{\theta}_{\Lambda,i}(t) = \frac{1}{\sigma_i(t)} \theta'_{\Lambda,i}(t) f_i(t) = k_{v\Lambda i} f_i(t), \quad (20)$$

where $k_{v\Lambda i} = \theta'_{\Lambda,i}(t) / \sigma_i(t)$. $\sigma_i(t)$ is the arc-length of tool tip point curve $C_i(t)$ at parameter t , $\sigma_i(t) = s'_i = |C'_i(t)|$.

The velocity limit of the axis Λ is denoted as $V_{\Lambda m}$. The velocity of the axis Λ should be restricted by:

$$|v_{\Lambda,i}(t)| = |k_{v\Lambda i} f_i(t)| = |k_{v\Lambda i}| f_i(t) \leq V_{\Lambda m}. \quad (21)$$

Then, the feedrate of the tool tip is restricted by:

$$f_i(t) \leq f_{vm}(t) = \min_{\Lambda=X,Y,Z} \left\{ \frac{V_{\Lambda m}}{|k_{v\Lambda i}|} \right\}. \quad (22)$$

Next, the acceleration of the axis Λ along the curve can be calculated by taking the derivative with respect to time of Eq. (20):

$$a_{\Lambda,i}(t) = \dot{v}_{\Lambda,i}(t) = k_{aa\Lambda i} a_{ti}(t) + k_{av\Lambda i} f_i(t)^2, \quad (23)$$

where $k_{aa\Lambda i} = \frac{\theta''_{\Lambda,i}(t)}{\sigma_i(t)}$, $k_{av\Lambda i} = \frac{\sigma_i(t)\theta''_{\Lambda,i}(t) - \sigma'_i(t)\theta'_{\Lambda,i}(t)}{\sigma_i(t)^3}$.

If the acceleration limit for axis Λ is specified as $-a_{\Lambda m}$ and $a_{\Lambda m}$, the feed rate $f_i(t)$ and tangent acceleration $a_{ti}(u)$ can be constrained as:

$$f_i(t) \leq f_{am}(t) = \max_{\substack{i,j,k \in \{X,Y,Z\} \\ i \neq j \neq k}} \left\{ f(i,j) \mid -a_{km} \leq k_{aak} a_f(i,j) + k_{avk} f^2(i,j) \leq a_{km} \right\}, \quad (24)$$

$$a_{f \min}(t, f) \leq a_f \leq a_{f \max}(t, f), \quad (25)$$

where

$$a_{f \max}(t, f) = \min \left\{ \begin{array}{l} \min_{\Lambda=X,Y,Z} \left\{ \frac{a_{\Lambda m} - k_{av\Lambda} f^2}{k_{aa\Lambda}(t)} \mid k_{av\Lambda} > 0 \right\}, \\ \min_{\Lambda=X,Y,Z} \left\{ \frac{-a_{\Lambda m} - k_{av\Lambda} f^2}{k_{aa\Lambda}(t)} \mid k_{av\Lambda} < 0 \right\} \end{array} \right\}, \quad (26)$$

$$a_{f \min}(t, f) = \min \left\{ \begin{array}{l} \max_{\Lambda=X,Y,Z} \left\{ \frac{-a_{\Lambda m} - k_{av\Lambda} f^2}{k_{aa\Lambda}(t)} \mid k_{av\Lambda} > 0 \right\}, \\ \max_{\Lambda=X,Y,Z} \left\{ \frac{a_{\Lambda m} - k_{av\Lambda} f^2}{k_{aa\Lambda}(t)} \mid k_{av\Lambda} < 0 \right\} \end{array} \right\}. \quad (27)$$

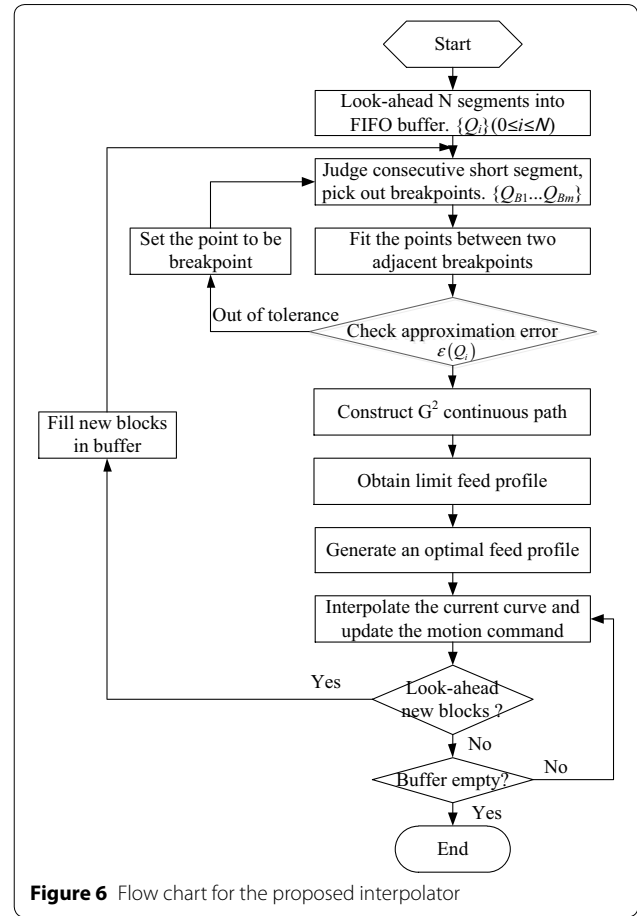


Figure 6 Flow chart for the proposed interpolator

3.1.4 The Maximum Feed Restricted by Comprehensive Constraints

Considering the command feed rate F_{cmd} at the same time, the maximum feasible feed $f_{lim}(t)$ is obtained as follows:

$$f_i(t) \leq f_{lim}(t) = \min \{ f_{ch}, f_{A\Lambda}, f_{vm}, f_{am}, F_{cmd} \}. \quad (28)$$

3.2 Optimal Feed Interpolator

The proposed interpolator respects both tool path contour constraints and machine dynamics constraints. A look-ahead scheme is utilized to improve the calculation performance for real-time application [27]. The proposed interpolator includes four tasks: fitting the linear tool path with a series of cubic Bézier curves, obtaining limit feed profile, generating optimal feedrate profile using jerk-limited acceleration method [28], and generating the interpolating point. The detailed procedures for the proposed interpolator are shown in Figure 6.

Task 1: The first task of the proposed interpolator is to look-ahead N segments into FIFO (First-In First-Out) buffer, and fit the linear tool path with a series of G^2 cubic Bézier curves.

Task 2: The second task of the proposed interpolator is to comprehensive constraints and obtain limit feed profile.

$$f_{lim}(t) = \min\{f_{ch}, f_{An}, f_{vm}, f_{am}, F_{cmd}\}.$$

Task 3: The third task of the proposed interpolator is to generate optimal feedrate profile using jerk-limited acceleration method. While the i th curve is being interpolated, the parameter, the feed and the acceleration of the j th sampling period are denoted as (t_i^j, f_i^j, a_i^j) . Task 3 includes the following three steps.

Step 1. A fixed jerk limit J_m is predefined by CNC system. Jerk-limited acceleration method is utilized to evaluate the feed of the $(j+1)$ th sampling period.

At the accelerating stage, the acceleration of the $(j+1)$ th sampling period is evaluated as:

$$a_i^{j+1} = \min\{a_{fmax}(t, f_i^j), a_i^j + J_m T_s, A_{max}\}. \tag{29}$$

At the decelerating stage, the acceleration of the $(j+1)$ th sampling period is evaluated as:

$$a_i^{j+1} = \max\{a_{fmin}(t, f_i^j), a_i^j - J_m T_s, -A_{max}\}. \tag{30}$$

Then, the feed of the $(j+1)$ th sampling period is evaluated as:

$$f_i^{j+1} = f_i^j + a_i^{j+1} T_s. \tag{31}$$

Step 2. The parameter t_i^{j+1} is calculated with the first-order Taylor's expansion.

$$t_i^{j+1} = t_i^j + \frac{f_i^{j+1} T_s}{\sigma(t_i^j)}. \tag{32}$$

Step 3. The feed limit $f_{lim}(t_i^{j+1})$ is calculated with Eq. (28). If $f_i^{j+1} \leq f_{lim}(t_i^{j+1})$, the optimal feed is obtained, otherwise, let $f_i^{j+1} = f_{lim}(t_i^{j+1})$ and go to Step 2.

Task 4: The fourth task of the proposed interpolator is to interpolate the current curve $C_i(t)$. Substituting Eq. (32) into Eq. (2), the interpolating point $C_i(t_i^{j+1})$ is obtained. Then the motion command for each axis is updated finally.

4 Simulation and Experiment

In this section, two examples are adopted to demonstrate the advantages of the proposed algorithm. The first example mainly validates the compression effect and the approximation error control effect of the proposed fitting method, while the second example verifies validity of the proposed interpolator to reduce the feed fluctuation and improve the machining efficiency.

4.1 Example 1

A 2-D butterfly shape linear profile is taken for example to the compression effect of proposed algorithm. Its original linear tool path consists of 8799 linear segments. The corner constraint is set to be 20° , and the chord error constraint is set to be 0.01 mm. The verification results are shown in Figure 7. After fitting, 158 Bézier curves are generated and the average compression ratio is 55.6:1.

Three different curvature areas are selected and enlarged respectively. As shown in Figure 7(b), when the curvature is $0.52\text{--}0.68 \text{ mm}^{-1}$, the compression ratio is 14:1. As shown in Figure 7(c), when the curvature is $0.05\text{--}0.8 \text{ mm}^{-1}$, the compression ratio can reach 30.5:1. As shown in Figure 7(d), when the curvature is $0.01\text{--}0.018 \text{ mm}^{-1}$, the compression ratio can reach 84:1. Accordingly, the compression ratio of the Bézier curve is associated with curvature. The profile with smaller curvature has higher compression ratio.

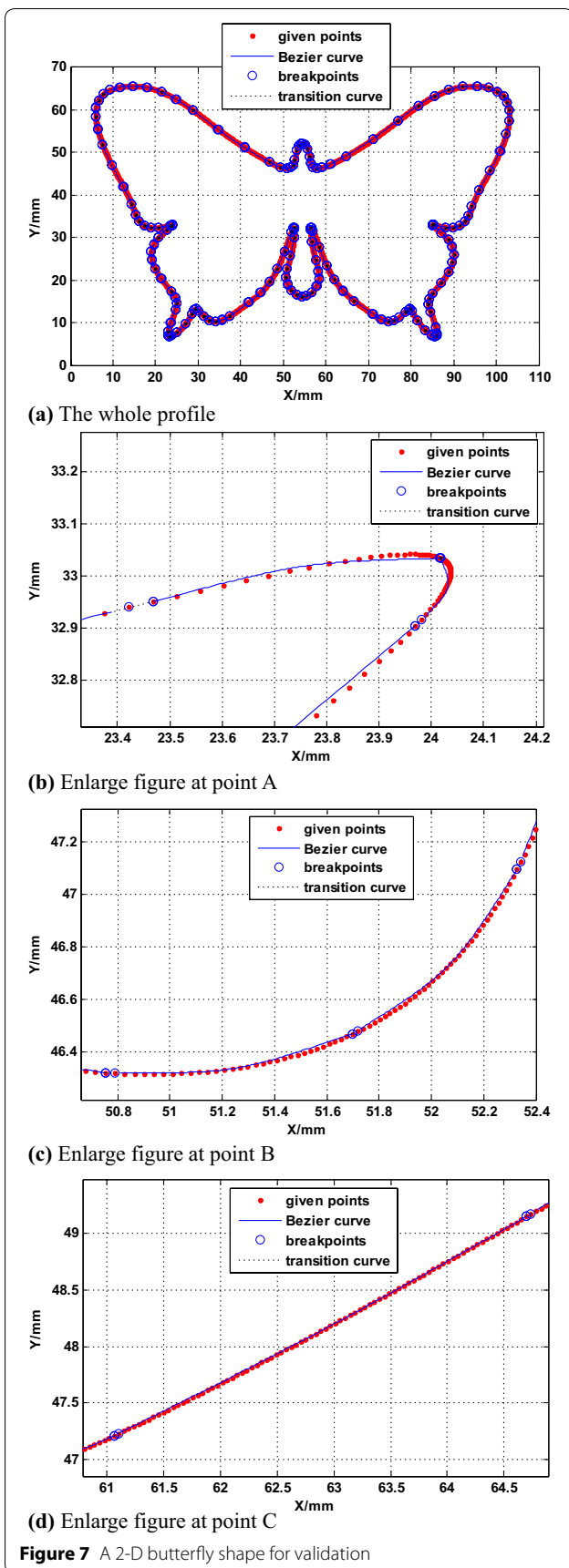
As shown in Figure 8, the approximation errors of 158 Bézier curves are all within the given tolerance 0.01 mm.

4.2 Example 2

Simulation and experiment are performed on a three-axis XK714 CNC milling machine tool. The in-house-developed research CNC, which is PC-based, has an open architecture, and allows rapid implementation of the proposed interpolation algorithm. The CNC utilizes windows XP operating system embedding Venturecom's RTX 6.1 which provides the real-time computation and multitasking.

This experiment demonstrates the advantages of the proposed interpolator by machining a scoop, shown in Figure 9. In the experiment, a cuboid stock made of aluminum is used. The command feedrate is set to 3000 mm/min , the maximum acceleration is set to $1.8 \times 10^6 \text{ mm/min}^2$, the maximum jerk is set to $1.35 \times 10^9 \text{ mm/min}^3$, and the sampling period T_s is 0.004 s. The corner constraint is set to be 20° , and the chord error constraint is set to be 0.01 mm. A flat end milling tool of diameter 10 mm is first used for rough machining. Then a 6-mm carbide ball-end cutter is used for finish machining.

There are 158665 segments for finish machining a scoop. A tool path consisted of 1308 linear segments is taken for example, as shown in Figure 10.



Two interpolation methods are integrated into the open CNC system of the machine tool. Both two methods utilize jerk-limited acceleration method for speed planning. The first method adopts the direct linear interpolation with the look-ahead scheme. In the second method, the linear tool path is first fitted by the proposed algorithm, and then the feedrate is planned with the proposed interpolator.

As shown in Figure 11(a), two feedrate profiles are generated with respect to displacement. When the linear interpolator is used for machining, the feedrate fluctuates severely and the values are far from 3000 mm/min. When the proposed interpolator is used for machining, the feedrate profile is much smoother and closer to 3000 mm/min. The reason is that the proposed algorithm compresses and smooths the linear path effectively. As marked in the Figure 11(a), a mount of segments are compressed and smoothed.

As shown in Figure 11(b), when the linear interpolator is used for machining, it needs 13610 interpolation periods, 54.44 s. When the proposed interpolator is used for machining, it needs 3337 interpolation periods, 13.348 s. Machining efficiency with the proposed interpolator is four times as it with the method.

The acceleration profiles corresponding to the two interpolation methods are shown in Figure 12. The noise for the first interpolation method is the biggest. The noise is mainly caused by the frequent impact effect because the acceleration and declaration occur in every segment junction. The proposed algorithm is helpful to smooth the normal acceleration. Jerk-limited acceleration method can guarantee the continuity of the tangential acceleration. The acceleration is then finally smoothed by combining the two methods. Figure 12 shows that the acceleration curve of the proposed method is smoother than that of the first method.

4.3 Discussion

The results of simulation and experiment are similar, which shows that the proposed method can smooth the linear tool path with G^2 continuity path and decrease the amount of segments. What's more, the results of experiment demonstrate that the proposed method can reduce the fluctuations of feedrate and acceleration and improve machining efficiency for CNC machining of short line segments.

5 Summary and Conclusions

- (1) A G^2 continuous Bézier fitting algorithm based on Least Squares method is proposed to smooth the consecutive short segments, and the approximation

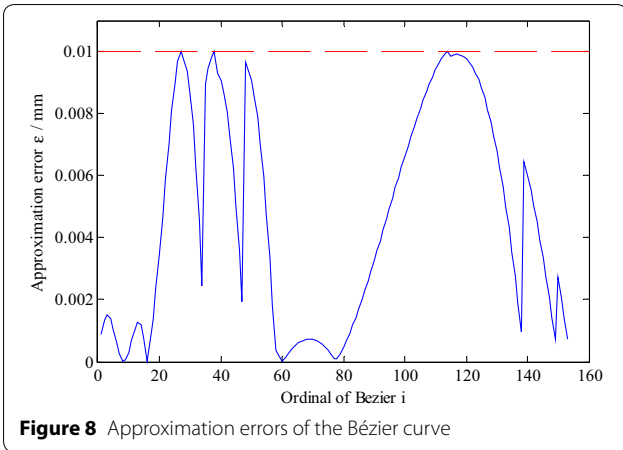


Figure 8 Approximation errors of the Bézier curve

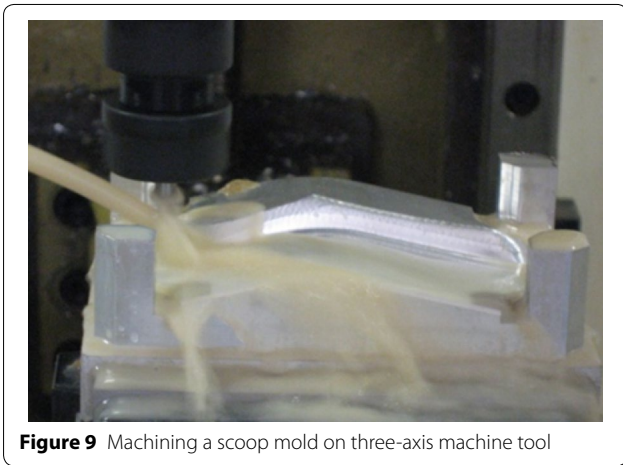


Figure 9 Machining a scoop mold on three-axis machine tool

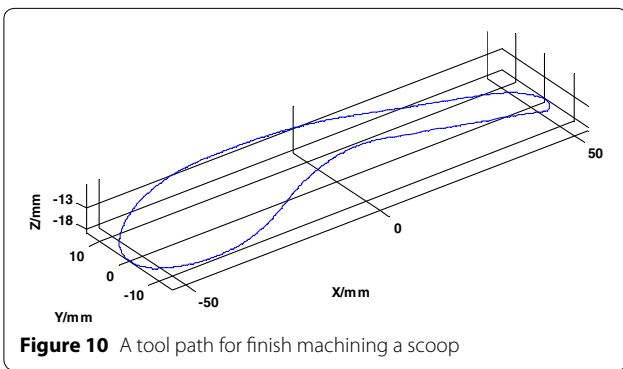
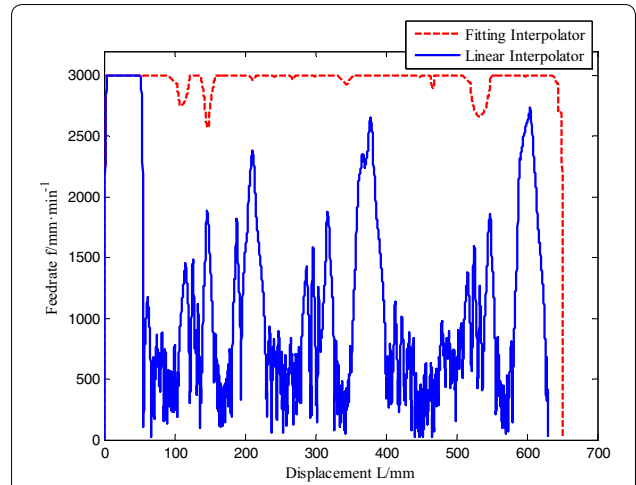


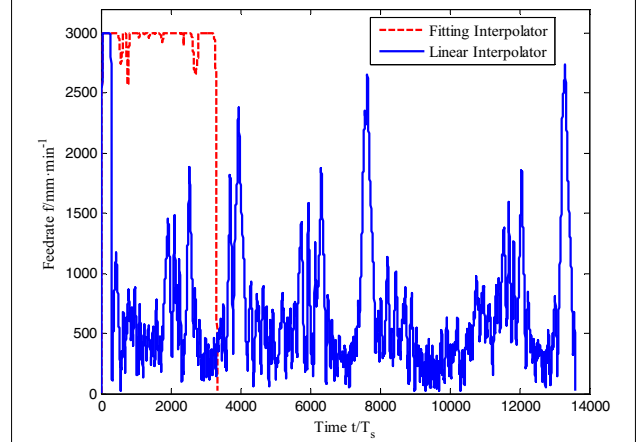
Figure 10 A tool path for finish machining a scoop

error is calculated and restricted within a given tolerance.

- (2) Cubic Bézier curve is used to smooth the junction between adjacent Bézier curves. And the linear tool path is transformed a new G^2 continuous tool path.



(a) Feedrate about displacement



(b) Feedrate about time

Figure 11 Feedrate profiles generated by two different interpolation methods

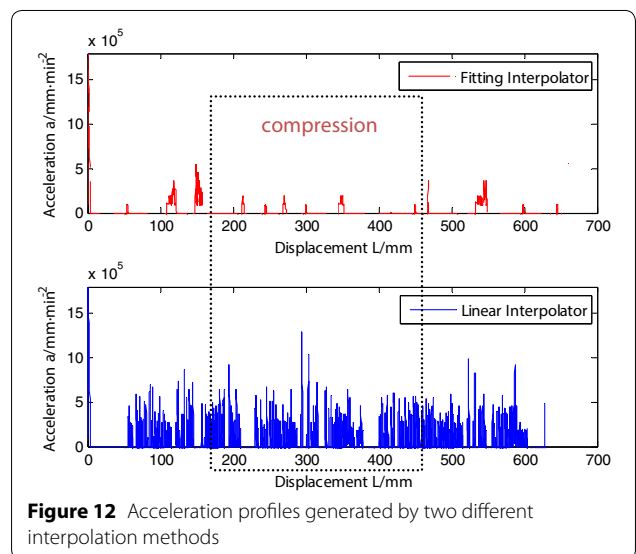


Figure 12 Acceleration profiles generated by two different interpolation methods

- (3) A real-time adaptive feedrate scheduling method considering the comprehensive constraints is proposed to interpolate the new tool path.
- (4) Simulation and experiment are conducted. With the proposed methodology, the linear tool path can be compressed and fitted into the G^2 continuous tool path. And, the feedrate and acceleration with the optimal feed interpolator are both smoothed, and the machining time is reduced.
- (5) In the future work, the real-time curvature-continuity transition algorithm for five-axis linear tool path will be studied. Besides the constraints of curvature-continuity and approximation error, the parametric synchronization of the position curve and the orientation curve must also be considered in five-axis NC machining.

Authors' Contributions

YJ was in charge of the whole trial; YJ wrote the manuscript; SZ and YW assisted with sampling and laboratory analyses. All authors read and approved the final manuscript.

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Competing Interests

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