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Active Damping of Milling Vibration Using Operational Amplifier Circuit

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Abstract

The problem of chatter vibration is associated with adverse consequences that often lead to tool impairment and poor surface finished in a workpiece, and thus, controlling or suppressing chatter vibrations is of great significance to improve machining quality. In this paper, a workpiece and an actuator dynamics are considered in modeling and controller design. A proportional-integral controller (PI) is presented to control and actively damp the chatter vibration of a workpiece in the milling process. The controller is chosen on the basis of its highly stable output and a smaller amount of steady-state error. The controller is realized using analog operational amplifier circuit. The work has contributed to planning a novel approach that addresses the problem of chatter vibration in spite of technical hitches in modeling and controller design. The method can also lead to considerable reduction in vibrations and can be beneficial in industries in term of cost reduction and energy saving. The application of this method is verified using active damping device actuator (ADD) in the milling of steel.

Keywords: Active damping, Anti-windup, Bending moment, Milling process, Workpiece vibration, Operational amplifier, Proportional integral controller

1 Introduction

The modification of metal part into desired shape and size, for a different purpose in industries, are produced by milling. Such process can certainly be disturbed by chatter vibration, which leads to negative effects ranging from the poor surface finish, reduced work quality and high production cost to machine impairment [1, 2]. Chatter vibration severely disturbs the milling practice and it is caused by undesirable relative movement between a tool and a workpiece. Thus, to improve the quality of the workpiece, it is of great importance to study the chatter generation mechanism together with its control process. Therefore, this research will focus more attention on milling chatter vibration detection and its control process.

Primarily, identification of chatter using stability lobe diagram (SLD) is very critical in machining. In machine tools, chatter and non-chatter vibrations separate from each other using SLD, which is a series of interconnected lobes in term of axial depth of cut and spindle speed that demarcates the regions of chatter vibrations. Regions below the lobes mean stable chatter vibrations [3]. SLD can be developed in either time or frequency domains. It is much faster to develop SLD in the frequency domain than in time domain. Frequency response function (FRF) [4–6], linear time-periodic function [7], and zero order approximation of the characteristics equation [8] are commonly employed to predict chatter vibrations in literature. In FRF, Fast Fourier transform (FFT) can determine the actual machining boundary where unstable cut show high amplitude with high vibration and stable cut show lower vibration with low amplitude [9]. Owing to the above deliberations, two important points, which increased chatter vibration in machining [4–6, 10–17] were identified as follows: an increase in depth of cut which move up vibration frequency and the effects of high spindle speed on shifting machine dynamic toward unstable cutting vibrations. Dos Santos and Coelho [4] improved the accuracy of chatter prediction in machine tools using the stability lobe diagram. Gagnol et al. [5] develop a spindle dynamic model for rotor equation using Timoshenko beam theory. Eynian [6] used Nyquist contour to predict the stability through dominant poles

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of the closed loop delay-differential equation for machining systems. Li et al. [10] established a comprehensive dynamic model of milling for stability analysis by considering regeneration, helix angle, and process damping. Solis et al. [11] used nondestructive trials to determine a new transfer function of the milling process for stability analysis. Altintas and Budak [12] and Altintas et al. [13] developed analytical methods for the predictions of chatter in milling. Soliman and Ismail [14] used PD fuzzy controller to limit the chatter indication. Quintana and Ciurana [15] experimentally determined stability lobe by increasing the depth of cut in feed direction. Tsai et al. [16] proposed acoustic signal feedback and spindle speed compensation for intelligent control application to avoid chatter vibration in milling process. Abele and Fiedler [17] created stability lobe of the milling process by considering the dynamic behavior of the system. In the past, efforts have been made by a quite number of researchers to study the problems of machining chatter vibrations [3]. Experimental and finite element (FE) model of machine tool were combined to investigate the vibrations [18, 19]. Kersting et al. [19] highlighted the effects of vibration on the workpiece, which focused on the feedback of workpiece displacement using FE. Dankena et al. [20, 21] related workpiece vibration to the workpiece material and workpiece structure. They pointed out that compound workpiece was a major cause of unfavorable workpiece performance, such as material height deviations, transition deviation at material joint, and surface roughness deviation, as compared to a single workpiece. Ma et al. [22–25] attributed vibration of the flexible workpiece to the workpiece fixture, and they improved the stiffness of the workpiece by using magnetorheological (MR) fluid. Ma et al. [22] designed a flexible fixture to investigate the regenerative chatter based on the MR fluids. Ma et al. [23] also proposed a dynamic analytical model by considering fixture constraints and the damping factor. Zeng et al. [24] constructed the dynamic model of the workpiece-fixture-cutter system by using the cutting force and the fixture element as the disturbance input and control input, respectively [24]. Similarly, retrofittable intelligent active fixtures, capable of observing the process in real-time and exert adequate counter-excitations was reported in Ref. [25]. In spindle speed variation, workpiece vibrations are curtailed by selecting suitable spindle speed so that the excitation frequency corresponds to the workpiece resonance [26]. Active control is a process that utilized the actuator force to provide adequate damping force that suppresses chatter vibration [27]. It consumes considerable energy to derive the actuator.

Moreover, efforts were made to decrease the negative effects of regenerative chatter in machining [3, 28–30]. Regenerative chatter occurs due to effects of interaction between previous and current cut [3]. Hayasaka et al. [28] employed the regenerative effects cancellation diagram in varied-helix end milling. Zhou et al. [29] related regenerative chatter to harmonic excitation. Moradi et al. [30] combined extremal algorithm and optimal control to suppress the regenerative chatter in machining.

Some techniques on active control were reported in Refs. [31–35]. Control of vibrating actuator was considered using a PID controller [31]. Active control was proposed based on delayed state feedback control and discrete optimal control [32]. The effect of a fuzzy logic controller on active magnetic bearing was verified to regulate the spindle position in milling [33]. Long et al. [34] designed feedback control through a robust mixed sensitivity method by using two degrees of freedom (TDOF) workpiece holder. A proportional integral controller was employed for nonlinear stiffness function [35]. Astrom and Hagglund described detail description of proportional integral and derivative (PID) controllers and its application in control system [36]. Information describing the operational amplifier circuit and its application in filter, amplification, and control capabilities were explained [37–39].

This is the major approach to resolve chatter vibration problem in the milling process via the operational amplifier circuit. Encouraged by the above consideration, a proportional-integral controller is proposed to suppress the chatter vibrations in the milling process. Milling test will be considered to investigate the effects of the proposed controller. This work contributes to minimizing chatter vibration and its negative consequences.

2 Modeling

2.1 Motion Equation of the System

This work developed control strategies that consider the chatter vibration of a flexible workpiece under both bending moment and cutting forces. The vibration of the workpiece is proportional to the perpendicular distance between the cutting force and the support point. Depending on the workpiece flexibility, the larger the perpendicular distance γ, the more the vibration. Figure 1 illustratively shows the bending displacement in the x-direction with the applied force. Eq. (1) relates the bending moment to the vibration.

$$\begin{aligned} m_x\ddot{\lambda}_x e^{-\alpha t} + c_x\dot{\lambda}_x e^{-\alpha t} + k_x\lambda_x e^{-\alpha t} &= \frac{M_x}{\beta_x}, \\ m_y\ddot{\lambda}_y e^{-\alpha t} + c_y\dot{\lambda}_y e^{-\alpha t} + k_y\lambda_y e^{-\alpha t} &= \frac{M_y}{\beta_y}, \end{aligned} \tag{1}$$

where M_x and M_y are the bending moments in x- and y-axes. β_x is the workpiece thickness thru x-axis, while β_y is the workpiece thickness thru y-axis. α is the exponential decay constant. λ_x is the bending

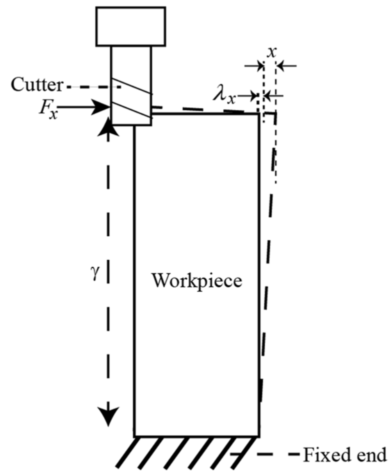

Figure 1 Effect of bending displacement λ_x and force displacement x. Here, γ is the perpendicular distance between applied force and a support point

displacement in x-axis, and λ_y is the bending displacement in y-axis. c_x and c_y are modal damping in x- and y-directions.

The vibration between cutting forces can be expressed as follows.

$$\begin{aligned} m_x\ddot{x} + c_x\dot{x} + k_x x &= F_x, \\ m_y\ddot{y} + c_y\dot{y} + k_y y &= F_y, \end{aligned} \tag{2}$$

where F_x and F_y are the cutting forces in x- and y-directions. m_x and m_y are modal masses in x- and y-directions. k_x and k_y are modal stiffness in x- and y-directions.

Combining Eq. (1) with Eq. (2) gives

$$\begin{aligned} &\boldsymbol{M}\begin{bmatrix} \ddot{x}+\ddot{\lambda}_x e^{-\alpha t} & 0 \\ 0 & \ddot{y}+\ddot{\lambda}_y e^{-\alpha t} \end{bmatrix} + \boldsymbol{C}\begin{bmatrix} \dot{x}+\dot{\lambda}_x e^{-\alpha t} & 0 \\ 0 & \dot{y}+\dot{\lambda}_y e^{-\alpha t} \end{bmatrix} \\ &+\boldsymbol{K}\begin{bmatrix} x+\lambda_x e^{-\alpha t} & 0 \\ 0 & y+\lambda_y e^{-\alpha t} \end{bmatrix} = \begin{bmatrix} F_x & 0 \\ 0 & F_y \end{bmatrix} + \begin{bmatrix} \frac{M_x}{\beta_x} & 0 \\ 0 & \frac{M_y}{\beta_y} \end{bmatrix}, \end{aligned} \tag{3}$$

where $\boldsymbol{M}=\begin{bmatrix} m_x & 0 \\ 0 & m_y \end{bmatrix}$, $\boldsymbol{C}=\begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix}$, $\boldsymbol{K}=\begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$.

2.2 Controller Design

Actuator action is regulated by the controller. The designed controller depends on the property of the system model and a vibration signal from the sensor. During the dynamic milling process, the controller receives a dynamic vibration signal from the sensor, which allows the controller to initiate a direct control action. This means an increase in positive error result to increase positive control output, and likewise, increase negative error result to increase negative control output. The stability of the controller is obtained by determining the close loop stability of characteristics equation, which has a negative real part. The range of controller value is determined from the controller gains that give stabilizing controller for the smooth running of the control system.

$$G = \frac{\sum_{i=1}^{n} a_i s^{-i}}{\sum_{i=1}^{p} b_i s^{-i}}, \tag{4}$$

$$\frac{\sum_{i=1}^{n} a_i s^{-i}}{\sum_{i=1}^{p} b_i s^{-i}} \left[\frac{K_p M_x M_y s + k_i}{s}\right] = -1. \tag{5}$$

The system transfer function is represented in the simplified form in Eq. (4). Proportional gain K_p and integral gain k_i are obtained from the system Eq. (5) as follows:

$$K_p = -\frac{\sum_{i=1}^{p} b_i s^{-i}}{M_x M_y \sum_{i=1}^{n} a_i s^{-i}}, \tag{6}$$

$$k_i = -\frac{s\sum_{i=1}^{p} b_i s^{-i}}{\sum_{i=1}^{n} a_i s^{-i}}[1 + \frac{1}{M_y}]. \tag{7}$$

These gains provide regulatory force to compensate the vibration of the workpiece.

2.3 Proportional Integral Controller Circuit Realization Using Operational Amplifier

An operational amplifier is an integrated circuit made of transistors and resistors to amplify or attenuate the input signal [38]. Firstly, two basic amplifiers will be described as follows.

2.3.1 Proportional Inverting Amplifier

The proportional inverting amplifier is an amplifier that amplifies the input signal based on the resistance gain. The signal is applied through inverting input amplified by the gain. The non-inverting input is grounded.

2.3.2 Integral Amplifier

In the integral amplifier, the capacitor is used instead of a resistor in the feedback path. This results in an operational amplifier that is time integral of the input signal.

A proportional integral controller is organized using the above amplifiers configuration: This is the combination of proportional and integral control action. The

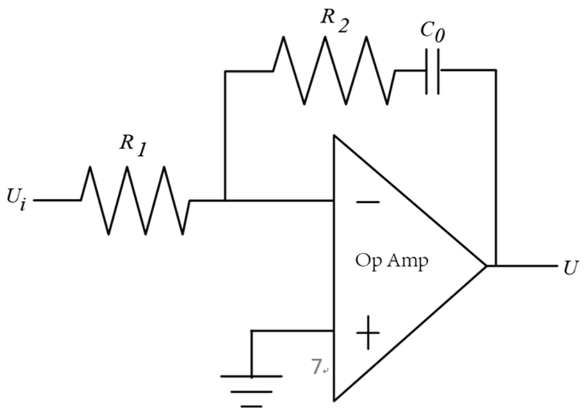

Figure 2 Proportional integral controller circuit

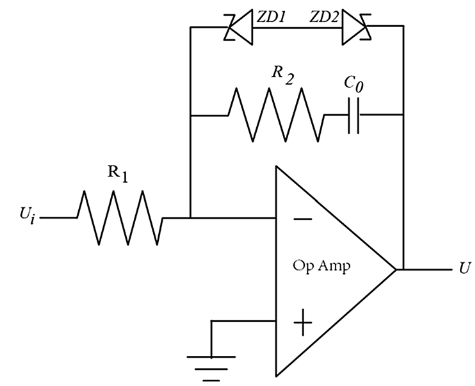

Figure 3 Proportional integral controller with Anti-windup configuration

circuit representation of the proportional integral controller based on the operational amplifier is presented in Figure 2.

In Figure 2, R_1 and R_2 are resistors, c_0 is a capacitor, U is the output voltage, and U_i is the input voltage.

$$U = -\frac{R_2}{R_1}U_i - \frac{1}{sC_0R_1}U_i \tag{8}$$

Proportional integral amplifier gain is given by:

$$\frac{U}{U_i} \triangleq G_c = -\frac{R_2 + 1/sc_0}{R_1}. \tag{9}$$

The proportional gain and integral gain are represented as Eq. (10):

$$K_p = \frac{R_2}{R_1},\ k_i = \frac{1}{sc_0R_1} \tag{10}$$

2.3.3 Anti-Windup

Properties of Zener diode was described [40]. Zener diode acts as low resistance in a forward path and built-up in reverse path till it goes into breakdown voltage U_z. Here, U_z is Zener breakdown voltage. In this work, a Zener diode will be employed to regulate the output voltage of the proportional integral controller from exceeding the actuator limits (saturation). When a positive voltage from the sensor output is applied across the controller input U_i, $ZD1$ will be open up to 10 V (with $ZD2$ low resistance) and the voltage across the amplifier output will not exceed 10V ($U \leq 10$V). Likewise when the negative voltage is applied across the terminalU_i, $ZD2$ will be open up to -10 V (with $ZD1$ low resistance) and the voltage across the output Uwill not exceed -10 V ($U \leq -10$ V). $ZD1$ and $ZD2$ are Zener diodes with U_z $=10$ V (shown in Figure 3).

2.4 Active Damping Actuator

The active damping actuator (ADD) consists of an accelerometer and actuator. The accelerometer detects the vibration signal of the structure and sends it to a control unit, which generates the control force according to the designed control law. The actuator is equipped with actuating mass, spring and damper. The actuating control force from the controller enhances the stiffness and damping to resist the vibration of the workpiece. During dynamic control process, an accelerometer detects the displacement of the workpiece and sends the signals to the external control unit, which generates the control force that derived the actuator. Figure 4 shows the schematic diagram of the active damping actuator. Where G_c is the controller; γ is the displacement vector; F_a is the control force vector; m_1, k_1 and c_1 are the mass, stiffness and damping coefficient.

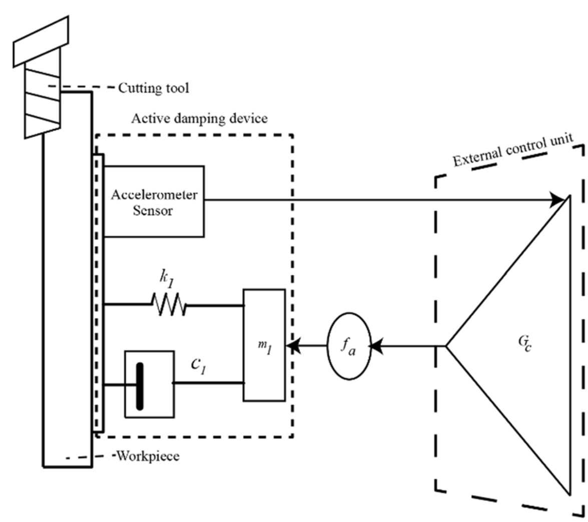

Figure 4 Schematic diagram of the active damping actuator

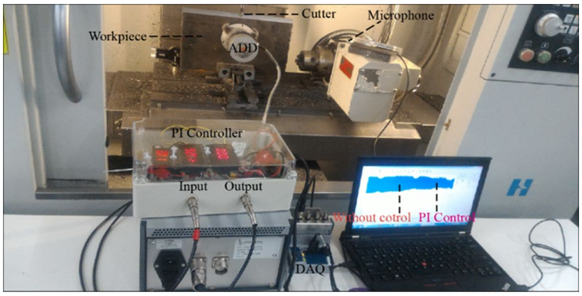

Figure 5 Experimental setup

3 Experimental Verification

This experiment is equipped with a data acquisition system (DAQ), a microphone, a proportional integral control circuit, and a personal computer. Actuator damping force is obtained from the active damping device (ADD-45N) based on the designed control low. An accelerometer present in the ADD allows the detection of displacement signals from the workpiece. The experimental setup is shown in Figure 5. The experiment consists of three separate processes, which are milling, control, and data acquisition process. The milling process is conducted at three different spindle speeds and using different depths of cut. The spindle speeds are at 1500 r/min, 2500 r/min, and 3000 r/min. The depths of cut are 1 mm, 1.3 mm, and 2.5 mm. The milling parameters are as follows: N (number of teeth) = 4, a_e (radial depth of cut) = 1 mm, feed rate=0.0750 mm/tooth. The data acquisition system records the signals showing the difference between controlled and uncontrolled vibrations of the system. The proportional integral control circuit is realized using an operational amplifier. The main aim of this control is damping the workpiece vibration signal. In Data acquisition process, CutPro$^{\mathrm{TM}}$ software, which was used in Ref. [41], is also used to acquire the sound signal (i.e., workpiece vibration signal) using data acquisition system (DAQ) at a sampling rate of 5120 Hz. Figure 5 shows the input and output of the control circuit. The input is from the sensor that detects workpiece vibration while the output is control signal that goes to the damping actuator. Table 1 shows the dynamic parameters of the system. In Table 1, the modal stiffness of the workpiece is 2.03×10^7 N/m, and the stiffness of the actuator, k_1=6130 N/m. Modal mass of the workpiece is 4.4725 kg, while mass of the actuator, i.e., m_1=2.2 kg.

Table 1 Dynamic parameters of the experimental system

Serial number	Frequency (Hz)	Damping ratio	Stiffness (N/m)	Mass (kg)
1	339	0.006865	2.03×10^7	4.4725
2	-	0.15	6130	2.2

Note: '1' represents modal parameter of the workpiece, while '2' represents actuator parameter from datasheet

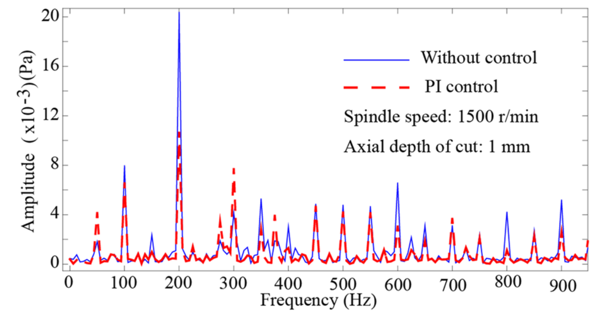

Figure 6 FFT of displacement signals at a spindle speed of 1500 r/min, radial depth of cut of 1 mm and axial depth of cut of 1 mm

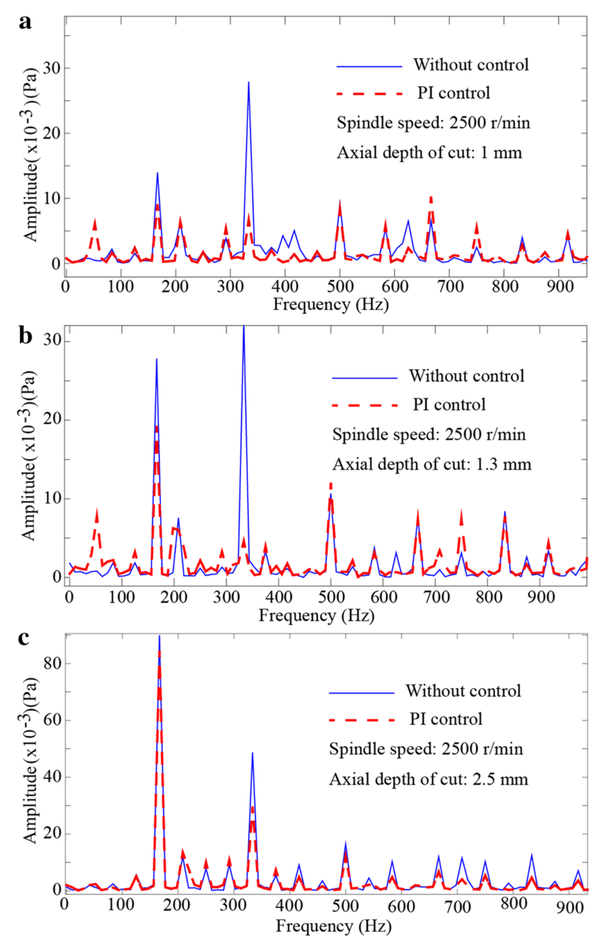

Figure 7 FFT of displacement signals at a spindle speed of 2500 r/min: (**a**) at an axial depth of cut of 1 mm, (**b**) at an axial depth of cut of 1.3 mm, and (**c**) at an axial depth of cut of 2.5 mm

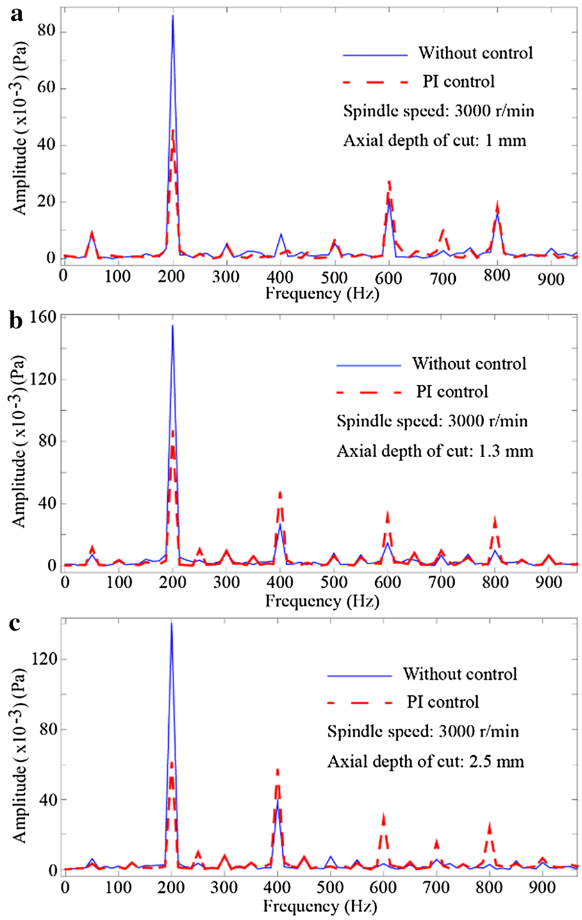

Figure 8 FFT of displacement signals at a spindle speed of 3000 r/min: (**a**) at an axial depth of cut of 1 mm, (**b**) at an axial depth of cut of 1.3 mm, and (**c**) at an axial depth of cut of 2.5 mm

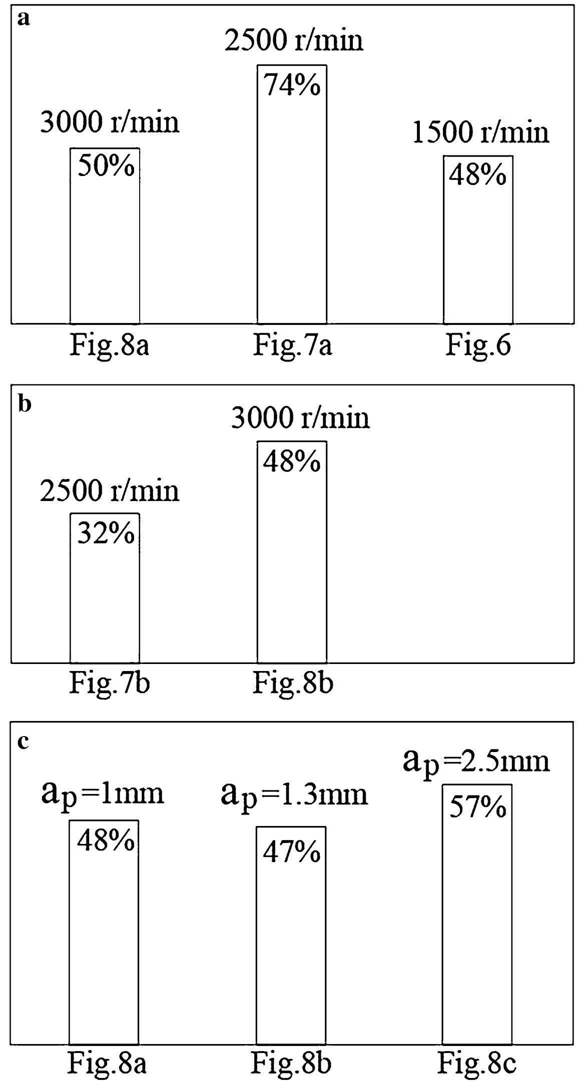

Figure 9 Bar chart showing percentages of vibration reduction at different cutting conditions: (**a**) at an axial depth of 1 mm, (**b**) at an axial depth of cut of 1.3 mm, (**c**) at a spindle speed of 3000 r/min

4 Results

Figures 6, 7, 8, 9 show the comparisons before and after usage of the proposed method. Figure 6 displays the effect of the planned scheme at spindle speed of 1500 r/min. Figure 7 described the outcome of the recommended system at spindle speed of 2500 r/min. In this figure, axial depths of cut of 1 mm, 1.3 mm and 2.5 mm are considered in Figures 7(a), (b) and (c). Figure 8 presents the result of the suggested technique at spindle speed of 3000 r/min and different axial depth of cut. Figure 9 illustrates the overall effect of the projected process under different cutting conditions. The magnitude of the FFT of the measured signals could reflect the vibration intensity. Consequently, the magnitude of the FFT signals can be selected as the criteria to characterize the dynamic vibration. To obviously show the effect of the proposed method, the percentages of vibration reduction under different cutting parameters was calculated and pictured in Figure 9.

From the above comparisons, it can be seen that the proposed method can well reduce the vibrations under different cutting conditions. Chatter due to vibration of a flexible workpiece resulting from the bending displacement is a major obstacle affecting the quality of a workpiece in the milling process. However, bending moment has not been given attention in the literature. This work has provided a means of reducing the severity of the situation. The work considered the spindle speed variation and depth of cut variation. The actuator provides a counterbalance force that minimizes the displacement due to the bending moment, as shown in the figures.

5 Conclusions

The work has presented the active control method to suppress chatter vibration in the milling process. The effect of the proposed method in active control of the workpiece vibration has been verified at different depths of cut and spindle speeds. The chatter vibration is suppressed in the milling experiment based on the designed method. The work has contributed to planning a novel approach that addresses the problem of chatter vibration in spite of technical hitches in modeling and controller design. This method can also lead to considerable reduction in chatter vibration that can be beneficial in industries in term of cost reduction and energy saving. Milling at a different depth of cut and spindle speed is accomplished to illustrate the effect of the proposed method. The research has provided a novel method of improving workpiece quality; this is achieved through reduction of workpiece vibration by using an analog proportional-integral controller.

Authors' Contributions

M. Wan was in charge of the whole trial; B.B. Muhammad wrote the manuscript; Y. Liu and H. Yuan assisted with sampling and laboratory analyses. All authors read and approved the final manuscript.

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Competing Interests

The authors declare no competing financial interests.

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