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Novel Accelerating Life Test Method and Its Application by Combining Constant Stress and Progressive Stress

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Abstract

Constant stress accelerated life tests (ALTs) can be applied to obtain a high estimation accuracy of reliability measurements, but these are time-consuming tests. Progressive stress ALTs can yield failures more quickly but cannot guarantee the estimation accuracy of reliability measurements. In this paper, a progressive-constant combination stress ALT is proposed to combine the merits of both tests. The optimal plan, in which the design variables are the initial progressive stress level, the progressive stress ramp rate, the sample allocation proportion of the progressive stress and the constant stress level, is determined using the principle of minimizing the asymptotic variance of the maximum likelihood estimator of the natural log reliable life for the connectors. A comparison between the optimal PCCSALT plan and the CSALT plan with the same sample size and estimation accuracy shows that the test time is reduced by 13.59% by applying the PCCSALT.

Keywords: Constant stress, Progressive stress, Accelerated life test, Optimal test plan, Reliability test

1 Introduction

Accelerated life tests (ALTs) can yield life information of a product in a short time [1–3]. Constant stress accelerated life tests (CSALTs) have the advantages of theory maturity and high statistical precision, but they require many samples and a long test time. Progressive stress accelerated life tests (PSALTs) [4] can generate results in a shorter test time and require a smaller sample size, but they are not widely used in practice because of the immature statistical method and the poor estimation accuracy [5, 6]. To yield accurate estimates of reliability measurements (e.g., reliable life, hazard rate), many studies have been performed on the design of an optimum PSALT plan have. Prot [7] proposed the PSALT optimal design method for a Weibull distribution. Bai et al. [8] optimized a time-censored simple ramp stress test plan for the power-Weibull model and then proposed an optimum single ramp stress test plan [9], which is more accurate and efficient than the plan in Ref. [8]. Liao and Elsayed [10] developed an optimum method for the log-normal distribution. Ma and Meeker [11] presented a new approach for computing approximate variances and used it to convert and optimize the plan in a previous study [10]. Zhu and Elsayed [12] developed an optimal model for exponential distribution with time censored and progressive stress based on the cumulative exposure model. Srivastava and Mittal [13] developed the model of the optimum multi-objective PSALT with a stress upper bound for a Burr type-XII distribution under time censoring. These studies have demonstrated that the design of the optimum test plan can contribute to improving the life estimation accuracy, but the improvement degree is not obvious.

To synthesize the characteristics of the CSALT and PSALT, a progressive-constant combination stress ALT (PCCSALT) is proposed. The example of the Y11X-1832 electrical connector is used to demonstrate the new test method. In the test plan optimization, the minimum asymptotic variance of the maximum likelihood (ML) estimator of the reliable life at design stress is treated as the optimality principle. The initial stress level, the stress

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ramp rate, the sample allocation proportion of progressive stress and the constant stress level should be determined to improve the test efficiency and maintain the same estimation accuracy as that of the CSALT.

2 Statistical Model of the Electrical Connectors

The life of electrical connectors t basically depends on the temperature and follows a two-parameter Weibull distribution under storage and working conditions [14]. The cumulative distribution function (CDF) of t is

$$F(t) = 1 - \exp[-(t/\eta)^m], \tag{1}$$

where m is the shape parameter, η is the characteristic life.

Generally, the reliability of the electrical connector is represented with the Pth quartile life at a certain working temperature. If failure mechanisms that expose at different temperature stresses are the same as those that expose in field use, then the statistical model of the electrical connector can be expressed as follows [14].

(1) The product lifetime is statistically independent and follows a two-parameter Weibull distribution.
(2) The parameter m is a constant at different temperature stresses.
(3) The relationship between η and the working temperature, T, satisfies the Arrhenius model, namely,

$$\eta = A \exp\left({}^{\Delta E}/_{kT}\right), \tag{2}$$

where A is the constant, ΔE is the activation energy for the reaction (eV), k is Boltzmann's constant, $k = 0.8617 \times 10^{-4}$ eV/K, T is the thermodynamic temperature (K).

If y equals the logarithmic lifetimes, ln t, then Eq. (1) can be transformed into an extreme value distribution from a Weibull distribution. The CDF is

$$F(y) = 1 - \exp\{-\exp[(y - \mu)/\sigma]\}, \tag{3}$$

where μ is the location parameter, $\mu = \ln \eta$, σ is the scale parameter, $\sigma = 1/m$.

Therefore, the Arrhenius-Weibull model described by Eq. (1) and Eq. (2) can be transformed into the linear extreme value model shown in Eq. (3), which can be described as follows:

(1) y is statistically independent and follows an extreme value distribution.
(2) σ is a constant and independent of the stress.
(3) μ is a function of the stress (generally, the transformed stresses) x, that is,

$$\ln \eta = \mu(x) = \gamma_0 + \gamma_1 x,$$

where $\gamma_0 = \ln A$, $\gamma_1 = \Delta E/10^3\,k$, $x = 10^3/T$.

To simplify the calculation and make the results more intuitive, the transformed stress should be standardized, as follows:

$$\xi(x) = \frac{x - x_0}{x_m - x_0},$$

where x_0 is the normal working stress level, $x_{\rm m}$ is the highest test stress level.

The normal working temperature, T_0, corresponds to $\xi_0 = 0$, whereas the highest test temperature, $T_{\rm m}$, corresponds to $\xi_{\rm m} = 0$. The log scale parameter ln η is

$$\ln \eta = \mu(\xi) = \beta_0 + \beta_1 \xi, \tag{4}$$

where $\beta_0 = \gamma_0 + \gamma_1 x_0$, $\beta_1 = (x_{\rm m} - x_0)\gamma_1$, ξ is the standardized stress.

The discussions below are based on Eq. (4).

3 Design Principle for the PCCSALT Plan

3.1 Stress Loading

To evaluate the life of the connector under normal stress, the temperature stress is used to conduct the ALT [14]. Figure 1 shows the PCCSALT stress loading profile, where the abscissa denotes t, and the ordinate denotes ξ.

The plan has two types of temperature stress loadings: a progressive one ($\xi_{\rm p} = \alpha t$) and a constant one ($\xi_{\rm c}$). The temperature stresses range from 0 to 1. The 0 denotes the normal working stress and the 1 denotes the highest test stress. The parameter $\xi_{\rm p}$ is the initial value of the progressive stress, and α is the stress ramp rate; $\xi_{\rm c}$ is a constant stress. The constant stress is always higher than the progressive stress. The parameter n denotes the number of samples, and $\pi_{\rm p}$ denotes the allocation proportion of the samples at the progressive stress. The test continues until time τ.

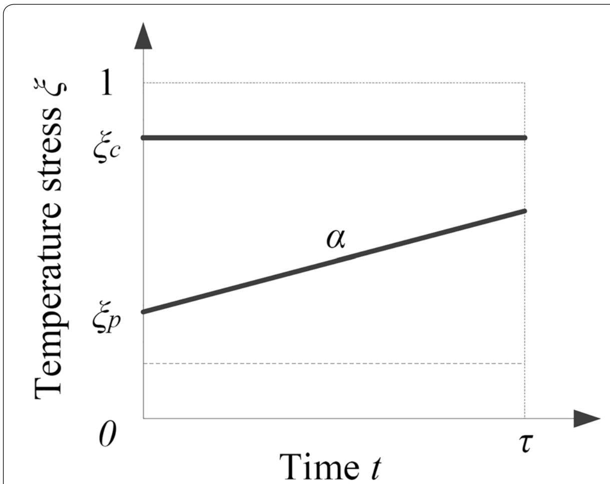

Figure 1 Stress loading profile

3.2 Test Design Criterion

To optimize the test plan, suppose that

(1) The test is stopped at τ or when all the samples fail;
(2) T_m is determined based on the principle of not changing the failure mechanism; and
(3) To guarantee quick failures, the test stress should be higher than the normal working stress.

According to Ref. [15], a higher upper test temperature limit will provide a small asymptotic variance of the ML estimator of the Pth quantile of the natural logarithm life. Therefore, T_m should be as high as possible.

3.3 Test Plan Optimization Criterion

To improve the life estimation accuracy and balance the test efficiency, the PCCSALT plan could be optimized by using the criteria of minimizing the asymptotic variance of the ML estimator of the Pth quantile of the natural logarithm life.

At stress ξ_0, the Pth quantile of the linear extreme value model is

$$y_P(\xi_0) = \mu(\xi_0) + z_P\sigma = \beta_0 + \beta_1\xi_0 + z_P\sigma, \tag{5}$$

where z_P is the Pth quantile of the standard extreme value distribution.

Here $P = 0.01$ [16], the corresponding quantile of the product is the log reliable life with a reliability of 99%.

3.4 Estimation Method

There are many methods for analyzing censored data such as ML methods, linear estimation methods based on order statistics and graphical estimation methods. The ML method is the most widely used because of the following reasons [16]:

(1) Analyzing the censored data by linear estimation methods causes a loss of test information, and these method are not statistically rigorous. However, ML methods are exactly correct.
(2) The optimal test plan obtained by ML methods is close to that obtained using the other estimation methods, and ML methods are easy to apply in plan optimization.
(3) The standard deviation of an ML estimator has the minimum value relative to the others.

Therefore, the ML method is used to compute the Pth quantile and its variance of the product when optimizing ALT plans.

Before optimizing the test plans for the PCCSALT, the model parameters β_0, β_1 and σ must be known. The values of the parameters are usually estimated by experience, similar product data or the results of an early test. In this manner, the optimal test plans may not be optimum, but it is assured that a crude estimator is better than no estimator.

4 Statistical Analysis Method for Designing the Plan

4.1 Sample Likelihood Function from a Constant Stress

In the CSALT, suppose sample i fails at $t_i \le \tau$. Then, for the linear extreme value model, its likelihood function is

$$L'_{ci} = \ln f(y_i) = -\ln\sigma + (y_{ci} - \mu)/\sigma - \exp[(y_{ci} - \mu)/\sigma],$$

where $y_{ci} = \ln t_i$.

Suppose sample i survives at τ. It is right censored, and its failure time is greater than τ. The likelihood function is

$$L''_{ci} = \ln R_c(y_\tau) = -\exp[(y_\tau - \mu)/\sigma],$$

where $R_c\,(y_\tau)$ is the reliability of the sample from constant stress at τ, $y_\tau = \ln\tau$.

Define an indicator function I_i such that $I_i = 1$ if $t_i \le \tau$ and $I_i = 0$ if $t_i > \tau$. Let $z_{ci} = (y_{ci} - \mu)/\sigma$ and $z_{c\tau} = (y_\tau - \mu)/\sigma$. The log likelihood function for sample i could be expressed as

$$\begin{aligned} L_{ci} &= I_i L'_{ci} + (1 - I_i)L''_{ci} \\ &= I_i[-\ln\sigma + z_{ci} - \exp(z_{ci})] + (1 - I_i)[-\exp(z_{c\tau})]. \end{aligned}$$

For n_c samples tested under constant stress, the sample log likelihood function is

$$L_c = \sum_{i=1}^{n_c} L_{ci}. \tag{6}$$

4.2 PSALT Theory and Statistical Analysis Method

4.2.1 PSALT Theory

The detailed form of the life distribution for the electrical connectors under progressive stress is derived based on the cumulative exposure model [16]. This model can be expressed as follows [17]: the remaining life of the sample only subject to the current stress and the current cumulative portion failed, regardless of how the portion accumulated. The model ignores the effect of the change in stress. For the samples under varying stress (e.g., the step stress, the progressive stress), at a given time, the failure probability of a survivor is according to the cumulative distribution for the stress at that time, but beginning when the previously accumulated portion failed [18]. The determination of starting time follows the equivalence principle in which the same cumulative exposure means the same failure probability.

In the PSALT, the stress ξ has a relationship with t.

$$\xi(t) = \xi_p + \alpha t. \tag{7}$$

The progressive stress is considered a limit of the step stress. The progressive stress above can be approximated to a step stress that has K equally spaced test stresses with equal time shifts [19], as shown in Figure 2.

$$\xi_i = \xi_{\mathrm{p}} + (i - 0.5) \cdot \alpha \cdot \tau/K, \quad i = 1, 2, \ldots, K.$$

Step 1 runs at stress ξ_1, starting at time 0, and stops at Δt. According to Eq. (1), the CDF of the samples could be written as

$$F(t) = 1 - \exp\{-[t/\eta(\xi_1)]^m\}, \quad 0 \le t < \Delta t.$$

Step 2 runs at stress ξ_2, and its equivalent start time is determined by the cumulative exposure at stress ξ_1. On the basis of the equivalence principle, the equivalent start time t_1 is just the solution of

$$1 - \exp\{-[t_1/\eta(\xi_2)]^m\} = 1 - \exp\{-[\Delta t/\eta(\xi_1)]^m\}.$$

Therefore, t_1 equals to $\eta(\xi_2)\ \Delta t/\eta(\xi_1)$ at stress ξ_2. Then, the CDF of the samples at stress ξ_2 by total time t is

$$F(t) = 1 - \exp\{-[(t - \Delta t + t_1)/\eta(\xi_2)]^m\}, \quad \Delta t \le t < 2\Delta t.$$

In general, the equivalent start time for step i is given by $t_{i-1} = \eta(\xi_i) \sum_{j=1}^{i-1} \Delta t/\eta(\xi_j)$ at stress ξ_i. The CDF of the samples at stress ξ_i by total time t is

$$F(t) = 1 - \exp\{-[\varepsilon(t)]^m\}, \quad (i - 1)\Delta t \le t < i\Delta t. \tag{8}$$

where $\varepsilon(t)$ is the cumulative exposure,

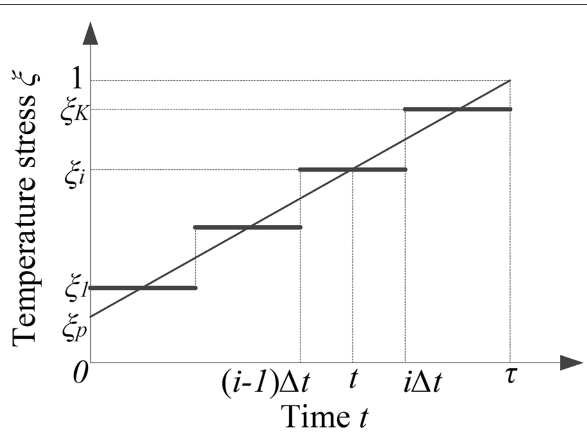

Figure 2 Approximation of a progressive stress by a step stress

$$\varepsilon(t) = \frac{t - (i - 1)\Delta t}{\eta(\xi_i)} + \sum_{j=1}^{i-1} \frac{\Delta t}{\eta(\xi_j)}$$

As $K \to \infty$, $i \to \infty$. For an infinitesimal time $\mathrm{d}t$, $t - (i-1)\Delta t \to \mathrm{d}t$, $\Delta t \to \mathrm{d}t$. At t, the stress can be described as $\xi(t)$. Then,

$$\begin{aligned}\lim_{i\to\infty} \varepsilon(t) &= \lim_{i\to\infty} \left[\frac{t - (i - 1)\Delta t}{\eta(\xi_i)} + \sum_{j=1}^{i-1} \frac{\Delta t}{\eta(\xi_j)}\right] \\ &= \int_{(i-1)\Delta t}^{t} \frac{\mathrm{d}t}{\eta[\xi(t)]} + \int_0^{(i-1)\Delta t} \frac{\mathrm{d}t}{\eta[\xi(t)]} \\ &= \int_0^t \frac{\mathrm{d}t}{\eta[\xi(t)]}\end{aligned}$$

Thus, the cumulative exposure as a sum (formed under the step stress) transforms into an integral (formed under the progressive stress). At the progressive stress, the cumulative exposure is

$$\begin{aligned}\varepsilon(t) &= \int_0^t \frac{\mathrm{d}t}{\eta[\xi(u)]} = \int_0^t \frac{1}{\exp[\beta_0 + \beta_1\xi(t)]}\mathrm{d}u \\ &= \frac{1 - \exp(-\beta_1\alpha t)}{\beta_1\alpha \exp(\beta_0 + \beta_1\xi_p)}.\end{aligned} \tag{9}$$

If a sample fails at t, then Eq. (9) is its cumulative exposure. Based on the equivalence principle, the equivalent time at initial stress ξ_{p} is

$$t_{\mathrm{p}} = \varepsilon(t) \cdot \eta(\xi_{\mathrm{p}}) = [1 - \exp(-\beta_1\alpha t)]/\beta_1\alpha,$$

where $\eta(\xi_{\mathrm{p}})$ is the characteristic life at initial stress ξ_{p},

$$\eta(\xi_{\mathrm{p}}) = \exp(\beta_0 + \beta_1\xi_{\mathrm{p}}).$$

The corresponding failure probability is

$$F(t_{\mathrm{p}}) = 1 - \exp\{-[t_{\mathrm{p}}/\eta(\xi_{\mathrm{p}})]^m\}.$$

The equation above has proven that after being transformed, the life of the products from the progressive stress still follows the Weibull distribution at the constant stress.

Combining Eqs. (7)–(9), the CDF of the products under progressive stress is

$$\begin{aligned}F(t) &= 1 - \exp\{-[\varepsilon(t)]^m\} \\ &= 1 - \exp\left\{-\left[\frac{1 - \exp(-\beta_1\alpha t)}{\beta_1\alpha \exp(\beta_0 + \beta_1\xi_{\mathrm{p}})}\right]^m\right\}.\end{aligned} \tag{10}$$

The probability density function (PDF) is the partial derivative of the CDF with respect to t, that is,

$$f(t)=\frac{m}{\exp(\beta_0+\beta_1\xi_p+\beta_1\alpha t)}\left[\frac{1-\exp(-\beta_1\alpha t)}{\beta_1\alpha\exp(\beta_0+\beta_1\xi_p)}\right]^{m-1}\times\exp\left\{-\left[\frac{1-\exp(-\beta_1\alpha t)}{\beta_1\alpha\exp(\beta_0+\beta_1\xi_p)}\right]^m\right\}. \tag{11}$$

4.2.2 Sample Likelihood Function from the Progressive Stress

Under progressive stress, suppose sample j fails at $t_j \le \tau$. Based on Eq. (11), its likelihood function is

$$L'_{pj}=\ln f(t_j) = -\beta_1(\xi_p+\alpha t_j)-y_{pj}-\ln\sigma+z_{pj}-\exp(z_{pj}),$$

where

$$y_{pj}=\ln c(t_j),$$

$$z_{pj}=(y_{pj}-\beta_0)/\sigma,$$

$$c(t_j)=[1-\exp(-\beta_1\alpha t_j)]/[\beta_1\alpha\exp(\beta_1\xi_p)].$$

Suppose sample j survives at τ. It is right censored, and its failure time is greater than τ. The likelihood function is

$$L''_{pj}=\ln R_p(\tau)=-\exp(z_{p\tau}),$$

where $R_p(\tau)$ is the reliability of the sample from progressive stress at τ, $z_{p\tau}=[\ln c(\tau)-\beta_0]/\sigma$.

Define an indicator function I_j such that $I_j=1$ if $t_j \le \tau$ and $I_j=0$ if $t_j>\tau$. The log likelihood function for sample j is

$$L_{pj}=I_jL'_{pj}+(1-I_j)L''_{pj} = I_j[-\beta_1(\xi_p+\alpha t_j)-y_{pj}-\ln\sigma+z_{pj}-\exp(z_{pj})]+(1-I_j)[-\exp(z_{p\tau})].$$

For n_p samples tested at the progressive stress, the sample log likelihood function is

$$L_p=\sum_{j=1}^{n_p}L_{pj}. \tag{12}$$

4.3 Sample Likelihood Function

The PCCSALT has n samples that are divided into two groups, and the number of samples allocated to the progressive stress is $\pi_p n$. The sample likelihood function Eq. (13) can be obtained by adding Eq. (6) and Eq. (12) which are both used to estimate the $\hat{y}_P(\xi_0)$:

$$L=L_c+L_p=\sum_{i=1}^{(1-\pi_p)n}L_{ci}+\sum_{j=1}^{n\pi_p}L_{pj}, \tag{13}$$

4.4 Information and Covariance Matrix

Based on the ML theory, the covariance matrix $\boldsymbol{\Sigma}$ for the model parameters $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}$ is

$$\boldsymbol{\Sigma}=\begin{pmatrix}\mathrm{Var}(\hat{\beta}_0) & \mathrm{Cov}(\hat{\beta}_0,\hat{\beta}_1) & \mathrm{Cov}(\hat{\beta}_0,\hat{\sigma})\\ \mathrm{Cov}(\hat{\beta}_1,\hat{\beta}_0) & \mathrm{Var}(\hat{\beta}_1) & \mathrm{Cov}(\hat{\beta}_1,\hat{\sigma})\\ \mathrm{Cov}(\hat{\sigma},\hat{\beta}_0) & \mathrm{Cov}(\hat{\sigma},\hat{\beta}_1) & \mathrm{Var}(\hat{\sigma})\end{pmatrix}=\boldsymbol{F}^{-1}, \tag{14}$$

where $\boldsymbol{F}$ is the information matrix, and it is the mathematical expectation of the negative second partial derivatives for L, that is

$$\boldsymbol{F}=\begin{bmatrix}E(-\partial^2L/\partial\beta_0^2) & E(-\partial^2L/\partial\beta_0\partial\beta_1) & E(-\partial^2L/\partial\beta_0\partial\sigma)\\ E(-\partial^2L/\partial\beta_0\partial\beta_1) & E(-\partial^2L/\partial\beta_1^2) & E(-\partial^2L/\partial\beta_1\partial\sigma)\\ E(-\partial^2L/\partial\beta_0\partial\sigma) & E(-\partial^2L/\partial\beta_1\partial\sigma) & E(-\partial^2L/\partial\sigma^2)\end{bmatrix}. \tag{15}$$

According to Eq. (13),

$$\boldsymbol{F}=\frac{1}{\sigma^2}\left[\sum_{i=1}^{(1-\pi_p)n}\begin{pmatrix}A_{1i} & \xi_cA_{1i} & A_{2i}\\ \xi_cA_{1i} & \xi_c^2A_{1i} & \xi_cA_{2i}\\ A_{2i} & \xi_cA_{2i} & A_{3i}\end{pmatrix}+\sum_{i=1}^{\pi_p n}\begin{pmatrix}B_1 & B_2 & B_3\\ B_2 & B_4 & B_5\\ B_3 & B_5 & B_6\end{pmatrix}\right] = \frac{n}{\sigma^2}\begin{pmatrix}(1-\pi_p)A_1+\pi_pB_1 & (1-\pi_p)\xi_cA_1+\pi_pB_2 & (1-\pi_p)A_2+\pi_pB_3\\ (1-\pi_p)\xi_cA_1+\pi_pB_2 & (1-\pi_p)\xi_c^2A_1+\pi_pB_4 & (1-\pi_p)\xi_cA_2+\pi_pB_5\\ (1-\pi_p)A_2+\pi_pB_3 & (1-\pi_p)\xi_cA_2+\pi_pB_5 & (1-\pi_p)A_3+\pi_pB_6\end{pmatrix}.$$

where $A_1 = 1 - \exp[-\exp(z_{c\tau})]$, $A_2 = \int_{-\infty}^{z_{c\tau}} z\exp(2z)$ $\exp[-\exp(z)]dz + z_{c\tau}\exp[z_{c\tau} - \exp(z_{c\tau})]$,

$$A_3 = \int_{-\infty}^{z_{c\tau}} z\exp(z)\exp[-\exp(z)][z\exp(z)+ \exp(z) - 1]dz + z_{c\tau}(z_{c\tau}+1)\exp[z_{c\tau} - \exp(z_{c\tau})],$$

$$B_1 = 1 - \exp[-\exp(z_{p\tau})],$$

$$B_2 = -\int_0^{\tau} \exp(z)y'f(t)dt - y'_\tau \exp[z_{p\tau} - \exp(z_{p\tau})],$$

$$B_3 = \int_0^{z_{p\tau}} z\exp(z)\exp[z - \exp(z)]dz + z_{p\tau}\exp[z_{p\tau} - \exp(z_{p\tau})],$$

$$B_4 = \int_0^{\tau} \{\sigma y''[\exp(z) + \sigma - 1] + y'^2\exp(z)\}f(t)dt + (y'^2_\tau + \sigma y''_\tau)\exp[z_{p\tau} - \exp(z_{p\tau})],$$

$$B_5 = \int_0^{\tau} y'[1 - \exp(z) - z\exp(z)]f(t)dt - y'_\tau(1 + z_{p\tau})\exp[z_{p\tau} - \exp(z_{p\tau})],$$

$$B_6 = \int_0^{z_{p\tau}} z^2\exp(z)\exp[z - \exp(z)]dz + B1 + z_{p\tau}^2\exp[z_{p\tau} - \exp(z_{p\tau})],$$

$z_{c\tau} = (\ln\tau - \mu)/\sigma, \mu = \ln\eta(\xi_c), z = (y - \beta_0)/\sigma, y = c(t)$,
$c(t) = [1 - \exp(-\beta_1\alpha t)]/\left[\beta_1\alpha\exp(\beta_1\xi_p)\right]$,
$z_{p\tau} = [\ln c(\tau) - \beta_0]/\sigma$,
$c(\tau) = [1 - \exp(-\beta_1\alpha\tau)]/\left[\beta_1\alpha\exp(\beta_1\xi_p)\right]$,

$$f(t) = \frac{m}{\exp(\beta_0 + \beta_1\xi_p + \beta_1\alpha t)}\left[\frac{1 - \exp(-\beta_1\alpha t)}{\beta_1\alpha\exp(\beta_0 + \beta_1\xi_p)}\right]^{m-1} \times \exp\left\{-\left[\frac{1 - \exp(-\beta_1\alpha t)}{\beta_1\alpha\exp(\beta_0 + \beta_1\xi_p)}\right]^m\right\},$$

$y' = \alpha t/[\exp(\beta_1\alpha t) - 1] - 1/\beta_1 - \xi_p$,
$y'_\tau = \alpha\tau/[\exp(\beta_1\alpha\tau) - 1] - 1/\beta_1 - \xi_p$,
$y'' = 1/\beta_1^2 - \alpha^2 t^2\exp(\alpha\beta_1 t)/[\exp(\alpha\beta_1 t) - 1]^2$,
$y''_\tau = 1/\beta_1^2 - \alpha^2\tau^2\exp(\alpha\beta_1\tau)/[\exp(\alpha\beta_1\tau) - 1]^2$.

4.5 Estimate of the Function Variance

Based on Eq. (5), for the linear extreme value model, the ML estimate of the Pth quantile $\hat{y}_P(\xi_0)$ at the normal working stress is a function of $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}$, that is,

$$\hat{y}_P(\xi_0) = \hat{\beta}_0 + \hat{\beta}_1\xi_0 + z_P\hat{\sigma}.$$

Since $\hat{y}_P(\xi_0)$ is close to a normal distribution [16], the asymptotic variance for $\hat{y}_P(\xi_0)$ is

$$\mathrm{Avar}\left[\hat{y}_P(\xi_0)\right] = (1, 0, z_P)\boldsymbol{F}^{-1}(1, 0, z_P)^{\mathrm{T}} = \sigma^2 V/n, \tag{16}$$

where V is the variance factor for the PCCSALT plan.

5 Model for Designing the Optimal Plan

5.1 Objective Function

The values of T_0, T_m, and τ are given before planning the optimal PCCSALT. The initial estimators of the model parameters $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}$ are determined by experience, similar product data or the results of an early test. The value of n is determined by V and $\hat{\sigma}$. Thus, the ML estimator V is treated as the objective function according to Eq. (16) and the criteria of minimizing $\mathrm{Avar}\left[\hat{y}_P(\xi_0)\right]$.

5.2 Design Variables

Since T_0, T_m, $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\sigma}$ are given, V depends on ξ_p, α, π_p, and ξ_c. Therefore, the design variables are

$$\xi_p, \alpha, \pi_p, \xi_c.$$

5.3 Constraints

(1) The value of ξ_c should range from 0 to 1, namely,

$$0 \le \xi_c \le 1.$$

(2) Considering the test design principle the progressive stress should always be less than the constant stress to ensure quick failures for the constant stress. That is,

$$\xi_p + \alpha t \le \xi_c.$$

(3) ξ_p and α are chosen to guarantee that the progressive stress values range from 0 to 1. Moreover, to ensure enough failures for the progressive stress, the progressive initial stress should be set 20% higher than the normal working stress, that is,

$$0.2 \le \xi_p + \alpha t \le 1.$$

(4) The value of π_p should also be set to range from 0 to 1. To make the plan feasible, two stresses should be allocated to a certain amount of samples. Set the π_p values to range between 0.3 and 0.7,

$$0.3 \le \pi_p \le 0.7.$$

5.4 Sample Size

Based on the ML theory, the MLE of the Pth quantile of $\hat{y}_P(\xi_0)$ under normal working stress is close to the normal distribution, with a mean of $\hat{y}_P(\xi_0)$ and a variance of $\mathrm{Var}[y_P(\xi_0)]$. If $\hat{y}_P(\xi_0)$ replaces $y_P(\xi_0)$, then the confidence bounds of $\hat{y}_P(\xi_0)$ for confident values γ are

$$[y_P(\xi_0)]_{\mathrm{U}} = \hat{y}_P(\xi_0) + K_\gamma\sqrt{\sigma^2 V/n},$$

$$[y_P(\xi_0)]_{\mathrm{L}} = \hat{y}_P(\xi_0) - K_\gamma\sqrt{\sigma^2 V/n}.$$

where K_γ is the $(100(1+\gamma)/2)$th standard normal quantile.

If $\hat{y}_P(\xi_0)$ is within $y_P(\xi_0) \pm W$ with probability γ, then

$$W = K_\gamma \sqrt{\sigma^2 V/n}.$$

Thus, the sample size n is

$$n = V\left(K_\gamma \sigma/W\right)^2. \tag{17}$$

6 Optimal Design of the Electrical Connectors PCCSALT Plan

6.1 Test Parameters

6.1.1 Temperature Stress

The working temperature of the Y11X series electrical connectors is 218.15–401.15 K according to a previous study [18]. In accordance with the usage conditions of the connectors, $T_0 = 298.15$ K is selected as the normal working temperature.

The electrical contact failure caused by the accumulation of the oxide film is the main mode [14] for the Y11X series electrical connectors. Since the highest test temperature should be close to the upper working temperature limit (431.15 K) with the requirement of avoiding introducing failure modes that will not be encountered in normal use, $T_m = 431.15$ K is chosen as the highest temperature stress in the test.

6.1.2 Censored Time

As the connectors fail fast at the progressive stress, considering the censored time for the CSALT [20] and the step stress ALT (SSALT) [21], this paper takes $\tau = 1000$ h as the censored time.

6.1.3 Parameter Estimators for the Statistical Model

By using the statistical analysis results [14] of the data from the previous CSALT, the crude estimate values of the model parameters for the Y11X-1832 electrical connector are $\hat{\gamma}_0 = -21.2813$, $\hat{\gamma}_1 = 9.7579$, and $\hat{\sigma} = 0.9867$. Using the transforms in Section 2, the standardized parameters are

Table 2 Optimal CSALT plan

No.	Stress ξ, Temperature T (K)	Sample allocation proportion π	Censored time t (h)	Sample size n	Variance factor V
1	0.4313, 338.45	0.7	1000	40	24.1501
2	1, 436.15	0.3			

$$\hat{\beta}_0 = 11.4467,\ \hat{\beta}_1 = -8.0340,\ \hat{\sigma} = 0.9867.$$

6.2 Optimal Results

MATLAB's fmincon solver is used to address the optimization problem. The objective function converges to $V_m = 23.6837$, and the corresponding test plan is given in Table 1.

Given the confidence $\gamma = 40\%$, $K_\gamma = 0.6745$ and confidence interval width 2 $W = 0.8$, substitute them into Eq. (17), and the sample size is $n = 40$.

A comparison between the PCCSALT plan (see Table 1) and the CSALT plan (see Table 2) is applied to verify that the proposed plan is better, as it has a higher estimation accuracy for y_p. The compared simple CSALT plan is defined in the following manner: the mid value of the progressive stress, namely, $\xi_1 = \xi_p + \alpha \cdot t_m/2$, is used as the low stress, while the sample allocation proportion, the high stress and the censored time are the same as those in the proposed plan.

In general, the estimation accuracy depends on the sample size and test time. To improve the estimation accuracy, more test time is required if the sample size is fixed. To compare the efficiency of the two plans quantitatively, the test time for the CSALT is elongated to when its variance factor equals that of the proposed plan. Here, 1157 h are obtained. Therefore, the test time could be shortened by 13.59% if using the PCCSALT plan under the same requirements of estimation accuracy and sample size.

Table 1 Optimal PCCSALT plan

Stress type	Stress ξ, Temperature T (K)	Stress change rate α (h^{-1})	Sample allocation proportion π	Censored time t (h)	Variance factor V
Progressive	0.3048, 322.26	2.5298×10^{-4}	0.7000	1000	23.6837
Constant	1, 436.15	–	0.3000		

7 Conclusions

(1) An PCCSALT method combining progressive stress and constant stress is proposed.
(2) With the same estimation accuracy and sample size, the time of the optimal PCCSALT plan is 13.59% less than that of the optimal CSALT plan.

Authors' Contributions
W-HC was in charge of the whole trial; FY wrote the manuscript; PQ, JP and Q-CH assisted with sampling and laboratory analyses. All authors read and approved the final manuscript.

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Competing Interests
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