ORIGINAL ARTICLE

Quasi-Static and Dynamic Behaviors of Helical Gear System with Manufacturing Errors

Bing Yuan[1] , Shan Chang[1,2], Geng Liu[1*] and Li-Yan Wu[1]

Abstract

Time-varying mesh stiffness (TVMS) and gear errors include short-term and long-term components are the two main internal dynamic excitations for gear transmission. The coupling relationship between the two factors is usually neglected in the traditional quasi-static and dynamic behaviors analysis of gear system. This paper investigates the influence of short-term and long-term components of manufacturing errors on quasi-static and dynamic behaviors of helical gear system considering the coupling relationship between TVMS and gear errors. The TVMS, loaded static transmission error (LSTE) and loaded composite mesh error (LCMS) are determined using an improved loaded tooth contact analysis (LTCA) model. Considering the structure of shaft, as well as the direction of power flow and bearing location, a precise generalized finite element dynamic model of helical gear system is developed, and the dynamic responses of the system are obtained by numerical integration method. The results suggest that lighter loading conditions result in smaller mesh stiffness and stronger vibration, and the corresponding resonance speeds of the system become lower. Long-term components of manufacturing errors lead to the appearance of sideband frequency components in frequency spectrum of dynamic responses. The sideband frequency components are predominant under light loading conditions. With the increase of output torque, the mesh frequency and its harmonics components tend to be enhanced relative to sideband frequency components. This study can provide effective reference for low noise design of gear transmission.

Keywords: Manufacturing error, Mesh stiffness, Transmission error, Loaded composite mesh error, Vibration acceleration, Sideband frequency

1 Introduction

Gear transmission systems are widely used in many industry applications. The prediction and control of gear vibration and noise are always important considerations in recent years. Due to manufacturing and heat treatment process, it is inevitable that gears contain manufacturing errors with different types and magnitudes. As the same as mesh stiffness, manufacturing errors are also one of the two main internal excitations that generate unwanted vibration in gear transmission. To accurately predict vibration of gear system, it is crucial to investigate the

coupling relationships between mesh stiffness and manufacturing errors.

As the existence of manufacturing errors, the quasi-static engagement process of mating gear teeth will be not identical with the ideal one. The contact regions will come into contact earlier or later, and overloading or contact loss of mating gear teeth will occur in some engagement positions, which will affect mesh stiffness significantly in different loading conditions, and influence the dynamic behaviors of gear system.

Manufacturing errors include short-term and long-term components. The short-term components mainly refer to profile deviation, helix deviation, as well as pitch deviation and tooth surface modification. Conry and Seireg [1] proposed an evaluation method of load distribution and optimal modifications for cylindrical

*Correspondence: npuliug@nwpu.edu.cn
[1] Shaanxi Engineering Laboratory for Transmissions and Controls, School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an 710072, China
Full list of author information is available at the end of the article

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gears based on flexibility analysis method and elastic contact theory. Kubo and Kiyono [2] investigated the effects of different kinds of tooth form errors on vibration of spur and helical gear system, and found that convex tooth form error leads to minimum vibration, but concave and waving tooth form error result in relatively stronger vibration. Later, Kubo et al. [3] investigated the relationship between tooth contact pattern and transmission error of gears having errors, and developed the fast calculation method through observing the actual tooth contact pattern. Umezawa et al. [4, 5] developed a torsional dynamic model of spur gear system and analyzed the effects of pressure angle error, normal pitch error, and waved form error on vibration of gear system. It is found that the influences of gear errors on system vibration are significant. Vedmar and Andersson [6] developed a dynamic contact model to investigate the dynamic contact characteristics and vibration of spur and helical gears. Mattar and Velex [7, 8] presented an analytical model for calculating mesh stiffness of cylindrical gear using length of contact line, and developed a dynamic contact model to investigate the effects of shape deviations on quasi-static and dynamic behaviors of narrow-faced helical gears. Matsumura et al. [9] developed a torsional dynamic model of helical gear system, and studied the gear errors on system vibration under lighter loading condition. The results showed that the partial contact loss due to gear errors in light loading condition has significant effect on system vibration. Munro et al. [10] presented an approximate formula of transmission error considering corner contact due to manufacturing and assembly errors. Ogawa et al. [11] performed the theoretical and experimental investigations about the dynamic behaviors of a spur gear pair having helix deviation. The results showed that helix deviation will result in decreased mesh stiffness and lead to lower resonance speed. Wei et al. [12] employed the finite element method to analyze the effects of five types of flank deviation on load distribution of helical gears, and found the superposition property of the influences of individual flank deviation on load distribution. Fernández-del-Rincón et al. [13, 14] used a global finite element model and a partial finite element model to develop the TVMS calculation model of spur gears based on flexibility analysis method, and investigated the effects of tooth profile deviation and support flexibility on the dynamic behaviors of spur gear system. Wang et al. [15] employed the thin slice theory and potential energy method to develop the TVMS and contact stress calculation method for helical gears having tooth profile errors. Li [16] developed a finite element method programs to investigate the influences of manufacturing errors, gear misalignment, as well as assembly

errors and gear modifications on TVMS of a spur gear pair. Lin and He [17] used the finite element method to determine the static transmission error of a spur gear pair with machining errors, assembly errors and modifications, and then established a bending-torsional-axial coupling dynamic model to calculate the dynamic transmission error.

Long-term components of manufacturing errors mainly include eccentricity and accumulative pitch error. Yu et al. [18] employed a dynamic model of cylindrical geared rotor system to investigate the dynamic coupling behavior of transverse and rotational motions of gears subjected to gear eccentricities. The results indicated that the dynamic coupling behavior will become apparent in the low speed range when the resonances are excited by TVMS or profile errors. Wang et al. [19] presented theoretical formulas of no loaded static transmission error and time-varying backlash due to gear eccentricity, and developed a calculation method of dynamic transmission error for spur gears in consideration of gear eccentricities based on LS-DYNA3D. Xiang and Gao [20] studied the coupled torsion-bending vibration of a gear-rotor-bearing system in consideration of TVMS, gear eccentricity and nonlinear bearing force. The results suggested that the eccentricity has more significant effects on system vibration when the rotational speed is relatively lower. Umezawa and Sato [21] investigated the effect of accumulative pitch error on vibration acceleration of a spur gear pair, and drew the influence chart related with speed and contact ratio when accumulative pitch error is combined with other errors. Fernández-del-Rincón et al. [22] studied the loaded transmission error, pressure angle, as well as meshing force and bearing force of spur gears having index and run out errors under several transmitted torques. The results indicated that index errors will influence vibration behavior of the system and result in high overloads. Index errors will bring higher amplitude at the rotation frequency of shaft but run out errors will lead to lower values. Handschuh et al. [23], Talbot et al. [24] and Inalpolat et al. [25] performed numerical simulations and experiments to investigate the effect of tooth spacing errors on the root stress, dynamic factors and dynamic transmission error of a spur gear pair, respectively. The results suggested that tooth spacing errors have a direct impact on the root stress and significantly alter the baseline dynamic response. Meanwhile, the frequency spectra of dynamic response are enriched due to amplitude and frequency modulation.

As aforementioned published works, some of them neglected the nonlinear relationship between TVMS and manufacturing errors, which may be not accurate enough for predicting the vibration of gear system, especially in
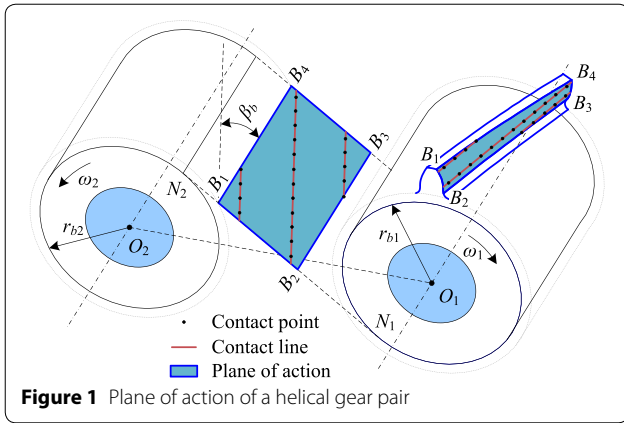
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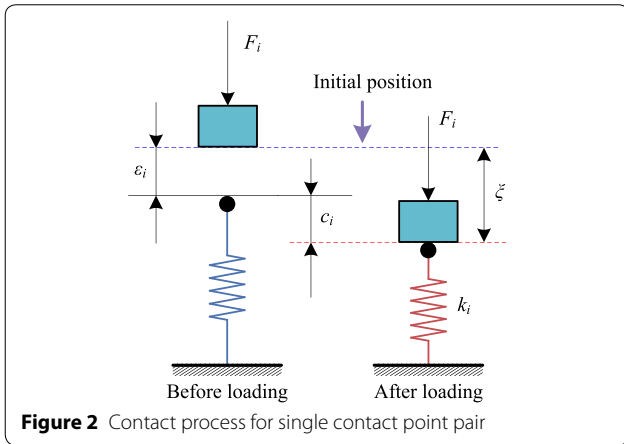**Figure 1** Plane of action of a helical gear pair

Figure 2 Contact process for single contact point pair

light loading condition. Some models of TVMS and manufacturing errors are too complicated to be widely used, for instance, the contact finite element model. Also, few researches were done on the effect of accumulate pitch error on quasi-static and dynamic behaviors of gear system, and most of them focused on spur gear system.

This study presents an improved LTCA model based on sub-structure technique and elastic contact theory to determine TVMS, LSTE and LCMS of helical gears having manufacturing errors. This model provides sufficient precision and high computation efficiency. Considering the structure of shaft, as well as the direction of power flow and bearing location, a precise generalized finite element dynamic model of helical gear system is developed to obtain the dynamic responses. In order to find out how the manufacturing errors affect the loaded tooth contact characteristics and dynamic behaviors of helical gear system, quasi-static and dynamic behaviors analysis are performed in consideration of short-term and long-term components of gear errors (Additional file 1).

2 Time-Varying Mesh Stiffness and Loaded Composite Mesh Error

2.1 Improved LTCA Model

The generation process of plane of action for a helical gear pair is shown in Figure 1. The three lines on the plane of action are the contact lines in the same engagement position. β_b is the helix angle of base circle. B_1B_2 and N_1N_2 are the theoretical and actual line of action in the transverse plane. r_{b1} and r_{b2} refer to the radius of base circle of driving and driven gear. ω_1 and ω_2 denote the rotational speed of the driving and driven gear.

The contact process of mating gear teeth after loading for helical gears can be considered as a three-dimensional line contact problem. All of the continuous contact lines can be discretized into several contact points, as shown in Figure 1. Gear errors, as well as tooth modification and misalignment can be regarded as the initial clearance of each contact point pair, the contact condition of all potential contact points for each engagement position can be written as

$$
\begin{cases}
-[\lambda]_{\text{Global}}\{F\} - \{u\}_{\text{Local}} + \xi + \{d\} = \{\varepsilon\}, \\
\sum_{i=1}^{n} F_i = \{I\}\{F\} = P, \\
\text{If } F_i > 0, \quad d_i = 0, \\
\text{If } F_i = 0, \quad d_i > 0,
\end{cases}
\tag{1}
$$

where $[\lambda]_{\text{Global}}$ is the flexibility matrix of global deformation of potential contact point pairs, $\{F\}$ is the external load vector, $\{u\}_{\text{Local}}$ is the contact deformation vector of potential contact point pairs, $\{d\}$ is the odd clearance vector of potential contact points, $\{\varepsilon\}$ is the initial clearance vector of potential contact points. ξ is the rigid body approach, it denotes the LSTE for a gear pair.

The deformation of mating gears consist of global and local contact deformation. The first one is linearly related to applied force, but the second one is nonlinearly related to applied force. A global finite element model and a partial finite element model are used to obtain the global flexibility matrix of potential contact points in the same engagement position based on the sub-structure method [26]. Considering the nonlinear relationship between local contact deformation and applied force, the local contact deformation of interested contact point can be calculated using the analytical formula [27]. The load distribution F and LSTE can be obtained using the iteration algorithm to solve the nonlinear matrix equation [26].

2.2 Loaded Composite Mesh Error

The contact process for single contact point pair i can be described as in the Figure 2. F_i is the force applied on the contact point pair i, ε_i is the initial clearance before

loading, c_i and k_i are the deformation and stiffness of the contact point pair i, respectively.

The mesh stiffness of a helical gear pair with manufacturing errors in one engagement position can be determined as

$$k_m = \sum_{i=1}^{N} k_i = \sum_{i=1}^{N} \frac{F_i}{\xi - \varepsilon_i}. \tag{2}$$

According to the deformation compatibility condition, the static load balance equation for one engagement position can be written as

$$\sum_{i=1}^{N} k_i(\xi - \varepsilon_i) = \sum_{i=1}^{N} k_i\xi - \sum_{i=1}^{N} k_i\varepsilon_i$$
$$= k_m\xi - \sum_{i=1}^{N} k_i\varepsilon_i = P, \tag{3}$$

where N denotes the number of potential contact points in the same engagement position.

Divide the left and right side of Eq. (3) by k_m, the following equation can be obtained

$$\xi - \frac{1}{k_m} \sum_{i=1}^{N} k_i\varepsilon_i = \frac{P}{k_m}. \tag{4}$$

Hence, the LCMS of a helical gear pair can be defined as

$$e_m = \frac{1}{k_m} \sum_{i=1}^{N} k_i\varepsilon_i. \tag{5}$$

Therefore, Eq. (3) can be rewritten as

$$k_m(\xi - e_m) = P. \tag{6}$$

It can be observed that the LCMS is related to TVMS, distribution of gear errors and applied force.

3 Finite Element Dynamic Model

A helical gear-rotor-bearing system can be divided into three elements that include shaft element, mesh element and bearing element. The dynamic model of shaft element is established using Timoshenko beam element with 2 nodes and 12 degrees of freedom. The dynamic model of mesh element is developed using spring-damping-error model. The stiffness matrix theory of bearing is employed to establish the dynamic model of bearing element. Assembling all of the dynamic models of these elements based on the finite element method, the global dynamic model of system can be obtained [28]. The matrix form of global motion equation of the system can be written as

Figure 3 Schematic diagram of a helical gear-rotor-bearing system and corresponding finite element dynamic model
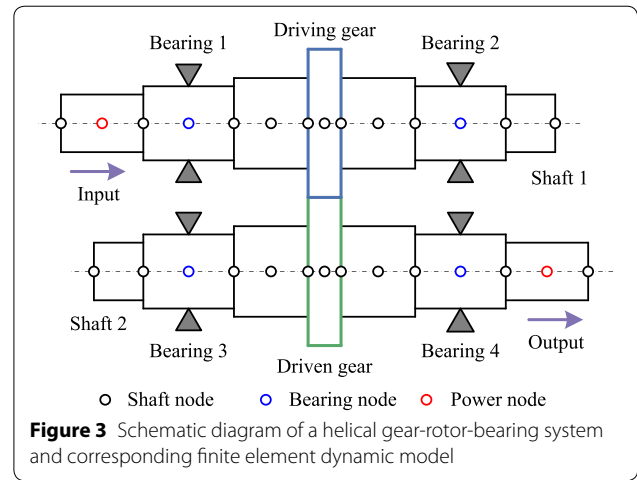
Table 1 Parameters of the driving and driven gear

Parameter	Driving gear	Driven gear
Tooth number	20	20
Normal module (mm)	10	10
Normal pressure angle (°)	20	20
Helix angle (°)	25	− 25
Addendum coefficient	1	1
Bottom clearance coefficient	0.25	0.25
Tooth width (mm)	60	60

$$M\ddot{x}(t) + C\dot{x}(t) + K(t)[x(t) - e(t)] = F, \tag{7}$$

where M is the global mass matrix, C is the global damping matrix, $K(t)$ is the global stiffness matrix, $x(t)$ is the generalized coordinates of finite element nodes, $e(t)$ refers to the LCMS vector. F is the external force vector.
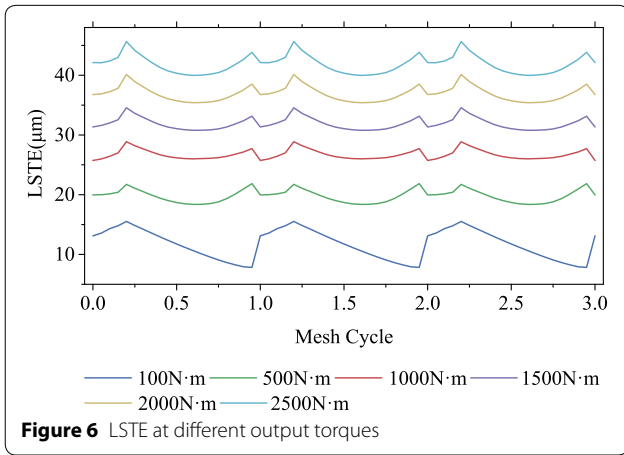
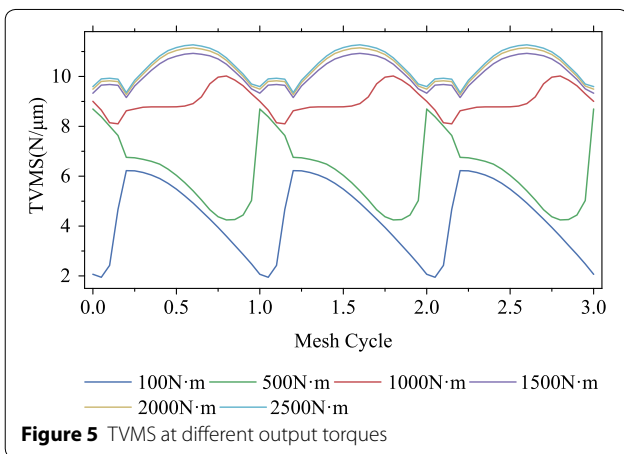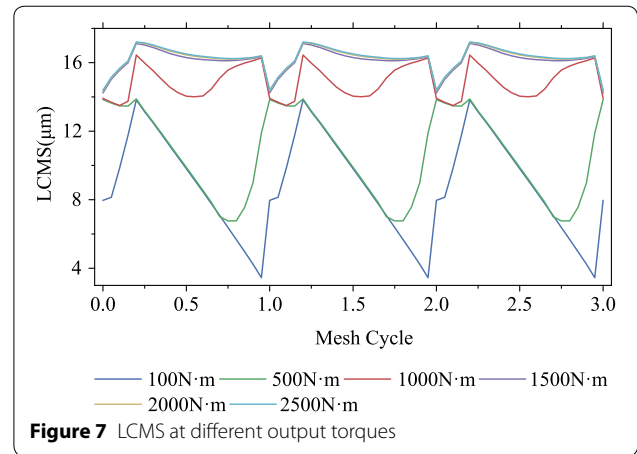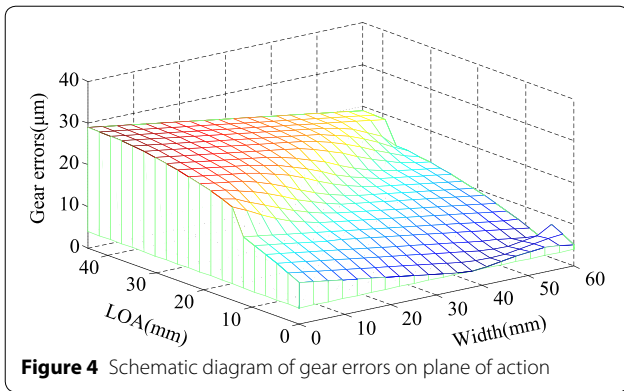Hence, the static balance equation of system can be written as

$$K(t)[x_s(t) - e(t)] = F, \tag{8}$$

where $x_s(t)$ is the static displacement vector.

4 Numerical Results and Discussion

The schematic diagram of a helical gear-rotor-bearing system and the corresponding finite element dynamic model are shown in Figure 3. The power inflow from the left side of shaft 1, and outflow from the right side of shaft 2. The finite element model of system include three types of nodes which are shaft nodes, bearing nodes and power nodes. The driving and driven gear are coupled using TVMS and LCMS. The basic parameters of driving and driven gear are given in Table 1.

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Figure 4 Schematic diagram of gear errors on plane of action

Figure 7 LCMS at different output torques

Figure 5 TVMS at different output torques

Figure 6 LSTE at different output torques

4.1 Short-term Components of Gear Errors
4.1.1 Description of Gear Errors
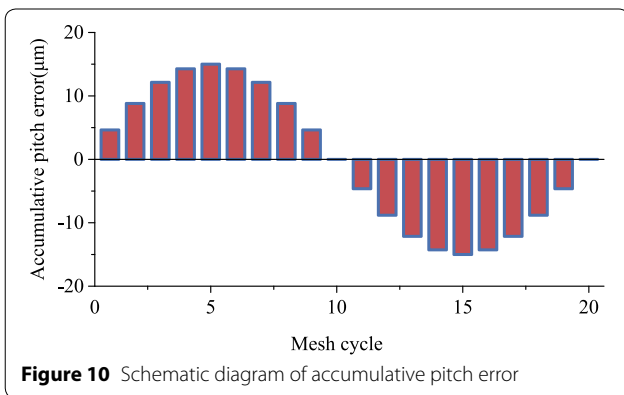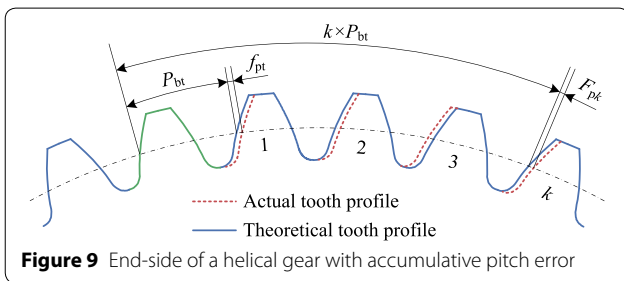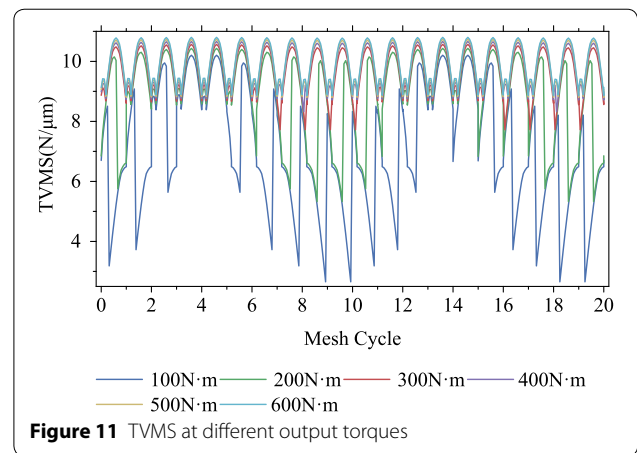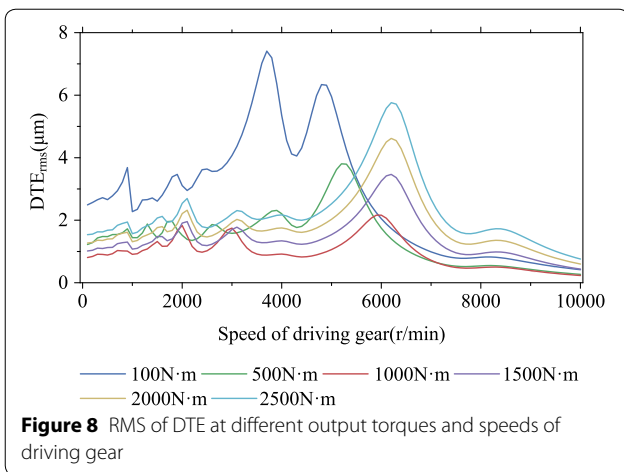The short-term components of gear errors, which include profile deviation, helix deviation and pitch deviation, are considered in this section. It is assumed that profile deviation are distributed along the tooth profile as a parabolic curve. As the inevitable shaft deformation and mounting errors can be regarded as helix deviation, it is defined that helix deviation is distributed along the gear width as a straight line. Neglecting the indexing errors, the pitch deviation alters positive and negative.

As the same as the calculation method of dynamic factor in ISO6336, only the deviations of driving gear are introduced into the LTCA model in this section. Gear errors on the plane of action are shown in Figure 4. The amplitude of gear errors is 26.71 μm when the accuracy of driving gear is sixth grade.

4.1.2 Quasi-Static Analysis
TVMS and LSTE at different output torques are shown in Figure 5 and Figure 6, respectively. It can be observed that the mesh stiffness at low torque is smaller than that at high torque and the shapes of curves are very different from each other at low torques. That is because light loading condition result in the appearance of contact loss in some theoretical contact regions. When the output torque is higher than 1500 Nm, the curves of mesh stiffness will be identical with each other. In these loading conditions, the actual contact region extend to full tooth surface. Due to local contact deformation nonlinearly related to applied force, with the increase of output torque, the mesh stiffness tend to slightly increase at high torques. The increase of output torque will also result in increase of LSTE. The amplitude of LSTE at 100 Nm output torque is larger than that at other loading conditions.

The curves of LCMS at different output torques are illustrated in Figure 7. It can be seen that the curves of LCMS are different from each other when applied torque is less than 1500 Nm. The LCMS increases with the output torques and will not change until the output torques are higher than 1500 Nm. The amplitudes of LCMS at

Figure 8 RMS of DTE at different output torques and speeds of driving gear

Figure 11 TVMS at different output torques

Figure 9 End-side of a helical gear with accumulative pitch error

Figure 10 Schematic diagram of accumulative pitch error

low torques are much larger than that at high torques. Obviously, the amplitudes of LCMS are always far smaller than the original amplitude of gear errors at all output torque levels.

4.1.3 Dynamic Analysis
Figure 8 shows the root mean square (RMS) of dynamic transmission error (DTE) at different output torques and speeds of driving gear. It can be observed that the resonance speed of the system become lower due to the reduction of mesh stiffness at light loading conditions.

When the output torque is higher than 1500 Nm, the resonance speed will not change anymore. It also can be found that lower output torque will bring stronger vibration compared with higher output torque. This is because the fluctuation of LSTE and LCMS are larger in light loading conditions.

4.2 Long-term Components of Gear Grrors
4.2.1 Description of Gear Errors
The schematic diagram of accumulative pitch error for a helical gear is shown as Figure 9, where, f_{pt} denotes the pitch deviation, P_{bt} refers to the theoretical tooth pitch. $k \times P_{bt}$ is the theoretical tooth pitch of k teeth, and F_{pk} is the accumulative pitch error of k teeth. Because the gear errors which are introduced into LTCA model are measured along the line of action, the gear error values of helical gears must be converted to the normal direction as follows:

$$f_{pbn} = f_{pt} \cos \alpha_t \cos \beta_b, \tag{9}$$

where f_{pbn} is the pitch deviation in the direction of line of action. α_t is the transverse pressure angle, β_b is the helix angle of base circle of helical gear.

The accumulative pitch error typically has a sine wave distribution over a rotational cycle of shaft. The sine function is employed to simulate the accumulative pitch error or long-term component of transmission error excitation in some published works [22, 29]. Therefore, it is assumed that the accumulative pitch error is distributed as a sine wave in this study, as shown in Figure 10.

4.2.2 Quasi-Static Analysis
Figures 11, 12, 13 shows four cases of TVMS, LSTE and LCMS of a helical gear pair at different output torques. The output torque is from 100 Nm to 600 Nm with step of 100 Nm. It can be observed that the accumulative
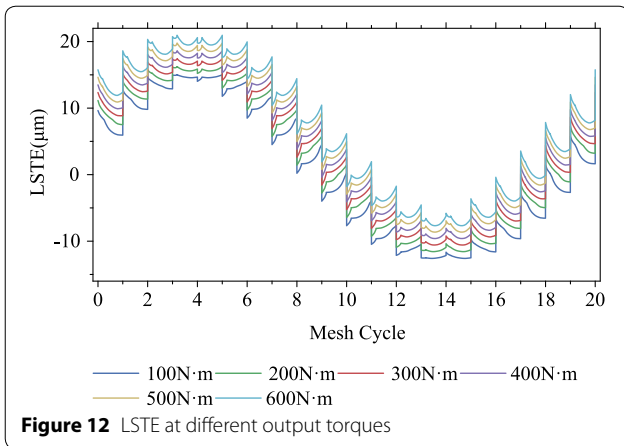
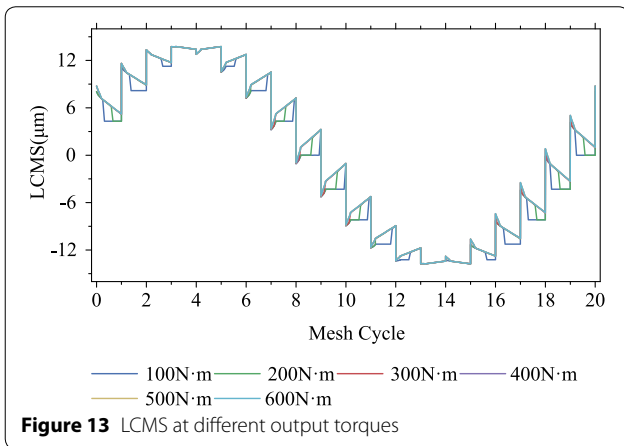**Figure 12** LSTE at different output torques

Figure 13 LCMS at different output torques

Figure 14 DTE of helical gear system with accumulative pitch error at 300 Nm output torque
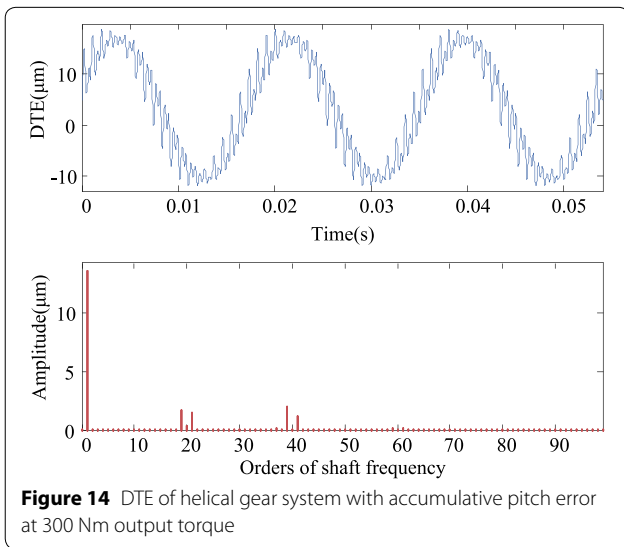

Figure 15 DTE of helical gear system with accumulative pitch error at 2500 Nm output torque

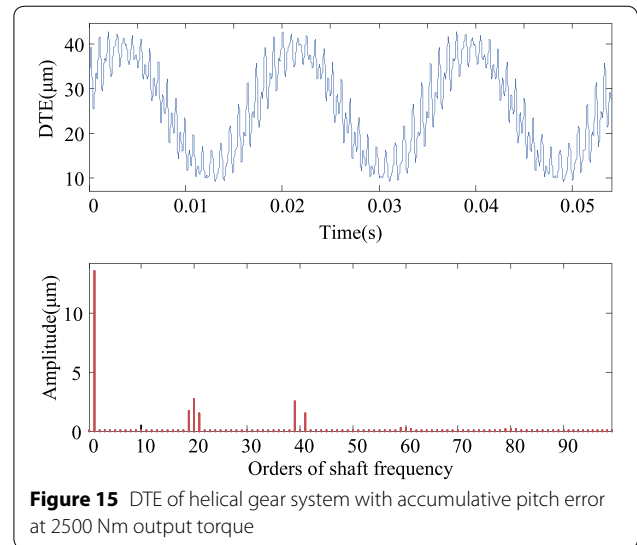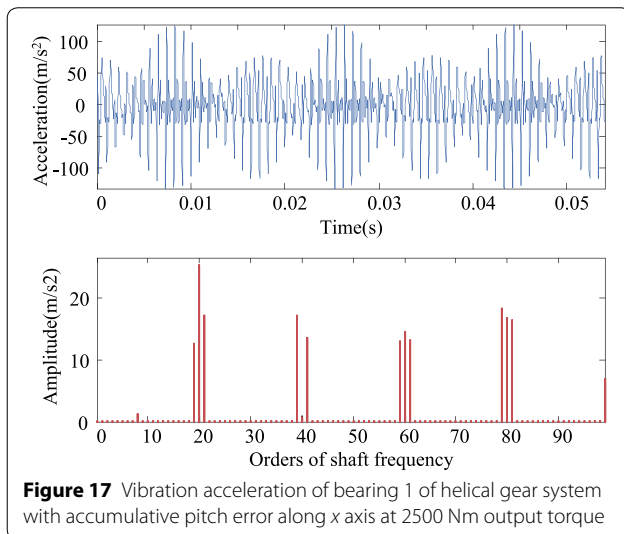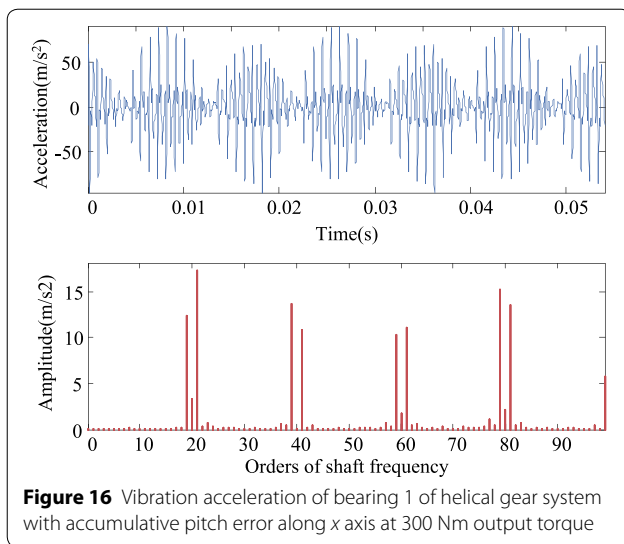pitch error has great influence on TVMS and LSTE in some mesh cycles when output torque is less than 400 Nm. This is because the partial contact loss in mating gear teeth occurs in this loading condition. As a result of the increase of output torque, the partial contact loss region become smaller. The contact pattern extend to full tooth surface when output torque is higher than 400 Nm.

4.2.3 Dynamic Analysis

The time domain and frequency domain of DTE of helical gear system with accumulative pitch error at 300 Nm and 2500 Nm output torque are shown in Figure 14 and Figure 15, respectively. The operating speed of driving gear is 5000 r/min. It can be observed that shaft frequency component is the predominant component and the sideband frequency components at the two sides of mesh frequency and its harmonics appear in frequency domain of dynamic transmission error of the system. The amplitude of mesh frequency component is less than the sideband frequency component when the output torque is 300 Nm, and the amplitude of mesh frequency component will relatively increase when output torque is 2500 Nm.

The time domain and frequency domain of the vibration acceleration of bearing 1 in *y* direction at 300 Nm and 2500 Nm output torque are given in Figure 16 and Figure 17, respectively. Different with the frequency domain of dynamic transmission error, the predominant frequency component of vibration acceleration of bearing 1 is not the shaft frequency which is very weak. As the same as the frequency domain of dynamic transmission error, the sideband components around mesh frequency and its harmonics are observed, and the amplitudes of mesh frequency and its harmonics will increase with the increase of output torque. The predominant frequency component is sideband frequency when output torque is 300 Nm, and the mesh frequency and its harmonics

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Figure 16 Vibration acceleration of bearing 1 of helical gear system with accumulative pitch error along *x* axis at 300 Nm output torque

Figure 17 Vibration acceleration of bearing 1 of helical gear system with accumulative pitch error along *x* axis at 2500 Nm output torque

components will be enhanced significantly when the output torque is 2500 Nm.

5 Conclusions

In this paper, the effects of manufacturing errors include short-term and long-term components on quasi-static and dynamic behaviors of helical gear system have been studied. The TVMS, LSTE and LCMS are determined using the improved LTCA model which was established based on the sub-structure technique and elastic contact theory. The dynamic responses of helical gear system are obtained by solving the generalized finite element dynamic model of the system which was developed based on the finite element method and Timoshenko beam theory. The main conclusions are summarized as follows.

(1) Both short-term and long-term components of manufacturing errors have notable influence on TVMS, LSTE and LCMS of helical gears. Lighter loading conditions lead to smaller mesh stiffness and stronger vibration. The corresponding resonance speed of the system become lower.

(2) Long-term components of manufacturing errors lead to the appearance of sideband frequency components in dynamic responses of the system. The sideband frequency components are predominant under lighter loading condition. The increase of output torque result in the increase of mesh frequency and its harmonics components.

Additional file

Additional file 1. Brief introduction of paper.

Authors' Contributions
SC was in charge of the whole trial; BY wrote the manuscript; GL and LYW assisted with sampling and laboratory analyses. All authors read and approved the final manuscript.

Author details
[1] Shaanxi Engineering Laboratory for Transmissions and Controls, School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an 710072, China. [2] China Shipbuilding Industry Corporation 703 Institute, Harbin 150078, China.

Authors' Information
Bing Yuan, born in 1987, is currently a PhD candidate at *Northwestern Polytechnical University* (*NWPU), China*. He received his bachelor degree from *Central South University, China*, in 2010. His research interests include gear dynamics, vibration and control. Tel: +86–15389359689; E-mail: 307504259@qq.com.

Shan Chang, born in 1965, is currently a professor and supervisor of PhD candidates and director of the *Institute for Shaanxi Engineering Laboratory for Transmissions and Controls, Northwestern Polytechnical University* (*NWPU), China*. He received his MS and PhD degrees from *Harbin Institute of Technology, China*. His research interests include gear modification, gear dynamic and load carrying capacity of gears. E-mail: changshan1@163.com.

Geng Liu, born in 1961, is currently a professor and supervisor of PhD candidates and director of *Shaanxi Engineering Laboratory for Transmissions and Controls, Northwestern Polytechnical University* (*NWPU*), *China*. He received his MS degree from *NWPU* and PhD degree from *Xi'an Jiao Tong University*. His research interests include mechanical dynamic design, mechanical systems dynamics, simulation and virtual prototype design, tribology, contact mechanics and numerical methods. Tel: +86–13891999032; E-mail: npuliug@nwpu.edu.cn.

Li-Yan Wu, born in 1958, is currently a professor at *Shaanxi Engineering Laboratory for Transmissions and Controls at Northwestern Polytechnical University* (*NWPU*), *China*. He received his BS degree from *NWPU*. His research interests include mechanical design and mechanical reliability. E-mail: wuliyan@nwpu.edu.cn.

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