Effect of magnetic field on terahertz generation via laser interaction with a carbon nanotube array

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Abstract

An amplitude-modulated laser, in the presence of a static magnetic field, interacts with an array of carbon nanotubes embedded on a metallic surface. The laser exerts a ponderomotive force on the free electrons of carbon nanotubes at twice the modulation frequency that falls in the terahertz (THz) range. Each nanotube acts as an oscillatory electric dipole, producing THz radiations. The presence of magnetic field shifts the resonance condition and provides tunability. The THz radiation power increases with magnetic field strength.

Keywords: Terahertz radiation, Carbon nanotubes, Cyclotron resonance, Modulation frequency

Findings

Background

Over the last decade, there has been significant research activity in the area of terahertz (THz) radiation generation and detection. The THz region of the electromagnetic spectrum lies in the gap between microwaves and infrared with a frequency range of 0.1 to 10 THz and corresponding wavelength range of 30 to 3,000 μm. THz radiations have applications in defense, medical imaging, sensors, spectroscopy, etc. THz radiations can be generated via several mechanisms, viz. optical rectification of a short pulse laser or a surface plasma wave, photoconduction, Cherenkov radiation, etc. [1-9]. Presently, there is an increased interest in employing metallic nanostructures and carbon nanotubes to generate THz radiations as these structures offer the advantage of compactness with reasonable efficiency. A metallic nanostructured surface has been employed by Welsh and Wyne [10] to generate ultrafast THz radiation pulses via excitation of surface plasmon. Garwe et al. [11] have observed THz radiations on either side of a gold-coated nanograting (500 nm), employing a 785-mm, 150-ps, 1-KHz Ti:sapphire laser. Studies on nanoscale periodic arrays of rectangular-shaped holes revealed band formation of THz nanoresonators [12]. Nemilentsau et al. [13] have investigated the application of a finite-length isolated carbon nanotube as a thermal nanoscale antenna in THz range. Batrakev et al. [14] and Portnoi et al. [15] have given elegant reviews of terahertz processes in carbon nanotubes. Another promising scheme of THz radiation generation is by exploiting excellent metallic properties of carbon nanotubes [16]. Wang and Wu [17] have experimentally studied the properties of THz radiation pulses emitted by a metallic, large-aspect ratio carbon nanotube antenna. Ramakrishnan et al. [18] have observed generation of a sub-picosecond THz pulse by illuminating a graphite surface with femtosecond infrared laser pulses. Dragoman and Dragoman [19] have given the physical model and simulation results of semiconducting carbon nanotube resonant-tunneling diodes for THz radiation generation. Zhao et al. [19] have studied the characteristics of metallic, single-walled carbon nanotubes as THz antenna. Wang et al. [20] have observed antenna-like effect in an array of aligned carbon nanotubes. Dagher et al. [21] have studied amplification of radiation in metallic carbon nanotubes under the influence of a d.c. magnetic field. Parashar and Sharma [22] have studied THz radiation generation via optical rectification of a laser in an array of carbon nanotubes. In this work, we extend their work to study the effect of static magnetic field on THz radiation generation from an array of carbon nanotubes. An amplitude-modulated laser interacts with an array of cylindrical carbon nanotubes. The laser exerts a ponderomotive force on the free electrons on the nanotubes, inducing on them a current at twice the modulation

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frequency. The image charges in the metallic base constitute a supplementary array with currents out of phase by π with respect to the primary array. These oscillating nanotube arrays act as antennas to produce THz radiation.

Analysis

Consider a metal surface ($x = 0$), embedded with normally mounted carbon nanotubes of radius r_c, length ℓ_c $\hat{x}$, and surface density N_c (per unit area), where $\ell_c \gg r_c$. The free electron density inside a nanotube is n_0 *c.f.* Figure 1.

An amplitude-modulated laser is impinged on the nanotube array with electric and magnetic fields

$$\vec{E}_L = A_L(\hat{x} - i\hat{y})(1 + \mu \cos[\Omega(t - z/c)]) e^{-i(\omega t - kz)},$$

$$\vec{B}_L = \frac{c\, \vec{k} \times \vec{E}_L}{\omega}, \tag{1}$$

$$A_L^2 = A_0^2 e^{-(x - \ell_c)^2 / r_0^2}, \tag{2}$$

where $k = \omega/c$, μ is the modulation index of laser, and Ω is in terahertz range.

There also exists a static magnetic field given by

$$\vec{B}_s = B_0 \hat{z}. \tag{3}$$

Under the influence of electric field, the electrons of nanotubes are displaced by a distance Δ, and the displacement is governed by the equation of motion

$$m \frac{d^2 \vec{\Delta}}{dt^2} = -e\vec{E}_L - m \frac{\omega_p^2 \Delta}{2\varepsilon_L} \hat{y} - e \frac{\partial \vec{\Delta}}{\partial t} \times \vec{B}_s. \tag{4}$$

Here, $\omega_p = (n_0\, e^2/m\, \varepsilon_0)^{1/2}$ is the plasma frequency of electrons in a nanotube, $-e$ and m are the charge and effective mass of the electron, ε_0 is the free space permittivity, and ε_L is the relative permittivity of the lattice.

The x and y components of the equation of motion can be written as

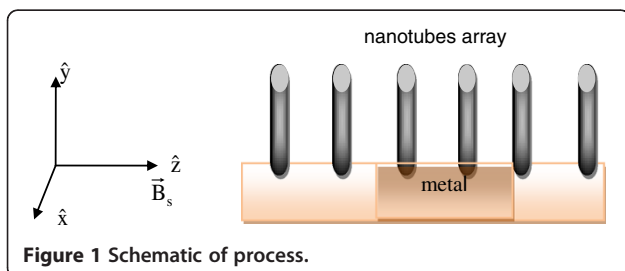$$\omega^2 \Delta_x + i\omega \omega_c \Delta_y = (e/m) E_{Lx} \tag{5}$$

and

Figure 1 Schematic of process.

$$\left(\omega^2 - \omega'_{pe} 2/2\right)\Delta_y - i\omega \omega_c \Delta_x = (e/m) E_{Ly}, \tag{6}$$

where

$$\omega'_p = \omega_p / \sqrt{\varepsilon_L}.$$

Here, it is to be mentioned that the restoring force due to the displacement of the electron cloud of nanotubes is finite for the field perpendicular to the nanotube axis and zero for the field along the axis.

On simplifying Equations 5 and 6, we obtain the x and y components for the displacement of the electron cloud as

$$\Delta_x = \frac{e}{m} \left[\frac{E_{Ly}}{\omega^2} - \frac{i(\omega/\omega_c)E_{Ly} - (\omega_c/\omega)E_{Lx}}{\omega^2 - \omega'^2_{pe}/2 - \omega_c^2} \right] \tag{7}$$

and

$$\Delta_y = \frac{e}{m} \left[\frac{E_{Ly} + i(\omega_c/\omega)E_{Lx}}{\omega^2 - \omega'^2_{pe}/2 - \omega_c^2} \right], \tag{8}$$

respectively.

The electron velocity can be written as

$$\vec{v} = -i\omega \left(\Delta_x \hat{x} + \Delta_y \hat{y}\right). \tag{9}$$

The laser exerts a ponderomotive force on electrons at the modulation frequency

$$\vec{F}_p = -\frac{e}{2c} \vec{v} \times \vec{B}_L^*$$

$$\cong -\hat{z} \frac{e^2 A_L^2 k\, i}{2m\omega^2} \frac{\omega \omega_c}{\omega^2 - \omega'^2_{pe} - \omega_c^2} e^{-2i\Omega(t - z/c)}. \tag{10}$$

The oscillatory velocity of electrons due to the ponderomotive force following Equation 5 can be written as

$$\vec{v}_{2\Omega} = \frac{i\, \vec{F}_p\, 2\Omega}{m\left(4\Omega^2 - \omega'^2_p/2\right)}. \tag{11}$$

The current density at Ω can be written as

$$\vec{J}_{2\Omega} = -n_0 e \vec{v}_{2\Omega} = -\frac{2n_0\, e\, \Omega\, i\, \vec{F}_p}{m\left(4\Omega^2 - \omega'^2_p/2\right)}. \tag{12}$$

$\vec{J}_{2\Omega}$ is finite over the cross section $\pi\, r_c^2$ of a nanotube and zero over the area $a^2 = 1/N$ (where a is the separation between the nanotubes). Thus, the average terahertz current density due to the array is
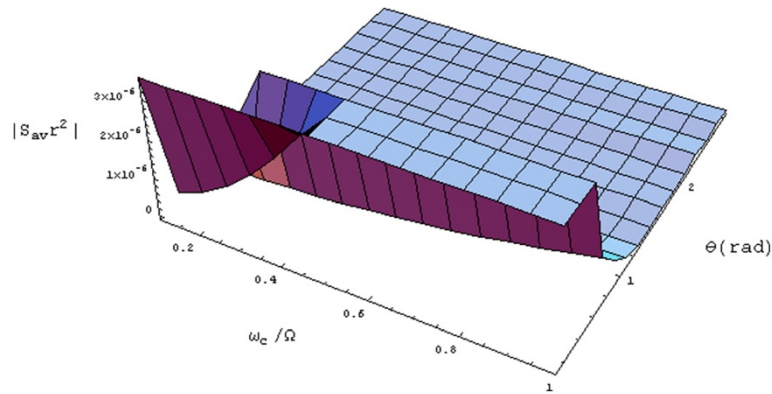
Figure 2 Variation of $|r^2 S_{av}|$ with ω_c/Ω at different values of θ for different parameters. The parameters are as follows: $\omega_c/\Omega = 20$, $r_c = 25$ nm, $\mu = 0.01$, inter-nanotube separation $a = 30$ nm, $\ell_c = 500$ nm, $n_0 = 10^{21}$ cm^{-3}, $\varphi = 1$ rad, $\varepsilon_L = 16$, $L \times L = 2 \times 2$cm, $eA_L/m\omega c = 0.03$.

$$\vec{J}_{2\Omega} = -\frac{\pi r_c^2 N 2 n_0 e \Omega i}{m\left(4\Omega^2 - \omega_p'^2/2\right)} \vec{F}_p$$
$$= \hat{z}\, \alpha\, e^{-(x-l_c)^2/r_0^2}\, e^{-2i(\Omega t - \Omega z/c)}, \qquad (13)$$

where

$$\alpha = \frac{\pi r_c^2 N n_0 e \mu}{\left(4\Omega^2 - \omega_p'^2/2\right)}\, \frac{\Omega^2 e^2 A_L^2}{m^2 \omega^2 c}\, \frac{\omega \omega_c}{\omega^2 - \omega_{pe}'^2/2 - \omega_c^2}.$$

As the base of the carbon nanotube is metallic, there will be an image current density underneath the metal surface,

$$\vec{J}'_{2\Omega} = -\hat{z}\, \alpha\, e^{-(x+\ell_c)^2/r_0^2}\, e^{-2i(\Omega t - \Omega z/c)}. \qquad (14)$$

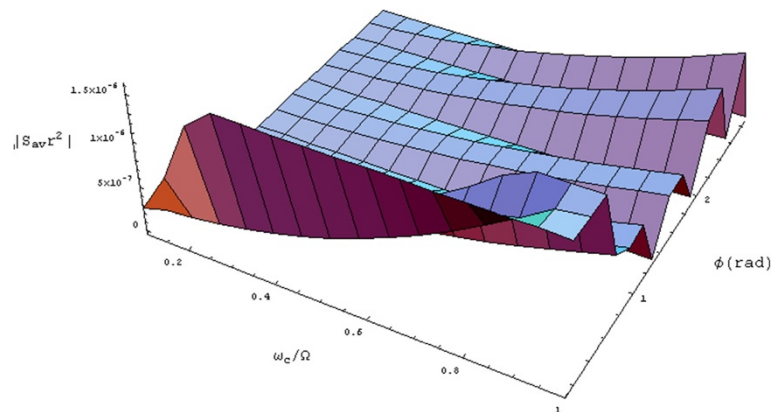Let the y and z dimensions of the array be $L \times L$.

The vector potential at a far point due to the current distributions is

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi}\left[\int \{\vec{J}_{2\Omega}(\vec{r}', t - R/c)/R)\}dV' + \int \{\vec{J}'_{2\Omega}(\vec{r}', t - R/c)/R)\}dV'\right], \qquad (15)$$

where $R = |\vec{r} - \vec{r}'| \approx r(1 - \vec{r}.\vec{r}'/r^2) = r - \sin\theta \cos\phi\, x' - \sin\theta \sin\phi\, y' - \cos\theta\, z'$; r, θ, and ϕ are the spherical polar coordinates of the point of observation, and we shall assume $\Omega \ell_c/c$, $\Omega\, r_c/c \ll 1$ The above Equation 15 can be written as

$$\vec{A} = \hat{z}\, \frac{\alpha \mu_0}{4\pi r}\, e^{-2i(\Omega t - \Omega r/c)}(I_1 - I_2), \qquad (16)$$

where

$$I_1 = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \int_{-\infty}^{\infty} e^{-(x'-\ell_c)^2/r_0^2}\, e^{i\psi}\, dx'\, dy'\, dz'$$

Figure 3 Variation of $|r^2 S_{av}|$ with ω_c/Ω at different values of φ for $\theta = 1$ rad. Other parameters are similar to those of Figure 2.

$$I_2 = \int\limits_{-1/2}^{1/2} \int\limits_{-1/2}^{1/2} \int\limits_{-\infty}^{\infty} e^{-(x'+\ell_c)^2/r_0^2} e^{i\psi} dx' dy' dz'$$

$$\psi' = 2\frac{\Omega}{c}[z'(1-\cos\theta) + x'\sin\theta\cos\phi + y'\sin\theta\sin\phi].$$

On carrying out integration, I_1 and I_2 reduce to

$$I_1 = \frac{\sin[(\Omega L/c)(1-\cos\theta)]}{i\,(\Omega/c)(1-\cos\theta)\sin\theta\sin\phi}$$
$$\sin[(\Omega L/c)\sin\theta\sin\phi]\sqrt{\pi}\,r_c\,e^{2i\,\Omega\,\ell_c/c}$$

(17)

and

$$I_2 = \frac{\sin[(\Omega L/c)(1-\cos\theta)]}{i\,(\Omega/c)(1-\cos\theta)\sin\theta\sin\phi}$$
$$\sin[(\Omega L/c)\sin\theta\sin\phi]\sqrt{\pi}\,r_c\,e^{-2i\,\Omega\,\ell_c/c},$$

(18)

respectively.

From Equations 17 and 18, we obtain

$$I_1 - I_2 \approx \sqrt{\pi}\,r_c \cos\left(\frac{\Omega\,\ell_c}{c}\right)\sin[(\Omega L/c)(1-\cos\theta)].$$
$$\frac{\sin[(\Omega L/c)\sin\theta\sin\phi]}{(\Omega/c)(1-\cos\theta)\sin\theta\sin\phi}.$$

(19)

The magnetic field at far distance is

$$\vec{B} = \vec{\nabla} \times \vec{A} \approx i\,2(\Omega/c)\hat{r} \times \vec{A}.$$ (20)

The time-average Poynting vector using Equation 20 can be written as

$$\vec{S}_{av} = \frac{|B|^2}{2\mu_0 c}\hat{r}\ W/m^2$$
$$= \hat{r}\,\frac{r_c^4 \alpha^2 \mu_0^2 \mu^2}{r^2}\left(\frac{4\Omega\,\ell_c}{c}\right)^2 \sin^2[(\Omega L/c)(1-\cos\theta)]$$
$$\sin^2\left[\frac{(\Omega L/c)(\sin\theta\sin\phi]}{(1-\cos\theta)^2\quad \sin^2\theta\ \sin^2\phi}\right.W/m^2.$$

(21)

In Figure 2, we have shown the variation of $|r^2\,S_{av}|$ with ω_c/Ω at different values of θ for the following set of parameters: ω_c/Ω = 20, nanotube radius r_c = 25 nm, μ = 0.01, inter-nanotube separation a = 30 nm, length of nanotube ℓ_c = 500 nm [20], $n_0 = 10^{21}$ cm^{-3}, ϕ =1 rad, ε_L = 16, $L \times L = 2 \times 2$ cm, and $eA_L/m\omega c$ = 0.03. The efficiency of the generated THz power shows a maximum near θ = 1 rad. The efficiency increases with magnetic field strength, and for the above-mentioned parameters, ω_c/Ω = 0.1 corresponds to a magnetic field strength of 100 kG. The above parameter of $eA_L/m\omega c$ = 0.03 can be realized using a Nd: YAG laser of 1.06 μm with an intensity of 10^{15} W/cm^2. In Figure 3, we have shown the variation of $|r^2\,S_{av}|$ with ω_c/Ω at different values of ϕ for θ = 1 rad. The other parameters are similar to those of Figure 2. The power radiated shows oscillatory behavior with ϕ.

Conclusions

An array of carbon nanotubes mounted on a metallic base can be employed to generate THz radiations. The radiations' pattern is directional with a peak value of power radiated at around 5 μW, which is quite reasonable. Wang et al. [16] have predicted a maximum efficiency of 5 μW for THz radiations from the armchair carbon nanotube dipole antenna. On comparing with THz radiation generation from a GaP crystal via the difference frequency generation scheme [3], the efficiency is quite good. One can exploit the resonance condition $\Omega \cong \omega_p/\sqrt{8\varepsilon_L}$ for enhancement of efficiency of THz radiation generation. The application of magnetic field reduces the requirement on the resonance condition. The efficiency of THz radiation increases with the strength of the applied magnetic field. The analysis is limited to a laser of moderate powers only.

Authors' contributions
SJ did the numerical analysis part of the work. JP did the analytical part of the work. RK conceived the idea and helped in designing and drafting the manuscript. All authors read and approved the final manuscript.

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SJ did his masters degree in Physics from Barkatullah University, Bhopal (M. P.), India, and at present, he is working towards his Ph.D. degree in Physics. His research interests are in the area of interaction of electromagnetic and electrostatic waves with nanoparticles and nanotubes.
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