Advances in Difference Equations
a SpringerOpen Journal

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Existence of solution to a class of boundary value problem for impulsive fractional differential equations

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Abstract

By means of the Green function, the boundary value problem of a fractional differential equation can be reduced to the equivalent integral equation. Recently, this method has been used successfully to discuss the existence of the solution to the boundary value problem of a nonlinear fractional differential equation. By applying the nonlinear alternative of the Leray-Schauder type and the Krasnoselskii fixed point theorem, we investigate the boundary value problem of a nonlinear impulsive fractional differential equation, and we obtain two existence results for the solution.

MSC: 26A33; 34B15

Keywords: boundary value problem; impulsive fractional differential equations; Caputo fractional derivative; existence of solutions; fixed point theorem

1 Introduction

Boundary value problems for nonlinear fractional differential equations have recently been addressed by several researchers. The interest in the study of differential equations of fractional order lies in the fact that fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes [1–5]. For some recent development on the topic, see [6–11] and the references therein.

Impulsive differential equations, which provide a natural description of observed evolution processes, are regarded as important mathematical tools for the better understanding of several real world problems in the applied sciences. The theory of impulsive differential equations of integer order has found extensive applications in realistic mathematical modeling of a wide variety of practical situations and has emerged as an important area of investigation in recent years. For the general theory and applications of impulsive differential equations, we refer the reader to references [12–15]. The impulsive differential equations of fractional order have also attracted considerable attention and a variety of results can be found in [16–24] and the references therein.

In [16], Ahmad *et al.* considered the following the impulsive fractional differential equations:

$$\begin{cases} {}^cD^q x(t) = f(t, x(t)), & t \in J = [0,1] \setminus \{t_1, t_2, t_3, \dots, t_p\}, \\ \Delta x(t_k) = I_k(x(t_k^-)), \quad \Delta x'(t_k) = J_k(x(t_k^-)), & t_k \in (0,1), k = 1, \dots, p, \\ x(0) + x'(0) = 0, \quad x(1) + x'(1) = 0, \end{cases}$$

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where ${}^cD^q$ is the Caputo fractional derivative. The results are based on the contraction mapping principle and Krasnoselskii's fixed point theorem.

In [17], Tian *et al.* considered the following the impulsive fractional differential equations:

$$\begin{cases} {}^cD^q u(t) = f(t,u), \quad 0 < t < 1, t \neq t_k, k = 1,\dots,p, 1 < q \leq 2, \\ \Delta u(t_k) = I_k(u(t_k^-)), \qquad \Delta u'(t_k) = \overline{I}_k(u(t_k^-)), \quad k = 1,\dots,p, \\ u(0) + u'(0) = 0, \qquad u(1) + u'(\xi) = 0, \quad \xi \in (0,1), \xi \neq t_k, k = 1,\dots,p. \end{cases}$$

The results are based on the contraction mapping principle and Schauder's fixed point theorem.

In [18], Zhang *et al.* considered the following impulsive fractional differential equations:

$$\begin{cases} {}^cD^q y(t) = f(t,y), \quad \forall t \in J = [0,T], t \neq t_k, k = 1,\dots,m, 1 < q \leq 2, \\ \Delta u(t_k) = I_k(u(t_k^-)), \qquad \Delta u'(t_k) = \overline{I}_k(u(t_k^-)), \quad k = 1,\dots,p, \\ y(0) = -y(T), \qquad y'(0) = -y'(T). \end{cases}$$

The results are based on the Altman fixed point theorem and Leray-Schauder fixed point theorem.

On the other hand, the impulsive boundary value problems for nonlinear fractional differential equations have not been addressed so extensively and many aspects of these problems are yet to be explored. For example, we observed that in the above-mentioned work [16–24], the authors all require that the nonlinear term f is bounded and continuous; if the impulse functions I_k and $\overline{I}_k$ are bounded, it is easy to see that these conditions are very strongly restrictive and difficult to satisfy in applications. Motivated by the above-mentioned work [16–24], this article is mainly concerned with the existence of a solution for the boundary value problems for the nonlinear impulsive fractional differential equations

$$\begin{cases} {}^cD^q u(t) = f(t,u(t)), \quad 1 < q \leq 2, t \in J', \\ \Delta u(t_k) = I_k(u(t_k^-)), \qquad \Delta u'(t_k) = \overline{I}_k(u(t_k^-)), \quad k = 1,\dots,m, \\ au(0) - bu'(0) = x_0, \qquad cu(1) + du'(1) = x_1, \end{cases} \tag{1.1}$$

where ${}^cD^q$ is the Caputo fractional derivative, $a \geq 0$, $b > 0$, $c \geq 0$, $d > 0$, $\delta = ac + ad + bc \neq 0$, and $x_0, x_1 \in \mathbb{R}$. $f \in C(I \times \mathbb{R}, \mathbb{R})$, $I_k, \overline{I}_k :\in C(\mathbb{R},\mathbb{R})$, $J = [0,1]$, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$, $J' = J \setminus \{t_1, t_2, \dots, t_m\}$, $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$, $u(t_k^+) = \lim_{h \to 0^+} u(t_k + h)$ and $u(t_k^-) = \lim_{h \to 0^-} u(t_k + h)$ represent the right and left limits of $u(t)$ at $t = t_k$, $k = 1,\dots,m$. $\Delta u'(t_k)$ has a similar meaning for $u'(t)$.

Evidently, problem (1.1) not only includes the boundary value problems mentioned above [16] but also extends them to a much wider case. Our main tools are the nonlinear alternative of Leray-Schauder type and the Krasnoselskii fixed point theorem. Some recent results in [16–24] are generalized and significantly improved (see Remark 3.1).

The remainder of this article is organized as follows. In Section 2, we provide some basic definitions, preliminaries facts, and various lemmas which will be used throughout this paper. In Section 3, we give two main results of problem (1.1). The last section is devoted to an example illustrating the applicability of the imposed conditions.

2 Preliminaries and lemmas

Let $J_0 = [0, t_1]$, $J_1 = (t_1, t_2], \dots, J_{p-1} = (t_{p-1}, t_p]$, $J_p = (t_p, 1]$, and let us introduce the spaces: $L^1(J, \mathbb{R})$ denotes the Banach space of measurable functions $u : J \mapsto \mathbb{R}$ which are Bochner integrable, equipped with the norm $\|u\|_{L^1} := \int_J \|u(t)\|\,dt$; $PC(J, \mathbb{R}) = \{u : J \to \mathbb{R} : u \in C(J_k), k = 0, 1, \dots, m, \text{and } u(t_k^+) \text{ exists}, k = 1, \dots, m\}$ is a Banach space with the norm $\|u\|_{PC} := \sup_{t\in J} \|u(t)\|$, and $PC^1(J, \mathbb{R}) = \{u : J \to \mathbb{R} : u \in C^1(J_k), k = 0, 1, \dots, m, \text{and } u(t_k^+), u'(t_k^+) \text{ exists}, k = 1, \dots, m\}$ is a Banach space with the norm $\|u\|_{PC^1} := \max_{t\in J}\{\|u\|, \|u'\|\}$.

Definition 2.1 [1] The Riemann-Liouville fractional integral of order r for a continuous function h is defined as

$$I^r h(t) = \int_0^t \frac{(t-s)^{r-1}}{\Gamma(r)} h(s)\,ds, \quad r > 0,$$

provided the integral exists.

Definition 2.2 [1] For an at least n-times continuously differentiable function $h : [0, \infty) \to R$, the Caputo derivative of fractional order r is defined as

$$^cD^r h(t) = \frac{1}{\Gamma(n-r)} \int_0^t (t-s)^{n-r-1} h^{(n)}(s)\,ds, \quad n-1 < r < n, n = [r] + 1,$$

where $[r]$ denotes the integer part of the real number r.

Lemma 2.1 *Let $r > 0$, $h \in C[0,1] \cap L(0,1)$, then the differential equation $^cD^r h(t) = 0$ has solutions*

$$h(t) = c_0 + c_1 t + c_2 t^2 + \cdots + c_{n-1} t^{n-1},$$

where $c_i \in \mathbb{R}$, $i = 0, 1, 2, \dots, n-1$, $n = [r] + 1$.

Lemma 2.2 *Assume that $h \in C[0,1] \cap L(0,1)$ with a derivative of order r that belongs to $C[0,1] \cap L(0,1)$. Then*

$$I_{0+}^r {}^cD_{0+}^r h(t) = h(t) + c_0 + c_1 t + c_2 t^2 + \cdots + c_{n-1} t^{n-1},$$

where $c_i \in \mathbb{R}$, $i = 0, 1, 2, \dots, n-1$, $n = [r] + 1$.

Lemma 2.3 [22] *For a given $h \in C[0,1]$, a function u is a solution of the following impulsive boundary value problem*:

$$\begin{cases} ^cD^q u(t) = h(t), \quad 1 < q \le 2, t \in J', \\ \Delta u(t_k) = I_k(u(t_k^-)), \qquad \Delta u'(t_k) = \overline{I}_k(u(t_k^-)), \quad k = 1, \dots, m, \\ au(0) - bu'(0) = x_0, \qquad cu(1) + du'(1) = x_1, \end{cases} \tag{2.1}$$

if and only if u is a solution of the impulsive fractional integral equation

$$u(t) = \begin{cases} \frac{1}{\Gamma(q)}\int_0^t (t-s)^{q-1}h(s)\,ds + C_1 + C_2 t, & \text{if } t \in J_0, \\ \frac{1}{\Gamma(q)}\int_{t_k}^t (t-s)^{q-1}h(s)\,ds + \frac{1}{\Gamma(q)}\sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-1}h(s)\,ds & \\ \quad + \frac{1}{\Gamma(q-1)}\sum_{i=1}^{k}(t-t_k)\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds & \\ \quad + \frac{1}{\Gamma(q-1)}\sum_{i=1}^{k-1}(t_k-t_i)\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds & \\ \quad + \sum_{i=1}^{k} I_i(y(t_i^-)) + \sum_{i=1}^{k}(t-t_k)\overline{I}_i(u(t_i^-)) & \\ \quad + \sum_{i=1}^{k-1}(t_k-t_i)\overline{I}_i(u(t_i^-)) + C_1 + C_2 t, & \text{if } t \in J_k, \end{cases} \tag{2.2}$$

where

$$\begin{aligned} C_1 = -\Bigg\{ & \sum_{i=1}^{m+1}\frac{bc}{\delta\Gamma(q)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-1}h(s)\,ds + \sum_{i=1}^{m}\frac{bc(1-t_m)}{\delta\Gamma(q-1)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds \\ & + \sum_{i=1}^{m-1}\frac{bc(t_m-t_i)}{\delta\Gamma(q-1)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds + \sum_{i=1}^{m+1}\frac{bd}{\delta\Gamma(q-1)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds \\ & + \sum_{i=1}^{m}\frac{bc}{\delta}I_i\big(u(t_i^-)\big) + \sum_{i=1}^{m}\frac{bc(1-t_p)}{\delta}\overline{I}_i\big(u(t_i^-)\big) + \sum_{i=1}^{m-1}\frac{bc(t_p-t_i)}{\delta}\overline{I}_i\big(u(t_i^-)\big) \\ & + \sum_{i=1}^{m}\frac{bd}{\delta}\overline{I}_i\big(u(t_i^-)\big) + \frac{(bc-\delta)x_0 - abx_1}{a\delta}\Bigg\}, \\ C_2 = -\Bigg\{ & \sum_{i=1}^{m+1}\frac{ac}{\delta\Gamma(q)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-1}h(s)\,ds + \sum_{i=1}^{m}\frac{ac(1-t_m)}{\delta\Gamma(q-1)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds \\ & + \sum_{i=1}^{m-1}\frac{ac(t_m-t_i)}{\delta\Gamma(q-1)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds + \sum_{i=1}^{m+1}\frac{ad}{\delta\Gamma(q-1)}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}h(s)\,ds \\ & + \sum_{i=1}^{m}\frac{ac}{\delta}I_i\big(u(t_i^-)\big) + \sum_{i=1}^{m}\frac{ac(1-t_p)}{\delta}\overline{I}_i\big(u(t_i^-)\big) + \sum_{i=1}^{m-1}\frac{ac(t_p-t_i)}{\delta}\overline{I}_i\big(u(t_i^-)\big) \\ & + \sum_{i=1}^{m}\frac{ad}{\delta}\overline{I}_i\big(u(t_i^-)\big) + \frac{cx_0 - ax_1}{a\delta}\Bigg\}. \end{aligned}$$

Now we state some well-known fixed point theorems which are needed to prove the existence of solutions for equation (1.1).

Lemma 2.4 [25] (the nonlinear alternative of Leray-Schauder) *Let E be a Banach space, C a convex subset of E, U an open subset of C and* $0 \in U$. *Suppose* $F : \overline{U} \to C$ (*here* $\overline{U}$ *denotes the closure of U in C*) *is a continuous, compact map. Then either*

(A1) *F has a fixed point in U*; *or*

(A2) *there exists* $u \in \partial U$ (*the boundary of U in C*) *and* $\lambda \in (0,1)$ *with* $u = \lambda F(u)$.

Lemma 2.5 [25] (Krasnoselskii fixed point theorem) *Let* $\mathcal{D}$ *be a closed convex and nonempty subset of a Banach space X. Let* $\mathcal{A}_1$, $\mathcal{A}_2$ *be the operators such that*

(i) $\mathcal{A}_1 x + \mathcal{A}_2 y \in \mathcal{D}$ *whenever* $x, y \in \mathcal{D}$;

(ii) $\mathcal{A}_1$ *is completely continuous*;

(iii) $\mathcal{A}_2$ *is a contraction mapping.*
Then there exists $z \in \mathcal{D}$ *such that* $z = \mathcal{A}_1 z + \mathcal{A}_2 z$.

3 Main results

Define an operator $\mathcal{A} : PC(J, \mathbb{R}) \to PC(J, \mathbb{R})$ as

$$\begin{aligned}
(\mathcal{A}u)(t) ={}& \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} f\bigl(s, u(s)\bigr)\,ds + \frac{1}{\Gamma(q)} \sum_{i=1}^{k} \int_{t_{i-1}}^{t_i} (t_i - s)^{q-1} f\bigl(s, u(s)\bigr)\,ds \\
&+ \frac{1}{\Gamma(q-1)} \sum_{i=1}^{k} (t - t_k) \int_{t_{i-1}}^{t_i} (t_i - s)^{q-2} f\bigl(s, u(s)\bigr)\,ds \\
&+ \frac{1}{\Gamma(q-1)} \sum_{i=1}^{k-1} (t_k - t_i) \int_{t_{i-1}}^{t_i} (t_i - s)^{q-2} f\bigl(s, u(s)\bigr)\,ds + \sum_{i=1}^{k} I_i\bigl(u\bigl(t_i^-\bigr)\bigr) \\
&+ \sum_{i=1}^{k} (t - t_k) \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) + \sum_{i=1}^{k-1} (t_k - t_i) \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) + M_1 + M_2 t,
\end{aligned} \tag{3.1}$$

where

$$\begin{aligned}
M_1 = -\Biggl\{& \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{bc(t_i - s)^{q-1}}{\delta\Gamma(q)} f\bigl(s, u(s)\bigr)\,ds + \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} \frac{bc(1 - t_m)(t_i - s)^{q-2}}{\delta\Gamma(q-1)} f\bigl(s, u(s)\bigr)\,ds \\
&+ \sum_{i=1}^{m-1} \int_{t_{i-1}}^{t_i} \frac{bc(t_m - t_i)(t_i - s)^{q-2}}{\delta\Gamma(q-1)} f\bigl(s, u(s)\bigr)\,ds + \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{bd(t_i - s)^{q-2}}{\delta\Gamma(q-1)} f\bigl(s, u(s)\bigr)\,ds \\
&+ \sum_{i=1}^{m} \frac{bc}{\delta} I_i\bigl(u\bigl(t_i^-\bigr)\bigr) + \sum_{i=1}^{m} \frac{bc(1 - t_p)}{\delta} \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) + \sum_{i=1}^{m-1} \frac{bc(t_p - t_i)}{\delta} \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) \\
&+ \sum_{i=1}^{m} \frac{bd}{\delta} \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) + \frac{(bc - \delta)x_0 - abx_1}{a\delta} \Biggr\},
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
M_2 = -\Biggl\{& \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{ac(t_i - s)^{q-1}}{\delta\Gamma(q)} f\bigl(s, u(s)\bigr)\,ds + \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} \frac{ac(1 - t_m)(t_i - s)^{q-2}}{\delta\Gamma(q-1)} f\bigl(s, u(s)\bigr)\,ds \\
&+ \sum_{i=1}^{m-1} \int_{t_{i-1}}^{t_i} \frac{ac(t_m - t_i)(t_i - s)^{q-2}}{\delta\Gamma(q-1)} f\bigl(s, u(s)\bigr)\,ds + \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{ad(t_i - s)^{q-2}}{\delta\Gamma(q-1)} f\bigl(s, u(s)\bigr)\,ds \\
&+ \sum_{i=1}^{m} \frac{ac}{\delta} I_i\bigl(u\bigl(t_i^-\bigr)\bigr) + \sum_{i=1}^{m} \frac{ac(1 - t_p)}{\delta} \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) + \sum_{i=1}^{m-1} \frac{bc(t_p - t_i)}{\delta} \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) \\
&+ \sum_{i=1}^{m} \frac{ad}{\delta} \overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr) + \frac{cx_0 - ax_1}{a\delta} \Biggr\}.
\end{aligned} \tag{3.3}$$

Lemma 3.1 [22] *Let* $f \in C(I \times \mathbb{R}, \mathbb{R})$, $I_k, \overline{I}_k : C(\mathbb{R}, \mathbb{R})$, *then* $\mathcal{A} : PC(J, \mathbb{R}) \to PC(J, \mathbb{R})$ *is completely continuous.*

Theorem 3.1 *Assume that the following conditions hold.*

(H1) *The function* $f \in C(I \times \mathbb{R}, \mathbb{R})$, $I_k, \overline{I}_k : \mathbb{R} \to \mathbb{R}$ *is continuous.*

(H2) *There exist* $\phi_f \in C(J, [0, +\infty))$ *and* $\varphi : [0, +\infty) \to [0, +\infty)$ *continuous and nondecreasing such that*

$$|f(t,u)| \le \phi_f(t)\varphi(|u|), \quad (t,u) \in J \times \mathbb{R}.$$

(H3) *There exist* $\varphi^\star, \overline{\varphi}^\star : [0, +\infty) \to [0, +\infty)$ *continuous and nondecreasing such that*

$$|I_k(u)| \le \varphi^\star(|u|), \qquad |\overline{I}_k(u)| \le \overline{\varphi}^\star(|u|), \quad u \in \mathbb{R}.$$

(H4) *There exists a number* $\overline{M} > 0$ *such that*

$$\frac{\overline{M}}{A\phi_f^0 \varphi(\overline{M}) + B\varphi^\star(\overline{M}) + C\overline{\varphi}^\star(\overline{M}) + D} > 1, \tag{3.4}$$

where

$$A = \frac{(m+1)[c(a+b)+\delta]}{\delta\Gamma(q+1)} + \frac{(2m-1)[c(a+b)+\delta] + (m+1)(a+b)d}{\delta\Gamma(q)},$$
$$B = \frac{m[c(a+b)+\delta]}{\delta},$$
$$C = \frac{(2m-1)[c(a+b)+\delta] + md(a+b)}{\delta}, \qquad D = \frac{(2c+d)|x_0| + (a+b)|x_1|}{\delta}.$$

Then problem (1.1) *has at least one solution on* J.

Proof Consider the operator $\mathcal{A}$ defined by (3.1). By Lemma 3.1, it can easily be shown that $\mathcal{A}$ is continuous and completely continuous. For $0 \le \lambda \le 1$, let u be such that for each $t \in J$ we have $u(t) = \lambda(\mathcal{A}u)(t)$. Then from (H2)-(H3) we have for each $t \in J$,

$$\begin{aligned}
|M_1| \le{} & \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{bc(t_i - s)^{q-1}}{\delta\Gamma(q)} |f(s,u(s))|\,ds + \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} \frac{bc(1-t_m)(t_i - s)^{q-2}}{\delta\Gamma(q-1)} |f(s,u(s))|\,ds \\
& + \sum_{i=1}^{m-1} \int_{t_{i-1}}^{t_i} \frac{bc(t_m - t_i)(t_i - s)^{q-2}}{\delta\Gamma(q-1)} |f(s,u(s))|\,ds \\
& + \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{bd(t_i - s)^{q-2}}{\delta\Gamma(q-1)} |f(s,u(s))|\,ds \\
& + \sum_{i=1}^{m} \frac{bc}{\delta} |I_i(u(t_i))| + \sum_{i=1}^{m} \frac{bc(1-t_p)}{\delta} |\overline{I}_i(u(t_i))| + \sum_{i=1}^{m-1} \frac{bc(t_p - t_i)}{\delta} |\overline{I}_i(u(t_i))| \\
& + \sum_{i=1}^{m} \frac{bd}{\delta} |\overline{I}_i(u(t_i))| + \frac{(c+d)|x_0| + b|x_1|}{\delta} \\
\le{} & \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{bc(t_i - s)^{q-1}}{\delta\Gamma(q)} \phi_f(s)\varphi(|u(s)|)\,ds + \sum_{i=1}^{m} \frac{bc}{\delta} \varphi^\star(|u(t_i)|) \\
& + \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} \frac{bc(1-t_m)(t_i - s)^{q-2}}{\delta\Gamma(q-1)} \phi_f(s)\varphi(|u(s)|)\,ds + \sum_{i=1}^{m} \frac{bc(1-t_p)}{\delta} \overline{\varphi}^\star(|u(t_i)|)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{m-1}\int_{t_{i-1}}^{t_i}\frac{bc(t_m-t_i)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}\phi_f(s)\varphi\bigl(|u(s)|\bigr)\,ds+\sum_{i=1}^{m-1}\frac{bc(t_p-t_i)}{\delta}\overline{\varphi}^\star\bigl(|u(t_i)|\bigr)\\
&+\sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{bd(t_i-s)^{q-2}}{\delta\Gamma(q-1)}\phi_f(s)\varphi\bigl(|u(s)|\bigr)\,ds+\sum_{i=1}^{m}\frac{bd}{\delta}\overline{\varphi}^\star\bigl(|u(t_i)|\bigr)\\
&+\frac{(c+d)|x_0|+b|x_1|}{\delta}\\
\le{}&\biggl[\frac{(m+1)bc}{\delta\Gamma(q+1)}+\frac{mbc}{\delta\Gamma(q)}+\frac{(m-1)bc}{\delta\Gamma(q)}+\frac{(m+1)bd}{\delta\Gamma(q)}\biggr]\phi_f^0\varphi\bigl(\|u\|_\infty\bigr)+\frac{mbc}{\delta}\varphi^\star\bigl(\|u\|_\infty\bigr)\\
&+\biggl[\frac{mbc}{\delta}+\frac{(m-1)bc}{\delta}+\frac{mbd}{\delta}\biggr]\overline{\varphi}^\star\bigl(\|u\|_\infty\bigr)+\frac{(c+d)|x_0|+b|x_1|}{\delta}.
\end{aligned}
\tag{3.5}
$$

Similarly, we have

$$
\begin{aligned}
|M_2|\le{}&\sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{ac(t_i-s)^{q-1}}{\delta\Gamma(q)}\phi_f(s)\varphi\bigl(|u(s)|\bigr)\,ds+\sum_{i=1}^{m}\frac{bc}{\delta}\varphi^\star\bigl(|u(t_i)|\bigr)\\
&+\sum_{i=1}^{m}\int_{t_{i-1}}^{t_i}\frac{ac(1-t_m)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}\phi_f(s)\varphi\bigl(|u(s)|\bigr)\,ds+\sum_{i=1}^{m}\frac{ac(1-t_p)}{\delta}\overline{\varphi}^\star\bigl(|u(t_i)|\bigr)\\
&+\sum_{i=1}^{m-1}\int_{t_{i-1}}^{t_i}\frac{ac(t_m-t_i)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}\phi_f(s)\varphi\bigl(|u(s)|\bigr)\,ds+\sum_{i=1}^{m-1}\frac{ac(t_p-t_i)}{\delta}\overline{\varphi}^\star\bigl(|u(t_i)|\bigr)\\
&+\sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{ad(t_i-s)^{q-2}}{\delta\Gamma(q-1)}\phi_f(s)\varphi\bigl(|u(s)|\bigr)\,ds+\sum_{i=1}^{m}\frac{ad}{\delta}\overline{\varphi}^\star\bigl(|u(t_i)|\bigr)+\frac{c|x_0|+a|x_1|}{\delta}\\
\le{}&\biggl[\frac{(m+1)ac}{\delta\Gamma(q+1)}+\frac{mac}{\delta\Gamma(q)}+\frac{(m-1)ac}{\delta\Gamma(q)}+\frac{(m+1)ad}{\delta\Gamma(q)}\biggr]\phi_f^0\varphi\bigl(\|u\|_\infty\bigr)+\frac{mac}{\delta}\varphi^\star\bigl(\|u\|_\infty\bigr)\\
&+\biggl[\frac{mac}{\delta}+\frac{(m-1)ac}{\delta}+\frac{mbd}{\delta}\biggr]\overline{\varphi}^\star\bigl(\|u\|_\infty\bigr)+\frac{c|x_0|+a|x_1|}{\delta}.
\end{aligned}
\tag{3.6}
$$

Therefore

$$
\begin{aligned}
\bigl|u(t)\bigr|\le{}&\frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1}\bigl|f\bigl(s,u(s)\bigr)\bigr|\,ds+\frac{1}{\Gamma(q)}\sum_{i=1}^{k}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-1}\bigl|f\bigl(s,u(s)\bigr)\bigr|\,ds\\
&+\frac{1}{\Gamma(q-1)}\sum_{i=1}^{k}(t-t_k)\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}\bigl|f\bigl(s,u(s)\bigr)\bigr|\,ds\\
&+\frac{1}{\Gamma(q-1)}\sum_{i=1}^{k-1}(t_k-t_i)\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}\bigl|f\bigl(s,u(s)\bigr)\bigr|\,ds+\sum_{i=1}^{k}\bigl|I_i\bigl(u\bigl(t_i^-\bigr)\bigr)\bigr|\\
&+\sum_{i=1}^{k}(t-t_k)\bigl|\overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr)\bigr|+\sum_{i=1}^{k-1}(t_k-t_i)\bigl|\overline{I}_i\bigl(u\bigl(t_i^-\bigr)\bigr)\bigr|+|M_1|+|M_2|\\
\le{}&\biggl[\int_{t_k}^{t}\frac{(t-s)^{q-1}}{\Gamma(q)}\,ds+\sum_{i=1}^{m}\int_{t_{i-1}}^{t_i}\frac{(t_i-s)^{q-1}}{\Gamma(q)}\,ds+|M_1|+|M_2|\\
&+\sum_{i=1}^{m}\int_{t_{i-1}}^{t_i}\frac{(t-t_k)(t_i-s)^{q-2}}{\Gamma(q-1)}\,ds+\sum_{i=1}^{m-1}\int_{t_{i-1}}^{t_i}\frac{(t_k-t_i)(t_i-s)^{q-2}}{\Gamma(q-1)}\,ds\biggr]\phi_f^0\varphi\bigl(\|u\|_\infty\bigr)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{i=1}^{m}\varphi^{\star}\left(\left|u(t_k)\right|\right)+\sum_{i=1}^{m}(t-t_k)\overline{\varphi}^{\star}\left(\left|u(t_k)\right|\right)+\sum_{i=1}^{m-1}(t_k-t_i)\overline{\varphi}^{\star}\left(\left|u(t_k)\right|\right)\\
\le{}&\frac{(m+1)[c(a+b)+\delta]}{\delta\Gamma(q+1)}\phi_f^0\varphi\left(\|u\|_\infty\right)\\
&+\frac{(2m-1)[c(a+b)+\delta]+(m+1)(a+b)d}{\delta\Gamma(q)}\phi_f^0\varphi\left(\|u\|_\infty\right)\\
&+\frac{m[c(a+b)+\delta]}{\delta}\varphi^{\star}\left(\|u\|_\infty\right)+\frac{(2m-1)[c(a+b)+\delta]+md(a+b)}{\delta}\overline{\varphi}^{\star}\left(\|u\|_\infty\right)\\
&+\frac{(2c+d)|x_0|+(a+b)|x_1|}{\delta},
\end{aligned}
\tag{3.7}
$$

which implies that

$$
\frac{\|u\|_\infty}{A\phi_f^0\varphi(\|u\|_\infty)+B\varphi^{\star}(\|u\|_\infty)+C\overline{\varphi}^{\star}(\|u\|_\infty)+D}\le 1.
\tag{3.8}
$$

Then by the condition (3.4) there exists $\overline{M}$ such that $\|u\|_\infty\neq\overline{M}$.

Let

$$
U=\left\{u\in PC(J,\mathbb{R}):\|u\|_\infty<\overline{M}\right\}.
$$

The operator $\mathcal{A}:\overline{U}\to PC(J,\mathbb{R})$ is continuous and completely continuous. From the choice of U, there is no $u\in\partial U$ such that $u=\lambda\mathcal{A}(u)$ for some $\lambda\in[0,1]$. As a consequence of the nonlinear alternative of Leray-Schauder type [25] we deduce that $\mathcal{A}$ has a fixed point u in $\overline{U}$ which is a solution of problem (1.1). This completes the proof. □

Remark 3.1 Compared with Theorem 3.2 in [16–22], our Theorem 3.1 does not need conditions $|f(t,u)|\le L_1$, $|I_k(u)|\le L_2$, $|\overline{I}_k(u)|\le L_3$, clearly, these conditions are very strong. Thus, the results of the above-mentioned works are generalized and significantly improved.

Theorem 3.2 *Let* $f\in C(I\times\mathbb{R},\mathbb{R})$, $I_k,\overline{I}_k:C(\mathbb{R},\mathbb{R})$, *and they satisfy*

(H5) *there exists a positive constant* $\gamma_1>0$, *such that*

$$
\left|f(t,u)-f(t,v)\right|\le\gamma_1|u-v|,\quad\forall t\in J,u,v\in\mathbb{R};
\tag{3.9}
$$

(H6) *there exist positive constants* $\gamma_2,\gamma_3,\gamma_4,\gamma_5>0$, *for* $\forall u,v\in\mathbb{R}$, *such that*

$$
\begin{aligned}
&\left|I_k(u)-I_k(v)\right|\le\gamma_2|u-v|,\qquad\left|\overline{I}_k(u)-\overline{I}_k(v)\right|\le\gamma_3|u-v|,\\
&\left|I_k(u)\right|\le\gamma_4,\qquad\left|\overline{I}_k(u)\right|\le\gamma_5,\quad k=1,\dots,m;
\end{aligned}
\tag{3.10}
$$

(H7) *for* $(t,x)\in J\times\mathbb{R}$ *and* $\mu\in PC(J,\mathbb{R}^+)$, *one has*

$$
\left|f\left(t,x(t)\right)\right|\le\mu(t),
$$

and

$$\lambda := \frac{m[c(a+b)+\delta]}{\delta}\gamma_4 + \frac{(2m-1)[c(a+b)+\delta]+md(a+b)}{\delta}\gamma_5 + \frac{(2c+d)|x_0|+(a+b)|x_1|}{\delta} < 1. \tag{3.11}$$

Then problem (1.1) *has at least one solution.*

Proof Let us fix

$$r \geq \left\{\frac{(m+1)[c(a+b)+\delta]}{\delta\Gamma(q+1)} + \frac{(2m-1)[c(a+b)+\delta]+(m+1)(a+b)d}{\delta\Gamma(q)}\right\}\|\mu\|_{PC} + \frac{m[c(a+b)+\delta]}{\delta}\gamma_4 + \frac{(2m-1)[c(a+b)+\delta]+md(a+b)}{\delta}\gamma_5 + \frac{(2c+d)|x_0|+(a+b)|x_1|}{\delta}.$$

Let $\mathbb{C} = PC(J,\mathbb{R})$, and consider $B_r = \{u \in \mathbb{C} : \|u\| \leq r\}$; then B_r is a bounded, closed, convex set in $\mathbb{C}$.

Now define the operators $\mathcal{A}_1$ and $\mathcal{A}_2$ on B_r as

$$\begin{aligned}
&(\mathcal{A}_1u)(t) \\
&= \frac{1}{\Gamma(q)}\int_{t_k}^{t}(t-s)^{q-1}f\bigl(s,u(s)\bigr)\,ds + \frac{1}{\Gamma(q)}\sum_{i=1}^{m}\int_{t_{i-1}}^{t_i}(t_i-s)^{q-1}f\bigl(s,u(s)\bigr)\,ds \\
&\quad + \frac{1}{\Gamma(q-1)}\sum_{i=1}^{m}(t-t_k)\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}f\bigl(s,u(s)\bigr)\,ds \\
&\quad + \frac{1}{\Gamma(q-1)}\sum_{i=1}^{m-1}(t_k-t_i)\int_{t_{i-1}}^{t_i}(t_i-s)^{q-2}f\bigl(s,u(s)\bigr)\,ds \\
&\quad - \Biggl\{\sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{bc(t_i-s)^{q-1}}{\delta\Gamma(q)}f\bigl(s,u(s)\bigr)\,ds + \sum_{i=1}^{m}\int_{t_{i-1}}^{t_i}\frac{bc(1-t_m)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds \\
&\quad + \sum_{i=1}^{m-1}\int_{t_{i-1}}^{t_i}\frac{bc(t_m-t_i)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds \\
&\quad + \sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{bd(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds\Biggr\} \\
&\quad - \Biggl\{\sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{ac(t_i-s)^{q-1}}{\delta\Gamma(q)}f\bigl(s,u(s)\bigr)\,ds + \sum_{i=1}^{m}\int_{t_{i-1}}^{t_i}\frac{ac(1-t_m)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds \\
&\quad + \sum_{i=1}^{m-1}\int_{t_{i-1}}^{t_i}\frac{ac(t_m-t_i)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds \\
&\quad + \sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{ad(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds\Biggr\}t
\end{aligned} \tag{3.12}$$

and

$$
\begin{aligned}
(\mathcal{A}_2u)(t) = {}& \sum_{i=1}^{m} I_i\big(u(t_i^-)\big) + \sum_{i=1}^{m}(t-t_k)\bar{I}_i\big(u(t_i^-)\big) + \sum_{i=1}^{m-1}(t_k-t_i)\bar{I}_i\big(u(t_i^-)\big) \\
& - \Bigg\{ \sum_{i=1}^{m} \frac{bc}{\delta} I_i\big(u(t_i^-)\big) + \sum_{i=1}^{m} \frac{bc(1-t_p)}{\delta} \bar{I}_i\big(u(t_i^-)\big) \\
& + \sum_{i=1}^{m-1} \frac{bc(t_p-t_i)}{\delta} \bar{I}_i\big(u(t_i^-)\big) + \sum_{i=1}^{m} \frac{bd}{\delta} \bar{I}_i\big(u(t_i^-)\big) + \frac{(bc-\delta)x_0 - abx_1}{a\delta} \Bigg\} \\
& - \Bigg\{ \sum_{i=1}^{m} \frac{ac}{\delta} I_i\big(u(t_i^-)\big) + \sum_{i=1}^{m} \frac{ac(1-t_p)}{\delta} \bar{I}_i\big(u(t_i^-)\big) \\
& + \sum_{i=1}^{m-1} \frac{bc(t_p-t_i)}{\delta} \bar{I}_i\big(u(t_i^-)\big) + \sum_{i=1}^{m} \frac{ad}{\delta} \bar{I}_i\big(u(t_i^-)\big) + \frac{cx_0 - ax_1}{a\delta} \Bigg\} t. \qquad (3.13)
\end{aligned}
$$

For $u, v \in B_r$, by (H7), we find that

$$
\begin{aligned}
&\|\mathcal{A}_1u + \mathcal{A}_2v\| \\
&\quad \le \left\{ \frac{(m+1)[c(a+b)+\delta]}{\delta\Gamma(q+1)} + \frac{(2m-1)[c(a+b)+\delta] + (m+1)(a+b)d}{\delta\Gamma(q)} \right\} \|\mu\|_{PC} \\
&\qquad + \frac{m[c(a+b)+\delta]}{\delta}\gamma_4 + \frac{(2m-1)[c(a+b)+\delta] + md(a+b)}{\delta}\gamma_5 \\
&\qquad + \frac{(2c+d)|x_0| + (a+b)|x_1|}{\delta} \\
&\quad \le r.
\end{aligned}
$$

Thus, $\|\mathcal{A}_1u + \mathcal{A}_2v\| \le r$, so $\mathcal{A}_1u + \mathcal{A}_2v \in B_r$.

For $\forall u, v \in B_r$ and for each $t \in J$, it follows from the assumption (H6) that $\mathcal{A}_2$ is a contraction mapping for $\lambda < 1$. Continuity of f implies that the operator $\mathcal{A}_1$ is continuous. Also, $\mathcal{A}_1$ is uniformly bounded on B_r. In fact,

$$
\begin{aligned}
\|\mathcal{A}_1x\| \le {}& \Bigg\{ \frac{(m+1)[c(a+b)+\delta]}{\delta\Gamma(q+1)} \\
& + \frac{(2m-1)[c(a+b)+\delta] + (m+1)(a+b)d}{\delta\Gamma(q)} \Bigg\} \|\mu\|_{PC}.
\end{aligned}
$$

On the other hand, for $\forall t \in J_k$, $0 \le k \le m$, we have

$$
\begin{aligned}
&\big|(\mathcal{A}_1u)'(t)\big| \\
&\quad \le \frac{1}{\Gamma(q-1)} \int_{t_k}^{t} (t-s)^{q-2} \big|f\big(s,u(s)\big)\big|\,ds + \frac{1}{\Gamma(q-1)} \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} (t_i-s)^{q-2} \big|f\big(s,u(s)\big)\big|\,ds \\
&\qquad + \Bigg| \Bigg\{ \sum_{i=1}^{m+1} \int_{t_{i-1}}^{t_i} \frac{ac(t_i-s)^{q-1}}{\delta\Gamma(q)} f\big(s,u(s)\big)\,ds + \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} \frac{ac(1-t_m)(t_i-s)^{q-2}}{\delta\Gamma(q-1)} f\big(s,u(s)\big)\,ds
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{i=1}^{m-1}\int_{t_{i-1}}^{t_i}\frac{ac(t_m-t_i)(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds+\sum_{i=1}^{m+1}\int_{t_{i-1}}^{t_i}\frac{ad(t_i-s)^{q-2}}{\delta\Gamma(q-1)}f\bigl(s,u(s)\bigr)\,ds\Biggr\}\Biggr| \\
&\le\left\{\frac{(m+1)ac}{\delta\Gamma(q+1)}+\frac{(2m-1)ac+(m+1)(ad+\delta)}{\delta\Gamma(q)}\right\}\|\mu\|_{PC}:=\mathcal{M}. \qquad (3.14)
\end{aligned}
$$

If $t_1, t_2 \in J_k$, and $t_1 < t_2$, $0 \le k \le m$, then

$$
\bigl|(\mathcal{A}_1u)(t_2)-(\mathcal{A}_1u)(t_1)\bigr| \le \int_{t_1}^{t_2}\bigl|(\mathcal{A}_1u)'(s)\bigr|\,ds \le \mathcal{M}(t_2-t_1).
$$

Thus, $\mathcal{A}_1$ is equicontinuous. Using the fact that f maps bounded subsets into relatively compact subsets, it follows that $\mathcal{A}_1$ is relatively compact on B_r. Hence, by the Ascoli-Arzelà theorem, $\mathcal{A}_1$ is compact on B_r. Thus all the assumptions of Lemma 2.5 are satisfied. Hence, by the conclusion of Lemma 2.5, the impulsive fractional boundary value problem (1.1) has at least one solution on J. □

In the sequel we present an example which illustrates Theorem 3.2.

4 An example

Example 4.1 Consider the following boundary value problem:

$$
\begin{cases}
{}^cD_{0+}^q u(t)=\frac{\sin t}{(t+3)^2}\frac{|u(t)|}{1+|u(t)|}, & 0\le t\le 1, t\ne\frac{1}{2},\\
\Delta u(\frac{1}{2})=\frac{1}{(t+5)^2}\frac{|u(t)|}{16+|u(t)|}, & \Delta u'(\frac{1}{2})=\frac{1}{(t+7)^2}\frac{|u(t)|}{25+|u(t)|},\\
u(0)-u'(0)=0.03, & u(1)+u'(1)=0.06,
\end{cases} \qquad (4.1)
$$

where $q=\frac{3}{2}$, $a=1$, $b=1$, $c=1$, $d=1$, $m=1$, $\delta=ac+ad+bc=3$. Clearly $\gamma_1=\frac{1}{9}$, $\gamma_2=\frac{1}{25}$, $\gamma_3=\frac{1}{49}$, $\gamma_4=\frac{1}{25}$, $\gamma_5=\frac{1}{49}$, $x_0=0.03$, $x_1=0.06$.

Moreover, we have

$$
\begin{aligned}
\lambda &:= \frac{m[c(a+b)+\delta]}{\delta}\gamma_4+\frac{(2m-1)[c(a+b)+\delta]+md(a+b)}{\delta}\gamma_5 \\
&\quad+\frac{(2c+d)|x_0|+(a+b)|x_1|}{\delta}\approx 0.1743<1. \qquad (4.2)
\end{aligned}
$$

Thus, all the assumptions of Theorem 3.2 are satisfied. Hence, by the conclusion of Theorem 3.2, the impulsive fractional boundary value problem (4.1) has at least one solution on J.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the manuscript. All authors read and approved the final manuscript.

Acknowledgements
This research was supported by the National Natural Science Foundation of China (11161027, 11262009); by the Scientific Research Projects in Colleges and Universities of Gansu Province of China (2013A-043); by the Fundamental Research Funds for the Gansu Universities (2013); by the Fundamental Research Funds for the Gansu Universities (212084); by the Youth Science Foundation of Lanzhou Jiaotong University (2012019); by the National Natural Science Foundation of China (11226132). The authors are thankful to the referees for their careful reading of the manuscript and insightful comments.

Received: 2 July 2013 Accepted: 17 December 2013 Published: 09 Jan 2014

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10.1186/1687-1847-2014-12
Cite this article as: Zhou and Liu: Existence of solution to a class of boundary value problem for impulsive fractional differential equations. *Advances in Difference Equations* 2014, 2014:12

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