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Existence results for p-Laplacian boundary value problems of impulsive dynamic equations on time scales

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Abstract
In this paper, Bai-Ge's fixed point theorem is used to investigate the existence of positive solutions for second-order boundary value problems of p-Laplacian impulsive dynamic equations on time scales. As an application, we give an example to demonstrate our results.

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Keywords: impulsive dynamic equation; p-Laplacian; positive solutions; fixed point theorems; time scales

1 Introduction

Impulse differential equations describe processes which experience a sudden change of state at certain moments; see the monographs of Lakshmikantham *et al.* [1] and Samoilenko and Perestyuk [2]. Impulsive differential equations can be used to describe a lot of natural phenomena such as the dynamics of populations subject to abrupt changes (harvesting, diseases, *etc.*), which cannot be described using classical differential equations. That is why in recent years they have attracted much attention of investigators (*cf.*, *e.g.*, [3, 4]).

The study of dynamic equations on time scales goes back to Stefan Hilger [5]. Now it is still a new area of fairly theoretical exploration in mathematics. We refer to the books by Bohner and Peterson [6, 7]. There are a lot of works concerning the p-Laplacian problems on time scales; see, for example, [8–10]. Few works have been done on the existence of solutions to boundary value problems (BVP) for p-Laplacian impulsive dynamic equations on time scales; see [11–14]. Moreover, there is not much work on m-point boundary value problems for the p-Laplacian impulsive dynamic equations on time scales except for that in [15] by Li *et al.* Our aim in this paper is to fill the gap.

Motivated by the above mentioned works, in this paper we consider the existence of positive solutions of the following m-point boundary value problems for p-Laplacian impulsive dynamic equation on time scales:

$$\begin{cases} (\phi_p(u^\Delta(t)))^\nabla + q(t)f(t,u(t),u^\Delta(t)) = 0, \quad t \in [0,1]_{\mathbb{T}}, t \neq t_k, k = 1,2,\ldots,n, \\ \Delta u(t_k) = -I_k(u(t_k)), \quad k = 1,2,\ldots,n, \\ \Delta\phi_p(u^\Delta(t_k)) = -\bar{I}_k(u(t_k),u^\Delta(t_k)), \quad k = 1,2,\ldots,n, \\ \phi_p(u^\Delta(0)) = \sum_{j=1}^{m-2}\alpha_j\phi_p(u^\Delta(\xi_j)), \qquad u(1) = \sum_{j=1}^{m-2}\beta_j u(\eta_j), \end{cases} \tag{1.1}$$

where $\mathbb{T}$ is a time scale, $0,1 \in \mathbb{T}$, $[0,1]_{\mathbb{T}} = [0,1] \cap \mathbb{T}$, $t_k \in (0,1)_{\mathbb{T}}$, $k = 1,2,\ldots,n$, with $0 < t_1 < t_2 < \cdots < t_n < 1$, $\xi_j,\eta_j \in (0,1)_{\mathbb{T}}$ $(j = 1,2,\ldots,m-2)$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < 1$, $0 < \eta_1 < \eta_2 < \cdots < \eta_{m-2} < 1$ and $\xi_j,\eta_j \neq t_k$, $j = 1,2,\ldots,m-2$, $k = 1,2,\ldots,n$. $\phi_p(s)$ is a p-Laplacian operator, *i.e.*, $\phi_p(s) = |s|^{p-2}s$ for $p > 1$, $(\phi_p)^{-1}(s) = \phi_q(s)$, where $\frac{1}{p} + \frac{1}{q} = 1$ and $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$, $\Delta\phi_p(u^\Delta(t_k)) = \phi_p(u^\Delta(t_k^+)) - \phi_p(u^\Delta(t_k^-))$, $u(t_k^+)$ and $u(t_k^-)$ represent the right-hand limit and left-hand limit of the function $u(t)$ at $t = t_k$, $k = 1,2,\ldots,n$.

In this paper we assume that

(A1) $f \in \mathcal{C}([0,1]_{\mathbb{T}} \times \mathbb{R}^+ \times \mathbb{R}, \mathbb{R}^+)$,

(A2) $\alpha_j \in [0,\infty)$, $\beta_j \in [0,\infty)$, $j = 1,2,\ldots,m-2$, with $0 < \sum_{j=1}^{m-2}\alpha_j < 1$ and $0 < \sum_{j=1}^{m-2}\beta_j < 1$,

(A3) $q \in \mathcal{C}([0,1]_{\mathbb{T}})$ is nonnegative and there exists an integer $l \geq 3$, $\frac{1}{l}, 1 - \frac{1}{l} \in \mathbb{T}$ such that $\int_{\frac{1}{l}}^{1-\frac{1}{l}} q(t)\nabla t > 0$,

(A4) $I_k \in \mathcal{C}(\mathbb{R}^+,\mathbb{R}^+)$ is a bounded function, $\bar{I}_k \in \mathcal{C}(\mathbb{R}^+ \times \mathbb{R}, \mathbb{R}^+)$, $k = 1,2,\ldots,n$,

(A5) $\max\{G_1,G_2,\ldots,G_n\} \leq \frac{lb_1}{n}$, where $G_k = \sup_{u\in[0,\infty)} I_k(u)$ and $b_1 > 0$ is a constant which is given by Theorem 3.1.

In this study, by employing Bai-Ge's fixed point theorem [16], we get the existence of at least three positive solutions for boundary value problem (1.1). In fact, our result is also new when $\mathbb{T} = \mathbb{R}$ (the differential case) and $\mathbb{Z}$ (the discrete case). Therefore, the result can be considered as a contribution to this field.

This paper is organized as follows. In Section 2, we give some definitions and preliminary lemmas which are key tools for our proof. The main results are given in Section 3. Finally, in Section 4, we give an example to demonstrate our results.

2 Preliminaries

In this section, we give some lemmas which are useful for our main results.

Throughout the rest of this paper, we always assume that the points of impulse t_k are right dense for each $k = 1,2,\ldots,n$. Let $J = [0,1]_{\mathbb{T}}, J_0 = [0,t_1]_{\mathbb{T}}, J_1 = (t_1,t_2]_{\mathbb{T}},\ldots,J_{n-1} = (t_{n-1},t_n]_{\mathbb{T}}, J_n = (t_n,1]_{\mathbb{T}}, J' = J \setminus \{t_1,t_2,\ldots,t_n\}$.

Set

$$\begin{aligned} PC(J) = \bigl\{&u : [0,1]_{\mathbb{T}} \longrightarrow \mathbb{R}; u \in C(J'), u(t_k^+) \text{ and } u(t_k^-) \text{ exist;} \\ &\text{and } u(t_k^-) = u(t_k), 1 \leq k \leq n\bigr\}, \\ PC^1(J) = \bigl\{&u \in PC(J) : u^\Delta \in C(J'), u^\Delta(t_k^+) \text{ and } u^\Delta(t_k^-) \text{ exist;} \\ &\text{and } u^\Delta(t_k^-) = u^\Delta(t_k), 1 \leq k \leq n\bigr\}. \end{aligned}$$

Obviously, $PC(J)$ and $PC^1(J)$ are Banach spaces with the norms

$$\|u\|_{PC} = \max_{t\in[0,1]_{\mathbb{T}}} \bigl|u(t)\bigr|, \qquad \|u\|_{PC^1} = \max\bigl\{\|u\|_{PC}, \bigl\|u^{\Delta}\bigr\|_{PC}\bigr\},$$

respectively. A function $u \in PC^1(J) \cap C^2(J')$ is called a solution to (1.1) if it satisfies all the equations of (1.1).

Define the cone $P \subset PC^1(J)$ by

$$\begin{aligned} P = \bigl\{&u \in PC^1(J) : u(t) \geq 0, u \text{ is concave on } J_k\ (0 \leq k \leq n) \\ &\text{and } u(t), u^{\Delta}(t) \text{ are non-increasing on } [0,1]_{\mathbb{T}}\bigr\}. \end{aligned}$$

Lemma 2.1 *If u is continuous, nonnegative and concave on $[0,1]_{\mathbb{T}}$, then*

$$u\left(1 - \frac{1}{l}\right) \geq \frac{1}{l} \max_{t\in[0,1]_{\mathbb{T}}} \bigl|u(t)\bigr|,$$

where $l \geq 3$.

Proof Suppose that $\max_{t\in[0,1]_{\mathbb{T}}} |u(t)| = u(\sigma)$, where $\sigma \in [0,1]_{\mathbb{T}}$. Since u is concave and non-negative,

$$\begin{aligned} u\left(1 - \frac{1}{l}\right) &= u\left(\frac{1+\sigma}{l(1+\sigma)}(l-1)(1-\sigma) + \frac{l-1}{l}\sigma\right) \\ &\geq \frac{1+\sigma}{l(1+\sigma)} u\bigl((l-1)(1-\sigma)\bigr) + \left(\frac{l-1}{l}\right)u(\sigma) \\ &\geq \left(\frac{l-1}{l}\right)u(\sigma) \\ &\geq \frac{1}{l}u(\sigma) \\ &\geq \frac{1}{l} \max_{t\in[0,1]_{\mathbb{T}}} \bigl|u(t)\bigr|. \end{aligned}$$

□

Lemma 2.2 *If $u \in P$, $l \geq 3$, then*

$$\min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} \bigl|u(t)\bigr| \geq \frac{1}{l} \max_{t\in[0,1]_{\mathbb{T}}} \bigl|u(t)\bigr| - \frac{1}{l} \sum_{i=1}^{n} \bigl|\Delta u(t_i)\bigr|.$$

Proof Let

$$v(t) = \begin{cases} u(t) - \sum_{k=1}^{n} |\Delta u(t_k)|, & t \in J_0, \\ u(t) - \sum_{k=2}^{n} |\Delta u(t_k)|, & t \in J_1, \\ \vdots & \\ u(t) - |\Delta u(t_n)|, & t \in J_{n-1}, \\ u(t), & t \in J_n. \end{cases}$$

Note that $u^{\Delta}(t)$ is non-increasing on $[0,1]_{\mathbb{T}}$, hence $v \in \mathcal{C}[0,1]_{\mathbb{T}}$ and v is concave on $[0,1]_{\mathbb{T}}$. By Lemma 2.1, we have

$$\min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} |v(t)| = v\left(1-\frac{1}{l}\right) \geq \frac{1}{l}\max_{t\in[0,1]_{\mathbb{T}}} |v(t)|.$$

Moreover, $u(t)$ is non-increasing on $[0,1]_{\mathbb{T}}$, and we have

$$\begin{aligned}
\max_{t\in[0,1]_{\mathbb{T}}} |v(t)| &= v(0) \\
&= u(0) - \sum_{k=1}^{n} |\Delta u(t_k)| \\
&= \max_{t\in[0,1]_{\mathbb{T}}} |u(t)| - \sum_{k=1}^{n} |\Delta u(t_k)|, \\
\min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} |v(t)| &= v\left(1-\frac{1}{l}\right) \\
&= u\left(1-\frac{1}{l}\right) - \sum_{1-\frac{1}{l}<t_k<1} |\Delta u(t_k)| \\
&= \min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} |u(t)| - \sum_{1-\frac{1}{l}<t_k<1} |\Delta u(t_k)|.
\end{aligned}$$

Hence,

$$\begin{aligned}
\min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} |u(t)| &= \min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} |v(t)| + \sum_{0<t_k<\frac{1}{l}} |\Delta u(t_k)| \\
&\geq \frac{1}{l}\max_{t\in[0,1]_{\mathbb{T}}} |v(t)| \\
&= \frac{1}{l}\max_{t\in[0,1]_{\mathbb{T}}} |u(t)| - \frac{1}{l}\sum_{k=1}^{n} |\Delta u(t_k)|.
\end{aligned}$$

□

Lemma 2.3 *Assume that* (A1)-(A4) *hold. Then* $u \in PC^1(J)\cap C^2(J')$ *is a solution to problem* (1.1) *if and only if* $u \in PC^1(J)$ *is a solution to the integral equation*:

$$\begin{aligned}
u(t) = {}& \int_t^1 \phi_q\left(\int_0^s q(\tau)f\left(\tau,u(\tau),u^{\Delta}(\tau)\right)\nabla\tau + \sum_{0<t_k<s}\bar{I}_k\left(u(t_k),u^{\Delta}(t_k)\right) + A\right)\Delta s \\
&+ \frac{\sum_{j=1}^{m-2}\beta_j}{1-\sum_{i=1}^{m-2}\beta_i}\int_{\eta_j}^1 \phi_q\left(\int_0^s q(\tau)f\left(\tau,u(\tau),u^{\Delta}(\tau)\right)\nabla\tau\right. \\
&\left. + \sum_{0<t_k<s}\bar{I}_k\left(u(t_k),u^{\Delta}(t_k)\right) + A\right)\Delta s \\
&+ \frac{1}{1-\sum_{i=1}^{m-2}\beta_i}\sum_{j=1}^{m-2}\beta_j\sum_{\eta_j<t_k<1} I_k\left(u(t_k)\right) + \sum_{t<t_k<1} I_k\left(u(t_k)\right),
\end{aligned} \tag{2.1}$$

where

$$A = \frac{1}{1-\sum_{i=1}^{m-2}\alpha_i}\sum_{j=1}^{m-2}\alpha_j\left[\int_0^{\xi_j} q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s + \sum_{0<t_k<\xi_j}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big)\right].$$

Proof First, suppose that $u \in PC^1(J) \cap C^2(J')$ is a solution to problem (1.1). Then

$$\big(\phi_p\big(u^{\Delta}(t)\big)\big)^{\nabla} + q(t)f\big(t,u(t),u^{\Delta}(t)\big) = 0, \quad t \neq t_k, k = 1,2,\ldots,n.$$

So,

$$\begin{aligned}
\phi_p\big(u^{\Delta}\big(t_1^-\big)\big) - \phi_p\big(u^{\Delta}(0)\big) &= -\int_0^{t_1} q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s, \\
\phi_p\big(u^{\Delta}(t)\big) - \phi_p\big(u^{\Delta}\big(t_1^+\big)\big) &= -\int_{t_1}^{t} q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s, \quad t \in J_1.
\end{aligned}$$

Thus,

$$\phi_p\big(u^{\Delta}(t)\big) = \phi_p\big(u^{\Delta}(0)\big) - \int_0^t q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s - \bar{I}_1\big(u(t_1),u^{\Delta}(t_1)\big), \quad t \in J_1.$$

Repeating the above process, for $t \in [0,1]_{\mathbb{T}}$, we have

$$\phi_p\big(u^{\Delta}(t)\big) = \phi_p\big(u^{\Delta}(0)\big) - \int_0^t q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s - \sum_{0<t_k<t}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big), \tag{2.2}$$

and taking $t = \xi_j$ in (2.2), we have

$$\phi_p\big(u^{\Delta}(\xi_j)\big) = \phi_p\big(u^{\Delta}(0)\big) - \int_0^{\xi_j} q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s - \sum_{0<t_k<\xi_j}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big).$$

So, we get

$$\begin{aligned}
\sum_{j=1}^{m-2}\alpha_j\phi_p\big(u^{\Delta}(\xi_j)\big) &= \sum_{j=1}^{m-2}\alpha_j\phi_p\big(u^{\Delta}(0)\big) - \sum_{j=1}^{m-2}\alpha_j\int_0^{\xi_j} q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s \\
&\quad - \sum_{j=1}^{m-2}\alpha_j\sum_{0<t_k<\xi_j}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big).
\end{aligned}$$

Since $\phi_p(u^{\Delta}(0)) = \sum_{j=1}^{m-2}\alpha_j\phi_p(u^{\Delta}(\xi_j))$, we have

$$\begin{aligned}
\phi_p\big(u^{\Delta}(0)\big) &= -\frac{\sum_{j=1}^{m-2}\alpha_j}{1-\sum_{i=1}^{m-2}\alpha_i}\left[\int_0^{\xi_j} q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s + \sum_{0<t_k<\xi_j}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big)\right] \\
&= -A.
\end{aligned} \tag{2.3}$$

Substituting (2.3) into (2.2), we get

$$\phi_p\bigl(u^\Delta(t)\bigr) = -\biggl[\int_0^t q(s)f\bigl(s,u(s),u^\Delta(s)\bigr)\nabla s + \sum_{0<t_k<t} \bar{I}_k\bigl(u(t_k),u^\Delta(t_k)\bigr) + A\biggr],$$

which implies that

$$u^\Delta(t) = -\phi_q\biggl(\int_0^t q(s)f\bigl(s,u(s),u^\Delta(s)\bigr)\nabla s + \sum_{0<t_k<t} \bar{I}_k\bigl(u(t_k),u^\Delta(t_k)\bigr) + A\biggr). \tag{2.4}$$

On the other hand, note that

$$\begin{aligned} u\bigl(t_n^-\bigr) - u(t) &= \int_t^{t_n} u^\Delta(s)\Delta s, \\ u(1) - u\bigl(t_n^+\bigr) &= \int_{t_n}^1 u^\Delta(s)\Delta s, \quad t \in J_{n-1}. \end{aligned}$$

So that we have

$$u(t) = u(1) - \int_t^1 u^\Delta(s)\Delta s + I_n\bigl(u(t_n)\bigr), \quad t \in J_{n-1}.$$

Repeating the above process again for $t \in [0,1]_{\mathbb{T}}$, we obtain

$$u(t) = u(1) - \int_t^1 u^\Delta(s)\Delta s + \sum_{t<t_k<1} I_k\bigl(u(t_k)\bigr). \tag{2.5}$$

Substituting (2.4) into (2.5), we get

$$\begin{aligned} u(t) = {}& u(1) + \sum_{t<t_k<1} I_k\bigl(u(t_k)\bigr) \\ &+ \int_t^1 \phi_q\biggl(\int_0^s q(\tau)f\bigl(\tau,u(\tau),u^\Delta(\tau)\bigr)\nabla\tau + \sum_{0<t_k<s} \bar{I}_k\bigl(u(t_k),u^\Delta(t_k)\bigr) + A\biggr)\Delta s, \end{aligned} \tag{2.6}$$

and taking $t = \eta_j$ in (2.6), we get

$$\begin{aligned} u(\eta_j) = {}& u(1) + \sum_{\eta_j<t_k<1} I_k\bigl(u(t_k)\bigr) \\ &+ \int_{\eta_j}^1 \phi_q\biggl(\int_0^s q(\tau)f\bigl(\tau,u(\tau),u^\Delta(\tau)\bigr)\nabla\tau + \sum_{0<t_k<s} \bar{I}_k\bigl(u(t_k),u^\Delta(t_k)\bigr) + A\biggr)\Delta s. \end{aligned}$$

So,

$$\begin{aligned}
&\sum_{j=1}^{m-2}\beta_j u(\eta_j)\\
&\quad= u(1)\sum_{j=1}^{m-2}\beta_j+\sum_{j=1}^{m-2}\beta_j\sum_{\eta_j<t_k<1}I_k\big(u(t_k)\big)\\
&\qquad+\sum_{j=1}^{m-2}\beta_j\int_{\eta_j}^1\phi_q\biggl(\int_0^s q(\tau)f\big(\tau,u(\tau),u^\Delta(\tau)\big)\nabla\tau+\sum_{0<t_k<s}\bar{I}_k\big(u(t_k),u^\Delta(t_k)\big)+A\biggr)\Delta s.
\end{aligned}$$

Since

$$\begin{aligned}
u(1)&=\sum_{j=1}^{m-2}\beta_j u(\eta_j),\\
&=\frac{\sum_{j=1}^{m-2}\beta_j}{1-\sum_{i=1}^{m-2}\beta_i}\int_{\eta_j}^1\phi_q\biggl(\int_0^s q(\tau)f\big(\tau,u(\tau),u^\Delta(\tau)\big)\nabla\tau\\
&\quad+\sum_{0<t_k<s}\bar{I}_k\big(u(t_k),u^\Delta(t_k)\big)+A\biggr)\Delta s+\frac{1}{1-\sum_{i=1}^{m-2}\beta_i}\sum_{j=1}^{m-2}\beta_j\sum_{\eta_j<t_k<1}I_k\big(u(t_k)\big).
\end{aligned}\tag{2.7}$$

Substituting (2.7) into (2.6), we get (2.1), which completes the proof of sufficiency.

Conversely, if $u(t)\in PC^1(J)$ is a solution to (2.1), apparently

$$\Delta u(t_k)=-I_k\big(u(t_k)\big),\quad k=1,2,\dots,n.$$

The Δ-derivative of (2.1) implies that for $t\neq t_k$,

$$\begin{aligned}
&u^\Delta(t)=-\phi_q\biggl(\int_0^t q(s)f\big(s,u(s),u^\Delta(s)\big)\nabla s+\sum_{0<t_k<t}\bar{I}_k\big(u(t_k),u^\Delta(t_k)\big)+A\biggr),\\
&\big(\phi_p\big(u^\Delta(t)\big)\big)^\nabla=-q(t)f\big(t,u(t),u^\Delta(t)\big).
\end{aligned}$$

Hence $u\in C^2(J')$, and

$$\begin{aligned}
&\Delta\phi_p\big(u^\Delta(t_k)\big)=-\bar{I}_k\big(u(t_k),u^\Delta(t_k)\big),\quad k=1,2,\dots,n,\\
&\phi_p\big(u^\Delta(0)\big)=\sum_{j=1}^{m-2}\alpha_j\phi_p\big(u^\Delta(\xi_j)\big),\qquad u(1)=\sum_{j=1}^{m-2}\beta_j u(\eta_j).
\end{aligned}$$

The proof is complete. □

Now define an operator $T:P\longrightarrow PC^1(J)$ by

$$\begin{aligned}
Tu(t)={}&\int_t^1\phi_q\biggl(\int_0^s q(\tau)f\big(\tau,u(\tau),u^\Delta(\tau)\big)\nabla\tau+\sum_{0<t_k<s}\bar{I}_k\big(u(t_k),u^\Delta(t_k)\big)+A\biggr)\Delta s\\
&+\frac{\sum_{j=1}^{m-2}\beta_j}{1-\sum_{i=1}^{m-2}\beta_i}\int_{\eta_j}^1\phi_q\biggl(\int_0^s q(\tau)f\big(\tau,u(\tau),u^\Delta(\tau)\big)\nabla\tau
\end{aligned}$$

$$+ \sum_{0<t_k<s} \bar{I}_k\big(u(t_k), u^{\Delta}(t_k)\big) + A\Bigg)\Delta s$$

$$+ \frac{1}{1-\sum_{i=1}^{m-2}\beta_i}\sum_{j=1}^{m-2}\beta_j \sum_{\eta_j<t_k<1} I_k\big(u(t_k)\big) + \sum_{t<t_k<1} I_k\big(u(t_k)\big). \tag{2.8}$$

Lemma 2.4 *Assume that* (A1)-(A4) *hold. Then* $T: P \to P$ *is a completely continuous operator.*

Proof From the definition of T, it is clear that $T(P) \subset P$. On the other hand, by conditions (A1)-(A4) and the definition of $Tu(t)$, it is clear that $T: P \to P$ is continuous.

Let $\Omega \subset P$ be bounded, *i.e.*, there exists a positive constant R such that

$$\Omega \subset \big\{u \in P : \|u\|_{PC^1} \le R\big\}.$$

Let

$$B_1 = \max_{(t,u,v)\in[0,1]_{\mathbb{T}}\times[0,R]\times[0,R]} f(t,u,v),$$

$$B_2 = \max_{1\le k\le n}\Big\{\max_{u\in[0,R]} I_k(u)\Big\},$$

$$B_3 = \max_{1\le k\le n}\Big\{\max_{(u,v)\in[0,R]\times[0,R]} \bar{I}_k(u,v)\Big\},$$

$$R_1 = \max_{t\in[0,1]_{\mathbb{T}}} q(t).$$

For all $u \in \Omega$, we have

$$A \le \frac{1}{1-\sum_{j=1}^{m-2}\alpha_j}\sum_{j=1}^{m-2}\alpha_j[B_1R_1 + nB_3].$$

Hence,

$$\big|Tu(t)\big| \le \frac{1}{1-\sum_{j=1}^{m-2}\beta_j}\left[\phi_q\left(\frac{B_1R_1 + nB_3}{1-\sum_{j=1}^{m-2}\alpha_j}\right) + nB_2\right],$$

$$\big|(Tu)^{\Delta}(t)\big| \le \phi_q\left[\frac{B_1R_1 + nB_3}{1-\sum_{j=1}^{m-2}\alpha_j}\right],$$

$$\Big|\big(\phi_p(Tu)^{\Delta}(t)\big)^{\nabla}\Big| \le R_1B_1, \quad t \ne t_k, k = 1,2,\ldots,n.$$

So, Tu and $(Tu)^{\Delta}$ are bounded on J and equicontinuous on each J_k ($k = 0,1,\ldots,n$). This implies that $T\Omega$ is relatively compact. Therefore, the operator $T: P \to P$ is completely continuous. □

3 Main results

In this section we state and prove our main result. Define the following convex sets:

$$P(\varphi, r; \omega, L) = \big\{u \in P : \varphi(u) < r, \omega(u) < L\big\},$$

$$\bar{P}(\varphi, r; \omega, L) = \big\{u \in P : \varphi(u) \le r, \omega(u) \le L\big\},$$

$$P(\varphi,r;\omega,L;\psi,a)=\{u\in P:\varphi(u)<r,\omega(u)<L,\psi(u)>a\},$$
$$\bar{P}(\varphi,r;\omega,L;\psi,a)=\{u\in P:\varphi(u)\le r,\omega(u)\le L,\psi(u)\ge a\}.$$

The following assumptions as regards the nonnegative continuous convex functions φ, ω are used:

(B1) there exists $M>0$ such that $\|x\|\le M\max\{\varphi(x),\omega(x)\}$ for all $x\in P$;

(B2) $P(\varphi,r;\omega,L)\neq\emptyset$ for any $r>0$ and $L>0$.

To prove our main result, we need the following fixed point theorem due to Bai and Ge in [16].

Lemma 3.1 [16] *Let P be a cone in a real Banach space $\mathbb{E}$, and let $r_2\ge d>b>r_1>0$, $L_2\ge L_1>0$. Assume that φ and ω are nonnegative continuous convex functions satisfying* (B1) *and* (B2), *ψ is a nonnegative continuous concave function on P such that $\psi(u)\le\varphi(u)$ for all $u\in\bar{P}(\varphi,r_2;\omega,L_2)$ and $T:\bar{P}(\varphi,r_2;\omega,L_2)\to\bar{P}(\varphi,r_2;\omega,L_2)$ is a completely continuous operator. Suppose that*

(B3) $\{u\in\bar{P}(\varphi,d;\omega,L_2;\psi,b):\psi(u)>b\}\neq\emptyset$, $\psi(Tu)>b$ *for* $u\in\bar{P}(\varphi,d;\omega,L_2;\psi,b)$,

(B4) $\varphi(Tu)<r_1$, $\omega(Tu)<L_1$ *for all* $u\in\bar{P}(\varphi,r_1;\omega,L_1)$,

(B5) $\psi(Tu)>b$ *for all* $u\in\bar{P}(\varphi,r_2;\omega,L_2;\psi,b)$ *with* $\varphi(Tu)>d$.

Then T has at least three fixed points u_1, u_2 and $u_3\in\bar{P}(\varphi,r_2;\omega,L_2)$ with

$$u_1\in P(\varphi,r_1;\omega,L_1),\qquad u_2\in\{\bar{P}(\varphi,r_2;\omega,L_2;\psi,b):\psi(u)>b\},$$
$$u_3\in\bar{P}(\varphi,r_2;\omega,L_2)\setminus\big(\bar{P}(\varphi,r_2;\omega,L_2;\psi,b)\cup\bar{P}(\varphi,r_1;\omega,L_1)\big).$$

Define nonnegative continuous functionals φ, ω and ψ by

$$\varphi(u)=\max_{t\in[0,1]_{\mathbb{T}}}|u(t)|,\qquad\omega(u)=\max_{t\in[0,1]_{\mathbb{T}}}|u^{\Delta}(t)|,\qquad\psi(u)=\min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}}|u(t)|\quad\text{for } u\in P.$$

Then, on the cone P, ψ is a concave functional, φ and ω are convex functionals satisfying (B1) and (B2).

Let

$$H=\frac{1}{1-\sum_{j=1}^{m-2}\beta_j}\left[\phi_q\left(\frac{R_1+n}{1-\sum_{j=1}^{m-2}\alpha_j}\right)+n\right],$$
$$L=\phi_q\left(\frac{R_1+n}{1-\sum_{j=1}^{m-2}\alpha_j}\right),$$
$$N=\phi_q\left(\int_{1/l}^{1-1/l}q(r)\nabla r\right).$$

Theorem 3.1 *Assume that* (A1)-(A4) *hold. There exist constants $r_2\ge 2lb_1>b_1>r_1>0$, $L_2\ge L_1>0$ and the following assumptions are satisfied*:

(A5) $\max\{f(t,u,v),\bar{I}_k(u,v)\}<\min\{\phi_p(\frac{r_1}{H}),\phi_p(\frac{L_1}{L})\}$, $I_k(u)\le\frac{r_1}{H}$ *for* $(t,u,v)\in[0,1]_{\mathbb{T}}\times[0,r_1]\times[0,L_1]$, $1\le k\le n$;

(A6) $f(t,u,v)>\phi_p(\frac{lb_1}{N})$ *for* $(t,u,v)\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}\times[b_1,2lb_1]\times[0,L_2]$;

(A7) $\max\{f(t,u,v),\bar{I}_k(u,v)\}<\min\{\phi_p(\frac{r_2}{H}),\phi_p(\frac{L_2}{L})\}$, $I_k(u)\le\frac{r_2}{H}$ *for* $(t,u,v)\in[0,1]_{\mathbb{T}}\times[0,r_2]\times[0,L_2]$, $1\le k\le n$.

Then problem (1.1) *possesses at least three positive solutions* u_1, u_2 *and* u_3 *such that*

$$\max_{t\in[0,1]_{\mathbb{T}}} u_1(t) < r_1, \qquad \max_{t\in[0,1]_{\mathbb{T}}} \left|u_1^{\Delta}(t)\right| < L_1;$$

$$b_1 < \min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} u_2(t) \le \max_{t\in[0,1]_{\mathbb{T}}} u_2(t) \le r_2, \qquad \max_{t\in[0,1]_{\mathbb{T}}} \left|u_2^{\Delta}(t)\right| \le L_2;$$

$$r_1 < \max_{t\in[0,1]_{\mathbb{T}}} u_3(t) < r_2, \qquad \min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}} u_3(t) < b_1, \qquad \max_{t\in[0,1]_{\mathbb{T}}} \left|u_3^{\Delta}(t)\right| \le L_2.$$

Proof Problem (1.1) has a solution $u = u(t)$ if and only if u solves the operator equation $u = Tu$. We have shown $T : P \to P$ is completely continuous by Lemma 2.4. We now verify that all the conditions of Lemma 3.1 are satisfied. The proof is divided into four steps.

Step 1. First we show that

$$T : \bar{P}(\varphi, r_2; \omega, L_2) \to \bar{P}(\varphi, r_2; \omega, L_2). \tag{3.1}$$

If $u \in \bar{P}(\varphi, r_2; \omega, L_2)$, then $\varphi(u) \le r_2$, $\omega(u) \le L_2$, and by assumption (A7), we have

$$\begin{aligned} A &\le \frac{\sum_{j=1}^{m-2}\alpha_j}{1-\sum_{i=1}^{m-2}\alpha_i}\left[\int_0^1 q(s)f\big(s,u(s),u^{\Delta}(s)\big)\nabla s + \sum_{k=1}^{n}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big)\right] \\ &\le \frac{\sum_{j=1}^{m-2}\alpha_j}{1-\sum_{i=1}^{m-2}\alpha_i}\min\left\{\phi_p\left(\frac{r_2}{H}\right),\phi_p\left(\frac{L_2}{L}\right)\right\}(R_1+n). \end{aligned}$$

Hence,

$$\begin{aligned} \varphi(Tu) &= \max_{t\in[0,1]_{\mathbb{T}}}\left|u(t)\right| = (Tu)(0) \\ &= \int_0^1 \phi_q\left(\int_0^s q(\tau)f\big(\tau,u(\tau),u^{\Delta}(\tau)\big)\nabla\tau + \sum_{0<t_k<s}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big) + A\right)\Delta s \\ &\quad + \sum_{k=1}^{n} I_k\big(u(t_k)\big) \\ &\quad + \frac{\sum_{j=1}^{m-2}\beta_j}{1-\sum_{i=1}^{m-2}\beta_i}\int_{\eta_j}^1 \phi_q\left(\int_0^s q(\tau)f\big(\tau,u(\tau),u^{\Delta}(\tau)\big)\nabla\tau\right. \\ &\quad \left. + \sum_{0<t_k<s}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big) + A\right)\Delta s \\ &\quad + \frac{1}{1-\sum_{i=1}^{m-2}\beta_i}\sum_{j=1}^{m-2}\beta_j\sum_{\eta_j<t_k<1} I_k\big(u(t_k)\big) \\ &\le \frac{1}{1-\sum_{j=1}^{m-2}\beta_j}\int_0^1 \phi_q\left(\int_0^1 q(\tau)f\big(\tau,u(\tau),u^{\Delta}(\tau)\big)\nabla\tau\right. \\ &\quad \left. + \sum_{k=1}^{n}\bar{I}_k\big(u(t_k),u^{\Delta}(t_k)\big) + A\right)\Delta s \\ &\quad + \frac{1}{1-\sum_{j=1}^{m-2}\beta_j}\sum_{k=1}^{n} I_k\big(u(t_k)\big) \end{aligned}$$

$$\begin{aligned}
&\leq \frac{r_2}{H}\frac{1}{1-\sum_{j=1}^{m-2}\beta_j}\left[\phi_q\left(\frac{R_1+n}{1-\sum_{j=1}^{m-2}\alpha_j}\right)+n\right] \\
&= r_2, \\
\omega(Tu) &= \max_{t\in[0,1]_{\mathbb{T}}}\left|(Tu)^{\Delta}(t)\right| = \left|(Tu)^{\Delta}(1)\right| \\
&\leq \frac{L_2}{L}\phi_q\left(\frac{R_1+n}{1-\sum_{j=1}^{m-2}\alpha_j}\right) \\
&= L_2.
\end{aligned}$$

So, (3.1) holds.

Step 2. We show that condition (B3) in Lemma 3.1 holds. We choose $u(t) = \frac{2l+1}{2}b_1$ for $t \in [0,1]_{\mathbb{T}}$. It is easy to see that $u(t) \in \bar{P}(\varphi, 2lb_1; \omega, L_2; \psi, b_1)$, $\psi(u) = u(1-\frac{1}{l}) > b_1$ and consequently $\{u \in \bar{P}(\varphi, 2lb_1; \omega, L_2; \psi, b_1) : \psi(u) > b_1\} \neq \emptyset$. Thus, for $u \in \bar{P}(\varphi, 2lb_1; \omega, L_2; \psi, b_1)$, there is $b_1 \leq u(t) \leq 2lb_1$ for $t \in [\frac{1}{l}, 1-\frac{1}{l}]_{\mathbb{T}}$. By condition (A6), we have

$$\begin{aligned}
\psi(Tu) &= \min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}}\left|Tu(t)\right| = (Tu)\left(1-\frac{1}{l}\right) \\
&\geq \int_{1-1/l}^{1}\phi_q\left(\int_0^s q(\tau)f\left(\tau,u(\tau),u^{\Delta}(\tau)\right)\nabla\tau\right)\Delta s \\
&\geq \int_{1-1/l}^{1}\phi_q\left(\int_0^{1-1/l} q(\tau)f\left(\tau,u(\tau),u^{\Delta}(\tau)\right)\nabla\tau\right)\Delta s \\
&\geq \frac{1}{l}\phi_q\left(\int_{1/l}^{1-1/l} q(\tau)f\left(\tau,u(\tau),u^{\Delta}(\tau)\right)\nabla\tau\right) \\
&\geq \frac{b_1}{N}\phi_q\left(\int_{1/l}^{1-1/l} q(r)\nabla r\right) \\
&= b_1.
\end{aligned}$$

Therefore,

$$\psi(Tu) > b_1, \quad \forall u \in \bar{P}(\varphi, 2lb_1; \omega, L_2; \psi, b_1).$$

Step 3. We now show that (B4) in Lemma 3.1 is satisfied. If $u \in \bar{P}(\varphi, r_1; \omega, L_1)$, by condition (A5), in the same way as in Step 1, we can obtain that $T : \bar{P}(\varphi, r_1; \omega, L_1) \to P(\varphi, r_1; \omega, L_1)$. Hence, condition (B4) in Lemma 3.1 is satisfied.

Step 4. Finally, we verify condition (B5) in Lemma 3.1 for $u \in \bar{P}(\varphi, r_2; \omega, L_2; \psi, b)$ with $\varphi(Tu) > 2lb_1$. Then, by Lemma 2.2 and condition (A4), we have

$$\begin{aligned}
\psi(Tu) &= \min_{t\in[\frac{1}{l},1-\frac{1}{l}]_{\mathbb{T}}}\left|Tu(t)\right| \\
&\geq \frac{1}{l}\max_{t\in[0,1]_{\mathbb{T}}}\left|Tu(t)\right| - \frac{1}{l}\sum_{k=1}^{n}\left|\Delta(Tu)(t_k)\right| \\
&\geq \frac{1}{l}\max_{t\in[0,1]_{\mathbb{T}}}\left|Tu(t)\right| - \frac{1}{l}\sum_{k=1}^{n}\left|I_k\left(u(t_k)\right)\right|
\end{aligned}$$

$$
\begin{aligned}
&> \frac{1}{l}2lb_1 - \frac{1}{l}\frac{lb_1}{n}n \\
&= b_1.
\end{aligned}
$$

Thus, condition (B5) in Lemma 3.1 is satisfied.

Consequently, from Lemma 3.1, boundary value problem (1.1) has at least three positive solutions $u_1, u_2, u_3 \in \bar{P}(\varphi, r_2; \omega, L_2)$ with

$$
\begin{aligned}
&u_1 \in P(\varphi, r_1; \omega, L_1), \qquad u_2 \in \bigl\{\bar{P}(\varphi, r_2; \omega, L_2; \psi, b_1) : \psi(u) > b_1\bigr\}, \\
&u_3 \in \bar{P}(\varphi, r_2; \omega, L_2) \setminus \bigl(\bar{P}(\varphi, r_2; \omega, L_2; \psi, b_1) \cup \bar{P}(\varphi, r_1; \omega, L_1)\bigr).
\end{aligned}
$$

The proof is complete. □

From the proof of Theorem 3.1, it is easy to see that if conditions like (A5)-(A7) are appropriately combined, we can obtain an arbitrary number of positive solutions of problem (1.1).

Corollary 3.1 *Assume that* (A1)-(A4) *hold. There exist constants* $0 < r_1 < b_1 < 2lb_1 \le r_2 < b_2 < 2lb_2 \le \cdots \le r_i$, $0 < L_1 \le L_2 \le \cdots \le L_i$, $i \in \mathbb{N}$, *and the following conditions are satisfied*:

(A8) $\max\{f(t,u,v), \bar{I}_k(u,v)\} < \min\{\phi_p(\frac{r_h}{H}), \phi_p(\frac{L_h}{L})\}$, $I_k(u) \le \frac{r_h}{H}$ *for* $(t,u,v) \in [0,1]_{\mathbb{T}} \times [0,r_h] \times [0,L_h]$, $1 \le h \le i$, $1 \le k \le n$;

(A9) $f(t,u,v) > \phi_p(\frac{lb_h}{N})$ *for* $(t,u,v) \in [\frac{1}{l}, 1-\frac{1}{l}]_{\mathbb{T}} \times [b_h, 2lb_h] \times [0, L_{h+1}]$, $1 \le h \le i-1$.

Then problem (1.1) *possesses at least* $2i-1$ *positive solutions*.

4 An example

Example 4.1 Let $\mathbb{T} = \{2^{-n}\}_{n\in\mathbb{N}^*} \cup \{0\}$. We consider the boundary value problem

$$
\begin{cases}
(\phi_{5/2}(u^\Delta(t)))^\nabla + f(t,u(t),u^\Delta(t)) = 0, \quad t \ne \frac{1}{4}, t \in [0,1]_{\mathbb{T}}, \\
\Delta u(\frac{1}{4}) = -I_1(u(\frac{1}{4})), \\
\Delta \phi_{5/2}(u^\Delta(\frac{1}{4})) = -\bar{I}_1(u(\frac{1}{4}), u^\Delta(\frac{1}{4})), \\
\phi_{5/2}(u^\Delta(0)) = \frac{1}{5}\phi_{5/2}(u^\Delta(\frac{1}{8})), \qquad u(1) = \frac{1}{10}u(\frac{1}{2}),
\end{cases}
\tag{4.1}
$$

where

$$
\begin{aligned}
&f(t,u,v) = \begin{cases} \frac{1}{16}t^2 + 60(\frac{u}{2})^{11} + \frac{1}{2}(\frac{v}{100})^4, & u < 2, \\ \frac{1}{16}t^2 + 60 + \frac{1}{2}(\frac{v}{100})^4, & u \ge 2, \end{cases} \\
&I(u) = \begin{cases} \frac{1}{16}u, & 0 \le u \le 1, \\ \frac{1}{16}, & u > 1, \end{cases} \\
&\bar{I}(u,v) = \frac{1}{100}u + \frac{1}{200}v, \quad u \ge 0, v \le 0.
\end{aligned}
$$

Here $q(t) \equiv 1$, $p = \frac{5}{2}$, $n = 1$, $m = 3$, $t_1 = \frac{1}{4}$, $\alpha_1 = \frac{1}{5}$, $\beta_1 = \frac{1}{10}$, $\xi_1 = \frac{1}{8}$, $\eta_1 = \frac{1}{2}$.

Choose $l = 5$, $r_1 = 1$, $b_1 = 2$, $r_2 = 49$, $L_1 = 1$, $L_2 = 100$.

Then we have

$$
H = 3.15779, \qquad L = 1.84201, \qquad N = \left(\frac{3}{5}\right)^{2/3}.
$$

It is easy to verify that (A1)-(A4) hold and $f(t,u,v)$ satisfies

$$\max\{f(t,u,v),\bar{I}_1(u,v)\} \leq 0.015 < \min\left\{\phi_{5/2}\left(\frac{r_1}{H}\right),\phi_{5/2}\left(\frac{L_1}{L}\right)\right\} \approx 0.178207;$$

$$I_1(u) \leq 0.0625 < \frac{r_1}{H} = 0.316677 \quad \text{for } (t,u,v) \in [0,1]_{\mathbb{T}} \times [0,1] \times [0,1];$$

$$f(t,u,v) > 60 > \phi_{5/2}\left(\frac{lb_1}{N}\right) = 52.705 \quad \text{for } (t,u,v) \in \left[\frac{1}{5},\frac{4}{5}\right]_{\mathbb{T}} \times [2,20] \times [0,100];$$

$$\max\{f(t,u,v),\bar{I}_1(u,v)\} < 60.5625 < \min\left\{\phi_{5/2}\left(\frac{r_2}{H}\right),\phi_{5/2}\left(\frac{L_2}{L}\right)\right\} \approx 61.125;$$

$$I(u) \leq \frac{1}{16} < \frac{r_2}{H} = 15.517 \quad \text{for } (t,u,v) \in [0,1]_{\mathbb{T}} \times [0,49] \times [0,100].$$

Thus, all the conditions of Theorem 3.1 hold. By Theorem 3.1, problem (4.1) has at least three positive solutions u_1, u_2, and u_3 such that

$$\max_{t\in[0,1]_{\mathbb{T}}} u_1(t) < 1, \qquad \max_{t\in[0,1]_{\mathbb{T}}} \left|u_1^{\Delta}(t)\right| < 1;$$

$$2 < \min_{t\in[\frac{1}{5},\frac{4}{5}]_{\mathbb{T}}} u_2(t) \leq \max_{t\in[0,1]_{\mathbb{T}}} u_2(t) \leq 49, \qquad \max_{t\in[0,1]_{\mathbb{T}}} \left|u_2^{\Delta}(t)\right| \leq 100;$$

$$1 < \max_{t\in[0,1]_{\mathbb{T}}} u_3(t) \leq 49, \qquad \min_{t\in[\frac{1}{5},\frac{4}{5}]_{\mathbb{T}}} u_3(t) < 2, \qquad \max_{t\in[0,1]_{\mathbb{T}}} \left|u_3^{\Delta}\right| \leq 100.$$

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
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