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A simultaneous iterative method for split equality problems of two finite families of strictly pseudononspreading mappings without prior knowledge of operator norms

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Abstract

In this article, we first introduce the concept of T-mapping of a finite family of strictly pseudononspreading mapping $\{T_i\}_{i=1}^N$, and we show that the fixed point set of the T-mapping is the set of common fixed points of $\{T_i\}_{i=1}^N$ and T is a quasi-nonexpansive mapping. Based on the concept of a T-mapping, we propose a simultaneous iterative algorithm to solve the split equality problem with a way of selecting the stepsizes which does not need any prior information about the operator norms. The sequences generated by the algorithm weakly converge to a solution of the split equality problem of two finite families of strictly pseudononspreading mappings. Furthermore, we apply our iterative algorithms to some convex and nonlinear problems.

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1 Introduction

Due to their extraordinary utility and broad applicability in many areas of applied mathematics (most notably, fully discretized models of problems in image reconstruction from projections, in image processing, and in intensity-modulated radiation therapy), algorithms for solving convex feasibility problems continue to receive great attention; see for instance [1–11]. Recently, Moudafi [12] introduced a new convex feasibility problem (CFP). Let H_1, H_2, H_3 be real Hilbert spaces, let $C \subset H_1$, $Q \subset H_2$ be two nonempty closed convex sets, let $A: H_1 \to H_3$, $B: H_2 \to H_3$ be two bounded linear operators. The convex feasibility problem in [12] is to find

$$x \in C, y \in Q \text{ such that } Ax = By, \tag{1.1}$$

which allows asymmetric and partial relations between the variables x and y. The interest is to cover many situations, for instance in decomposition methods for PDEs, applications in game theory and in intensity-modulated radiation therapy (IMRT). In decision sciences, this allows one to consider agents who interplay only via some components of their decision variables, for further details, the interested reader is referred to [13]. In IMRT, this amounts to envisage a weak coupling between the vector of doses absorbed in all voxels and that of the radiation intensity, for further details, the interested reader is referred to [13, 14].

For solving the CFP (1.1), Moudafi [12] studied the fixed point formulation of the solutions of the CFP (1.1). Assume that the CFP (1.1) is consistent (*i.e.*, (1.1) has a solution), if (x, y) solves (1.1), then it solves the following fixed point equation system:

$$\begin{cases} x = P_C(x - \gamma A^*(Ax - By)), \\ y = P_Q(y + \beta B^*(Ax - By)), \end{cases} \tag{1.2}$$

where $\gamma, \beta > 0$ are any positive constants, and then Moudafi introduced the following alternating CQ algorithm:

$$\begin{cases} x_{k+1} = P_C(x_k - \gamma_k A^*(Ax_k - By_k)), \\ y_{k+1} = P_Q(y_k + \beta_k B^*(Ax_{k+1} - By_k)), \end{cases} \tag{1.3}$$

where $\gamma_k, \beta_k \in (\varepsilon, \min(\frac{1}{\lambda_A}, \frac{1}{\lambda_B}) - \varepsilon)$, λ_A and λ_B are the spectral radii of A^*A and B^*B, respectively. The weak convergence of the sequence (x_k, y_k) to a solution of (1.1) under some conditions was proved.

In [15], Moudafi and Al-Shemas considered the following problem:

$$x \in F(U), y \in F(T) \text{ such that } Ax = By, \tag{1.4}$$

and proposed the following simultaneous algorithm:

$$\begin{cases} x_{k+1} = U(x_k - \gamma_k A^*(Ax_k - By_k)), \\ y_{k+1} = T(y_k + \gamma_k B^*(Ax_k - By_k)), \end{cases} \tag{1.5}$$

for firmly quasi-nonexpansive operators U and T, where $\gamma_k \in (\varepsilon, \frac{2}{\lambda_A + \lambda_B}, -\varepsilon)$, λ_A and λ_B are the spectral radiuses of A^*A and B^*B, respectively.

Observe that in the algorithms (1.3) and (1.5) mentioned above, the determination of the stepsize $\{\gamma_k\}$ depends on the operator (matrix) norms $\|A\|$ and $\|B\|$ (or the largest eigenvalues of A^*A and B^*B). To implement the alternating algorithm (1.3) and the simultaneous algorithm (1.5), one has first to compute (or, at least, estimate) operator norms of A and B, which is in general not easy in practice.

To overcome this difficulty, Lopez *et al.* [16] and Zhao *et al.* [17] presented useful method for choosing the stepsizes which do not need prior knowledge of the operator norms for solving the split feasibility problems and multiple-set split feasibility problems, respectively.

Motivated by above results, we introduce a new choice of the stepsize sequence $\{\gamma_k\}$ for the simultaneous iterative algorithm to solve (1.4) governed by quasi-nonexpansive mapping as follows:

$$\gamma_k \in \left(0, \min\left\{\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2}, \frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2}\right\}\right). \tag{1.6}$$

The advantage of our choice (1.6) of the stepsizes lies in the fact that no prior information about the operator norms of A and B is required, and still convergence is guaranteed.

In this article, we propose the following simultaneous iterative algorithm where the stepsizes do not depend on the operator norms $\|A\|$ and $\|B\|$ and prove the weak convergence of the algorithm to solve (1.4). Let $U : H_1 \to H_1$ and $T : H_2 \to H_2$ be two quasi-nonexpansive mappings which are defined by (2.5). We denote by Γ be the set of solutions of (1.4), *i.e.*,

$$\Gamma = \bigl\{x \in F(U), y \in F(T) \text{ such that } Ax = By\bigr\}.$$

Algorithm 1.1 Let $x_0 \in H_1$, $y_0 \in H_2$ be arbitrary and $\{a_k\}$ be real number sequences in $[a, b] \subset (0, 1)$. Assume that the kth iterate $x_k \in H_1$, $y_k \in H_2$ has been constructed and $Ax_k - By_k \neq 0$, then we calculate $(k + 1)$th iterate (x_{k+1}, y_{k+1}) via the formula

$$\begin{cases} u_k = x_k - \gamma_k A^*(Ax_k - By_k), \\ x_{k+1} = a_k x_k + (1 - a_k)U(u_k), \\ v_k = y_k + \gamma_k B^*(Ax_k - By_k), \\ y_{k+1} = a_k y_k + (1 - a_k)T(v_k), \end{cases} \tag{1.7}$$

where the stepsize γ_k is chosen by (1.6). If $Ax_k - By_k = 0$, then $(x_k, y_k) = (x_{k+1}, y_{k+1})$ is a solution of the problem (1.4) and the iterative process stops. Otherwise, we set $k := k + 1$ and go on to (1.7) to evaluate the next iterate (x_{k+2}, y_{k+2}).

Remark 1.1 Notice that in (1.6) the choice of the stepsize γ_k is independent of the norms $\|A\|$ and $\|B\|$.

2 Preliminaries

Throughout this paper, we denote by H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and induced norm $\|\cdot\|$, and denote by C be a nonempty closed convex subset of H. Let $T : H \to H$ be a mapping. A point $x \in H$ is said to be a fixed point of T provided $x = Tx$. we use $F(T)$ to denote the fixed point set. We write $x_n \rightharpoonup x$ to indicate that the sequence $\{x_n\}$ converges weakly to x, $x_n \to x$ implies that $\{x_n\}$ converges strongly to x. We use $\omega_w(x_k) = \{x : \exists x_{k_j} \rightharpoonup x\}$ to stand for the weak ω-limit set of $\{x_k\}$. For any $x \in H$, there exists a unique nearest point in C, denoted by $P_C x$, such that

$$\|x - P_C x\| \le \|x - y\|, \quad \forall y \in C.$$

Before proceeding, we need to introduce a few concepts.

A mapping $T : C \to C$ belongs to the set Φ_q of quasi-nonexpansive, if

$$\|Tx - q\| \le \|x - q\|, \quad \forall (x, q) \in C \times F(T). \tag{2.1}$$

A mapping $T : C \to C$ belongs to the set Φ_n of nonexpansive, if

$$\|Tx - Ty\| \le \|x - y\|, \quad \forall (x, y) \in C \times C. \tag{2.2}$$

A mapping $T : C \to C$ belongs to the set Φ_f of firmly nonexpansive, if

$$\|Tx - Ty\|^2 \le \|x - y\|^2 - \bigl\|(x - y) - (Tx - Ty)\bigr\|^2, \quad \forall (x, y) \in C \times C. \tag{2.3}$$

A mapping $T : C \to C$ belongs to the set Φ_{fq} of firmly quasi-nonexpansive, if

$$\|Tx - q\| \leq \|x - q\| - \|x - Tx\|^2, \quad \forall (x, q) \in C \times F(T). \tag{2.4}$$

A mapping $T : C \to C$ is called nonspreading, if

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + 2\langle x - Tx, y - Ty\rangle, \quad \forall x, y \in K.$$

A mapping $T : C \to C$ is called k-strictly pseudononspreading if there exists $k \in [0, 1)$ such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + k\big\|x - Tx - (y - Ty)\big\|^2 + 2\langle x - Tx, y - Ty\rangle, \quad \forall x, y \in C.$$

Remark 2.1 It is easy to see that $\Phi_f \subset \Phi_n \subset \Phi_q$ and $\Phi_f \subset \Phi_{fq} \subset \Phi_q$. Furthermore, Φ_f is well known to contain resolvents and projection operators, and Φ_{fq} includes subgradient projection operators [18]. T is a nonspreading mapping if and only if T is a 0-strictly pseudononspreading mapping.

The so-called demiclosedness principle plays an important role in our argument.

A mapping $T : H \to H$ is called demiclosed at the origin if for any sequence $\{x_n\}$ which weakly converges to x, and if the sequence $\{x_n\}$ strongly converges to 0, then $Tx = 0$.

To establish our results, we need the following technical lemmas.

Lemma 2.1 ([19]) *If $x, y, z \in H$, then*:

(a) $\|x + y\|^2 \leq \|x\|^2 + 2\langle y, x + y\rangle$.

(b) *For any* $\lambda \in [0, 1]$,

$$\big\|\lambda x + (1 - \lambda)y\big\|^2 = \lambda\|x\|^2 + (1 - \lambda)\|y\|^2 - \lambda(1 - \lambda)\|x - y\|^2.$$

(c) *For* $a, b, c \in [0, 1]$ *with* $a + b + c = 1$,

$$\|ax + by + cz\|^2 = a\|x\|^2 + b\|y\|^2 + c\|z\|^2 - ab\|x - y\|^2 - ac\|x - z\|^2 - bc\|y - z\|^2.$$

The following definition will be useful for our results.

In 2009, Kangtunyakarn and Suantai [20] introduced T-mapping generated by $T_1, T_2, \ldots, T_N$ and $\lambda_1, \lambda_2, \ldots, \lambda_N$ as follows.

Definition 2.1 Let C be a nonempty convex subset of real Banach space. Let $\{T_i\}_{i=1}^N$ be a finite family of mappings of C into itself, and let $\lambda_1, \lambda_2, \ldots, \lambda_N$ be real numbers such that $0 < \lambda_i < 1$ for every $1, 2, \ldots, N$. We define a mapping $T : C \to C$ as follows:

$$\begin{cases} U_1 = \lambda_1 T_1 + (1 - \lambda_1)I, \\ U_2 = \lambda_2 T_2 U_1 + (1 - \lambda_2)U_1, \\ U_3 = \lambda_3 T_3 U_2 + (1 - \lambda_3)U_2, \\ \ldots, \\ U_{N-1} = \lambda_{N-1} T_{N-1} U_{N-2} + (1 - \lambda_{N-1})U_{N-2}, \\ T = U_N = \lambda_N T_N U_{N-1} + (1 - \lambda_N)U_{N-1}. \end{cases} \tag{2.5}$$

Such a mapping T is called the T-mapping generated by $T_1, T_2, \ldots, T_N$ and $\lambda_1, \lambda_2, \ldots, \lambda_N$.

Using the above definition, we have the following important lemma.

Lemma 2.2 *Let C be a nonempty convex subset of real Banach space. Let $\{T_i\}_{i=1}^N$ be a finite family of ρ_i-strictly pseudononspreading mappings of C into itself with $\bigcap_{i=1}^N F(T_i) \neq \emptyset$, and let $\lambda_1, \lambda_2, \ldots, \lambda_N$ be real numbers such that $0 < \lambda_i + \rho_i < 1$ for every $1, 2, \ldots, N$. If T is the T-mapping generated by $T_1, T_2, \ldots, T_N$ and $\lambda_1, \lambda_2, \ldots, \lambda_N$, then $F(T) = \bigcap_{i=1}^N F(T_i)$ and T is a quasi-nonexpansive mapping.*

Proof It is easy to deduce that $\bigcap_{i=1}^N F(T_i) \subset F(T)$. Next, we claim that $F(T) \subset \bigcap_{i=1}^N F(T_i)$. Let $x_0 \in F(T)$ and $x^* \in \bigcap_{i=1}^N F(T_i)$. Assume that $U_0 = I$, for $i = 1, 2, \ldots, N$, it follows from $\{T_i\}_{i=1}^N$ being a finite family of ρ_i-strictly pseudononspreading mappings of C into itself that

$$\begin{aligned}
&\langle U_{i-1}x_0 - T_iU_{i-1}x_0, U_{i-1}x_0 - x^*\rangle \\
&\quad = \frac{1}{2}\|U_{i-1}x_0 - T_iU_{i-1}x_0\|^2 + \frac{1}{2}\|U_{i-1}x_0 - x^*\|^2 - \frac{1}{2}\|T_iU_{i-1}x_0 - x^*\|^2 \\
&\quad = \frac{1-\rho_i}{2}\|U_{i-1}x_0 - T_iU_{i-1}x_0\|^2 - \frac{1}{2}\|T_iU_{i-1}x_0 - x^*\|^2 \\
&\qquad + \frac{1}{2}\bigl(\|U_{i-1}x_0 - x^*\|^2 + \rho_i\|U_{i-1}x_0 - T_iU_{i-1}x_0\|^2\bigr) \\
&\quad \geq \frac{1-\rho_i}{2}\|U_{i-1}x_0 - T_iU_{i-1}x_0\|^2.
\end{aligned} \tag{2.6}$$

From the definition of T and (2.6), we have

$$\begin{aligned}
\|x_0 - x^*\|^2 &= \|Tx_0 - x^*\|^2 \\
&= \|\lambda_N T_N U_{N-1}x_0 + (1-\lambda_N)U_{N-1}x_0 - x^*\|^2 \\
&= \|\lambda_N(T_N U_{N-1}x_0 - x^*) + (1-\lambda_N)(U_{N-1}x_0 - x^*)\|^2 \\
&= \lambda_N^2\|T_N U_{N-1}x_0 - x^*\|^2 + (1-\lambda_N)^2\|U_{N-1}x_0 - x^*\|^2 \\
&\quad + 2\lambda_N(1-\lambda_N)\langle T_N U_{N-1}x_0 - x^*, U_{N-1}x_0 - x^*\rangle \\
&= \lambda_N^2\|T_N U_{N-1}x_0 - x^*\|^2 + (1-\lambda_N)^2\|U_{N-1}x_0 - x^*\|^2 \\
&\quad + 2\lambda_N(1-\lambda_N)\langle T_N U_{N-1}x_0 - U_{N-1}x_0, U_{N-1}x_0 - x^*\rangle \\
&\quad + 2\lambda_N(1-\lambda_N)\|U_{N-1}x_0 - x^*\|^2 \\
&\leq \|U_{N-1}x_0 - x^*\|^2 - \lambda_N\bigl[1 - (\rho_N + \lambda_N)\bigr]\|U_{N-1}x_0 - T_N U_{N-1}x_0\|^2 \\
&\leq \|U_{N-1}x_0 - x^*\|^2 \\
&\quad \cdots \\
&\leq \|U_2x_0 - x^*\|^2 - \lambda_3\bigl[1 - (\rho_3 + \lambda_3)\bigr]\|U_2x_0 - T_3U_2x_0\|^2 \\
&\leq \|U_2x_0 - x^*\|^2 \\
&\leq \|U_1x_0 - x^*\|^2 - \lambda_2\bigl[1 - (\rho_2 + \lambda_2)\bigr]\|U_1x_0 - T_2U_1x_0\|^2 \\
&\leq \|U_1x_0 - x^*\|^2 \\
&\leq \|x_0 - x^*\|^2 - \lambda_1\bigl[1 - (\rho_1 + \lambda_1)\bigr]\|x_0 - T_1x_0\|^2 \\
&\leq \|x_0 - x^*\|^2,
\end{aligned} \tag{2.7}$$

which means $\|x_0 - T_1x_0\| = 0$, that is, $x_0 \in F(T_1)$. Furthermore,

$$U_1x_0 = \lambda_1 T_1x_0 + (1 - \lambda_1)x_0 = x_0,$$

it yields $x_0 \in F(U_1)$. Applying the same argument, we can conclude that $x_0 \in F(T_i)$ and $x_0 \in F(U_i)$, for $i = 1, 2, \dots, N - 1$.

Next, we claim that $x_0 \in F(T_N)$. Indeed,

$$\begin{aligned} 0 &= Tx_0 - x_0 \\ &= \lambda_N T_N U_{N-1}x_0 + (1 - \lambda_N)U_{N-1}x_0 - x_0 \\ &= \lambda_N(T_Nx_0 - x_0). \end{aligned}$$

It follows that $x_0 \in F(T_N)$. Therefore, $x_0 \in \bigcap_{i=1}^N F(T_i)$, that is, $F(T) \subset \bigcap_{i=1}^N F(T_i)$. Hence, $F(T) = \bigcap_{i=1}^N F(T_i)$. From the definition of T and (2.7), we find that T is a quasi-nonexpansive mapping. □

Proposition 2.1 *Let C be a closed convex subset of a real Hilbert space H. If T is a quasi-nonexpansive mapping from C into itself, then $F(T)$ is closed and convex.*

Proof Obviously, the continuity of T implies that $F(T)$ is closed. Now, we show that $F(T)$ is convex. For $x, y \in F(T)$ and $t \in (0, 1)$, put $z = tx + (1 - t)y$. Now, we claim that $z \in F(T)$. In fact,

$$\begin{aligned} \|z - Tz\|^2 &= \|z\|^2 - 2\langle z, Tz\rangle + \|Tz\|^2 \\ &= \|z\|^2 - 2\big\langle tx + (1 - t)y, Tz\big\rangle + \|Tz\|^2 \\ &= \|z\|^2 - 2t\langle x, Tz\rangle - 2(1 - t)\langle y, Tz\rangle + \|Tz\|^2 \\ &= \|z\|^2 + t\|x - Tz\|^2 + (1 - t)\|y - Tz\|^2 - t\|x\|^2 - (1 - t)\|y\|^2 \\ &\le \|z\|^2 + t\|x - z\|^2 + (1 - t)\|y - z\|^2 - t\|x\|^2 - (1 - t)\|y\|^2 \\ &= \big\|tx + (1 - t)y\big\|^2 + t\|x - z\|^2 + (1 - t)\|y - z\|^2 - t\|x\|^2 - (1 - t)\|y\|^2 \\ &= 0, \end{aligned}$$

which means that $\|z - Tz\| = 0$. Hence, $z \in F(T)$ and $F(T)$ is convex. □

3 Main results

Now, we are in a position to prove our convergence results in this section.

Theorem 3.1 *Let H_1, H_2, H_3 be real Hilbert spaces. Given two bounded linear operators $A : H_1 \to H_3$, $B : H_2 \to H_3$. Let $\{T_i\}_{i=1}^N$ be a finite family of ρ_i-strictly pseudononspreading mappings of C into itself with $\bigcap_{i=1}^N F(T_i) \neq \emptyset$, and Let $\{S_i\}_{i=1}^N$ be a finite family of τ_i-strictly pseudononspreading mappings of Q into itself with $\bigcap_{i=1}^N F(S_i) \neq \emptyset$. Suppose that U is defined by* (2.5) *which is generated by $T_1, T_2, \dots, T_N$ and $\lambda_1, \lambda_2, \dots, \lambda_N$, with $0 < \lambda_i + \rho_i < 1$ for every $1, 2, \dots, N$, and suppose that T is defined by* (2.5) *which is generated by $S_1, S_2, \dots, S_N$ and $\beta_1, \beta_2, \dots, \beta_N$, with $0 < \beta_i + \tau_i < 1$ for every $1, 2, \dots, N$, respectively. Assume that $U - I$ and*

$T-I$ are demiclosed at the origin. If the solution set Γ of (1.4) *is nonempty and for small enough $\varepsilon > 0$ and $\sigma > 0$,*

$$\gamma_k \in \left(\varepsilon, (1-\sigma)\min\left\{\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2}, \frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2}\right\}\right),$$

then the sequence $\{(x_k, y_k)\}$ generated by Algorithm 1.1 *weakly converges to a solution (x^*, y^*) of* (1.4). *Moreover, $\|Ax_k - By_k\| \to 0$, $\|x_k - x_{k+1}\| \to 0$, and $\|y_k - y_{k+1}\| \to 0$ as $k \to \infty$.*

Proof It follows from the condition on $\{\gamma_k\}$ that

$$\inf_{Ax_k \neq By_k}\left\{\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2} - \gamma_k\right\} > 0 \tag{3.1}$$

and

$$\inf_{Ax_k \neq By_k}\left\{\frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2} - \gamma_k\right\} > 0. \tag{3.2}$$

On the other hand, from

$$\left\|A^*(Ax_k - By_k)\right\|^2 \le \left\|A^*\right\|^2 \|Ax_k - By_k\|^2$$

and

$$\left\|B^*(Ax_k - By_k)\right\|^2 \le \left\|B^*\right\|^2 \|Ax_k - By_k\|^2,$$

we obtain $\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2} \ge \frac{1}{\|A^*\|^2}$ and $\frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2} \ge \frac{1}{\|B^*\|^2}$. Furthermore,

$$\min\left\{\frac{1}{\|A^*\|^2}, \frac{1}{\|B^*\|^2}\right\} \le \min\left\{\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2}, \frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2}\right\}.$$

Inequalities (3.1) and (3.2) lead to $\sup_{Ax_k \neq By_k} \gamma_k < +\infty$ and $\{\gamma_k\}$ is bounded.

For $(x, y) \in \Gamma$, by Algorithm 1.1, we obtain

$$\begin{aligned}\|u_k - x\|^2 &= \left\|x_k - \gamma_k A^*(Ax_k - By_k) - x\right\|^2 \\ &= \|x_k - x\|^2 - 2\gamma_k\left\langle x_k - x, A^*(Ax_k - By_k)\right\rangle + \gamma_k^2 \left\|A^*(Ax_k - By_k)\right\|^2.\end{aligned} \tag{3.3}$$

Notice that

$$\begin{aligned}-2\left\langle x_k - x, A^*(Ax_k - By_k)\right\rangle &= -2\langle Ax_k - Ax, Ax_k - By_k\rangle \\ &= -\|Ax_k - Ax\|^2 - \|Ax_k - By_k\|^2 + \|By_k - Ax\|^2.\end{aligned} \tag{3.4}$$

Substituting (3.4) into (3.3), one has

$$\begin{aligned}\|u_k - x\|^2 &= \|x_k - x\|^2 - \gamma_k\|Ax_k - By_k\|^2 - \gamma_k\|Ax_k - Ax\|^2 \\ &\quad + \gamma_k\|By_k - Ax\|^2 + \gamma_k^2\left\|A^*(Ax_k - By_k)\right\|^2.\end{aligned} \tag{3.5}$$

Similarly, by Algorithm 1.1, we deduce

$$\begin{aligned}\|v_k - y\|^2 &= \|y_k - y\|^2 - \gamma_k\|Ax_k - By_k\|^2 - \gamma_k\|By_k - By\|^2 \\ &\quad + \gamma_k\|By - Ax_k\|^2 + \gamma^2\big\|B^*(Ax_{k+1} - By_k)\big\|^2. \end{aligned} \tag{3.6}$$

Furthermore, adding the two last inequalities, following from the fact $Ax = By$, we have

$$\begin{aligned}\|u_k - x\|^2 + \|v_k - y\|^2 &\leq \|x_k - x\|^2 + \|y_k - y\|^2 \\ &\quad - \gamma_k\big(\|Ax_k - By_k\|^2 - \gamma_k\big\|A^*(Ax_k - By_k)\big\|^2\big) \\ &\quad - \gamma_k\big(\|Ax_k - By_k\|^2 - \gamma_k\big\|B^*(Ax_k - By_k)\big\|^2\big). \end{aligned} \tag{3.7}$$

Next, we will estimate $\|x_{k+1} - x\|$ and $\|y_{k+1} - y\|$. It follows from U and T being two quasi-nonexpansive mappings that

$$\begin{aligned}\|x_{k+1} - x\|^2 &= \big\|a_k x_k + (1 - a_k)U(u_k) - x\big\|^2 \\ &= \big\|a_k(x_k - x) + (1 - a_k)\big(U(u_k) - x\big)\big\|^2 \\ &= a_k\|x_k - x\|^2 + (1 - a_k)\big\|U(u_k) - x\big\|^2 - a_k(1 - a_k)\big\|U(u_k) - x_k\big\|^2 \\ &\leq a_k\|x_k - x\|^2 + (1 - a_k)\|u_k - x\|^2 - a_k(1 - a_k)\big\|U(u_k) - x_k\big\|^2 \end{aligned} \tag{3.8}$$

and

$$\|y_{k+1} - y\|^2 \leq a_k\|y_k - y\|^2 + (1 - a_k)\|v_k - y\|^2 - a_k(1 - a_k)\big\|T(v_k) - y_k\big\|^2. \tag{3.9}$$

Thus, (3.8) and (3.9) lead to

$$\begin{aligned}\|x_{k+1} - x\|^2 + \|y_{k+1} - x\|^2 &\leq a_k\big(\|x_k - x\|^2 + \|y_k - y\|^2\big) \\ &\quad + (1 - a_k)\big(\|u_k - x\|^2 + \|v_k - y\|^2\big) \\ &\quad - a_k(1 - a_k)\big(\big\|U(u_k) - x_k\big\|^2 + \big\|T(v_k) - y_k\big\|^2\big). \end{aligned} \tag{3.10}$$

Furthermore, it follows from (3.7) that

$$\begin{aligned}\|x_{k+1} - x\|^2 + \|y_{k+1} - x\|^2 &\leq \|x_k - x\|^2 + \|y_k - y\|^2 \\ &\quad - (1 - a_k)\gamma_k\big(\|Ax_k - Ax\|^2 - \gamma_k\big\|A^*(Ax_k - Ax)\big\|^2\big) \\ &\quad - (1 - a_k)\gamma_k\big(\|Ax_k - By_k\|^2 - \gamma_k\big\|B^*(Ax_k - By_k)\big\|^2\big) \\ &\quad - a_k(1 - a_k)\big(\big\|U(u_k) - x_k\big\|^2 + \big\|T(v_k) - y_k\big\|^2\big). \end{aligned} \tag{3.11}$$

Now, setting $\rho_k(x,y) = \|x_k - x\|^2 + \|y_k - y\|^2$, one has

$$\begin{aligned}\rho_{k+1}(x,y) &\leq \rho_k(x,y) - (1 - a_k)\gamma_k\big(\|Ax_k - Ax\|^2 - \gamma_k\big\|A^*(Ax_k - Ax)\big\|^2\big) \\ &\quad - (1 - a_k)\gamma_k\big(\|Ax_k - By_k\|^2 - \gamma_k\big\|B^*(Ax_k - By_k)\big\|^2\big) \\ &\quad - a_k(1 - a_k)\big(\big\|U(u_k) - x_k\big\|^2 + \big\|T(v_k) - y_k\big\|^2\big). \end{aligned} \tag{3.12}$$

On the other hand, note that

$$\rho_k(x,y) = \|x_k - x\|^2 + \|y_k - y\|^2 \geq 0.$$

From the assumptions on $\{a_k\}$ and $\{\gamma_k\}$, we see that the sequence $\rho_k(x,y)$ being decreasing and lower bounded by 0, consequently, converges to some finite limit, that is, $\lim_{k\to\infty} \rho_k(x,y) = \rho(x,y)$, which means the sequences $\{x_n\}$ and $\{y_n\}$ are bounded. Thus, we have

$$\lim_{k\to\infty} \gamma_k\bigl(\|Ax_k - By_k\|^2 - \gamma_k\bigl\|A^*(Ax_k - By_k)\bigr\|^2\bigr) = 0, \tag{3.13}$$

$$\lim_{k\to\infty} \gamma_k\bigl(\|Ax_k - By_k\|^2 - \gamma_k\bigl\|B^*(Ax_k - By_k)\bigr\|^2\bigr) = 0 \tag{3.14}$$

and

$$\lim_{k\to\infty} \bigl\|U(u_k) - x_k\bigr\|^2 = \lim_{k\to\infty} \bigl\|T(v_k) - y_k\bigr\|^2 = 0. \tag{3.15}$$

Now, we show that $\lim_{k\to\infty} \|Ax_k - By_k\| = 0$. Indeed, as is shown below, we break up the proof by distinguishing two cases.

Case 1. Suppose that there exists k_0 such that $\frac{\|Ax_k-By_k\|^2}{\|A^*(Ax_k-By_k)\|^2} \geq \frac{\|Ax_k-By_k\|^2}{\|B^*(Ax_k-By_k)\|^2}$, for all $k \geq k_0$, we obtain $\gamma \in (\varepsilon, (1-\sigma)\frac{\|Ax_k-By_k\|^2}{\|B^*(Ax_k-By_k)\|^2})$. It yields

$$\begin{aligned}
&\gamma_k\bigl(\|Ax_k - By_k\|^2 - \gamma_k\bigl\|A^*(Ax_k - By_k)\bigr\|^2\bigr) \\
&\quad\geq \gamma_k\left(\|Ax_k - By_k\|^2 - (1-\sigma)\frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2}\bigl\|A^*(Ax_k - By_k)\bigr\|^2\right) \\
&\quad\geq \gamma_k\left(\|Ax_k - By_k\|^2 - (1-\sigma)\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2}\bigl\|A^*(Ax_k - By_k)\bigr\|^2\right) \\
&\quad\geq \sigma\varepsilon\|Ax_k - By_k\|^2.
\end{aligned}$$

Furthermore, (3.13) leads to

$$\lim_{k\to\infty} \|Ax_k - By_k\| = 0. \tag{3.16}$$

Since

$$\bigl\|A^*(Ax_k - By_k)\bigr\|^2 \leq \bigl\|A^*\bigr\|^2\|Ax_k - By_k\|^2$$

and

$$\bigl\|B^*(Ax_k - By_k)\bigr\|^2 \leq \bigl\|B^*\bigr\|^2\|Ax_k - By_k\|^2,$$

we deduce

$$\lim_{k\to\infty} \bigl\|A^*(Ax_k - By_k)\bigr\| = \lim_{k\to\infty} \bigl\|B^*(Ax_k - By_k)\bigr\| = 0.$$

Conversely, suppose that there exists k_1 such that $\frac{\|Ax_k-By_k\|^2}{\|A^*(Ax_k-By_k)\|^2} \leq \frac{\|Ax_k-By_k\|^2}{\|B^*(Ax_k-By_k)\|^2}$, for all $k \geq k_1$, following the above process, we obtain the results.

Case 2. Suppose that there does not exist k_0 such that

$$\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2} \geq \frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2}$$

or

$$\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2} \leq \frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2},$$

for all $k \geq k_0$. We can divide the sequence $\frac{\|Ax_k-By_k\|^2}{\|A^*(Ax_k-By_k)\|^2}$ into two sequences: one satisfies $\frac{\|Ax_k-By_k\|^2}{\|A^*(Ax_k-By_k)\|^2} \geq \frac{\|Ax_k-By_k\|^2}{\|B^*(Ax_k-By_k)\|^2}$, which is denoted by $\{\frac{\|Ax_{k_n}-By_{k_n}\|^2}{\|A^*(Ax_{k_n}-By_{k_n})\|^2}\}$ and the other sequence satisfies $\frac{\|Ax_k-By_k\|^2}{\|A^*(Ax_k-By_k)\|^2} < \frac{\|Ax_k-By_k\|^2}{\|B^*(Ax_k-By_k)\|^2}$, which is denoted by $\{\frac{\|Ax_{k_m}-By_{k_m}\|^2}{\|A^*(Ax_{k_m}-By_{k_m})\|^2}\}$. Following the process of Case 1, we show that the results hold for the subsequences with k_n and k_m. Thus, we obtain $\lim_{k\to\infty} \|Ax_k - By_k\| = 0$.

Let us prove that $\{x_k\}$ and $\{y_k\}$ are asymptotically regular. Indeed, since

$$\|u_k - x_k\| = \gamma_k \big\|A^*(Ax_k - By_k)\big\|,$$

one has

$$\lim_{k\to\infty} \|u_k - x_k\| = 0. \tag{3.17}$$

Consequently,

$$\lim_{k\to\infty} \|x_{k+1} - x_k\| = \lim_{k\to\infty} (1 - a_k)\big\|U(u_k) - x_k\big\| = 0,$$

which yields $\{x_k\}$ is asymptotically regular. Similarly, $\lim_{k\to\infty} \|v_k - y_k\| = 0$ and $\{y_k\}$ is asymptotically regular, too.

Next, we show that $\|u_k - U(u_k)\| \to 0$ and $\|v_k - T(v_k)\| \to 0$ as $k \to \infty$. Indeed, since

$$\big\|u_k - U(u_k)\big\| = \big\|u_k - x_k + x_k - U(u_k)\big\| \leq \|u_k - x_k\| + \big\|x_k - U(u_k)\big\|,$$

(3.15) and (3.17) mean that $\lim_{k\to\infty} \|u_k - U(u_k)\| = 0$. In the same way as above, we can also show that $\|v_k - T(v_k)\| \to 0$ as $k \to \infty$.

Taking $(x^*, y^*) \in \omega_w(x_k, y_k)$, from $\lim_{k\to\infty} \|u_k - x_k\| = 0$ and $\lim_{k\to\infty} \|v_k - y_k\| = 0$, we obtain $x \in \omega_w(x_k)$ and $y \in \omega_w(y_k)$. Combining with the demiclosednesses of $U - I$ and $T - I$ at 0, one has

$$\lim_{k\to\infty} \big\|U(u_k) - u_k\big\| = \lim_{k\to\infty} \big\|T(v_k) - v_k\big\| = 0,$$

which yields $Ux^* = x^*$ and $Ty^* = y^*$. Thus, $x^* \in F(U)$ and $y^* \in F(T)$. On the other hand, $Ax^* - By^* \in \omega_w(Ax_k - By_k)$ and lower semicontinuity of the norm imply that

$$\big\|Ax^* - By^*\big\| \leq \lim_{k\to\infty} \inf \|Ax_k - By_k\| = 0,$$

hence $(x^*, y^*) \in \Gamma$.

Finally, we will show the uniqueness of the weak cluster points of $\{x_k\}$ and $\{y_k\}$. Indeed, let $\overline{x}$, $\overline{y}$ be other weak cluster points of $\{x_k\}$ and $\{y_k\}$, respectively. From the definition of $\rho_k(x,y)$, we have

$$\begin{aligned}\rho_k(x^*,y^*) &= \|x_k - x^*\|^2 + \|y_k - y^*\|^2 \\ &= \|x_k - \overline{x}\|^2 + \|\overline{x} - x^*\|^2 + 2\langle x_k - \overline{x}, \overline{x} - x^*\rangle \\ &\quad + \|y_k - \overline{y}\|^2 + \|\overline{y} - y^*\|^2 + 2\langle y_k - \overline{y}, \overline{y} - y^*\rangle \\ &= \rho_k(\overline{x},\overline{y}) + \|\overline{x} - x^*\|^2 + \|\overline{y} - y^*\|^2 \\ &\quad + 2\langle x_k - \overline{x}, \overline{x} - x^*\rangle + 2\langle y_k - \overline{y}, \overline{y} - y^*\rangle. \end{aligned} \tag{3.18}$$

Without loss of generality, we may assume that $x_k \rightharpoonup \overline{x}$, $y_k \rightharpoonup \overline{x}$, and then

$$\rho(x^*,y^*) = \rho(\overline{x},\overline{y}) + \|\overline{x} - x^*\|^2 + \|\overline{y} - y^*\|^2. \tag{3.19}$$

Reversing the role of (x^*,y^*) and $(\overline{x},\overline{y})$, we obtain

$$\rho(\overline{x},\overline{y}) = \rho(x^*,y^*) + \|x^* - \overline{x}\|^2 + \|y^* - \overline{y}\|^2. \tag{3.20}$$

Equations (3.19) and (3.20) yield

$$\|x^* - \overline{x}\|^2 + \|y^* - \overline{y}\|^2 = 0,$$

which means $x^* = \overline{x}$ and $y^* = \overline{y}$. Hence, the sequence $\{(x_k,y_k)\}$ weakly converges to a solution of the problem (1.4), which completes the proof. □

The following conclusions can be obtained from Theorem 3.1 immediately.

Theorem 3.2 *Let H_1, H_2, H_3 be real Hilbert spaces. Given two bounded linear operators $A: H_1 \to H_3$, $B: H_2 \to H_3$. Let U be a ρ-strictly pseudononspreading mapping of C into itself and Let T be a τ-strictly pseudononspreading mapping of Q into itself. Assume that $U - I$ and $T - I$ are demiclosed at the origin. If the solution set Γ of* (1.4) *is nonempty and for small enough $\varepsilon > 0$ and $\sigma > 0$,*

$$\gamma_k \in \left(\varepsilon, (1-\sigma)\min\left\{\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2}, \frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2}\right\}\right),$$

then the sequence $\{(x_k,y_k)\}$ generated by Algorithm 1.1 *weakly converges to a solution (x^*,y^*) of* (1.4). *Moreover, $\|Ax_k - By_k\| \to 0$, $\|x_k - x_{k+1}\| \to 0$, and $\|y_k - y_{k+1}\| \to 0$ as $k \to \infty$.*

Theorem 3.3 *Let H_1, H_2, H_3 be real Hilbert spaces. Given two bounded linear operators $A: H_1 \to H_3$, $B: H_2 \to H_3$. Let U be a nonspreading mapping of C into itself and let T be a nonspreading mapping of Q into itself. Assume that $U - I$ and $T - I$ are demiclosed at the origin. If the solution set Γ of* (1.4) *is nonempty and for small enough $\varepsilon > 0$ and $\sigma > 0$,*

$$\gamma_k \in \left(\varepsilon, (1-\sigma)\min\left\{\frac{\|Ax_k - By_k\|^2}{\|A^*(Ax_k - By_k)\|^2}, \frac{\|Ax_k - By_k\|^2}{\|B^*(Ax_k - By_k)\|^2}\right\}\right),$$

then the sequence $\{(x_k, y_k)\}$ *generated by Algorithm* 1.1 *weakly converges to a solution* (x^*, y^*) *of* (1.4). *Moreover*, $\|Ax_k - By_k\| \to 0$, $\|x_k - x_{k+1}\| \to 0$, *and* $\|y_k - y_{k+1}\| \to 0$ *as* $k \to \infty$.

4 Applications

We now pay attention to applying our simultaneous iterative algorithms to some convex and nonlinear analysis notions; see, for example, [21].

4.1 Split feasibility problem

Let C and Q be nonempty closed convex subset of real Hilbert spaces H_1 and H_2, respectively. The split feasibility problem (SFP) is to find a point

$$x \in C \text{ such that } Ax \in Q, \tag{4.1}$$

where $A : H_1 \to H_2$ is a bounded linear operator. The SFP was first introduced by Censor and Elfving [22] for modeling inverse problems which arise from phase retrievals and in medical image reconstruction [23].

If $B = I$, $H_2 = H_3$, then Algorithm 1.1 becomes:

Algorithm 4.1

$$\begin{cases} u_k = x_k - \gamma_k A^*(Ax_k - y_k), \\ x_{k+1} = a_k x_k + (1 - a_k)U(u_k), \\ v_k = y_k + \gamma_k(Ax_k - y_k), \\ y_{k+1} = a_k y_k + (1 - a_k)T(v_k), \end{cases} \tag{4.2}$$

where the stepsize γ_k is chosen by (1.6). If $Ax_k = y_k$, then $(x_k, y_k) = (x_{k+1}, y_{k+1})$ is a solution of the problem (4.1) and the iterative process stops. Otherwise, we set $k := k + 1$ and go on to (4.1) to evaluate the next iterate (x_{k+2}, y_{k+2}).

Furthermore, if $U = P_C$ and $T = P_Q$, then we obtain the following simultaneous iterative algorithm for solving SFP (4.1).

Algorithm 4.2

$$\begin{cases} u_k = x_k - \gamma_k A^*(Ax_k - y_k), \\ x_{k+1} = a_k x_k + (1 - a_k)P_C(u_k), \\ v_k = y_k + \gamma_k(Ax_k - y_k), \\ y_{k+1} = a_k y_k + (1 - a_k)P_Q(v_k), \end{cases} \tag{4.3}$$

where the stepsize γ_k is chosen by (1.6). If $Ax_k = y_k$, then $(x_k, y_k) = (x_{k+1}, y_{k+1})$ is a solution of the problem (4.1) and the iterative process stops. Otherwise, we set $k := k + 1$ and go on to (4.3) to evaluate the next iterate (x_{k+2}, y_{k+2}).

4.2 Variational problems via resolvent mappings

Given a maximal monotone operator $M : H_1 \to 2^{H_1}$, it is well known that its associated resolvent mapping, $J_\mu^M = (I + \mu M)^{-1}$, is quasi-nonexpansive and $0 \in M(x) \Leftrightarrow x = J_\mu^M(x)$, which implies that zeroes of M are exactly fixed points of its resolvent mapping. If $U^k = J_\mu^M$

and $T^k = J_\nu^N$, where $N : H_2 \to 2^{H_2}$ is another maximal monotone operator, the problem under consideration is nothing but

$$\text{find } x^* \in M^{-1}(0), y^* \in N^{-1}(0) \text{ such that } Ax^* = By^*, \tag{4.4}$$

and the algorithm is applied to the following form.

Algorithm 4.3

$$\begin{cases} u_k = x_k - \gamma_k A^*(Ax_k - By_k), \\ x_{k+1} = a_k x_k + (1 - a_k) J_\mu^M u_k, \\ v_k = y_k + \beta_k B^*(Ax_k - By_k), \\ y_{k+1} = a_k y_k + (1 - a_k) J_\nu^N v_k, \end{cases} \tag{4.5}$$

where the stepsize γ_k is chosen by (1.6). If $Ax_k - By_k = 0$, then $(x_k, y_k) = (x_{k+1}, y_{k+1})$ is a solution of the problem (4.4) and the iterative process stops. Otherwise, we set $k := k + 1$ and go on to (4.5) to evaluate the next iterate (x_{k+2}, y_{k+2}).

Competing interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' contributions
The authors have contributed in this work on an equal basis. All authors read and approved the final manuscript.

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