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A note on the values of weighted q-Bernstein polynomials and weighted q-Genocchi numbers

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Abstract

The rapid development of q-calculus has led to the discovery of new generalizations of Bernstein polynomials and Genocchi polynomials involving q-integers. The present paper deals with weighted q-Bernstein polynomials (or called q-Bernstein polynomials with weight α) and weighted q-Genocchi numbers (or called q-Genocchi numbers with weight α and β). We apply the method of generating function and p-adic q-integral representation on $\mathbb{Z}_p$, which are exploited to derive further classes of Bernstein polynomials and q-Genocchi numbers and polynomials. To be more precise, we summarize our results as follows: we obtain some combinatorial relations between q-Genocchi numbers and polynomials with weight α and β. Furthermore, we derive an integral representation of weighted q-Bernstein polynomials of degree n based on $\mathbb{Z}_p$. Also we deduce a fermionic p-adic q-integral representation of products of weighted q-Bernstein polynomials of different degrees $n_1, n_2, \ldots$ on $\mathbb{Z}_p$ and show that it can be in terms of q-Genocchi numbers with weight α and β, which yields a deeper insight into the effectiveness of this type of generalizations. We derive a new generating function which possesses a number of interesting properties which we state in this paper.

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1 Introduction

The q-calculus theory is a novel theory that is based on finite difference re-scaling. First results in q-calculus belong to Euler, who discovered Euler's identities for q-exponential functions, and Gauss, who discovered q-binomial formula. The systematic development of q-calculus begins from FH Jackson who 1908 reintroduced the Euler-Jackson q-difference operator (Jackson, 1908). One of the important branches of q-calculus is q-special orthogonal polynomials. Also p-adic numbers were invented by Kurt Hensel around the end of the nineteenth century, and these two branches of number theory joined in the link of p-adic integral and developed. In spite of them being already one hundred years old, these special numbers and polynomials, for instance, q-Bernstein polynomials, q-Genocchi numbers and polynomials, *etc.*, are still today enveloped in an aura of mystery within the scientific community. The p-adic integral was used in mathematical physics, for instance, the functional equation of the q-zeta function, q-Stirling numbers and q-Mahler theory of integration with respect to the ring $\mathbb{Z}_p$ together with Iwasawa's p-adic L functions. During the last ten years, the q-Bernstein polynomials and q-Genocchi polynomials have attracted a lot of interest and have been studied from different points of view along with some generalizations and modifications by a number of researchers. By using the p-adic invariant q-integral on $\mathbb{Z}_p$, Kim [1] constructed p-adic Bernoulli numbers and polynomials with weight α. He also gave the identities on the q-integral representation of the product of several q-Bernstein polynomials and constructed a link between q-Bernoulli polynomials and q-umbral calculus (*cf.* [2, 3]). Our aim of this paper is also to show that a fermionic p-adic q-integral representation of products of weighted q-Bernstein polynomials of different degrees $n_1, n_2, \ldots$ on $\mathbb{Z}_p$ can be written in terms of q-Genocchi numbers with weight α and β.

Suppose that p is chosen as an odd prime number. Throughout this paper, we make use of the following notations: $\mathbb{Z}_p$ denotes the ring of p-adic rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers and $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$. The normalized p-adic absolute value is defined by $|p|_p = \frac{1}{p}$. When one mentions q-extension, q can be variously considered as an indeterminate, a complex number $q \in \mathbb{C}$, or a p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, we assume $|q| < 1$. If $q \in \mathbb{C}_p$, we assume $|q-1|_p < p^{-\frac{1}{p-1}}$.

Suppose $UD(\mathbb{Z}_p)$ is the space of uniformly differentiable functions on $\mathbb{Z}_p$. For $f \in UD(\mathbb{Z}_p)$, the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by Kim (see [4, 5]):

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(\xi)\, d\mu_{-q}(\xi) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{\xi=0}^{p^N-1} q^{\xi} f(\xi)(-1)^{\xi}. \tag{1.1}$$

For $\alpha, k, n \in \mathbb{N}^*$ and $x \in [0,1]$, Kim *et al.* defined weighted q-Bernstein polynomials as follows:

$$B_{k,n}^{(\alpha)}(x,q) = \binom{n}{k} [x]_{q^{\alpha}}^{k} [1-x]_{q^{-\alpha}}^{n-k} \quad \text{(see [6] and [7])}. \tag{1.2}$$

If we put $q \to 1$ and $\alpha = 1$ in Eq. (1.2), since $[x]_{q^{\alpha}}^{k} \to x^k$, $[1-x]_{q^{-\alpha}}^{n-k} \to (1-x)^{n-k}$, it turns out to be the classical Bernstein polynomials (see [8] and [9]).

The q-extension of x, $[x]_q$, is defined by

$$[x]_q = \frac{1-q^x}{1-q}.$$

Note that $\lim_{q\to 1}[x]_q = x$ (for more information, see [1–24]).

In [11], for $n \in \mathbb{N}^*$, modified q-Genocchi numbers with weight α and β are defined by Araci *et al.* as follows:

$$\begin{aligned} \frac{g_{n+1,q}^{(\alpha,\beta)}(x)}{n+1} &= \int_{\mathbb{Z}_p} q^{-\beta\xi} [x+\xi]_{q^{\alpha}}^{n}\, d\mu_{-q^{\beta}}(\xi) \\ &= \frac{[2]_{q^{\beta}}}{[\alpha]_q^n (1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha \ell x} \frac{1}{1+q^{\alpha\ell}} \\ &= [2]_{q^{\beta}} \sum_{m=0}^{\infty} (-1)^m [m+x]_{q^{\alpha}}^{n}. \end{aligned} \tag{1.3}$$

In the case, for $x = 0$, we have $g_{n,q}^{(\alpha,\beta)}(0) = g_{n,q}^{(\alpha,\beta)}$ that are called q-Genocchi numbers with weight α and β.

In [11], for $\alpha \in \mathbb{N}^*$ and $n \in \mathbb{N}$, q-Genocchi numbers with weight α and β are defined by Araci *et al.* as follows:

$$g_{0,q}^{(\alpha,\beta)} = 0, \quad \text{and} \quad g_{n,q}^{(\alpha,\beta)}(1) + g_{n,q}^{(\alpha,\beta)} = \begin{cases} [2]_{q^\beta} & \text{if } n = 1, \\ 0 & \text{if } n > 1. \end{cases} \tag{1.4}$$

In this paper, we obtain some relations between the weighted q-Bernstein polynomials and the modified q-Genocchi numbers with weight α and β. From these relations, we derive some interesting identities on the q-Genocchi numbers with weight α and β.

2 On the weighted q-Genocchi numbers and polynomials

In this part, we will give some arithmetical properties of weighted q-Genocchi polynomials by using the techniques of p-adic integral and the method of generating functions. Thus, by utilizing the definition of weighted q-Genocchi polynomials, we have

$$\begin{aligned}
\frac{g_{n+1,q}^{(\alpha,\beta)}(x)}{n+1} &= \int_{\mathbb{Z}_p} q^{-\beta\xi} [x+\xi]_{q^\alpha}^n \, d\mu_{-q^\beta}(\xi) \\
&= \int_{\mathbb{Z}_p} q^{-\beta\xi} \left([x]_{q^\alpha} + q^{\alpha x}[\xi]_{q^\alpha}\right)^n d\mu_{-q}(\xi) \\
&= \sum_{k=0}^n \binom{n}{k} [x]_{q^\alpha}^{n-k} q^{\alpha k x} \int_{\mathbb{Z}_p} q^{-\beta\xi} [\xi]_{q^\alpha}^k \, d\mu_{-q}(\xi) \\
&= \sum_{k=0}^n \binom{n}{k} [x]_{q^\alpha}^{n-k} q^{\alpha k x} \frac{g_{k+1,q}^{(\alpha,\beta)}}{k+1}.
\end{aligned}$$

Thus we state the following theorem.

Theorem 1 *Suppose $n, \alpha, \beta \in \mathbb{N}^*$. Then we have*

$$g_{n,q}^{(\alpha,\beta)}(x) = q^{-\alpha x} \sum_{k=0}^n \binom{n}{k} q^{\alpha k x} g_{k,q}^{(\alpha,\beta)} [x]_{q^\alpha}^{n-k}. \tag{2.1}$$

Moreover,

$$g_{n,q}^{(\alpha,\beta)}(x) = q^{-\alpha x} \left(q^{\alpha x} g_q^{(\alpha,\beta)} + [x]_{q^\alpha}\right)^n, \tag{2.2}$$

by using the umbral (*symbolic*) *convention* $(g_q^{(\alpha,\beta)})^n = g_{n,q}^{(\alpha,\beta)}$.

By expression of (1.3), we get

$$\begin{aligned}
\frac{g_{n+1,q^{-1}}^{(\alpha,\beta)}(1-x)}{n+1} &= \int_{\mathbb{Z}_p} q^{\beta\xi} [1-x+\xi]_{q^{-\alpha}}^n \, d\mu_{-q^{-\beta}}(\xi) \\
&= \frac{[2]_{q^{-\beta}}}{(1-q^{-\alpha})^n} \sum_{l=0}^n \binom{n}{l} (-1)^l q^{-\alpha\ell(1-x)} \frac{1}{1+q^{-\alpha\ell}}
\end{aligned}$$

$$= (-1)^n q^{\alpha n-\beta}\left(\frac{[2]_{q^\beta}}{(1-q^\alpha)^n}\sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha lx}\frac{1}{1+q^{\alpha l}}\right)$$

$$= (-1)^n q^{\alpha n-\beta}\frac{g_{n+1,q}^{(\alpha,\beta)}(x)}{n+1}.$$

Consequently, we obtain the following theorem.

Theorem 2 *The following*

$$g_{n+1,q^{-1}}^{(\alpha,\beta)}(1-x) = (-1)^n q^{\alpha n-\beta} g_{n+1,q}^{(\alpha,\beta)}(x) \tag{2.3}$$

is true.

From expression of (2.2) and Theorem 1, we get the following theorem.

Theorem 3 *The following identity holds*:

$$g_{0,q}^{(\alpha,\beta)} = 0, \quad \textit{and} \quad q^{-\alpha}\left(q^\alpha g_q^{(\alpha,\beta)}+1\right)^n + g_{n,q}^{(\alpha,\beta)} = \begin{cases} [2]_{q^\beta} & \textit{if } n=1, \\ 0 & \textit{if } n>1, \end{cases}$$

with the usual convention about replacing $(g_q^{(\alpha,\beta)})^n$ *by* $g_{n,q}^{(\alpha,\beta)}$.

For $n, \alpha \in \mathbb{N}$, by Theorem 3, we note that

$$\begin{aligned} q^{2\alpha} g_{n,q}^{(\alpha,\beta)}(2) &= \left(q^\alpha\left(q^\alpha g_q^{(\alpha,\beta)}+1\right)+1\right)^n \\ &= \sum_{k=0}^{n}\binom{n}{k} q^{k\alpha}\left(q^\alpha g_q^{(\alpha,\beta)}+1\right)^k \\ &= \left(q^\alpha g_q^{(\alpha,\beta)}+1\right)^0 + nq^\alpha\left(q^\alpha g_q^{(\alpha,\beta)}+1\right)^1 + \sum_{k=2}^{n}\binom{n}{k} q^{k\alpha}\left(q^\alpha g_q^{(\alpha,\beta)}+1\right)^k \\ &= nq^{2\alpha}[2]_{q^\beta} - q^\alpha \sum_{k=0}^{n}\binom{n}{k} q^{\alpha k} g_{k,q}^{(\alpha,\beta)} \\ &= nq^{2\alpha}[2]_{q^\beta} + q^\alpha g_{n,q}^{(\alpha,\beta)} \quad \text{if } n>1. \end{aligned}$$

Consequently, we state the following theorem.

Theorem 4 *Suppose* $n \in \mathbb{N}$. *Then we have*

$$g_{n,q}^{(\alpha,\beta)}(2) = n[2]_{q^\beta} + \frac{g_{n,q}^{(\alpha,\beta)}}{q^\alpha}.$$

From expression of Theorem 2 and (2.3), we easily see that

$$\begin{aligned} &(n+1)q^{-\beta}\int_{\mathbb{Z}_p} q^{-\beta\xi}[1-\xi]_{q^{-\alpha}}^n \, d\mu_{-q^\beta}(\xi) \\ &\quad = (-1)^n q^{n\alpha-\beta}\int_{\mathbb{Z}_p} q^{-\beta\xi}[\xi-1]_{q^\alpha}^n \, d\mu_{-q^\beta}(\xi) \\ &\quad = (-1)^n q^{n\alpha-\beta} g_{n+1,q}^{(\alpha,\beta)}(-1) = g_{n+1,q^{-1}}^{(\alpha,\beta)}(2). \end{aligned} \tag{2.4}$$

Thus, we obtain the following theorem.

Theorem 5 *The following identity*

$$(n+1)q^{-\beta}\int_{\mathbb{Z}_p} q^{-\beta\xi}[1-\xi]_{q^{-\alpha}}^{n}\,d\mu_{-q^{\beta}}(\xi) = g_{n+1,q^{-1}}^{(\alpha,\beta)}(2)$$

is true.

Suppose $n,\alpha \in \mathbb{N}$. By expression of Theorem 4 and Theorem 5, we get

$$\begin{aligned}&(n+1)q^{-\beta}\int_{\mathbb{Z}_p} q^{-\beta\xi}[1-\xi]_{q^{-\alpha}}^{n}\,d\mu_{-q^{\beta}}(\xi)\\&\quad= (n+1)q^{-\beta}[2]_{q^{\beta}} + q^{\alpha}g_{n+1,q^{-1}}^{(\alpha,\beta)}.\end{aligned}\tag{2.5}$$

For (2.5), we obtain the corollary as follows.

Corollary 1 *Suppose* $n,\alpha \in \mathbb{N}^*$. *Then we have*

$$\int_{\mathbb{Z}_p} q^{-\beta\xi}[1-\xi]_{q^{-\alpha}}^{n}\,d\mu_{-q^{\beta}}(\xi) = [2]_{q^{\beta}} + q^{\alpha-\beta}\frac{g_{n+1,q^{-1}}^{(\alpha,\beta)}}{n+1}.$$

3 Novel identities on the weighted q-Genocchi numbers

In this section, we develop modified q-Genocchi numbers with weight α and β, namely we derive interesting and worthwhile relations in analytic number theory.

For $x \in \mathbb{Z}_p$, the p-adic analogues of weighted q-Bernstein polynomials are given by

$$B_{k,n}^{(\alpha)}(x,q) = \binom{n}{k}[x]_{q^{\alpha}}^{k}[1-x]_{q^{-\alpha}}^{n-k}, \quad \text{where } n,k,\alpha \in \mathbb{N}^*.\tag{3.1}$$

By expression of (3.1), Kim *et al.* get the symmetry of q-Bernstein polynomials weight α as follows:

$$B_{k,n}^{(\alpha)}(x,q) = B_{n-k,n}^{(\alpha)}\bigl(1-x,q^{-1}\bigr) \quad \text{(for details, see [7])}.\tag{3.2}$$

Thus, from Corollary 1, (3.1) and (3.2), we see that

$$\begin{aligned}&\int_{\mathbb{Z}_p} B_{k,n}^{(\alpha)}(\xi,q)q^{-\beta\xi}\,d\mu_{-q^{\beta}}(\xi)\\&\quad= \int_{\mathbb{Z}_p} B_{n-k,n}^{(\alpha)}\bigl(1-\xi,q^{-1}\bigr)q^{-\beta\xi}\,d\mu_{-q^{\beta}}(\xi)\\&\quad= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\int_{\mathbb{Z}_p} q^{-\beta\xi}[1-\xi]_{q^{-\alpha}}^{n-l}\,d\mu_{-q^{\beta}}(\xi)\\&\quad= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k+l}\left([2]_{q^{\beta}} + q^{\alpha-\beta}\frac{g_{n-l+1,q^{-1}}^{(\alpha,\beta)}}{n-l+1}\right).\end{aligned}$$

For $n, k \in \mathbb{N}^*$ and $\alpha \in \mathbb{N}$ with $n > k$, we obtain

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n}^{(\alpha)}(\xi, q) q^{-\beta\xi} \, d\mu_{-q^\beta}(\xi) \\
&= \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} \left([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n-l+1,q^{-1}}^{(\alpha,\beta)}}{n-l+1} \right) \\
&= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n+1,q^{-1}}^{(\alpha,\beta)}}{n+1} & \text{if } k = 0, \\ \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n-l+1,q^{-1}}^{(\alpha,\beta)}}{n-l+1}) & \text{if } k > 0. \end{cases}
\end{aligned} \tag{3.3}$$

Let us take the fermionic p-adic q-integral on $\mathbb{Z}_p$ on the weighted q-Bernstein polynomials of degree n as follows:

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n}^{(\alpha)}(\xi, q) q^{-\beta\xi} \, d\mu_{-q^\beta}(\xi) \\
&= \binom{n}{k} \int_{\mathbb{Z}_p} q^{-\beta\xi} [\xi]_{q^\alpha}^{k} [1-\xi]_{q^{-\alpha}}^{n-k} \, d\mu_{-q^\beta}(\xi) \\
&= \binom{n}{k} \sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{g_{l+k+1,q}^{(\alpha,\beta)}}{l+k+1}.
\end{aligned} \tag{3.4}$$

Consequently, by expression of (3.3) and (3.4), we state the following theorem.

Theorem 6 *The following identity holds*:

$$\begin{aligned}
&\sum_{l=0}^{n-k} \binom{n-k}{l} (-1)^l \frac{g_{l+k+1,q}^{(\alpha,\beta)}}{l+k+1} \\
&= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n+1,q^{-1}}^{(\alpha,\beta)}}{n+1} & \textit{if } k = 0, \\ \sum_{l=0}^{k} \binom{k}{l} (-1)^{k+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n-l+1,q^{-1}}^{(\alpha,\beta)}}{n-l+1}) & \textit{if } k > 0. \end{cases}
\end{aligned}$$

Suppose $n_1, n_2, k \in \mathbb{N}^*$ and $\alpha \in \mathbb{N}$ with $n_1 + n_2 > 2k$. It yields

$$\begin{aligned}
&\int_{\mathbb{Z}_p} B_{k,n_1}^{(\alpha)}(\xi, q) B_{k,n_2}^{(\alpha)}(\xi, q) q^{-\beta\xi} \, d\mu_{-q^\beta}(\xi) \\
&= \binom{n_1}{k} \binom{n_2}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \int_{\mathbb{Z}_p} q^{-\beta\xi} [1-\xi]_{q^{-\alpha}}^{n_1+n_2-l} \, d\mu_{-q^\beta}(\xi) \\
&= \left(\binom{n_1}{k} \binom{n_2}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} \left([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2-l+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2-l+1} \right) \right) \\
&= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2+1} & \text{if } k = 0, \\ \binom{n_1}{k} \binom{n_2}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2-l+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2-l+1}) & \text{if } k \neq 0. \end{cases}
\end{aligned}$$

Therefore, we obtain the following theorem.

Theorem 7 *Suppose $n_1, n_2, k \in \mathbb{N}^*$ and $\alpha, \beta \in \mathbb{N}$ with $n_1 + n_2 > 2k$, then we have*

$$\int_{\mathbb{Z}_p} q^{-\beta\xi} B_{k,n_1}^{(\alpha)}(\xi, q) B_{k,n_2}^{(\alpha)}(\xi, q)\, d\mu_{-q^\beta}(\xi)$$
$$= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2+1} & \text{if } k = 0, \\ \binom{n_1}{k}\binom{n_2}{k} \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2-l+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2-l+1}) & \text{if } k \neq 0. \end{cases}$$

By using the binomial theorem, we can derive the following equation:

$$\begin{aligned} &\int_{\mathbb{Z}_p} B_{k,n_1}^{(\alpha)}(\xi, q) B_{k,n_2}^{(\alpha)}(\xi, q) q^{-\beta\xi}\, d\mu_{-q^\beta}(\xi) \\ &= \prod_{i=1}^{2} \binom{n_i}{k} \sum_{l=0}^{n_1+n_2-2k} \binom{n_1+n_2-2k}{l} (-1)^l \int_{\mathbb{Z}_p} [\xi]_{q^\alpha}^{2k+l} q^{-\beta\xi}\, d\mu_{-q^\beta}(\xi) \\ &= \prod_{i=1}^{2} \binom{n_i}{k} \sum_{l=0}^{n_1+n_2-2k} \binom{n_1+n_2-2k}{l} (-1)^l \frac{g_{l+2k+1,q}^{(\alpha,\beta)}}{l+2k+1}. \end{aligned} \tag{3.5}$$

Thus, we can obtain the following corollary.

Corollary 2 *Suppose $n_1, n_2, k \in \mathbb{N}^*$ and $\alpha \in \mathbb{N}$ with $n_1 + n_2 > 2k$. Then we have*

$$\sum_{l=0}^{n_1+n_2-2k} \binom{n_1+n_2-2k}{l} (-1)^l \frac{g_{l+2k+1,q}^{(\alpha,\beta)}}{l+2k+1}$$
$$= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2+1} & \text{if } k = 0, \\ \sum_{l=0}^{2k} \binom{2k}{l} (-1)^{2k+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2-l+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2-l+1}) & \text{if } k \neq 0. \end{cases}$$

For $\xi \in \mathbb{Z}_p$ and $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, \ldots, n_s, k \in \mathbb{N}^*$ and $\alpha \in \mathbb{N}$ with $\sum_{l=1}^{s} n_l > sk$. Then we take the fermionic p-adic q-integral on $\mathbb{Z}_p$ for the weighted q-Bernstein polynomials of degree n as follows:

$$\begin{aligned} &\int_{\mathbb{Z}_p} \underbrace{B_{k,n_1}^{(\alpha)}(\xi, q) B_{k,n_2}^{(\alpha)}(\xi, q) \cdots B_{k,n_s}^{(\alpha)}(\xi, q)}_{s\text{-times}} q^{-\beta\xi}\, d\mu_{-q^\beta}(\xi) \\ &= \prod_{i=1}^{s} \binom{n_i}{k} \int_{\mathbb{Z}_p} [\xi]_{q^\alpha}^{sk} [1-\xi]_{q^{-\alpha}}^{n_1+n_2+\cdots+n_s-sk} q^{-\beta\xi}\, d\mu_{-q^\beta}(\xi) \\ &= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{l+sk} \int_{\mathbb{Z}_p} q^{-\beta\xi} [1-\xi]_{q^{-\alpha}}^{n_1+n_2+\cdots+n_s-sk}\, d\mu_{-q^\beta}(\xi) \\ &= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2+\cdots+n_s+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2+\cdots+n_s+1} & \text{if } k = 0, \\ \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g_{n_1+n_2+\cdots+n_s-l+1,q^{-1}}^{(\alpha,\beta)}}{n_1+n_2+\cdots+n_s-l+1}) & \text{if } k \neq 0. \end{cases} \end{aligned}$$

So from above, we have the following theorem.

Theorem 8 *Suppose $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, \ldots, n_s, k \in \mathbb{N}^*$ and $\alpha \in \mathbb{N}$ with $\sum_{l=1}^{s} n_l > sk$. Then we have*

$$\int_{\mathbb{Z}_p} q^{-\beta\xi} \prod_{i=1}^{s} B_{k,n_i}^{(\alpha)}(\xi)\, d\mu_{-q}(\xi)$$
$$= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g^{(\alpha,\beta)}_{n_1+n_2+\cdots+n_s+1,q^{-1}}}{n_1+n_2+\cdots+n_s+1} & \text{if } k = 0, \\ \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g^{(\alpha,\beta)}_{n_1+n_2+\cdots+n_s-l+1,q^{-1}}}{n_1+n_2+\cdots+n_s-l+1}) & \text{if } k \neq 0. \end{cases}$$

From the definition of weighted q-Bernstein polynomials and the binomial theorem, we easily get

$$\int_{\mathbb{Z}_p} q^{-\beta\xi} \underbrace{B_{k,n_1}^{(\alpha)}(\xi,q) B_{k,n_2}^{(\alpha)}(\xi,q) \cdots B_{k,n_s}^{(\alpha)}(\xi,q)}_{s\text{-times}}\, d\mu_{-q^\beta}(\xi)$$
$$= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{n_1+\cdots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \int_{\mathbb{Z}_p} q^{-\beta\xi} [\xi]_{q^\alpha}^{sk+l}\, d\mu_{-q^\beta}(\xi)$$
$$= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{n_1+\cdots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \frac{g^{(\alpha,\beta)}_{l+sk+1,q}}{l+sk+1}. \tag{3.6}$$

Therefore, from (3.6) and Theorem 8, we get an interesting corollary as follows.

Corollary 3 *Suppose $s \in \mathbb{N}$ with $s \geq 2$, let $n_1, n_2, \ldots, n_s, k \in \mathbb{N}^*$ and $\alpha \in \mathbb{N}$ with $\sum_{l=1}^{s} n_l > sk$. Then we have*

$$\sum_{l=0}^{n_1+\cdots+n_s-sk} \binom{\sum_{d=1}^{s}(n_d-k)}{l} (-1)^l \frac{g^{(\alpha,\beta)}_{l+sk+1,q}}{l+sk+1}$$
$$= \begin{cases} [2]_{q^\beta} + q^{\alpha-\beta} \frac{g^{(\alpha,\beta)}_{n_1+n_2+\cdots+n_s+1,q^{-1}}}{n_1+n_2+\cdots+n_s+1} & \text{if } k = 0, \\ \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk+l} ([2]_{q^\beta} + q^{\alpha-\beta} \frac{g^{(\alpha,\beta)}_{n_1+n_2+\cdots+n_s-l+1,q^{-1}}}{n_1+n_2+\cdots+n_s-l+1}) & \text{if } k \neq 0. \end{cases}$$

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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