Farkas *et al.*
European Transport Research Review (2023) 15:42
https://doi.org/10.1186/s12544-023-00615-z

European Transport Research Review

ORIGINAL PAPER

Open Access

Speed control with low complexity for multiple autonomous vehicles in roundabouts

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Abstract

The paper introduces a high level speed control method for the coordination of multiple autonomous vehicles (AVs) in roundabout scenarios. The aim of the control method is to guarantee collision-free motion of the AVs, and similarly, to minimize their traveling time. In the method a priority-based ordering process of the AVs is used, which enforces the time-efficient motion of the AVs. The collision-free motion is guaranteed through an optimization-based method including control input constraints. The ordering process and the optimization form a low complexity solution, which requires low computation effort. The proposed control strategy is involved in the high level of a hierarchical control structure. The effectiveness of the proposed control strategy is illustrated by simulation examples and Hardware-in-the-Loop demonstration.

Keywords Roundabout scenarios, Autonomous vehicle control, Multiple vehicles

1 Introduction

The emerging technology of autonomous vehicles and their expected growing proportion in road traffic raises plenty of challenging tasks, mainly related to safety and the corresponding public acceptance. Advances in automated vehicle design enables researchers to establish novel control methods for the trajectory planning of such vehicles, ensuring enhanced safety and efficiency in traffic environments such as intersections, roundabouts, on-ramps, in which the cooperation among participants can have a significant effect on the traffic performance. Centralized control methods are commonly used for the scenario of connected AVs on-ramp merging, for example formulated as a bi-objective optimization problem solved with Pontryagin's minimum principle [1]. A comprehensive review is given of the existing ramp merging strategies leveraging connected AVs, focusing on the latest developments in the field [2]. Many control strategies have been developed to facilitate collision-free driving of autonomous vehicles in situations where AVs and human participants coexist in traffic. For example, a trajectory tracking control algorithm is based on the state estimation of vehicles in order to achieve the collision-free crossing of vehicles at roundabouts [3]. Several safety conditions are built in the designed methods for AVs to pass through the roundabouts conflict areas, like merging points. Control strategies have also been developed to guarantee safe navigation of AVs in one- and multi-lane roundabouts as well [4]. A vehicle-to-vehicle (V2V) communication and intersection control have been designed for AVs to prevent accidents in complex traffic situations, and the proposed intersection management have also been proved in roundabout scenarios. An optimal analytical solution has been designed for roundabout control in mixed traffic environment for connected autonomous

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vehicles (CAVs), also analyzing the impact of different penetration rates of CAVs [5].

The scope of studying AVs and CAVs in traffic environments is deeply connected to the behavior analysis of human driven vehicles as well, thus designing control methods which can provide safer traffic at intersection and roundabout crossings. Hence, in the control design of AVs at intersections and roundabouts, the results of driver behavior analysis should be also considered [6]. An adaptive tactical behavior planner has also been introduced for AVs, mixing human behavior and tactical decision-making [7]. Moreover, the specific risks related to crossing roundabouts have also been studied with the aim to adapt a behavior for AVs by which the comfort and safety can be improved [8].

Optimal control strategy has also been introduced in order to enhance energy efficiency and to reduce traveling time for AVs at roundabouts using collision avoidance constraints [9]. Reviews on the effectiveness of car-following models and their impact on the performance of traffic flow can be found in [52, 53]. Concept of using virtual platooning strategy for AVs at roundabouts has also been developed with the aim to handle complex traffic scenarios [10]. This strategy integrates a map-based approach with curvilinear coordinates framework in order to ensure safety among AVs and human operated vehicles. Based on the Dynamic Bayesian Network, a classification method has been designed to define the intentions of traffic participants at roundabouts [11]. A decentralized control strategy has also been introduced with virtual vehicles, with the aim to monitor the states and interactions between AVs at roundabouts and to realize an appropriate balance among predefined performances [12].

Another state-of-the-art approach for the safe handling of AVs at roundabouts is using artificial intelligence (AI) and machine learning techniques. For example support vector machine, linear regression and deep learning algorithms have been evaluated and studied for the estimation of vehicle speed and steering angle at various types of roundabouts for human drivers, and rules of action to be applied have been designed for AVs to execute maneuvers at roundabouts [13]. Model-free reinforcement learning algorithms containing safety conditions have been also designed for AVs handling various traffic scenarios including roundabout crossings [14, 15]. Also, optimization embedded reinforcement learning has been introduced to coordinate multiple AVs at roundabouts [16], where the control strategy analyzes the behaviors of AVs for the evaluation and comparison of their efficiency. Methods for motion estimation of AVs are also applied combining dynamic Bayesian network and sequential neural network models [17]. Moreover, adversarial multi-agent reinforcement learning method is applied to coordinate the crossing of roundabouts by AVs considering behaviors, human-driving baseline [18]. This method improves the performances like traveling time and average speed of the vehicles. A fuzzy-behavior-based algorithm for roundabout intersection management is also designed to calculate speed profiles for different vehicles, in order to achieve more comfortable driving profiles, as well as to reduce congestion [19].

In the field of connected and automated vehicles (CAVs) various results for handling roundabout scenarios have been achieved. Paper [20] proposes a two-level optimization approach for CAVs, with which fuel consumption and vehicle delays can be significantly reduced. At the first level the entering scheduling of the vehicles is handled, and on the second level the trajectory optimization, considering the output of the first level. In the work of [21], analysis on the effectiveness of CAVs in roundabout and intersection scenarios, i.e., comparison of control strategies has been provided. The conclusion of the examination is that a roundabout can be more beneficial in the signal-free management of CAVs, compared to intersection-based control. The positive impact of CAVs on traffic flow at roundabout scenarios has also been studied by [22, 23]. It has been shown that 20% and 40% of AVs in the flow are able to increase leg capacities around 10% and 20%, respectively. Moreover, safety-oriented evaluation of CAVs on mixed traffic scenarios, i.e., CAVs and pedestrians, cyclists, can be found in [24], and for turbo-roundabouts in [25]. Optimization of trajectory for CAVs using convexified constraints on collision-avoidance has been proposed by [26]. In the presented method a distributed architecture has been provided with vehicle-level layers. Another method based on distributed control strategy can be found in [27]. The contribution of this work is smooth and collision-free trajectories for the AVs, by which positive impact on the traffic flow can be achieved. A hierarchical control strategy with two levels for providing a solution on merging problems at roundabouts is found in [28]. Due to the efficient merging maneuver of AVs with receding horizon control, the traffic flow can be improved, and moreover, average fuel consumption of the vehicles can be reduced. Personalized driving behavior, which is in the focus of control design for CAVs at roundabouts is found in the paper [29]. It has been shown that grand coalition game solutions can be beneficial from the viewpoint of the coordination of AVs, while a strategy through Stackelberg game can guarantee personalized driving objectives of individuals.

Several methods presented in the literature that utilize game theory approaches to model the behavior and decision making of autonomous vehicles at roundabouts. In paper [30] a game theory based representation of ego vehicle and opponent vehicle has been developed, in which online estimated driver type of the opponent vehicle has also been considered. Prisoner's Dilemma game strategy [31] is also selected as an approach for autonomous vehicle-to-vehicle (V2V) decision making, demonstrating that the roundabout entry problem can be solved efficiently by shortened waiting times for individual autonomous vehicles.

Although the previous brief overview illustrates that various methods have been developed in the topic of autonomous vehicle control in roundabout scenarios, finding a solution for handling high number of AVs is a challenge. The goal of this paper is to provide a systematic solution on the given problem within a previously developed robust hierarchical control framework, see [32]. The aim of this framework is to provide guarantees on safety performance requirements through the robust control, and similarly, to maintain further requirements, e.g., traveling comfort, economy-based performances, time requirements, etc. In some preliminary studies, the framework has been successfully applied to control AVs in intersections [33] or roundabouts [34]. Nevertheless, the limitations of these solutions are the number of vehicles while their interactions can be handled. This limitation is resulted by the coordination problem of the AVs, which has been carried out by learning-based techniques on the high-level of the control hierarchy, see e.g., [35, 47]. The problem of coordination has three main sources. First, in real-life scenarios the number of vehicles is varying, but during the formulation of the optimal control design problem, the consideration of fixed vehicle number is recommended. Second, in the reward of reinforcement learning process, the signals of all vehicles must be involved. Nevertheless, the positive impact of AVs with high sub-reward can be deteriorated by the negative impact of AVs with low sub-reward. A possible solution on this issue can be found in [36]. Third, the resulted learning-based agents request high computation effort on the high level of the control hierarchy.

In the presented solution of this paper a novel low complexity method on the high-level of the control hierarchy has been provided. The proposed method is based on low-complexity rules of the coordination, which focuses on the minimization of vehicle's traveling time. The resulted method provides a fast and effective solution on the improvement of traffic flow, which is an advantageous property of roundabouts, see e.g., [37]. Although the designed framework is briefly introduced, from the viewpoint of contribution, this paper focuses on the design of the high-level control. Nevertheless, the operation of the entire loop using HiL tests is demonstrated. The limitation of the method is that the high-level only considers fully autonomous vehicles with vehicle-to-vehicle communication capability [38], i.e., mixed traffic situations [3, 5] in the design of the high-level are not involved. The presence of human participants on the low-level is considered, and thus, the AV is able to operate in mixed traffic situation.

The paper is organized as follows. Section 2 describes the proposed control method for handling multiple vehicle scenarios in roundabout. The implementation of the method under simulation environment is found in Sect. 4.3. The efficiency of the proposed method is presented through simulation examples and a HiL test in Sect. 4. Finally, conclusion remarks are presented in Sect. 5.

2 Design of vehicle control in roundabouts with multiple vehicles

The control for autonomous vehicles in roundabout is based on the designed framework, which is illustrated in Fig. 1. In this section the framework is presented briefly, the details on the design process of the supervisor and robust controller, together with the modeling of the vehicle-traffic environment are found in [32, 33].

The control algorithm has two main levels, i.e., the high level and the vehicle level. In the concept of the proposed AV control, the aim of the high level control elements is to provide assistance for the motion of the vehicle. The assistance is based on the process of increased number of signals, which can be achieved through the traffic management system, e.g., measurements on the motion of further AVs. The low vehicle level control elements, such as the supervisor and the robust controller are implemented on the vehicle level. The supervisor has the responsibility to accept or ignore the assistance of the high level, in that process the robust controller helps its operation. The assistance of the high level is acceptable, if the safety performance requirements can be guaranteed.

The control signal $u_i(k)$ for AV_i is computed by the supervisor, which is equal to the longitudinal acceleration command. In the presented control structure the supervisor computes $u(k)$ based on two candidate control signals, which are $u_{L,i}(k)$ and $u_{K,i}(k)$. The candidate control signal $u_{L,i}(k)$ is computed by the high level control, which is responsible to the time-efficient and collision-free coordination of the AVs in the roundabout scenario. The values of $u_{L,i}(k)$ for all AVs are computed by one centralized high level control, and thus, these values are transmitted to the AVs via wireless communication platform

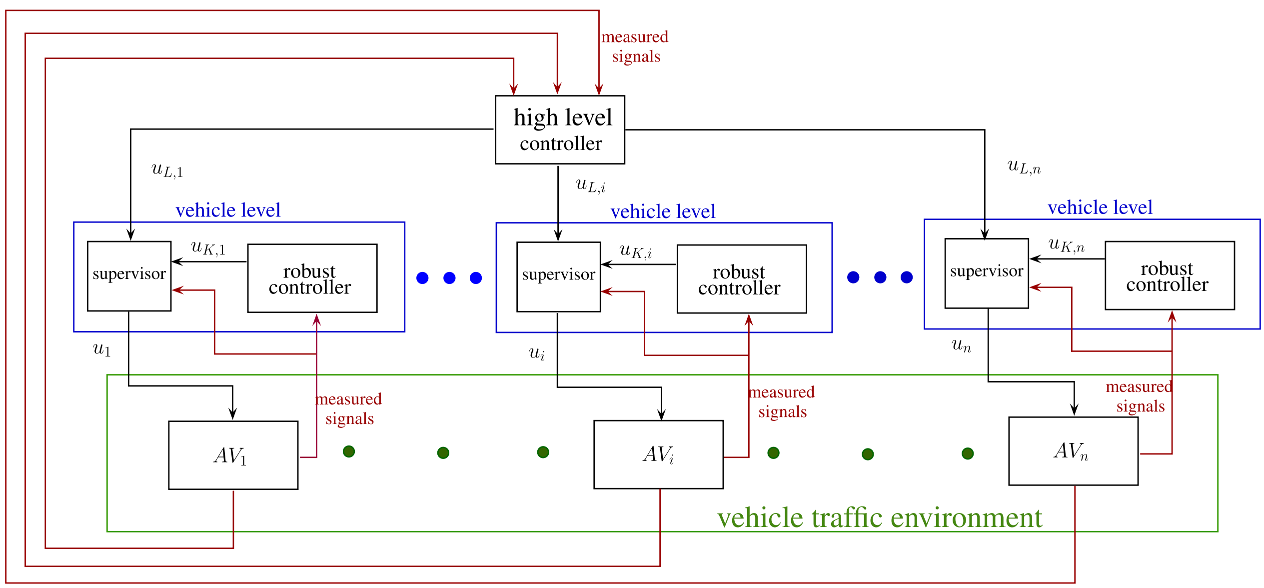


Fig. 1 Illustration of the control architecture for AVs

[34]. Due to safety reasons, e.g., avoiding collision at packet loss, $u_{L,i}(k)$ values are not used directly as control signals. Thus, second controller for each of the AV is designed based on the robust control approach [33]. The computed second candidate control input $u_{K,i}(k)$ is calculated on the vehicle level of each AV_i, which computation is able to guarantee collision-free motion profile for AV_i using onboard measurements. Nevertheless, $u_{K,i}(k)$ cannot be used directly as control signal for AV_i, because it does not guarantee time-efficient coordination of AVs. Consequently, the supervisor combines these two candidate control signals for reaching collision-free and time-efficient motion for AV_i.

The combination is carried out through the expression of $u_i(k) = u_{K,i}(k) + \Delta_i(k)$, where $\Delta_i(k) \in \Delta$ is a bounded addition to $u_{K,i}(k)$, which is computed by the supervisor. This bound represents a limit on the difference from the collision-free candidate control input $u_{K,i}$. The computation of Δ_i aims to minimize the difference between $u_i(k)$ and $u_{L,i}(k)$, i.e., goal of the control system is to accept $u_{L,i}(k)$ as much as possible during the computation of $\Delta_i(k)$. It leads to a constrained optimization problem that results $u_i(k)$, see [32].

In the rest of the paper the focus is located on the design of the high level controller, which assists the motion of AVs in the roundabout. It requests determining of their safe speed, defining priorities among AVs and providing their $u_{L,i}(k)$ acceleration command.

2.1 Constraints for the control design

A safe cornering speed is specified for all AVs based on the geometry of the intersection and simplified vehicle dynamics for the cornering. In addition, another recommendation against the speed profile is that it must result in a comfortable vehicle motion. Assuming that friction coefficient between the tire and the road surface μ is estimated [39–41], the centrifugal force working on the vehicle can be equalized by choosing an appropriate velocity.

Assuming identical road friction μ for longitudinal and lateral direction on each wheel, the tire force of the AV_i is limited by friction circle, i.e., a limit on the total acceleration of the vehicle $a_{tot,i}$ is found:

$$ma_{tot,i} \leq mg\mu, \tag{1a}$$

$$a_{tot,i}^2 = a_i^2 + \left(\frac{v^2}{R}\right)^2, \tag{1b}$$

where $g = 9.81$ m/s^2 is the gravitational constant, m is the vehicle mass and R denotes the radius of the roundabout. Considering that road geometry information can be available by on-board devices of AVs such as GPS, a maximal safe cornering velocity can be calculated by reorganizing (2), i.e., the lateral acceleration of the vehicle is limited as

$$\frac{v^2}{R} \leq \sqrt{(g\mu)^2 - a_i^2}, \tag{2}$$

which leads to the speed limit $v \leq \sqrt{R\sqrt{(g\mu)^2 - a_i^2}}$. Note that in case of constant speed ($a_i = 0$ m/s^2) it leads to the simplified form $v \leq \sqrt{Rg\mu}$.

At the same time, the motion of the vehicle must be comfortable for the passengers, which leads to another limitation on the speed. It has been shown through experimental analysis that the comfort threshold on acceleration for passenger vehicles is 0.4g, see [45, 48]. Therefore, the next limit on the lateral acceleration is formed as:

$$a_{tot,i} = \sqrt{a_i^2 + \left(\frac{v^2}{R}\right)^2} \leq 0.4g, \tag{3}$$

which leads to the speed limit $v \leq \sqrt{R\sqrt{(0.4g)^2 - a_i^2}}$.

Finally, the safe and comfortable speed of AV_i in the roundabout is bounded by the lower speed limit, resulted by the safety constraint or the comfort constraint. Since the speed limits have similar form, the speed limit is computed as

$$v_{round} \leq \sqrt{R\sqrt{(\min(\mu; 0.4)g)^2 - a_i^2}}. \tag{4}$$

As an example for constant longitudinal speed scenario, a conventional roundabout with a 12.5 m radius and with a road friction coefficient of $\mu = 0.8$, the maximal safe cornering speed is $v_{round} \approx 25$ km/h. In addition, longitudinal acceleration and deceleration constraints are also defined in order to ensure both passenger comfort and wheel traction. Here, maximal and minimal acceleration values of $a_{max} = 2.5$ m/s^2 and $a_{min} = -5$ m/s^2 are chosen [42].

2.2 Defining priorities for the vehicles

Handling of multiple vehicles in a roundabout requires the giving of priority for each vehicle. The priorities have impact on the motion of the vehicles, i.e., their ordering at the entrance of the roundabout.

In the priorities of AVs, the minimization of vehicles' traveling time, as a performance requirement is involved. Thus, it is necessary to provide priorities for the vehicles, with which traffic flow can be improved. Thus, the AV_i has higher priority as of AV_j, if their forthcoming routes are crossed and the predicted time of AV_i for reaching the exit of roundabout T_i is smaller than the same predicted time (T_j) for AV_j. It means that the traffic flow is improved, if AV_i has priority against AV j. For example, if AV_i is in the roundabout and similarly, AV_j approaches to the roundabout and their forthcoming routes are crossed, AV_j can enter into the roundabout when AV_i has left the entrance.

The prediction of time T_i for reaching the exit is formed through the consideration of the following assumptions. It is considered that the AV_i is able to move on its route without stopping, i.e., its motion is not disturbed by other vehicles. Moreover, on the prediction horizon the AV is considered to move with its maximum speed. Thus, the predicted time for reaching the exit is

$$T_i = \frac{s_{entr,i}}{v_{entr}} + \frac{s_{round,i}}{v_{round}}, \tag{5}$$

where $s_{entr,i}$ is the distance of AV_i until reaching the entrance of the roundabout, v_{entr} is its maximum speed on the entrance section. $s_{round,i}$ is the route length of AV in the roundabout and v_{round} is the maximum speed in the roundabout, which has been computed by (4). If a given AV is in the roundabout, the formula for time prediction (5) is reduced to

$$T_i = \frac{s_{round,i}}{v_{round}}. \tag{6}$$

The time values T_i, $i \in [1 \ldots n]$ are computed for all AVs in the region of interest, which have not left the exit of the roundabout. The n number of AVs in descending order are sorted, depending on their T_i time values. Thus, priorities for all of the AVs are defined, which is the basis for the computation of their acceleration command. Thus, the i index of an AV represents its priority, i.e., $i = 1$ has the highest priority and AV_i at $i = n$ has the lowest priority.

Remark, if one of the AVs has left the exit of the roundabout, but it is inside of the roundabout region of interest, it has not been taken part in the ordering. For that vehicle the maximum speed is given as a reference to improve traffic flow performance.

2.3 Providing vehicle acceleration command

In the computation process of the acceleration command $u_{L,i}$ for each AV the priorities are used. The goal of the computation is to find the speed profiles for AVs, by which their time-efficient and collision-free motion, i.e., safe distance s_{safe} between AVs, in the context of roundabout can be guaranteed. The value of s_{safe} depending on the speed of the vehicles can be selected, such as $s_{safe} = T_{safe} \cdot v_{round}$, where T_{safe} is a safety time value, whose value is between $1 - 2$s. The model on the computation of T_{safe} assumes that the deceleration braking for all vehicles, i.e., the maximum achievable deceleration, are the same. The value of $u_{L,i}$ is selected on a bounded range of $U_L = [u_{L,min}; u_{L,max}]$, where $u_{L,min}$ represents maximum braking and $u_{L,max}$ is related to maximum acceleration of the AVs. The computation process of $u_{L,i}$ for $i = 1 \dots n$ with T sampling time is performed.

The idea behind the computation of $u_{L,i}$ is to predict the motion of vehicle i, and all of AV_j, where $j = 1 \dots i-1$. Thus, the motion of AV_i must be adapted to the vehicles, which have higher priorities. It is necessary to find maximum $u_{L,i}$, by which s_{safe} can be guaranteed on T time horizon between AV_i and all of AV_j. The distance $s_{i,j}$ between AV_i and AV_j is composed as the distances on the straight sections and the distance on the circular section. The distance on the straight sections are computed through Pythagoras theorem formula and the distance on the circular section are computed through polar coordinates. This distinction helps to reduce the complexity of the computation. The longitudinal acceleration to keep safety distance, i.e., $s_{i,j} > s_{safe}$, on a T prediction time horizon is:

$$u_{L,i} \leq \frac{2(s_{i,j} - s_{safe} - v_i T)}{T^2}. \tag{7}$$

Thus, through (7) an upper bound is provided, by which the maximum of $u_{L,i}$ in relation of AV_i, AV_j can be computed.

Nevertheless, there can be some scenarios, in which the vehicles with lower priorities must be considered. The motion of AV_i must be adapted to the motion of AV_k, $k > i$, if AV_j poses an obstacle for AV_i within the predicted time T. For example, if AV_k with lower priority on the route of AV_i is a preceding vehicle, then upper bound on the longitudinal acceleration for AV_k using (7) must also be computed.

Finally, the computation of $u_{L,i}$ is formed as an optimization problem with constraints, such as:

$$\max\ u_{L,i} \tag{8a}$$

subject to

$$u_{L,i} \leq \frac{2(s_{i,m} - s_{safe} - v_i T)}{T^2}, \forall m \in AV_m, \tag{8b}$$

$$u_{L,i} \in U_L, \tag{8c}$$

where AV_m represents the sets of vehicles AV_j, $j = 1 \dots i-1$ and of all obstacle vehicles AV_k. The result of the optimization is $u_{L,i}$, which is the input for the supervisor, see Fig. 1. The initial value for $u_{L,i}$ in a given optimization process can be selected as the solution of the optimization process in the last computation. This selection assumes that slight modification of $u_{L,i}$ during a time step is required, which leads to faster convergence of the maximization process. Moreover, the maximization can be terminated, when a terminal constrain is fulfilled, i.e., $|u_{L,i}(k) - u_{L,i}(k-1)| < \epsilon$, where k represents the number of iteration in the maximization process and ϵ is a predefined small scalar value.

Although this paper provides a method for achieving collision-free and time-efficient speed profile, the fully autonomous control of AVs requires also the path planning, i.e., designing steering control. In the rest of this section to some existing results in this topic are referred. The method of Bézier curve control points generation for roundabout scenarios can be found in [43]. B-Spline type of curves have also been used for path planning, where relationship between roundabout intersection leg angle and path performance has been simultaneously considered [44]. A path planning strategy method for lane-free roundabouts can be found in [46], which method is also able to consider limits on steering angle. At the level of steering control, the application of control barrier functions can lead to safe motion of the vehicle [45].

3 Implementation of the control method in simulation environment

The advantage of the proposed method is that it requires low computation effort. In this section the structure of the implementation is presented, which can be used during simulation-based evaluations.

On the high level control it is recommended to store data on AVs in an array form, where each row is related to the AVs in the region of interest, i.e., the array is known as $\mathcal{S}$. The benefit of the form is that it can be dynamically modified, depending on the number of vehicles n. Moreover, this form provides a simple structure to give priorities for the vehicles, where the number of the row represents priority of an AV.

An example on the structure of $\mathcal{S}$ with $n = 3$ is found in Table 1. The first column of $\mathcal{S}$ contains the number of the roundabout entrance, the second column contains the exit, where AV_i leaves the roundabout. The positions of the AV is recommended to store in the measures of polar coordinate system, which is motivated by the circled shape of the roundabout. The origin of the polar coordinate system is the physical center of the roundabout. The distances of the AVs from the origin is found in column three, and the angle in column four. The angle is measured from entrance 1, i.e., 0° is related to entrance/exit 1, 90° is to entrance/exit 2, etc. The longitudinal speed of the AVS are involved in the fifth column, see Table 1. Column six contains the information, whether AV_i has left the roundabout, but it is inside of the region of interest. In the given example of Table 1, where the radius of

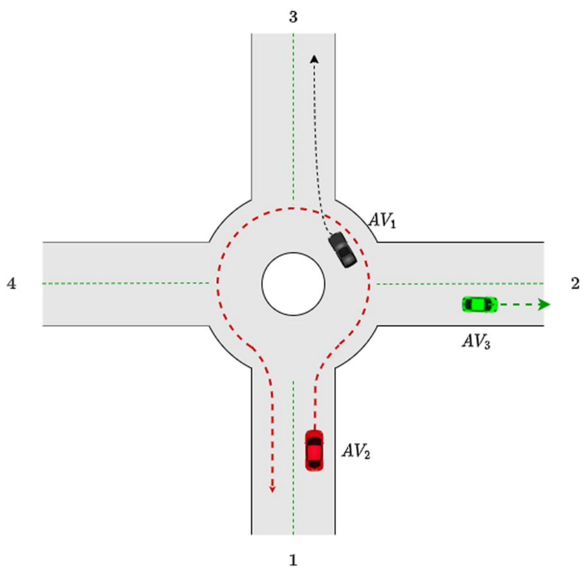


Fig. 2 Example on the motion of vehicles in a roundabout

the roundabout is $10m$, AV_1 is inside of the roundabout, between entrance 2 and exit 3. AV_2 is inside of the region of interest, but it has not entered into the roundabout. Moreover, AV_3 is also inside of the region of interest, but it has left exit 1. The given example is illustrated in Fig. 2. Finally, the last column contains information on T_i, which determines the ordering of AVs in $\mathcal{S}$. In this example AV_1 is predicted to leave exit 3 earlier than AV_2, and thus, AV_1 has priority against AV_2. Since AV_3 has left the roundabout, prediction of T_3 is unnecessary and it is out of the priority ordering. The values of T_i are computed through (5) and (6).

The generation of $\mathcal{S}$ with its ordering is processed as follows. In each simulation time step the positions and the speed values of the vehicles, together with their forthcoming trajectories are transmitted from the AVs. The positions of the vehicles are transformed to polar coordinates and it is determined from the direction of vehicle motion, whether AV_i has left the intersection or not. Then, the values of T_i are calculated and the priorities for the AVs through their ordering are also determined. In this way, table $\mathcal{S}$ for the vehicle control is generated.

After the generation and the ordering of $\mathcal{S}$, the control input for *AVs* are computed. The signal $u_{L,i}$ for each AV is calculated by (8). It requests the formulation of constraints (8b), which is different for each AV. For the formulation it is necessary to determine the AV_m set of AVs, which contains higher priority AVs and obstacle AVs. For AV_i, the vehicles with higher priorities in the set are AV_j, $j \in [1; i-1]$. The determination of obstacle AVs requests the examination of vehicle positions, that are on the route of AV_i, in a S_h length horizon ahead. The selection of S_h is influenced by the geometry of the roundabout, i.e., it is recommended to select at least one quarter of the roundabout circumference. In this example, $S_h = \frac{2R\pi}{4} = 15.7m$ is selected. Through the formulation of the constraints, the optimization problem (8) for each AV_i is formed, which result in $u_{L,i}$ for each vehicle. Remark that for AVs, whose *exited* variable is 1, $u_{L,i} = u_{L,max}$ is selected. Finally, the computed $u_{L,i}$ value for each vehicle is transmitted.

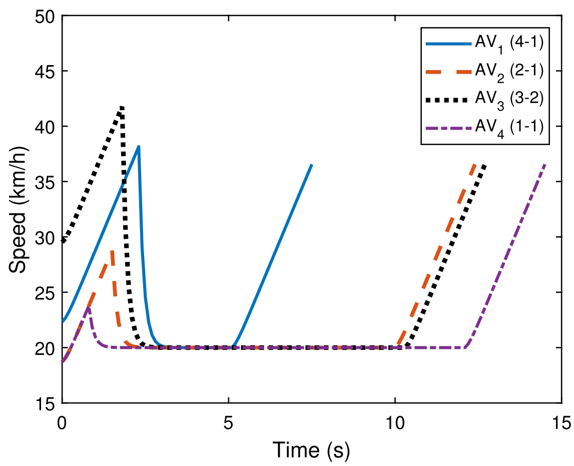
Although in this section the implementation is focused on simulation scenarios, it can be carried out for test vehicle scenarios. An important difference of the scenarios is the presence of time delay from the viewpoint of the implementation. It means that it is necessary to consider the time elapse between measurement of AVs' positions and time intervention in the selection of T prediction time horizon. In case of simulation scenarios, time delay is not considered during the implementation.

4 Illustrative simulation examples

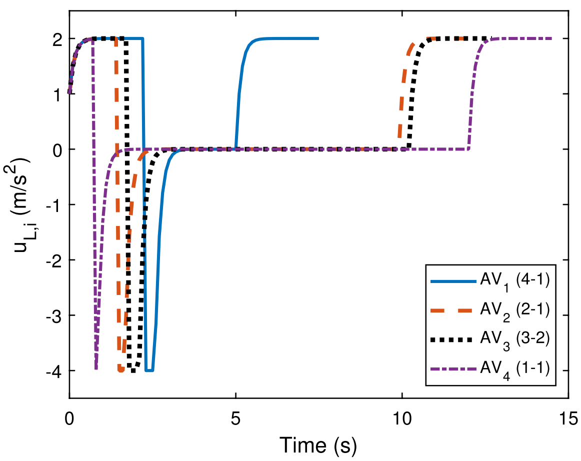
For demonstrating the effectiveness of the method, two scenarios are illustrated. First, a long-time simulation, in which high number of AVs are entranced in the roundabout, is presented. The aim of this simulation is to show that the proposed algorithm is able to provide high-efficient motion for the AVs even at continuous traffic flow. For illustrating the efficient operation of the proposed method, long-time simulation under various conditions has also been performed. Second, the proposed control strategy in an entire control architecture with vehicle low-level control is embedded, and thus, a Hardware-in-the-Loop scenario is analyzed.

Table 1 Example on the structure of $\mathcal{S}$

Entrance	Exit	Distance (m)	Angle (°)	Speed (km/h)	Exited	T_i (s)
2	3	10	100.6	25	0	3.8
1	1	17.3	0	25	0	6.2
3	2	35.2	180	50	1	[−]

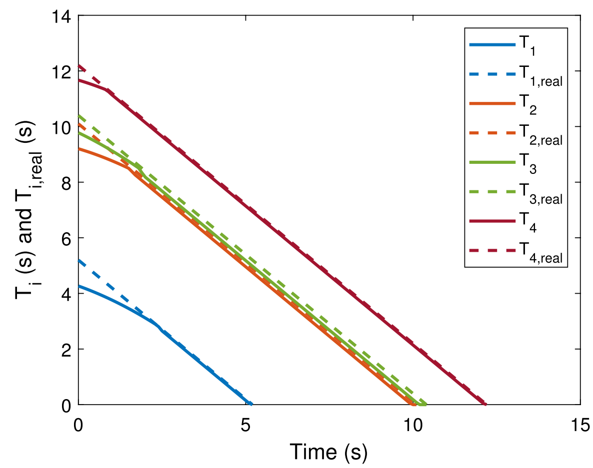


(a) Speed of AVs

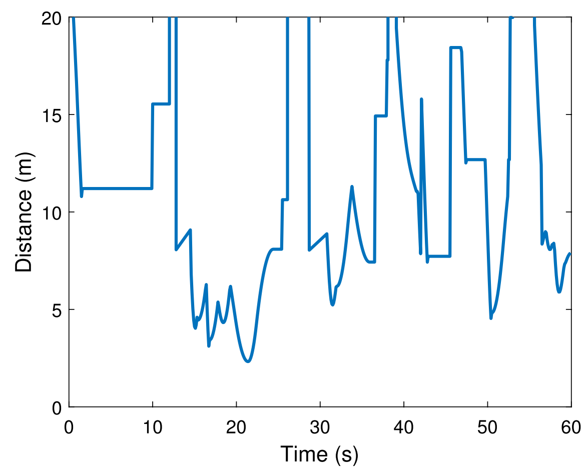


(b) $u_{L,i}$ acceleration command for AVs

Fig. 3 Results on the long-term simulation example



(a) Comparison of time values

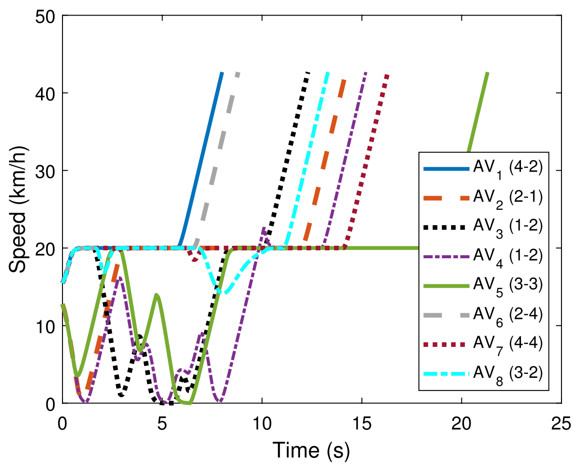


(b) Distance between AVs

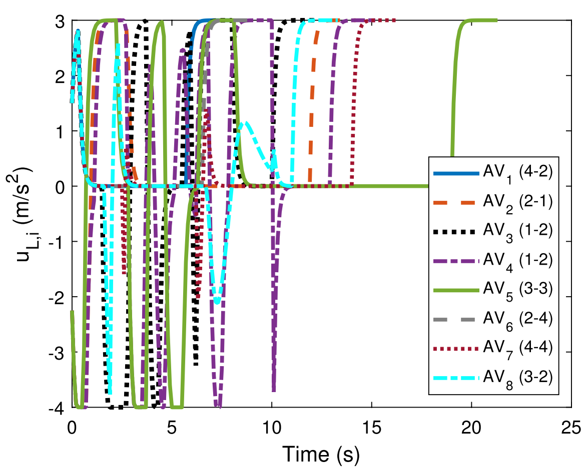
Fig. 4 Results on the long-term simulation example (cont.)

First, the simulation results for the scenario with continuous traffic flow are shown. During the 1 *min* long simulation scenario 21 AVs are taken part into the scenario, that have entered in the region of interest on random entrance road. The roundabout has four entrance/exit roads and $R = 10$ m radius, as it is illustrated in Fig. 2. Some results on the simulation are found in Figs. 3,4, i.e., Fig. 4b shows result on the entire simulation, while Figs. 3a, b and 4a focus on the initial term of the simulation, when four AVs are in the region of interest.

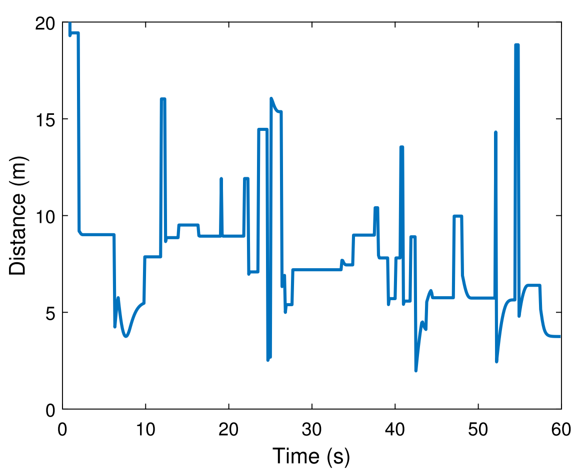
Figure 3a illustrates the speed of $AV_1 \dots AV_4$, until they stay in the region of interest of the roundabout. It can be seen that the vehicles keep $v_{lim} = 20$ km/h within the roundabout to avoid dangerous situation, while leaving the roundabout the AVs accelerate on the straight road section. The suggested $u_{L,i}$ candidate control input of the high level control for each AV is shown in Fig. 3b. Figure 4a illustrates the effectiveness of the time prediction T_i. In Fig. 4a $T_{i,real}$ is the real time value of AV_i until the exit from the roundabout, which has been determined after the simulation. It can be seen that T_i and $T_{i,real}$ signals are close to each other, which means that the prediction error is low. Moreover, T_i values are in relation with the priorities of the AVs: if AV_i has lower T_i value, it has higher priority. For example, AV_1, AV_2, AV_4 move to the same exit road (1), but the avoiding of collision is handled, i.e., AV_1 has the highest priority, and AV_4 has the



(a) Speed of AVs



(b) $u_{L,i}$ acceleration command for AVs



(c) Distance between AVs

Fig. 5 Simulation with 8 AVs

lowest. Finally, Fig. 4b illustrates the distance between the vehicles. It can be seen that distance between the vehicles during the entire simulation is higher as $s_{safe} = 2$ m, i.e., the safe motion of the vehicles are guaranteed.

4.1 Variations in the simulation example

Further simulation examples are shown below to illustrate the effectiveness of the control system under different conditions. Thus, the conditions of the long-term simulation example are varied, such as number of vehicles and the radius of the roundabout.

In Fig. 5 the results of long-term simulation with 8 AVs are presented. Since an increased number of AVs is taken part into this simulation example, more vehicles must reduce their speed to keep s_{safe}, see e.g., Fig. 5a, in which the speed profiles of the initial 8 AVs are illustrated. Although the rush traffic leads to varying acceleration commands (see Fig. 5b), minimum distance is always above s_{safe}, see Fig. 5c.

Another example in Fig. 6 is shown, in which the radius of the roundabout is reduced to $R = 5$m. Although the proposed control strategy is not dedicated to a special type of roundabout, i.e., it is independent from the value of R, the goal of the variation is to demonstrate that it is able to operate efficiently for roundabouts with different geometries. The consequence of the reduced radius is that the vehicles are closer to each other during their motion. Its consequences are the reduced speed (Fig. 6a) and the varied acceleration commands (Fig. 6b). The speed reduction leads to the reduction of prediction performance, see Fig. 6c, e.g., stopping of AV_3 and AV_4 leads to increased difference of T_i and $T_{i,real}$ at the first half of the simulation. Nevertheless, s_{safe} is kept during the entire simulation scenario, see Fig. 6d.

Another simulation example with $R = 5m$ and 8 AVs is presented in Fig. 7., which is a rush-hour traffic scenario. Due to the simultaneous presence of 8 AVs in the small area of the roundabout, their speed must be significantly varied, see e.g. AV_5 in Fig. 7a. In spite of the high intensity of traffic flow, the minimum distance of the vehicles is above s_{safe} during the entire scenario, see Fig. 7b.

Finally, Fig. 8 provides the results of a simulation with $R = 15$ m of the roundabout with 8 AVs. It can be seen that the most important performance measure, such as guaranteeing at least s_{safe} distance between AVs, is achieved in this scenario, see Fig. 8b. Thus, the conclusion of the various simulation examples is that the proposed control strategy is able to provide safe motion for the vehicles under various traffic conditions.

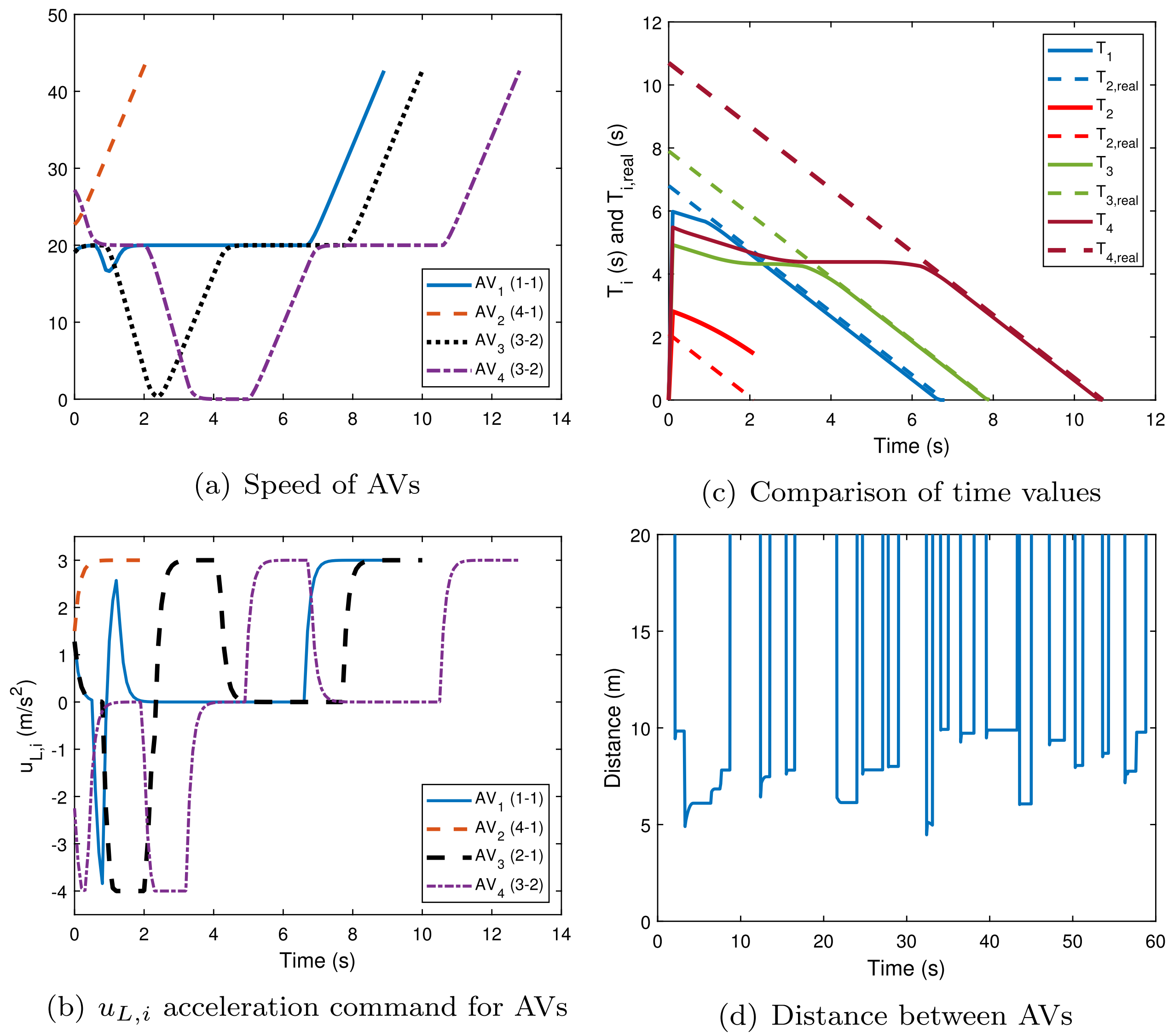
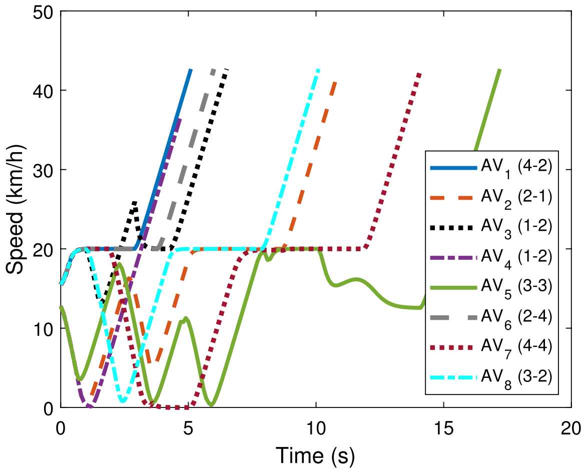


Fig. 6 Simulation with 4 AVs at $R = 5$m

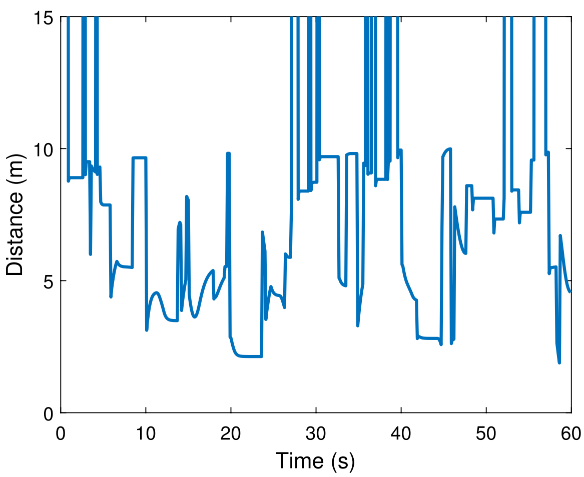
4.2 Evaluation of the method using SUMO traffic simulator

The operation of the proposed method has also been tested under SUMO (Simulation of Urban MObility) traffic simulator environment [49]. Moreover, the simulation results of the proposed method are compared to the results of three car-following models, such as the modified SUMO-Krauss model [50], Adaptive Cruise Control (ACC) and Cooperative ACC (CACC) car-following methods [51]. In the simulation example the roundabout with $R = 10$ m radius and 4 entrance/exit roads has been built, i.e., the similar scenario has been simulated as in the previous cases. The time length of the simulation is 15 min, during this time 225 vehicles are moved in the traffic network.

Table 1 presents the results in a comparative form. The simulations have been performed with two speed limit settings: in the first case the speed limit is 20 km/h in the entire network, but in the second case the speed limit on the straight sections is 50 km/h. In Table 1 the total time spent (TTS) of the vehicles in the network and the fuel consumption are compared. In the evaluation,

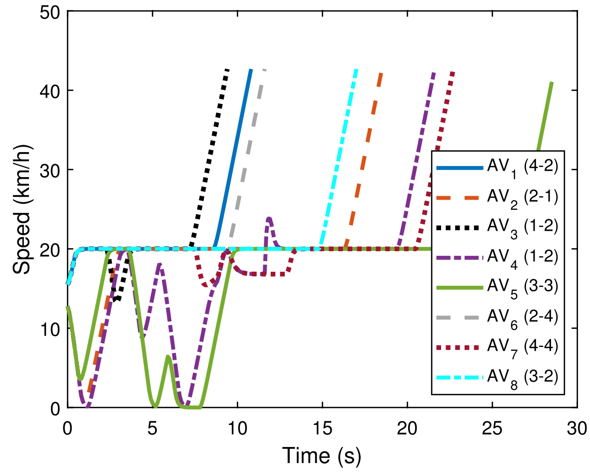


(a) Speed of AVs

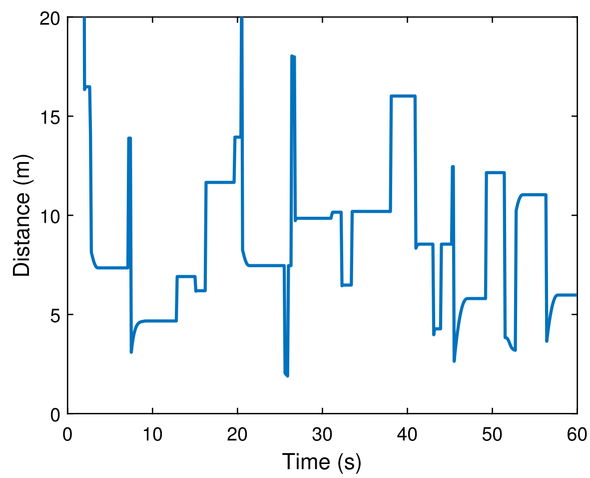


(b) Distance between AVs

Fig. 7 Simulation with 8 AVs at $R = 5$ m



(a) Speed of AVs



(b) Distance between AVs

Fig. 8 Simulation with 8 AVs at $R = 15$ m

the vehicles are separated on the basis of the distance that they travel within the roundabout. The traveled distances are represented by their circular motion, such as 90°–360°. The results show that the proposed method is able to effectively reduce TTS in case of all scenarios, and fuel consumption in most of the scenarios. Nevertheless, there is a trade-off between these measures, compare the two speed limit scenarios. For example, the increased reduction of TTS can lead to the slight reduction, or increase of fuel consumption, see the scenarios of 20 km/h speed limit to SUMO-Krauss model and of 50km/h speed limit to ACC method. Since the proposed method focuses on the reduction of TTS, it can lead to increased fuel consumption, see e.g., vehicles with 90° motion at 20km/h speed limit. But, in the 50km/h speed limit scenarios, the TTS reduction is balanced, resulting in lower fuel consumption for all vehicle groups.

Figures 9, 10, 11 and 12 illustrate TTS and fuel consumption in case of 50 km/h speed limit scenario in a histogram form. It can be seen that that proposed motion strategy is able to provide reduced TTS and fuel consumption in each vehicle group. Although the mean of TTS is slightly larger (0.7%) at 90° circular motion in case of SUMO-Krauss model, see Fig. 9a and Table 2. Nevertheless, this small increase is insignificant compared to the reductions in the other groups and in fuel

Table 2 Comparison of simulations in SUMO traffic simulator

Speed limit (km/h)	TTS reduction (%)				Fuel consumption reduction (%)			
	90°	180°	270°	360°	90°	180°	270°	360°
20 (to SUMO-Krauss)	11.4	20.6	24.9	39.5	-15.7	-7.9	4.6	31.2
50 (to SUMO-Krauss)	-0.7	21.6	29.2	38.3	15.2	21.1	32.7	39.1
50 (to ACC)	8.8	11.9	14.4	23.1	-2.1	6.4	10.6	20.8
50 (to CACC)	5.2	10.2	14.1	20.2	1.2	8.3	12.8	20.9

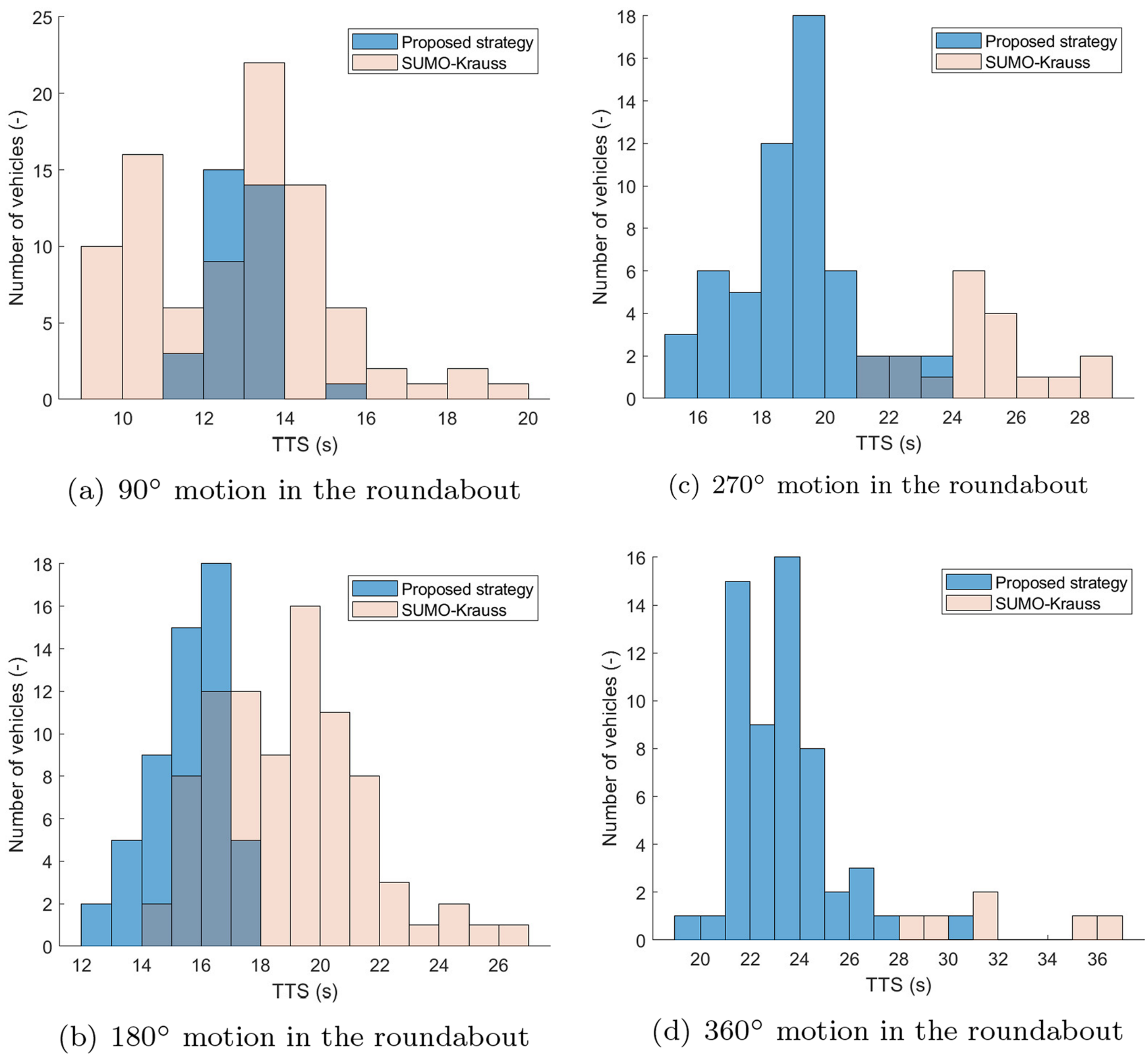
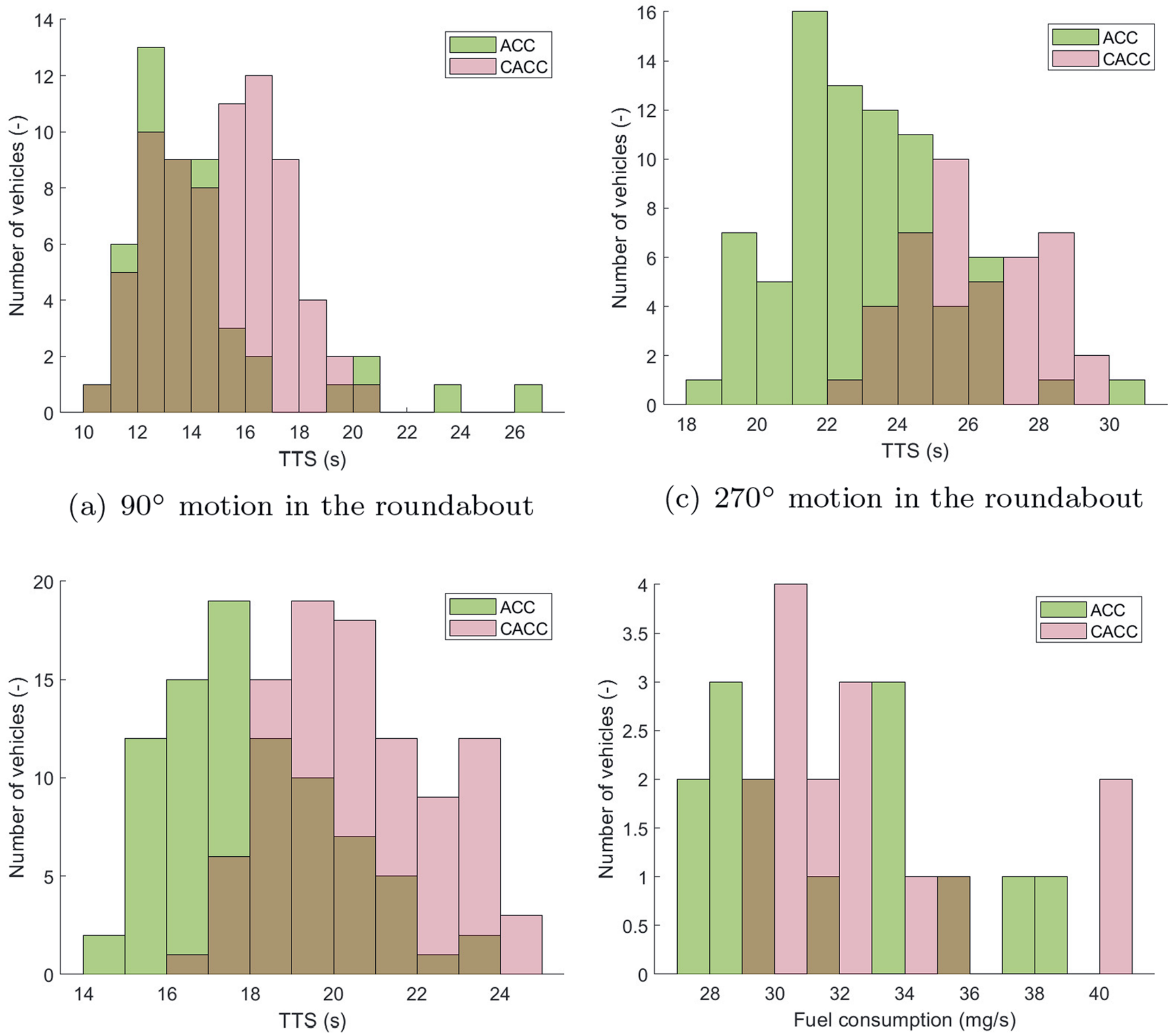


Fig. 9 Histogram of total time spent in the network (I)



(a) 90° motion in the roundabout

(c) 270° motion in the roundabout

(b) 180° motion in the roundabout

(d) 360° motion in the roundabout

Fig. 10 Histogram of total time spent in the network (II)

consumption. Similarly, the proposed method is able to provide the same or better performance level as ACC, and significantly better as CACC, see Figs. 10 and 12.

4.3 Implementation of the motion control algorithm

In the rest of this paper the effectiveness of the algorithm through its implementation on small-scaled test vehicles is demonstrated. In the demonstration a Hardware-in-the-Loop (HiL) environment has been used, in which augmented reality (AR) and multiple indoor vehicles are contained. The goal of the presented example, i.e., motion of automated vehicles in a roundabout scenario, is to show the safe motion of the automated vehicles, which use the proposed control algorithm. The roundabout has anticlockwise circulation and three entrance/exit connections, see Fig. 13a. The safety

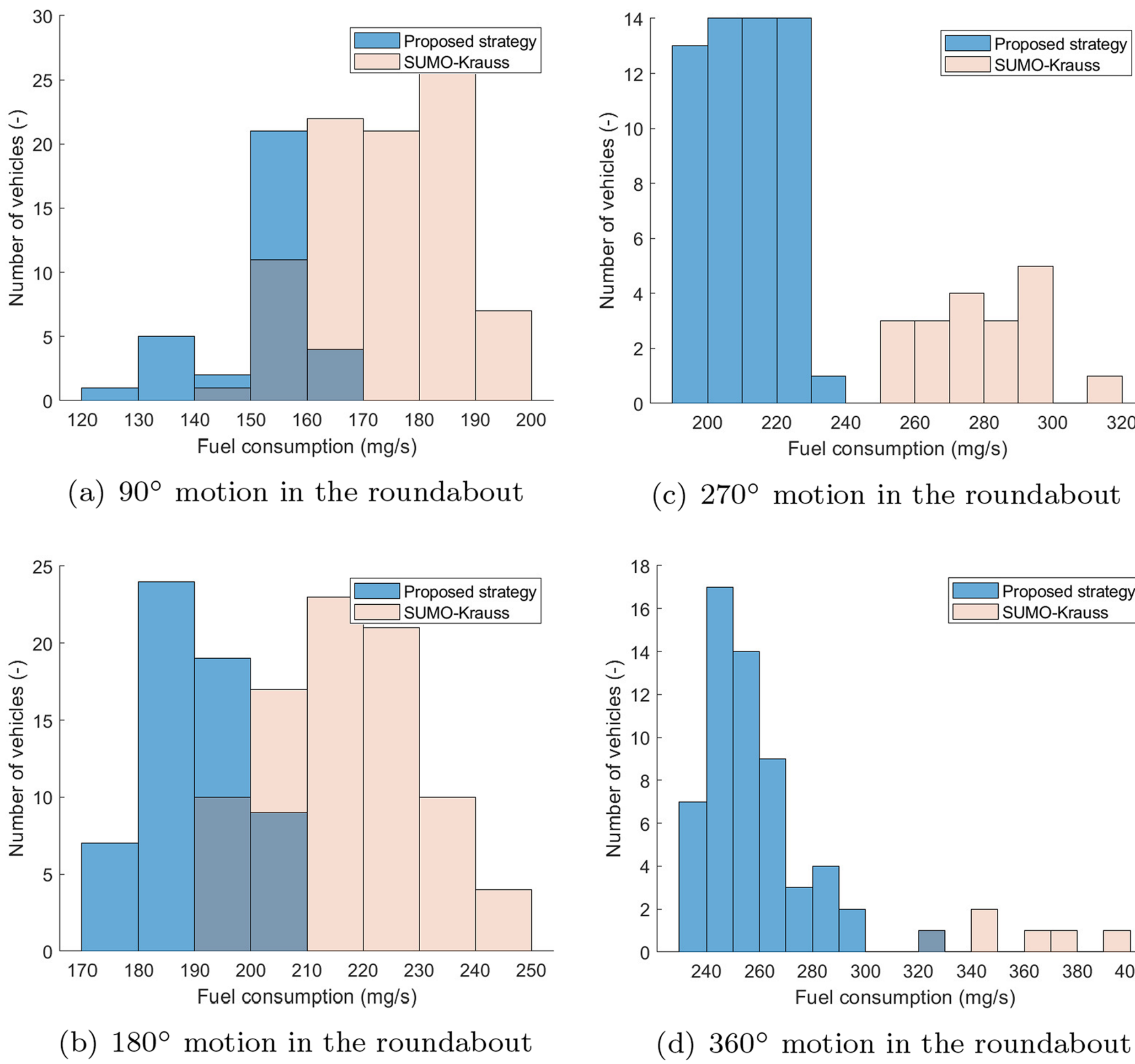


Fig. 11 Histogram of fuel consumption of vehicles (I)

performance requirement against the vehicles is to keep at least $s_{safe} = 1$ m distance from each other.

In the example three automated vehicles are involved, two of them are real physical small-scaled vehicles and one of them is virtual vehicle in the AR. The positions of the physical vehicles through OptiTrack motion capture system are measured and this information via ROS network is transferred. In the architecture the motions of the virtual vehicles on a PC, as a node of the ROS network, are simulated. The motion of virtual vehicles in the AR is visualized on a tablet. On the tablet the Android-based Unity environment with Vuforia AR engine is used, with which the pose of the tablet, related to a fixed marker on the floor is estimated. From the viewpoint of control implementation, the proposed low-complexity high-level control on the PC is found, and the robust control with supervisor on the physical vehicles (or on the PC for virtual vehicles) are installed.

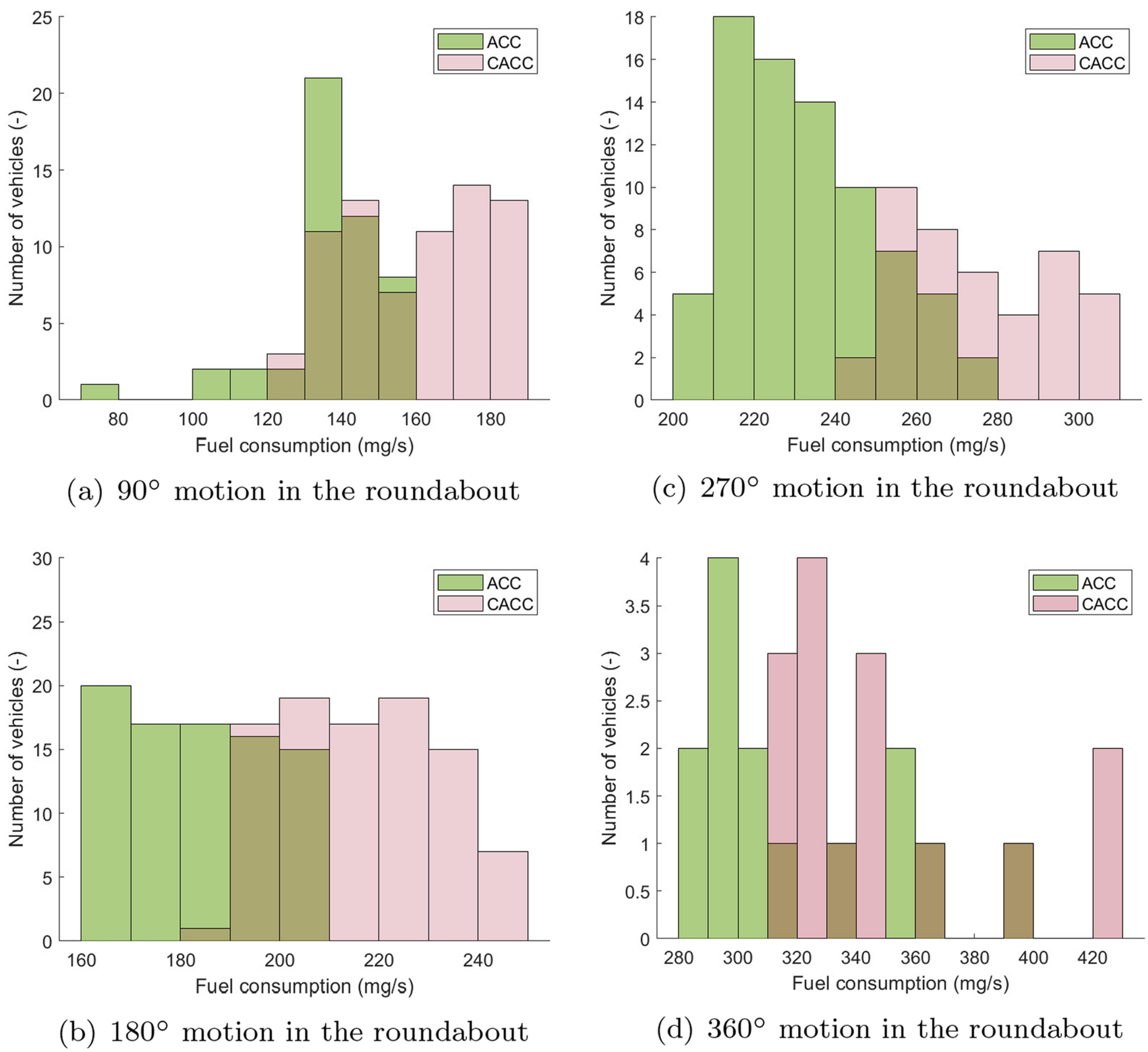
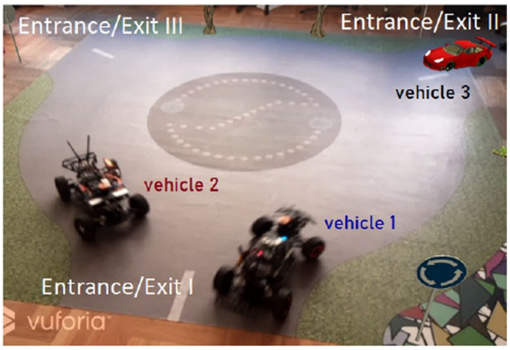


Fig. 12 Histogram of fuel consumption of vehicles (II)

The lateral motion of the physical vehicle based on their lateral error from the centerline through a PID controller is influenced.

Some scenes of the simulation scenario is illustrated in Fig. 13. At the beginning of the scenario vehicle 1 and vehicle 2 are in conflict, see Fig. 13a. Although vehicle 1 decides to enter into the roundabout at Entrance I., but the distance between vehicle 1 and vehicle 2 is kept above s_{safe}, see Fig. 14d around 1.5s. The avoidance of the collision is achieved by the reduction of u_2 (see Fig. 14b), which induces the reduction of v_2, as it is shown in Fig. 14c. In Fig. 13b the conflict of vehicle 1 and vehicle 3 is shown, which results in the speed reduction of vehicle 1, see Fig. 14c after 2s. For a short time between 2s–4s, until vehicle 2 does not leave the roundabout at Exit II. (see Fig. 13c), all of the



(a) Vehicle 1 enters into the roundabout

(b) Vehicle 3 enters into the roundabout

(c) Vehicle 2 leaves the roundabout

(d) Vehicle 3 leaves the roundabout

Fig. 13 Visualization of the HiL demonstration example

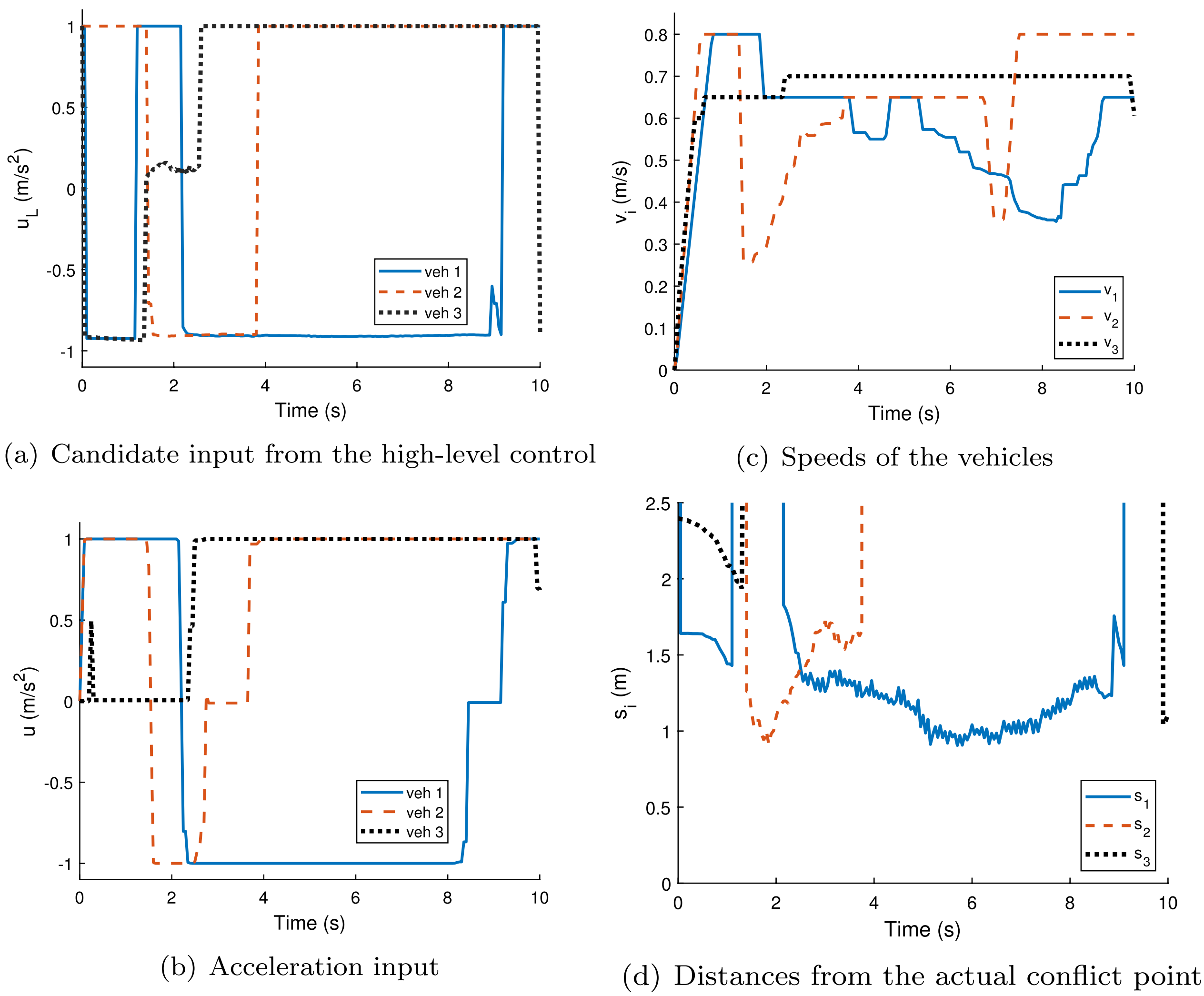
vehicles move together. In this phase of the scenario, s_1 and s_2 have small values, but s_{safe} has been kept, see Fig. 14d. At the last part of the scenario, vehicle 1 follows vehicle 3 and both vehicles leave the roundabout at Exit I. The motion of the vehicles together with the characteristics of s_1 (see Fig. 14d) demonstrate that the proposed motion control algorithm is able to guarantee safe vehicle following and the handling of vehicle interactions.

Finally, it is suggested to compare the signals of u_L and u for each vehicle, see Fig. 14a, b. In the objective of the supervisor the minimization of difference between u and u_L is formed [32, 33], i.e., the characteristics of u and u_L for all vehicles are close to each other. Nevertheless, the difference between u and u_L guarantees the safe motion of the automated vehicles.

5 Conclusion

The paper has proposed a low complexity control method for handling multiple autonomous vehicles in roundabout scenarios. The effectiveness of the method has been evaluated through simulation and HiL scenarios. The simulations have shown that the safe and continuous motion of the vehicles can be guaranteed by the control method. The low complexity, as an important contribution of the method is achieved by a priority-based ordering strategy of the autonomous vehicles. The efficient operation of the control system through the test measurements has been demonstrated.

The future challenge of the method is to extend the priority-based control strategy to further traffic scenarios, e.g., intersections and multiple lanes. Through the extension of the proposed method, autonomous vehicles can be controlled in large-scaled traffic networks with low computation effort. This extension can also request to consider time already spent in the traffic network, because it may be a legitimate request for traffic participants to receive priority after prolonged waiting. Nevertheless, it may lead to a multi-objective optimization problem with respect to the ordering process. Moreover, another challenge is the implementation of the method on real test vehicles, e.g., on proving ground context. A possible low-cost solution for providing multiple autonomous vehicles is the use of augmented reality [34]. The implementation requests the consideration of time delay in communication and vehicle actuation.



(a) Candidate input from the high-level control

(c) Speeds of the vehicles

(b) Acceleration input

(d) Distances from the actual conflict point

Fig. 14 Results on roundabout scenario

Acknowledgements
Not applicable.

Author contributions
ZSF: methodology, validation; BN: conceptualization; AM: software; PG: supervision. All authors have read and approved the manuscript.

Funding
The research was supported by the European Union within the framework of the National Laboratory for Autonomous Systems (RRF-2.3.1-21-2022-00002). The paper was partially funded by the National Research, Development and Innovation Office (NKFIH) under OTKA Grant Agreement No. K 135512.

Availability of data and materials
Not applicable.

Declarations

Competing interests
The authors declare that they have no competing interests.

Received: 23 February 2023 Accepted: 1 November 2023
Published online: 22 November 2023

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