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Total Lagrange duality for DC infinite optimization problems

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Abstract
We present some total Lagrange duality results for inequality systems involving infinitely many DC functions. By using properties of the subdifferentials of involved functions, we introduce some new notions of constraint qualifications. Under the new constraint qualifications, we provide necessary and/or sufficient conditions for the stable total Lagrange duality to hold.
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1 Introduction

Let X be a real locally convex Hausdorff topological vector space, whose dual space X^* is endowed with the weak* topologies $W^*(X^*, X)$. Let T be an arbitrary (possibly infinite) index set, C be a nonempty convex subset of X, and let $h, h_t : X \to \overline{\mathbb{R}} := \mathbb{R} \cup \{+\infty\}$, $t \in T$, be proper convex functions. Consider the following optimization problem (*cf.* [1–13] and the references therein):

$$(\mathcal{P}) \quad \begin{array}{ll} \text{Min} & h(x), \\ \text{s.t.} & h_t(x) \le 0, \quad t \in T, \\ & x \in C, \end{array} \tag{1.1}$$

and its Lagrange dual problem

$$(\mathcal{D}) \quad \sup_{\lambda \in \mathbb{R}_+^{(T)}} \inf_{x \in C} \left\{ h(x) + \sum_{t \in T} \lambda_t h_t(x) \right\}, \tag{1.2}$$

where $\mathbb{R}_+^{(T)}$ is the cone consisting of vector $(\lambda_t) \in \mathbb{R}^T$ with nonnegative and only finitely many nonzero coordinates, that is,

$$\mathbb{R}_+^{(T)} = \left\{ (\lambda_t) \in \mathbb{R}^T : \lambda_t \ge 0 \text{ for each } t \in T \text{ and only finitely many } \lambda \ne 0 \right\}.$$

The optimal values of problems $(\mathcal{P})$ and $(\mathcal{D})$ are denoted by $v(\mathcal{P})$ and $v(\mathcal{D})$, respectively.

Usually, there is a so-called duality gap between the optimal values of primal problem $(\mathcal{P})$ and its Lagrange dual problem $(\mathcal{D})$. A challenge in convex analysis is to give sufficient conditions which guarantee the strong Lagrange duality, that is, $v(\mathcal{P}) = v(\mathcal{D})$ and the dual problem $(\mathcal{D})$ has an optimal solution. Several sufficient and/or necessary conditions were given in the past in order to eliminate the above-mentioned duality gap, see, for example, [1–3, 5] and the references therein. In particular, the authors in [5] established a complete characterization for the strong Lagrange duality under assumption that f and f_t are not necessarily convex, and in [14], the authors considered the optimization problem $(\mathcal{P})$, but with $h := f - g$ and $h_t := f_t - g_t$, $t \in T$ being DC (difference of two convex functions) functions, and they obtained some complete characterizations for the weak and strong Lagrange dualities. As pointed in [15], problems of DC programming are highly important from both viewpoints of optimization theory and applications, and they have been extensively studied in the literature (*cf.* [15–24] and the references therein).

Inspired by the works mentioned above, we continue to study the optimization problem which was studied in [14], that is,

$$(P) \quad \begin{array}{ll} \text{Min} & f(x) - g(x), \\ \text{s.t.} & f_t(x) - g_t(x) \le 0, \quad t \in T, \\ & x \in C, \end{array} \tag{1.3}$$

where T, C are as in (1.1), $f, g, f_t, g_t : X \to \overline{\mathbb{R}}$, $t \in T$, are proper convex functions. Throughout this paper, we assume that

$$\emptyset \neq A := \{x \in C : f_t(x) - g_t(x) \le 0, \forall t \in T\}. \tag{1.4}$$

Following [14], we define the Lagrange function $L : H^* \times \mathbb{R}_+^{(T)} \to \overline{\mathbb{R}}$ for the DC optimization problem (1.3) by

$$L(w^*, \lambda) := g^*(u^*) + \sum_{t\in T} \lambda_t g_t^*(v_t^*) - \Big(f + \delta_C + \sum_{t\in T} \lambda_t f_t\Big)^* \Big(u^* + \sum_{t\in T} \lambda_t v_t^*\Big)$$

for any $(w^*, \lambda) \in H^* \times \mathbb{R}_+^{(T)}$ with $w^* = (u^*, (v_t^*)) \in H^* := \operatorname{dom} g^* \times \prod_{t\in T} \operatorname{dom} g_t^*$ and $\lambda = (\lambda_t) \in \mathbb{R}_+^{(T)}$. Then the Lagrange dual problem for the DC optimization problem (1.3) is defined by

$$(D) \quad \sup_{\lambda\in\mathbb{R}_+^{(T)}} \inf_{w^*\in H^*} L(w^*, \lambda), \tag{1.5}$$

where and throughout the whole paper, following [25, p.39], we adopt the convention that $(+\infty) + (-\infty) = (+\infty) - (+\infty) = +\infty$ and $0 \cdot (\infty) = 0$. Then, for any two proper convex functions $h_1, h_2 : X \to \overline{\mathbb{R}}$, we have that

$$h_1(x) - h_2(x) \begin{cases} \in \mathbb{R}, & x \in \operatorname{dom} h_1 \cap \operatorname{dom} h_2, \\ = -\infty, & x \in \operatorname{dom} h_1 \setminus \operatorname{dom} h_2, \\ = +\infty, & x \notin \operatorname{dom} h_1; \end{cases} \tag{1.6}$$

hence,

$$h_1 - h_2 \text{ is proper} \iff \operatorname{dom} h_1 \subseteq \operatorname{dom} h_2. \tag{1.7}$$

As mentioned in [14], in the case when g and g_t, $t \in T$ are lsc, then the dual problem (1.5) is equivalent to the following problem

$$\sup_{\lambda \in \mathbb{R}_+^{(T)}} \inf_{x \in C} \left\{ f(x) - g(x) + \sum_{t \in T} \lambda_t \bigl(f_t(x) - g_t(x)\bigr) \right\}. \tag{1.8}$$

However, without assuming the lower semicontinuity of g and g_t, problems (1.8) and (D) are in general not equivalent.

The present paper is centered around the total Lagrangian duality for the DC problem (P) and its dual problem (D). For the problem of total Lagrange duality, one seeks conditions ensuring that the following implication holds for $x_0 \in \operatorname{dom}(f - g) \cap A$:

$$\begin{aligned} &\Bigl[f(x_0) - g(x_0) = \min_{x \in A} \bigl\{ f(x) - g(x) \bigr\}\Bigr] \\ &\quad \Longrightarrow \quad \bigl[\exists \lambda \in \mathbb{R}_+^{(T)}, \forall w^* \in H^*, L\bigl(w^*, \lambda\bigr) = f(x_0) - g(x_0)\bigr]. \end{aligned} \tag{1.9}$$

Clearly, the strong Lagrange duality ensures the total Lagrange duality, but the converse does not necessarily hold in general. To our knowledge, not many results are known to provide complete characterizations for the total Lagrangian duality for the DC optimization problem (1.3). Except the works in paper [6] by Fang *et al.*, where, assuming in addition that $g = g_t = 0$, $t \in T$, a complete characterization was established for the stable total Lagrangian duality for problem (1.3), that is, the characterization for (1.9) to hold for $f + p$ in place of f with any $p \in X^*$. However, the approaches in [6] do not work for the DC optimization problem (1.3).

In this paper, we do not impose any topological assumption on C or on f, g, f_t and g_t, that is, C is not necessarily closed, and f, g, f_t, $t \in T$ are not necessarily lsc, and g_t, $t \in T$ are necessarily differentiable. One of our main aims in the present paper is to use these constraint qualifications (or their variations) involving subdifferentials, which have been studied and extensively used, see, for example, [2, 3, 6, 12, 26], to provide characterizations for the total Lagrangian duality. Most of results obtained in the present paper seem new and are proper extensions of the results in [6] in the special case when $g = g_t = 0$, $t \in T$. In particular, both our dual problem and the regularity conditions introduced here are defined in terms of subdifferential of the convex functions f, g, f_t and g_t rather than those of the DC functions $f - g$ and $f_t - g_t$, which are different from the consideration in [6].

The paper is organized as follows. The next section contains the necessary notations and preliminary results. In Section 3, we provide some characterizations for the weak Lagrange dualities and the total Lagrangian dualities to hold.

2 Notations and preliminaries

The notations used in this paper are standard (*cf.* [25]). In particular, we assume throughout the whole paper that X is a real locally convex space, and let X^* denote the dual space of X. For $x \in X$ and $x^* \in X^*$, we write $\langle x^*, x \rangle$ for the value of x^* at x, that is, $\langle x^*, x \rangle := x^*(x)$. Let Z be a set in X. The closure of Z is denoted by $\operatorname{cl} Z$. If $W \subseteq X^*$, then $\operatorname{cl} W$ denotes the weak* closure of W. For the whole paper, we endow $X^* \times \mathbb{R}$ with the product topology of $w^*(X^*, X)$ and the usual Euclidean topology.

Following [27], we use $\mathbb{R}^{(T)}$ to denote the space of real tuples $\lambda = (\lambda_t)_{t\in T}$ with only finitely many $\lambda_t \neq 0$, and let $\mathbb{R}_+^{(T)}$ denote the nonnegative cone in $\mathbb{R}^{(T)}$, that is,

$$\mathbb{R}_+^{(T)} := \{(\lambda_t)_{t\in T} \in \mathbb{R}^{(T)} : \lambda_t \geq 0 \text{ for each } t \in T\}.$$

The normal cone of the nonempty set Z at $z_0 \in Z$ is denoted by $N_Z(z_0)$ and is defined by

$$N_Z(z_0) = \{x^* \in X^* : \langle x^*, z - z_0\rangle \leq 0 \text{ for all } z \in Z\},$$

and the indicator function δ_Z of Z is defined by

$$\delta_Z(z) := \begin{cases} 0, & z \in Z, \\ +\infty, & \text{otherwise.} \end{cases}$$

Let f be a proper function defined on X. We use $\operatorname{dom} f$, $\operatorname{epi} f$ and f^* to denote respectively the effective domain, the epigraph and the conjugate function of f, that is,

$$\operatorname{dom} f := \{x \in X : f(x) < +\infty\},$$
$$\operatorname{epi} f := \{(x, r) \in X \times \mathbb{R} : f(x) \leq r\}$$

and

$$f^*(x^*) := \sup\{\langle x^*, x\rangle - f(x) : x \in X\} \quad \text{for each } x^* \in X^*.$$

Let $x \in X$. The subdifferential of f at x is defined by

$$\partial f(x) := \{x^* \in X^* : f(x) + \langle x^*, y - x\rangle \leq f(y) \text{ for each } y \in X\} \tag{2.1}$$

if $x \in \operatorname{dom} f$, and $\partial f(x) := \emptyset$, otherwise. Then by definition,

$$N_Z(x) = \partial\delta_Z(x) \quad \text{for each } x \in Z. \tag{2.2}$$

By [25, Theorems 2.3.1 and 2.4.2(iii)], the Young-Fenchel inequality below holds

$$f(x) + f^*(x^*) \geq \langle x, x^*\rangle \quad \text{for each pair } (x, x^*) \in X \times X^*, \tag{2.3}$$

and the Young equality holds

$$f(x) + f^*(x^*) = \langle x^*, x\rangle \quad \text{if and only if} \quad x^* \in \partial f(x). \tag{2.4}$$

Furthermore, if g, h are proper functions, then

$$g \leq h \quad \Rightarrow \quad g^* \geq h^* \quad \Leftrightarrow \quad \operatorname{epi} g^* \subseteq \operatorname{epi} h^* \tag{2.5}$$

and

$$\partial g(a) + \partial h(a) \subseteq \partial(g + h)(a) \quad \text{for each } a \in \operatorname{dom} g \cap \operatorname{dom} h. \tag{2.6}$$

The closure of f is denoted by $\mathrm{cl} f$, which is defined by

$$\mathrm{epi}(\mathrm{cl} f) = \mathrm{cl}(\mathrm{epi} f).$$

Then (*cf.* [25, Theorems 2.3.1]),

$$f^* = (\mathrm{cl} f)^*. \tag{2.7}$$

By [25, Theorem 2.3.4], if $\mathrm{cl} f$ is proper and convex, then the following equality holds:

$$f^{**} = \mathrm{cl} f. \tag{2.8}$$

Moreover, by [25, Theorem 2.4.1], if $\partial f(x) \neq \emptyset$, then

$$(\mathrm{cl} f)(x) = f(x) \quad \text{and} \quad \partial(\mathrm{cl} f)(x) = \partial f(x). \tag{2.9}$$

Finally, note that an element $p \in X^*$ can be naturally regarded as a function on X in such way that

$$p(x) := \langle p, x\rangle \quad \text{for each } x \in X. \tag{2.10}$$

Then the following facts are clear for any $a \in \mathbb{R}$ and a real-valued proper function f:

$$\mathrm{epi}(f + p + a)^* = \mathrm{epi} f^* + (p, -a), \tag{2.11}$$

and

$$\partial(f + p + a)(x) = \partial f(x) + p \quad \text{for each } x \in \mathrm{dom} f. \tag{2.12}$$

3 The total Lagrange dualities

Unless explicitly stated otherwise, let f, g, T, C, $\{f_t, g_t : t \in T\}$ and A be as in Section 1, namely, T is an index set, $C \subseteq X$ is a convex set, f, g, f_t, g_t, $t \in T$ are proper convex functions on X such that $f - g$ and $f_t - g_t$, $t \in T$ are proper, and A is the solution set of the following system:

$$x \in C; \quad f_t(x) - g_t(x) \leq 0 \quad \text{for each } t \in T. \tag{3.1}$$

Then by (1.7), we have that

$$\emptyset \neq \mathrm{dom} f \subseteq \mathrm{dom}\, g \quad \text{and} \quad \emptyset \neq \mathrm{dom} f_t \subseteq \mathrm{dom}\, g_t. \tag{3.2}$$

To avoid the triviality, we always assume that $A \cap \mathrm{dom}(f - g) \neq \emptyset$. For simplicity, we denote

$$H^* := \mathrm{dom}\, g^* \times \prod_{t \in T} \mathrm{dom}\, g_t^* \tag{3.3}$$

and

$$\partial H(x) := \partial g(x) \times \prod_{t\in T} \partial g_t(x) \quad \text{for each } x \in X. \tag{3.4}$$

To make the dual problem considered here well defined, we further assume that $\operatorname{cl} g$ and $\operatorname{cl} g_t$, $t \in T$ are proper. Then $H^* \neq \emptyset$. For the whole paper, any elements $\lambda \in \mathbb{R}^{(T)}$ and $v^* \in \prod_{t\in T} \operatorname{dom} g_t^*$ are understood as $\lambda = (\lambda_t) \in \mathbb{R}^{(T)}$ and $v^* = (v_t^*) \in \prod_{t\in T} \operatorname{dom} g_t^*$, respectively.

Replacing f by all of its linear perturbed functions $f - p$, where $p \in X^*$, we consider the following DC infinite optimization problem

$$(P_p) \quad \begin{array}{ll} \text{Min} & f(x) - g(x) - \langle p, x\rangle, \\ \text{s.t.} & f_t(x) - g_t(x) \le 0, \quad t \in T, \\ & x \in C \end{array} \tag{3.5}$$

and its dual problem

$$(D_p) \quad \sup_{\lambda\in\mathbb{R}_+^{(T)}} \inf_{w^*\in H^*} L_p\left(w^*, \lambda\right), \tag{3.6}$$

where the Lagrange function $L_p : H^* \times \mathbb{R}_+^{(T)} \to \overline{\mathbb{R}}$ is defined by

$$L_p\left(w^*, \lambda\right) := g^*\left(u^*\right) + \sum_{t\in T} \lambda_t g_t^*\left(v_t^*\right) - \left(f - p + \delta_C + \sum_{t\in T} \lambda_t f_t\right)^* \left(u^* + \sum_{t\in T} \lambda_t v_t^*\right) \tag{3.7}$$

for any $(w^*, \lambda) \in H^* \times \mathbb{R}_+^{(T)}$ with $w^* = (u^*, v^*) \in H^*$ and $\lambda = (\lambda_t) \in \mathbb{R}_+^{(T)}$. In particular, in the case when $p = 0$, problem (P_p), as well as its dual problem (D_p), are reduced to problem (P), and its dual problem (D) as defined in (1.3) and (1.5), respectively.

Let $v(P_p)$ and $v(D_p)$ denote the optimal values of (P_p) and (D_p), respectively. For each $p \in X^*$, we use $S(P_p)$ to denote the optimal solution set of (P_p). In particular, we write $S(P)$ for $S(P_0)$. Obviously, for each $p \in X^*$, $S(P_p) \subseteq A$. This section is devoted to the study of characterizing the total Lagrange dualities. Unlike the convexity case, the cases for DC optimization problems are more complicated. We begin with the following definition, where the notations of the weak Lagrange duality and the stable weak Lagrange duality were introduced in [14].

Definition 3.1 Let X_0 be a subset of X. Between problems (P) and (D), we say that

(i) the weak Lagrange duality holds if $v(D) \le v(P)$;
(ii) the stable weak Lagrange duality holds if $v(D_p) \le v(P_p)$ for each $p \in X^*$;
(iii) the stable X_0-total Lagrange duality holds if, for each $p \in X^*$, $v(P_p) = v(D_p)$ and problem (D_p) has an optimal solution provided that $S(P_p) \cap X_0 \neq \emptyset$. In particular, in the case when $X_0 = X$, the stable X_0-total duality is called the stable total duality.

Unlike the convexity case, the weak Lagrange duality does not necessarily hold in general as shown in [14, Example 3.1]. In order to provide some sufficient conditions ensuring the weak Lagrange duality, we consider the following optimization problem, which plays a bridging role for our study:

$$\left(P_p^{\text{cl}}\right) \quad \begin{array}{ll} \text{Min} & f(x) - (\text{cl}\, g)(x) - \langle p, x\rangle, \\ \text{s.t.} & f_t(x) - (\text{cl}\, g_t)(x) \le 0, \quad t \in T, \\ & x \in C, \end{array} \tag{3.8}$$

where $p \in X^*$. Let A^{cl} denote the solution set of the system $\{x \in C; f_t(x) - (\text{cl}\, g_t)(x) \le 0, t \in T\}$, that is,

$$A^{\text{cl}} := \left\{x \in C : f_t(x) - (\text{cl}\, g_t)(x) \le 0 \text{ for each } t \in T\right\}. \tag{3.9}$$

Then, $A^{\text{cl}} \subseteq A$. As usual, we use $\nu(P_p^{\text{cl}})$ to define the optimal value of problem (P_p^{cl}). Then,

$$\nu(P_p) \le \nu\left(P_p^{\text{cl}}\right) \quad \text{for each } p \in X^*. \tag{3.10}$$

Moreover, by [14, (1.5)], we see that

$$\nu(D_p) \le \nu\left(P_p^{\text{cl}}\right) \quad \text{for each } p \in X^*. \tag{3.11}$$

Thus, if g and g_t, $t \in T$, are lsc, then the weak Lagrange duality holds. The following proposition provides a weaker condition for the weak Lagrange duality to hold.

Proposition 3.1 *Let $x_0 \in S(P)$. Suppose that g and each g_t are lsc at x_0. Then the weak Lagrange duality holds.*

Proof Since $x_0 \in S(P)$, it follows that

$$\nu(P) = f(x_0) - g(x_0) + \delta_A(x_0) = f(x_0) - g(x_0). \tag{3.12}$$

Note that g and each g_t are lsc at x_0. Then for each $x \in X$,

$$\begin{aligned} f(x_0) - (\text{cl}\, g)(x_0) + \delta_{A^{\text{cl}}}(x_0) &= f(x_0) - g(x_0) + \delta_A(x_0) \\ &\le f(x) - g(x) + \delta_A(x) \\ &\le f(x) - (\text{cl}\, g)(x) + \delta_{A^{\text{cl}}}(x), \end{aligned}$$

the last inequality holds because $\text{cl}\, g \le g$ and $A^{\text{cl}} \subseteq A$. This implies that $\nu(P) = \nu(P^{\text{cl}})$. Hence, by (3.11), one gets $\nu(D) \le \nu(P)$ and the proof is complete. □

Let Ω_0 denote the set of all points $x \in X$ such that $\partial H(x) \ne \emptyset$. Below we will make use of the subdifferential $\partial h(x)$ for a general proper function (not necessarily convex) $h : X \to \overline{\mathbb{R}}$; see (2.1). Clearly, the following equivalence holds:

$$x_0 \text{ is a minimizer of } h \text{ if and only if } 0 \in \partial h(x_0). \tag{3.13}$$

Form (2.9), if $x_0 \in \Omega_0$, then g and each g_t are lsc at x_0. Hence, the following corollary follows from Proposition 3.1 directly.

Corollary 3.1 *Let $p \in X^*$. If $S(P_p) \cap \Omega_0 \neq \emptyset$, then $v(D_p) \leq v(P_p)$.*

Motivating by [20], we introduce the following condition (LSC) to characterize the relationships between (P) and (P^{cl}) and the weak Lagrange duality.

Definition 3.2 The family $\{f, g, \delta_C; f_t, g_t : t \in T\}$ is said to satisfy the lower semi-continuity closure (LSC) if

$$\operatorname{epi}(f - g + \delta_A)^* = \operatorname{epi}(f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}})^*. \tag{3.14}$$

Remark 3.1

(a) Since $\operatorname{cl} g \leq g$ and $A^{\mathrm{cl}} \subseteq A$, it follows that $f - g + \delta_A \leq f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}}$. Hence, by (2.5), the family $\{f, g, \delta_C; f_t, g_t : t \in T\}$ satisfies the (LSC) if and only if

$$\operatorname{epi}(f - g + \delta_A)^* \supseteq \operatorname{epi}(f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}})^*. \tag{3.15}$$

(b) Obviously, if g and g_t, $t \in T$ are lsc, then the (LSC) holds. But the converse is not true, in general, as to be shown by Example 3.1 below.

Example 3.1 Let $X = C := \mathbb{R}$, and let $T := \{1\}$. Let $f, g, f_1, g_1 : \mathbb{R} \to \overline{\mathbb{R}}$ be defined respectively by

$$f(x) := \begin{cases} 0, & x \geq 0, \\ 2, & x = 0, \\ +\infty, & x < 0, \end{cases} \qquad g(x) := \begin{cases} 0, & x > 0, \\ 1, & x = 0, \\ +\infty, & x < 0 \end{cases} \quad \text{for each } x \in \mathbb{R},$$

$f_1 := \delta_{[0,+\infty)}$ and $g_1 := 0$. Then f, g, f_1, g_1 are proper convex functions and

$$\operatorname{epi} f^* = \operatorname{epi} g^* = (-\infty, 0] \times [0, +\infty).$$

Moreover, it is easy to see that $A = A^{\mathrm{cl}} = [0, +\infty)$ and

$$f - g + \delta_A = g, \qquad f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}} = f.$$

Hence,

$$\operatorname{epi}(f - g + \delta_A)^* = \operatorname{epi}(f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}})^* = (-\infty, 0] \times [0, +\infty).$$

This implies that the (LSC) holds. However, the function g is not lsc at $x = 0$.

The following proposition gives an equivalent condition to ensure that

$$v(P_p) = v\big(P_p^{\mathrm{cl}}\big) \quad \text{for each } p \in X^*, \tag{3.16}$$

in terms of the (LSC). For this purpose, we first give the following lemma by the definition of conjugate functions. The proof is standard (*cf.* [14, Lemma 4.1]), and so we omit it.

Lemma 3.1 *Let $p \in X^*$, and let $r \in \mathbb{R}$. Then the following statements hold*:

$$(p,r) \in \operatorname{epi}(f - g + \delta_A)^* \iff \nu(P_p) \geq -r; \tag{3.17}$$

$$(p,r) \in \operatorname{epi}(f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}})^* \iff \nu\big(P_p^{\mathrm{cl}}\big) \geq -r. \tag{3.18}$$

Proposition 3.2 *The family $\{f, g, \delta_C; f_t, g_t : t \in T\}$ satisfies the (LSC) if and only if* (3.16) *holds. Consequently, if the (LSC) holds, then the weak Lagrange duality holds.*

Proof Suppose that the (LSC) holds. Then (3.14) holds. Let $p \in X^*$. To show that $\nu(P_p) = \nu(P_p^{\mathrm{cl}})$, it suffices to show that $\nu(P_p) \geq \nu(P_p^{\mathrm{cl}})$ by (3.10). To do this, suppose, on the contrary, that $\nu(P_p) < \nu(P_p^{\mathrm{cl}})$. Then there exists $r \in \mathbb{R}$ such that $\nu(P_p) < -r < \nu(P_p^{\mathrm{cl}})$. Thus, by (3.18), $(p,r) \in \operatorname{epi}(f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}})^*$, and so $(p,r) \in \operatorname{epi}(f - g + \delta_A)^*$ by (3.14). It follows from (3.17) that $\nu(P_p) \geq -r$. This contradicts $\nu(P_p) < -r$ and completes the proof of the inequality $\nu(P_p) \geq \nu(P_p^{\mathrm{cl}})$.

Conversely, suppose that (3.16) holds. By Remark 3.1(b), it suffices to show that (3.15) holds. To do this, let $(p,r) \in \operatorname{epi}(f - \operatorname{cl} g + \delta_{A^{\mathrm{cl}}})^*$. Then, by (3.18), $\nu(P_p^{\mathrm{cl}}) \geq -r$, and so $\nu(P_p) \geq -r$, thanks to (3.16). Hence, by (3.17), $(p,r) \in \operatorname{epi}(f - g + \delta_A)^*$. Therefore, (3.15) is proved. The proof is complete. □

The remainder of this paper is devoted to studying the stable total Lagrange duality between (P) and (D). For each $x \in X$, let $T(x)$ be the active index set of system (3.1), that is,

$$T(x) := \big\{t \in T : f_t(x) - g_t(x) = 0\big\}.$$

For simplicity, we define $N'(x)$ by

$$N'(x) := \bigcup_{\lambda \in \mathbb{R}_+^{(T)}} \left(\bigcap_{(u^*,\nu^*) \in H^*} \left(\partial \left(f + \delta_C + \sum_{t \in T(x)} \lambda_t f_t \right)(x) - u^* - \sum_{t \in T(x)} \lambda_t \nu_t^* \right) \right) \tag{3.19}$$

and $N_0'(x)$ by

$$N_0'(x) := \bigcup_{\lambda \in \mathbb{R}_+^{(T)}} \left(\bigcap_{(u^*,\nu^*) \in \partial H(x)} \left(\partial \left(f + \delta_C + \sum_{t \in T(x)} \lambda_t f_t \right)(x) - u^* - \sum_{t \in T(x)} \lambda_t \nu_t^* \right) \right), \tag{3.20}$$

where, following [25, p.2], we adapt the convention that $\bigcap_{t \in \emptyset} S_t = X$. Then for each $x \in X$,

$$N'(x) \subseteq N_0'(x).$$

The following proposition provides an estimate for the subdifferential of the DC function $f - g + \delta_A$ in terms of the subdifferentials of the convex functions involved.

Proposition 3.3 *Suppose that the family $\{f, g, \delta_C; f_t, g_t : t \in T\}$ satisfies the (LSC). Then for each $x_0 \in \Omega_0$,*

$$N'(x_0) \subseteq \partial(f - g + \delta_A)(x_0). \tag{3.21}$$

Proof Let $x_0 \in \Omega_0$ and $p \in N'(x_0)$. Then there exists $\lambda \in \mathbb{R}_+^{(T)}$ such that for each $(u^*, v^*) \in H^*$,

$$0 \in \partial\left(f + \delta_C + \sum_{t\in T(x_0)} \lambda_t f_t\right)(x_0) - p - u^* - \sum_{t\in T(x_0)} \lambda_t v_t^*.$$

Let $x \in X$. Then for each $(u^*, v^*) \in H^*$,

$$\begin{aligned}&\left(f + \delta_C + \sum_{t\in T(x_0)} \lambda_t f_t\right)(x_0) - \left\langle p + u^* + \sum_{t\in T(x_0)} \lambda_t v_t^*, x_0\right\rangle \\ &\quad \le \left(f + \delta_C + \sum_{t\in T(x_0)} \lambda_t f_t\right)(x) - \left\langle p + u^* + \sum_{t\in T(x_0)} \lambda_t v_t^*, x\right\rangle;\end{aligned}$$

hence,

$$\begin{aligned}&f(x_0) + \delta_C(x_0) + \sum_{t\in T(x_0)} \lambda_t f_t(x_0) - \langle p, x_0\rangle - \left\{\langle u^*, x_0\rangle - g^*\left(u^*\right)\right\} \\ &\quad - \sum_{t\in T(x_0)} \lambda_t\left\{\langle v_t^*, x_0\rangle - g_t^*\left(v_t^*\right)\right\} \\ &\quad \le f(x) + \delta_C(x) + \sum_{t\in T(x_0)} \lambda_t f_t(x) - \langle p, x\rangle - \left\{\langle u^*, x\rangle - g^*\left(u^*\right)\right\} \\ &\quad - \sum_{t\in T(x_0)} \lambda_t\left\{\langle v_t^*, x\rangle - g_t^*\left(v_t^*\right)\right\}.\end{aligned}$$

Taking the infimum over H^*, we get that

$$\begin{aligned}&f(x_0) + \delta_C(x_0) + \sum_{t\in T(x_0)} \lambda_t f_t(x_0) - \langle p, x_0\rangle - (\mathrm{cl}\, g)(x_0) - \sum_{t\in T(x_0)} \lambda_t(\mathrm{cl}\, g_t)(x_0) \\ &\quad \le f(x) + \delta_C(x) + \sum_{t\in T(x_0)} \lambda_t f_t(x) - \langle p, x_0\rangle - (\mathrm{cl}\, g)(x) - \sum_{t\in T(x_0)} \lambda_t(\mathrm{cl}\, g_t)(x).\end{aligned} \tag{3.22}$$

Since $x_0 \in \Omega_0$, it follows from (2.9) that $(\mathrm{cl}\, g)(x_0) = g(x_0)$ and $(\mathrm{cl}\, g_t)(x_0) = g_t(x_0)$ for each $t \in T$. Note that $x_0 \in A^{\mathrm{cl}}$ and $f_t(x_0) - g_t(x_0) = 0$ for each $t \in T(x_0)$. Then, by (3.22), one has that for each $x \in X$,

$$\begin{aligned}&f(x_0) - (\mathrm{cl}\, g)(x_0) - \langle p, x_0\rangle + \delta_{A^{\mathrm{cl}}}(x_0) \\ &\quad \le f(x) - (\mathrm{cl}\, g)(x) - \langle p, x\rangle + \delta_C(x) + \sum_{t\in T(x_0)} \lambda_t(f_t - \mathrm{cl}\, g_t)(x) \\ &\quad \le f(x) - (\mathrm{cl}\, g)(x) - \langle p, x\rangle + \delta_{A^{\mathrm{cl}}}(x).\end{aligned}$$

Hence,

$$v\left(P_p^{\mathrm{cl}}\right) = f(x_0) - g(x_0) - \langle p, x_0\rangle + \delta_A(x_0). \tag{3.23}$$

Moreover, by Proposition 3.2, the (LSC) implies that $v(P_p^{\mathrm{cl}}) = v(P_p)$. This together with (3.23) implies that for each $x \in X$,

$$f(x_0) - g(x_0) - \langle p, x_0\rangle + \delta_A(x_0) \le f(x) - g(x) - \langle p, x\rangle + \delta_A(x).$$

Hence, $p \in \partial(f - g + \delta_A)(x_0)$, and inclusion (3.21) holds. □

Considering the possible inclusions among $\partial(f-g+\delta_A)(x)$, $N'(x)$ and $N_0'(x)$, we introduce the following definition.

Definition 3.3 The family $\{f, g, \delta_C; f_t, g_t : t \in T\}$ is said to satisfy

(a) the quasi weakly basic constraint qualification (the quasi (WBCQ)) at $x \in A$ if

$$\partial(f - g + \delta_A)(x) \subseteq N_0'(x); \tag{3.24}$$

(b) the weakly basic constraint qualification (the (WBCQ)) at $x \in A$ if

$$\partial(f - g + \delta_A)(x) \subseteq N'(x). \tag{3.25}$$

We say that the family $\{f, g, \delta_C; f_t, g_t : t \in T\}$ satisfies the quasi (WBCQ) (resp. the (WBCQ)) if it satisfies the quasi (WBCQ) (resp. the (WBCQ)) at each point $x \in A$.

Remark 3.2

(a) The following implication holds:

$$\text{the (WBCQ)} \quad \Longrightarrow \quad \text{the quasi (WBCQ)}.$$

(b) In the special case, when $g = g_t = 0$, $t \in T$, the quasi (WBCQ) and (WBCQ) are reduced to the $(\mathrm{WBCQ})_f$ for the family $\{\delta_C; f_t : t \in T\}$ introduced in [6].

For our main theorems in this section, the following lemma is helpful.

Lemma 3.2 *Let $x_0 \in X$ and $p \in X^*$ with $x_0 \in S(P_p)$. If $p \in N'(x_0)$, then there exists $\overline{\lambda} \in \mathbb{R}_+^{(T)}$ such that for each $w^* = (u^*, v^*) \in H^*$,*

$$L_p\bigl(w^*, \overline{\lambda}\bigr) \ge v(P_p). \tag{3.26}$$

Proof Let $p \in N'(x_0)$. Then there exists $\overline{\lambda} = (\overline{\lambda}_t)_{t\in T} \in \mathbb{R}_+^{(T)}$ with

$$\sum_{t\in T} \overline{\lambda}_t (f_t - g_t)(x_0) = 0 \tag{3.27}$$

such that for each $(u^*, v^*) \in H^*$,

$$p \in \partial\left(f + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\right)(x_0) - u^* - \sum_{t\in T} \overline{\lambda}_t v_t^*.$$

Let $(u^*, v^*) \in H^*$. Then there exists $x^* \in \partial(f + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t)(x_0)$ such that

$$p = x^* - u^* - \sum_{t\in T} \overline{\lambda}_t v_t^*. \tag{3.28}$$

By the Young equality (2.6),

$$\langle x^*, x_0\rangle = \Big(f + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\Big)^*(x^*) + \Big(f + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\Big)(x_0), \tag{3.29}$$

and by the Young-Fenchel inequality (2.3),

$$\langle u^*, x_0\rangle \le g^*(u^*) + g(x_0) \quad \text{and} \quad \langle v_t^*, x_0\rangle \le g_t^*(u^*) + g_t(x_0) \quad \text{for each } t \in T. \tag{3.30}$$

Combining (3.28), (3.29) with (3.30), we have

$$\begin{aligned}
L_p(w^*, \overline{\lambda}) &= g^*(u^*) + \sum_{t\in T} \overline{\lambda}_t g_t^*(u_t^*) - \Big(f + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\Big)^*\Big(p + u^* + \sum_{t\in T} \overline{\lambda}_t v_t^*\Big) \\
&= g^*(u^*) + \sum_{t\in T} \overline{\lambda}_t g_t^*(u_t^*) - \Big(f + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\Big)^*(x^*) \\
&\ge \langle u^*, x_0\rangle - g(x_0) + \sum_{t\in T} \overline{\lambda}_t\big(\langle v_t^*, x_0\rangle - g_t(x_0)\big) - \langle x^*, x_0\rangle \\
&\quad + \Big(f + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\Big)(x_0) \\
&= f(x_0) - g(x_0) - \langle p, x_0\rangle + \delta_C(x_0) + \sum_{t\in T} \overline{\lambda}_t\big(f_t(x_0) - g_t(x_0)\big) \\
&= f(x_0) - g(x_0) - \langle p, x_0\rangle,
\end{aligned}$$

where the last equality holds because of (3.27) and $x_0 \in A$. Since $x_0 \in S(P_p)$, it follows that (3.26) holds. The proof is complete. □

The following theorem provides a sufficient condition and a necessary condition for the stable Ω_0-total Lagrange duality.

Theorem 3.1 *Consider the following assertions*:

(i) *The family* $\{f, g, \delta_C; f_t, g_t : t \in T\}$ *satisfies the* (*WBCQ*).

(ii) *The stable* Ω_0*-total Lagrange duality holds between* (*P*) *and* (*D*).

(iii) *The family* $\{f, g, \delta_C; f_t, g_t : t \in T\}$ *satisfies the quasi* (*WBCQ*).

Then (i) ⇒ (ii) ⇒ (iii).

Proof (i) ⇒ (ii) Suppose that (i) holds. Let $p \in X^*$ be such that $S(P_p) \cap \Omega_0 \neq \emptyset$. Take $x_0 \in S(P_p) \cap \Omega_0$. Then

$$p \in \partial(f - g + \delta_A)(x_0) \subseteq N'(x_0),$$

thanks to the assumed (WBCQ). Thus, by Lemma 3.2, we get that there exists $\overline{\lambda} \in \mathbb{R}_+^{(T)}$ such that (3.26) holds for each $w^* = (u^*, v^*) \in H^*$. Moreover, we have that $v(P_p) \ge v(D_p)$ by

Corollary 3.1. Thus, $v(D_p) = v(P_p)$ and $\overline{\lambda}$ is an optimal solution of (D_p). This implies that the stable Ω_0-total Lagrange duality holds.

(ii) ⇒ (iii) Suppose that (ii) holds. Let $x_0 \in A$. Obviously, if $\partial H(x_0) = \emptyset$, then the quasi (WBCQ) holds trivially because $N'_0(x_0) = X^*$. Below, we assume that $\partial H(x_0) \neq \emptyset$. Let $p \in \partial(f - g + \delta_A)(x_0)$. Then by (3.13), we have that $x_0 \in S(P_p)$, and hence $x_0 \in S(P_p) \cap \Omega_0$. By the assumed Ω_0-total Lagrange duality, there exists $\overline{\lambda} = (\overline{\lambda})_{t\in T} \in \mathbb{R}_+^{(T)}$ such that for each $(u^*, v^*) \in \partial H(x_0)$,

$$g^*(u^*) + \sum_{t\in T} \overline{\lambda}_t g_t^*(u_t^*) - \left(f - p + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\right)^* \left(u^* + \sum_{t\in T} \overline{\lambda}_t v_t^*\right) \geq v(P_p).$$

Noting that $v(P_p) = f(x_0) - g(x_0) - \langle p, x_0\rangle$, we have

$$\begin{aligned}&\left(f - p + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\right)^* \left(u^* + \sum_{t\in T} \overline{\lambda}_t v_t^*\right) + f(x_0) - \langle p, x_0\rangle \\ &\quad \leq g^*(u^*) + g(x_0) + \sum_{t\in T} \overline{\lambda}_t g_t^*(v_t^*). \end{aligned} \tag{3.31}$$

Since by the Young-Fenchel inequality (2.3),

$$\begin{aligned}&\left(f - p + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\right)^* \left(u^* + \sum_{t\in T} \overline{\lambda}_t v_t^*\right) \\ &\quad \geq \left\langle u^* + \sum_{t\in T} \overline{\lambda}_t v_t^* + p, x_0\right\rangle - f(x_0) - \sum_{t\in T} \overline{\lambda}_t f_t(x_0); \end{aligned} \tag{3.32}$$

and by the Young equality (2.6),

$$g^*(u^*) + g(x_0) = \langle u^*, x_0\rangle \quad \text{and} \quad g_t^*(v_t^*) + g_t(x_0) = \langle v_t^*, x_0\rangle \quad \text{for each } t \in T, \tag{3.33}$$

it follows that

$$\begin{aligned} v(P_p) &= f(x_0) - g(x_0) - \langle p, x_0\rangle \\ &\leq g^*(u^*) + \sum_{t\in T} \overline{\lambda}_t g_t^*(v_t^*) - \left(f - p + \delta_C + \sum_{t\in T} \overline{\lambda}_t f_t\right)^* \left(u^* + \sum_{t\in T} \overline{\lambda}_t v_t^*\right) \\ &\leq \left(\langle u^*, x_0\rangle - g(x_0)\right) + \sum_{t\in T} \overline{\lambda}_t\left(\langle v_t^*, x_0\rangle - g_t(x_0)\right) - \left\langle u^* + \sum_{t\in T} \overline{\lambda}_t v_t^* + p, x_0\right\rangle \\ &\quad + f(x_0) + \sum_{t\in T} \overline{\lambda}_t f_t(x_0) \\ &= f(x_0) - g(x_0) - \langle p, x_0\rangle + \sum_{t\in T} \overline{\lambda}_t\left(f_t(x_0) - g_t(x_0)\right) \\ &\leq f(x_0) - g(x_0) - \langle p, x_0\rangle, \end{aligned}$$

where the last inequality holds because $x_0 \in A$. Thus,

$$\sum_{t\in T} \overline{\lambda}_t\left(f_t(x_0) - g_t(x_0)\right) = 0. \tag{3.34}$$

Moreover, by (3.31) and (3.32), we have

$$\begin{aligned}
0 &\le \left(f+\delta_C+\sum_{t\in T}\overline{\lambda}_t f_t\right)^*\left(u^*+\sum_{t\in T}\overline{\lambda}_t v_t^*+p\right) \\
&\quad+\left(f+\delta_C+\sum_{t\in T}\overline{\lambda}_t f_t\right)(x_0)-\left\langle u^*+\sum_{t\in T}\overline{\lambda}_t v_t^*+p,x_0\right\rangle \\
&\le g^*(u^*)+g(x_0)+\sum_{t\in T}\overline{\lambda}_t g_t^*(v_t^*)+\sum_{t\in T}\overline{\lambda}_t f_t(x_0)-\langle u^*,x_0\rangle-\sum_{t\in T}\overline{\lambda}_t\langle v_t^*,x_0\rangle \\
&= g^*(u^*)+g(x_0)-\langle u^*,x_0\rangle+\sum_{t\in T}\overline{\lambda}_t\bigl(g_t^*(v_t^*)+g_t(x_0)-\langle v_t^*,x_0\rangle\bigr)+\sum_{t\in T}\overline{\lambda}_t\bigl(f_t(x_0)-g_t(x_0)\bigr) \\
&= 0,
\end{aligned}$$

where the last equality holds by (3.33) and (3.34). This implies that

$$\begin{aligned}
&\left(f+\delta_C+\sum_{t\in T}\overline{\lambda}_t f_t\right)^*\left(u^*+\sum_{t\in T}\overline{\lambda}_t v_t^*+p\right) \\
&\quad+\left(f+\delta_C+\sum_{t\in T}\overline{\lambda}_t f_t\right)(x_0)-\left\langle u^*+\sum_{t\in T}\overline{\lambda}_t v_t^*+p,x_0\right\rangle=0.
\end{aligned}$$

Hence, by the Young equality (2.4),

$$u^*+\sum_{t\in T}\overline{\lambda}_t v_t^*+p\in\partial\left(f+\delta_C+\sum_{t\in T}\overline{\lambda}_t f_t\right)(x_0).$$

Consequently,

$$p=\left(u^*+\sum_{t\in T}\overline{\lambda}_t v_t^*+p\right)-u^*-\sum_{t\in T}\overline{\lambda}_t v_t^*\in\partial\left(f+\delta_C+\sum_{t\in T}\overline{\lambda}_t f_t\right)(x_0)-u^*-\sum_{t\in T}\overline{\lambda}_t v_t^*$$

and

$$p\in\bigcap_{(u^*,v^*)\in\partial H(x_0)}\left(\partial\left(f+\delta_C+\sum_{t\in T}\overline{\lambda}_t f_t\right)(x_0)-u^*-\sum_{t\in T}\overline{\lambda}_t v_t^*\right)$$

as $(u^*,v^*)\in\partial H(x_0)$ is arbitrary. Hence, $p\in N_0'(x_0)$. Therefore, (3.24) holds, and the proof is complete. □

Theorem 3.2 below provides sufficient conditions ensuring the stable total Lagrange duality.

Theorem 3.2 *Suppose that the family $\{f,g,\delta_C;f_t,g_t:t\in T\}$ satisfies the (WBCQ), and that the stable weak Lagrange duality holds between (P) and (D). Then the stable total Lagrange duality holds.*

Proof Let $p\in X^*$. Suppose that $S(P_p)\neq\emptyset$. Let $x_0\in S(P_p)$. Then $p\in\partial(f-g+\delta_A)(x_0)$ and hence $p\in N'(x_0)$ by the assumed (WBCQ). Thus, Lemma 3.2 is applied to get that there exists $\overline{\lambda} \in \mathbb{R}_+^{(T)}$ such that (3.26) holds for each $w^* = (u^*, v^*) \in H^*$. This together with the stable weak Lagrange duality implies that $v(D_p) = v(P_p)$, and $\overline{\lambda}$ is an optimal solution of (D_p). Thus, the stable total Lagrange duality holds, and the proof is complete. □

In the case when $g = g_t = 0$, $t \in T$, by Theorem 3.1, we have the following corollary, which was given in [5, Theorem 5.2].

Corollary 3.2 *The family* $\{f, \delta_C; f_t : t \in T\}$ *satisfies the* $(WBCQ)_f$ *if and only if the following formula holds for each* $p \in X^*$ *satisfying* $S(P_p) \neq \emptyset$:

$$\min_{x \in A}\left\{f(x) - \langle p, x\rangle\right\} = \max_{\lambda \in \mathbb{R}_+^{(T)}} \inf_{x \in C}\left\{f(x) - \langle p, x\rangle + \sum_{t \in T} \lambda_t f_t(x)\right\}.$$

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
DF studied and researched the Total Lagrange duality for DC programming and also wrote this article. ZC participated in the process of the study and helped to draft the manuscript. All authors read and approved the final manuscript.

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