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Some results on continuous pseudo-contractions in a reflexive Banach space

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Abstract
In this paper, we investigate fixed point problems of a continuous pseudo-contraction based on a viscosity iterative scheme. Strong convergence theorems are established in a reflexive Banach space which also enjoys a weakly continuous duality mapping.

MSC: 47H05; 47H09; 47H10; 47J05; 47J25

Keywords: fixed point; Banach space; weakly continuous duality mapping; pseudo-contraction

1 Introduction and preliminaries

Fixed point problems of nonlinear operators, which include many important problems in nonlinear analysis and optimization such as the Nash equilibrium problem, variational inequalities, complementarity problems, vector optimization problems, and saddle point problems, recently have been studied as an effective and powerful tool for studying many real world problems which arise in economics, finance, medicine, image reconstruction, ecology, transportation, and network; see [1–20] and the references therein. Interest in pseudo-contractive operators, an important class of nonlinear operators, stems mainly from their firm connection with equations of evolution. It is known that many physically significant problems can be modeled by initial value problems of the form $x'(t) + x(t) - Ax(t) = 0$, $x(0) = x_0$, where A is a pseudo-contractive operator in the framework of Banach spaces. Typical examples where such evolution equations occur can be found in the heat, wave or Schrödinger equations. Fixed points of a pseudo-contractive operator have been investigated by many authors; see [21–32] and the references therein. For the existence of fixed points of pseudo-contractive operators, we refer readers to [31] and [32] and the references therein. In this paper, we consider the mean regularization viscosity method for treating fixed points of pseudo-contractive operators in a Banach space.

Let E be a real Banach space, E^* be the dual space of E. Let $\varphi : [0,\infty] := R^+ \to R^+$ be a continuous and strictly increasing function such that $\varphi(0) = 0$ and $\varphi(t) \to \infty$ as $t \to \infty$. This function φ is called a gauge function. The duality mapping $J_\varphi : E \to E^*$ associated with a gauge function φ is defined by

$$J_\varphi(x) = \{f^* \in E^* : \langle x, f^*\rangle = \|x\|\varphi(\|x\|), \|f^*\| = \varphi(\|x\|)\}, \quad \forall x \in E,$$

where $\langle\cdot,\cdot\rangle$ denotes the generalized duality pairing. In the case that $\varphi(t) = t$, we write J for J_φ and call J the normalized duality mapping.

Recall that a Banach space E is said to have a weakly continuous duality mapping if there exists a gauge φ for which the duality mapping $J_\varphi(x)$ is single-valued and weak-to-weak* sequentially continuous (*i.e.*, if $\{x_n\}$ is a sequence in E weakly convergent to a point x, then the sequence $J_\varphi(x_n)$ converges weakly* to J_φ). It is known that l^p has a weakly continuous duality mapping with a gauge function $\varphi(t) = t^{p-1}$ for all $1 < p < \infty$. Set $\Phi(t) = \int_0^t \varphi(\tau)\,d\tau$, $\forall t \geq 0$. Then $J_\varphi(x) = \partial\Phi(\|x\|)$, $\forall x \in X$, where ∂ denotes the sub-differential in the sense of convex analysis.

Let C be a nonempty closed and convex subset of E. Let D be a nonempty subset of C, and let Q be a mapping of C into D. Then Q is said to be sunny if

$$Q\big(Qx + t(x - Qx)\big) = Qx,$$

whenever $Qx + t(x - Qx) \in C$ for $x \in C$ and $t \geq 0$. A mapping Q of C into itself is called a retraction if $Q^2 = Q$. If a mapping Q of C into itself is a retraction, then $Qz = z$ for all $z \in R(Q)$, where $R(Q)$ is the range of Q. A nonempty subset D of C is called a sunny nonexpansive retract of C if there exists a sunny nonexpansive retraction from C onto D.

Let $T : C \to C$ be a nonlinear mapping. In this paper, we use $F(T)$ to denote the fixed point set of the mapping T. Recall that T is contractive with the coefficient $k \in (0,1)$ if

$$\|Tx - Ty\| \leq k\|x - y\|, \quad \forall x, y \in C.$$

The mapping T is said to be nonexpansive if the contractive coefficient $k = 1$.

One classical way to study nonexpansive mappings is to use contractions to approximate a nonexpansive mapping [21, 33, 34]. More precisely, take $t \in (0,1)$ and define a contraction $T_t : C \to C$ by

$$T_t x = tu + (1 - t)Tx, \quad \forall x \in C, \tag{1.1}$$

where $u \in C$ is a fixed point. Banach's contraction mapping principle guarantees that T_t has a unique fixed point x_t in C. In the case of T having a fixed point, Browder [1] proved that if E is a Hilbert space, then x_t converges strongly to a fixed point of T that is nearest to u. Reich [33] extended Browder's result to the setting of Banach spaces and proved that if E is a uniformly smooth Banach space, then x_t converges strongly to a fixed point of T and the limit defines the (unique) sunny nonexpansive retraction from C onto $F(T)$.

Xu [34] proved that Reich's results hold in reflexive Banach spaces which have a weakly continuous duality map. To be more precise, he proved the following theorem.

Theorem X *Let E be a reflexive Banach space and have a weakly continuous duality map J_φ with a gauge φ. Let C be a closed convex subset of E, and let $T : C \to C$ be a nonexpansive mapping. Fix $u \in C$ and $t \in (0,1)$. Let $x_t \in C$ be the unique solution in C to the equation*

$$x_t = tu + (1 - t)Tx_t.$$

Then T has a fixed point if and only if $\{x_t\}$ remains bounded as $t \to 0^+$, and in this case, $\{x_t\}$ converges as $t \to 0^+$ strongly to a fixed point of T.

Recall that the mapping $T : C \to C$ is strongly pseudo-contractive with the coefficient $k \in (0,1)$ if

$$\langle Tx - Ty, J_\varphi(x-y)\rangle \le k\|x-y\|\varphi(\|x-y\|), \quad \forall x,y \in C.$$

The mapping T is said to be pseudo-contractive if the coefficient $k = 1$.

Recently, Zegeye and Shahzad [35] improved Theorem X about the mapping T from nonexpansive mappings to pseudo-contractions by the viscosity approximation method which was first introduced by Moudafi [36]. More precisely, they proved the following results.

Theorem ZS *Let K be a nonempty closed and convex subset of a real Banach space E. Let $T : K \to E$ be a continuous pseudo-contractive mapping and $f : K \to E$ be a contraction (with constant β) both satisfying the weakly inward condition. Then, for $t \in (0,1)$, there exists a sequence $\{y_t\} \subset K$ satisfying the following condition:*

$$y_t = (1-t)f(y_t) + tTy_t.$$

Suppose further that $\{y_t\}$ is bounded or $(F(T) \neq \emptyset)$ and E is a reflexive Banach space having a weakly continuous duality mapping J_φ for some gauge φ. Then $\{y_t\}$ converges strongly to a fixed point of T.

We give some remarks about Moudafi's viscosity approximation method which was recently studied by many authors. From Suzuki [37], we know that Moudafi's viscosity approximation with a contraction is trivial. Since the mapping $P_C f$, where P_C is a metric projection from H onto its nonempty closed and convex subset C, is a contraction, we can get the conclusion if Browder's property is satisfied; see [37] for more details.

Next, we propose the following question.

What happens if the mapping f is a strong pseudo-contraction instead of a contraction? Does Theorem ZS still hold?

Since we do not know whether the mapping $P_C f$, where f is a strong pseudo-contraction, has a unique fixed point or not, we cannot answer the above question easily based on Suzuki's results.

It is our purpose in this paper to consider the convergence of paths for a continuous pseudo-contraction by Moudafi's viscosity approximation with continuous strong pseudo-contractions instead of contractions, which gives an answer to the above question.

To prove our main results, we need the following lemma and definitions.

Let C be a nonempty subset of a Banach space E. For $x \in C$, the inward set of x, $I_C(x)$, is defined by $I_C(x) := \{x + \lambda(u-x) : u \in C, \lambda \ge 1\}$. A mapping $T : C \to E$ is called weakly inward if $Tx \in cl[I_C(x)]$ for all $x \in C$, where $cl[I_C(x)]$ denotes the closure of the inward set. Every self-map is trivially weakly inward.

The first part of the next lemma is an immediate consequence of the subdifferential inequality and the proof of the second part can be found in [38].

Lemma 1.1 *Assume that a Banach space E has a weakly continuous duality mapping J_φ with a gauge φ.*

(i) *For all $x, y \in E$, the following inequality holds*:

$$\Phi(\|x+y\|) \le \Phi(\|x\|) + \langle y, J_\varphi(x+y)\rangle.$$

(ii) *Assume that a sequence $\{x_n\}$ in E converges weakly to a point $x \in E$. Then the following identity holds*:

$$\limsup_{n\to\infty} \Phi(\|x_n - y\|) = \limsup_{n\to\infty} \Phi(\|x_n - x\|) + \Phi(\|y-x\|), \quad \forall x, y \in E.$$

Lemma 1.2 [31] *Let E be a Banach space, C be a nonempty closed and convex subset of E and $T : C \to C$ be a continuous and strong pseudo-contraction. Then T has a unique fixed point in C.*

2 Main results

Lemma 2.1 *Let C be nonempty closed and convex subset of a real Banach space E, and let $T : C \to C$ be a continuous pseudo-contraction. Let $f : C \to C$ be a fixed continuous bounded and strong pseudo-contraction with the pseudo-contractive coefficient $k \in (0,1)$. For $t \in (0,1)$, define a mapping T_t^f by*

$$T_t^f x = tf(x) + (1-t)Tx, \quad x \in C. \tag{2.1}$$

Then

(i) (2.1) *has a unique solution $x_t \in C$ for every $t \in (0,1)$*;
(ii) *If $\{x_t\}$ is bounded, then $\|x_t - Tx_t\| \to 0$ as $t \to 0$*;
(iii) *If $F(T) \neq \emptyset$, where $F(T)$ denotes the fixed point set of T, then $\{x_t\}$ is bounded and satisfies*

$$\langle x_t - f(x_t), J_\varphi(x_t - p)\rangle \le 0, \quad \forall p \in F(T).$$

Proof (i) Indeed, for any $x, y \in C$ and $t \in (0,1)$, we have

$$\begin{aligned}
&\langle T_t^f x - T_t^f y, J_\varphi(x-y)\rangle \\
&\quad = t\langle f(x) - f(y), J_\varphi(x-y)\rangle + (1-t)\langle Tx - Ty, J_\varphi(x-y)\rangle \\
&\quad \le tk\|x-y\|\varphi(\|x-y\|) + (1-t)\|x-y\|\varphi(\|x-y\|) \\
&\quad = [1 - t(1-k)]\|x-y\|\varphi(\|x-y\|).
\end{aligned}$$

This shows that $T_t^f : C \to C$ is a continuous and strong pseudo-contraction for each $t \in (0,1)$. By Lemma 1.2, we obtain that T_t^f has a unique fixed point x_t in C for each $t \in (0,1)$. Therefore, one obtains that (i) holds.

(ii) It follows from (i) that

$$x_t = tf(x_t) + (1-t)Tx_t \tag{2.2}$$

for each $t \in (0,1)$. It follows from (2.2) that

$$\|x_t - Tx_t\| = t\|f(x_t) - Tx_t\|.$$

Noticing the boundedness of f and $\{x_t\}$, we have

$$\lim_{t\to 0}\|x_t - Tx_t\| = 0. \tag{2.3}$$

(iii) For $\forall p \in F(T)$, we see that

$$\begin{aligned}
&\|x_t - p\|\varphi\big(\|x_t - p\|\big)\\
&\quad = \big\langle x_t - p, J_\varphi(x_t - p)\big\rangle\\
&\quad = t\big\langle f(x_t) - p, J_\varphi(x_t - p)\big\rangle + (1-t)\big\langle Tx_t - p, J_\varphi(x_t - p)\big\rangle\\
&\quad \le t\big\langle f(x_t) - f(p), J_\varphi(x_t - p)\big\rangle + t\big\langle f(p) - p, J_\varphi(x_t - p)\big\rangle\\
&\qquad + (1-t)\big\langle Tx_t - p, J_\varphi(x_t - p)\big\rangle\\
&\quad \le tk\|x_t - p\|\varphi\big(\|x_t - p\|\big) + t\big\langle f(p) - p, J_\varphi(x_t - p)\big\rangle\\
&\qquad + (1-t)\|x_t - p\|\varphi\big(\|x_t - p\|\big)\\
&\quad = \big[1 - t(1-k)\big]\|x_t - p\|\varphi\big(\|x_t - p\|\big) + t\big\langle f(p) - p, J_\varphi(x_t - p)\big\rangle.
\end{aligned}$$

It follows that

$$\|x_t - p\|\varphi\big(\|x_t - p\|\big) \le \frac{1}{1-k}\big\langle f(p) - p, J_\varphi(x_t - p)\big\rangle. \tag{2.4}$$

That is, $\|x_t - p\| \le \frac{1}{1-k}\|f(p) - p\|$, $\forall t \in (0,1)$. This shows that $\{x_t\}$ is bounded. Noticing $x_t - f(x_t) = (1-t)(Tx_t - f(x_t))$, we arrive at

$$\begin{aligned}
&\big\langle x_t - f(x_t), J_\varphi\big(x_t - p\big)\big\rangle\\
&\quad = (1-t)\big\langle Tx_t - f(x_t), J_\varphi(x_t - p)\big\rangle\\
&\quad = (1-t)\big\langle Tx_t - x_t, J_\varphi(x_t - p)\big\rangle + (1-t)\big\langle x_t - f(x_t), J_\varphi(x_t - p)\big\rangle\\
&\quad = (1-t)\big(\big\langle Tx_t - p, J_\varphi(x_t - p)\big\rangle - \|x_t - p\|\varphi\big(\|x_t - p\|\big)\big)\\
&\qquad + (1-t)\big\langle x_t - f(x_t), J_\varphi(x_t - p)\big\rangle\\
&\quad \le (1-t)\big\langle x_t - f(x_t), J_\varphi(x_t - p)\big\rangle
\end{aligned}$$

and hence

$$\big\langle x_t - f(x_t), J_\varphi(x_t - p)\big\rangle \le 0, \quad \forall p \in F(T).$$

This completes the proof. □

Theorem 2.2 *Let E be a reflexive Banach space which has a weakly continuous duality mapping J_φ for some gauge φ, and let C be a nonempty closed and convex subset of E. Let $T : C \to C$ be a continuous pseudo-contraction and $f : C \to C$ be a fixed bounded, continuous and strong pseudo-contraction with the coefficient $k \in (0,1)$. Let $\{x_t\}$ be as in Lemma* 2.1.

$$x_t = tf(x_t) + (1-t)Tx_t.$$

If $\{x_t\}$ is bounded, then $\{x_t\}$ converges strongly to a fixed point p of T as $t \to 0$, which is the unique solution in $F(T)$ to the following variational inequality:

$$\langle f(p) - p, J_\varphi(y - p)\rangle \le 0, \quad \forall y \in F(T).$$

Proof Since $\{x_t\}$ is bounded and E is reflexive, there exists a subnet $\{x_{t_\alpha}\}$ of $\{x_t\}$ such that $x_{t_\alpha} \rightharpoonup p$ as $t_\alpha \to 0$. Put

$$g(x) = \limsup_{n\to\infty} \Phi\bigl(\|x_{t_\alpha} - x\|\bigr), \quad \forall x \in E. \tag{2.5}$$

It follows from Lemma 1.1 that

$$g(x) = g(p) + \Phi\bigl(\|x - p\|\bigr), \quad \forall x \in E. \tag{2.6}$$

For any $y \in C$, define a mapping $S : C \to C$ by

$$Sx = \frac{1}{2}y + \frac{1}{2}Tx, \quad \forall x \in C.$$

It is easy to see that S is a continuous strong pseudo-contraction. From Lemma 1.2, we see that S has a unique fixed point x in C, that is, $x = \frac{1}{2}y + \frac{1}{2}Tx$. This implies that

$$y = (2I - T)x \in (2I - T)(C).$$

This shows that $C \subseteq (2I - T)(C)$. Define another mapping $H : C \to C$ by

$$Hx = (2I - T)^{-1}x, \quad \forall x \in C.$$

We see that H is a nonexpansive mapping. Indeed, for any $x, y \in C$, we have

$$\begin{aligned}\|Hx - Hy\| &= \bigl\|(2I - T)^{-1}x - (2I - T)^{-1}y\bigr\| \\ &\le \bigl\|\bigl(I + (I - T)\bigr)^{-1}x - \bigl(I + (I - T)\bigr)^{-1}y\bigr\| \\ &\le \|x - y\|.\end{aligned}$$

We also see that $F(H) = F(T)$. Indeed,

$$x = Hx \quad \Longleftrightarrow \quad 2x - Tx = x \quad \Longleftrightarrow \quad x = Tx.$$

From Lemma 2.1, we obtain that

$$\|x_{t_\alpha} - Tx_{t_\alpha}\| \to 0 \tag{2.7}$$

as $t_\alpha \to 0$. On the other hand, for $\forall x \in C$, we have

$$\|x - Hx\| = \bigl\|HH^{-1}x - Hx\bigr\| \le \bigl\|H^{-1}x - x\bigr\| = \bigl\|(2I - T)x - x\bigr\| = \|x - Tx\|.$$

It follows from (2.7) that $\|x_{t_\alpha} - Hx_{t_\alpha}\| \to 0$, as $t_\alpha \to 0$. From (2.5), we arrive at

$$\begin{aligned} g(Hp) &= \limsup_{n\to\infty} \Phi\bigl(\|x_{t_\alpha} - Hp\|\bigr) = \limsup_{n\to\infty} \Phi\bigl(\|Hx_{t_\alpha} - Hp\|\bigr) \\ &\le \limsup_{n\to\infty} \Phi\bigl(\|x_{t_\alpha} - p\|\bigr) = g(p). \end{aligned} \tag{2.8}$$

On the other hand, from (2.6), we obtain that $g(Hp) = g(p) + \Phi(\|Hp - p\|)$. It follows from (2.8) that $\Phi(\|Hp - p\|) = 0$. Hence, $p \in F(H) = F(T)$.

Next, we show that x_{t_α} converges strongly to p. From (2.4), we see that

$$\|x_{t_\alpha} - p\|\varphi\bigl(\|x_{t_\alpha} - p\|\bigr) \le \frac{1}{1-k}\bigl\langle f(p) - p, J_\varphi(x_{t_\alpha} - p)\bigr\rangle.$$

It follows that $x_{t_\alpha} \to p$.

Finally, we show that the entire net $\{x_t\}$ converges strongly to p. If there exists another subset $\{x_{t_\beta}\}$ of $\{x_t\}$ such that $\{x_{t_\beta}\} \to q$, then q is also a fixed point of T. By using (iii) of Lemma 2.1, we have

$$\bigl\langle p - f(p), J_\varphi(p - q)\bigr\rangle \le 0 \quad \text{and} \quad \bigl\langle q - f(q), J_\varphi(q - p)\bigr\rangle \le 0.$$

Adding up the above inequalities, we obtain that $p = q$. This completes the proof. □

Remark 2.3 In the case that T and f are non-self mappings, we remark that the results of Lemma 2.1 and Theorem 2.2 still hold under the assumption that both T and f satisfy the weak inward condition. We give an affirmative answer to the question purposed in Section 1.

Remark 2.4 It is clear that every contraction is a continuous, bounded and strong pseudo-contraction. The main results in this paper develop the so-called viscosity approximation method which was first introduced by Moudafi [36] from the class of contractions to the class of strong pseudo-contractions.

If $f(x) = u \in C$ for all $x \in C$ in Theorem 2.2, we have the following result.

Corollary 2.5 *Let E be a reflexive Banach space which has a weakly continuous duality mapping J_φ for some gauge φ, and let C be a nonempty closed and convex subset of E. Let $T : C \to C$ be a continuous pseudo-contraction. Fix $u \in C$ and $t \in (0,1)$. Let $\{x_t\}$ be the unique solution of the equation $x_t = tu + (1-t)Tx_t$. If $\{x_t\}$ is bounded, then $\{x_t\}$ converges strongly to a fixed point p of T as $t \to 0$.*

Remark 2.6 Corollary 2.5 improves Theorem X (Theorem 3.1 of Xu [34]). To be more precise, it improves the mapping T from a nonexpansive mapping to continuous pseudo-contractions.

Remark 2.7 In the case that T is a non-self mapping, we remark that Corollary 2.5 still holds under the assumption that T satisfies the weak inward condition.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors read and approved the final manuscript.

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