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Signature of chaos and multistability in a Thomas-Fermi plasma

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Abstract. We propose a dynamical system governing from the nonlinear extension of the electrostatic ion-acoustic waves (IAWs) under the influence of external magnetic field in Thomas-Fermi (TM) consisting of hot electrons, cold electrons and mobile ions. The coupled lower dimensional model with a planar Hamiltonian exhibits periodic oscillations and conservative properties. The perturbed system with a trigonometric forcing can produce more rich phenomena in terms of quasiperiodic and chaotic patterns. The cosine excitation can also be observed as a source of multistability described by various coexisting attractors. The multistability in an external forcing TM plasma has never been reported before.

1 Introduction

Recently, quantum plasmas engaged huge awareness amongst the scientists because of their remarkable applications in various scientific fields of laboratory experiments and astrophysical environments. The leading exploratory implementations of dense quantum plasmas are their various applications in the formation of quantum wire [1], semiconductor materials [2], quantum diodes [3,4], biophotonic [5], microplasmas [6], ultra-cold plasmas [7], and laser [8]. From astrophysical point of view the physical phenomena of dense plasma are of rudimentary consequence for apprehension of the feature of dense astrophysical situations [9], for example, the Jupiter and neutron stars. Much attention was paid to gain and study the characteristics of quantum plasmas [10,11]. Haas et al. [10] implemented nonlinear wave features of IAWs in quantum plasmas with the help of Fermi-Dirac pressure and Bohm potential. Nonlinear oblique IAWs were studied against the superficial magnetic field [11]. TM estimation was carried out for relating the degenerate electrons [12]. Abdelsalam et al. [13] studied IAWs with degenerated electrons and positrons. Masood et al. [14] showed nonlinear characteristics of IAWs in quantum plasmas. Sahu et al. [15]

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investigated Ion Acoustic(IA) solitons in quantum plasmas. Nonlinear IAWs were further studied by some researchers in dense plasmas [16] and quantum e-p-i plasmas [17–19]. Sabry et al. [20] showed nonlinear excitations of solitary waves in a dense plasma. Recently, ion-coustic solitary waves (IASWs) were investigated in TM plasmas with arbitrary limit [21] as well as small amplitude limit [22]. Very recently, nonlinear ion-acoustic multi-solitons were investigated in dense TM plasmas [23].

To demonstrate the deportment of multidisciplinary physical systems, the dynamical posture of these systems was studied by various authors [24,25]. For example, dynamical posture of nonlinear waves through various nonlinear evolution equations (KdV, KP, ZK, and Schrodinger equations) was explored in distinct systems, such as in unmagnetized and magnetized plasmas [26,27], fluid mechanics [28], electric transmission lines [31], optical fibers [29] and nonlinear lattice vibrations [46]. Rahim et al. [32] explored nonplanar waves and its chaotic motions in an unmagnetized TM dusty plasma. Mandi et al. [33] explored dynamical posture of IAWs in a TM plasma composing of source term. Very recently, Prasad et al. [34] explored qualitative feature of nonlinear ion-acoustic superperiodic waves in dense TM plasmas.

In the investigation for dynamics of physical systems one can discover that some nonlinear systems can show more than one solution for a special set of parametric space and different initial conditions [35,36]. This important fact is termed as multistability behavior. More interestingly, it is seen that multistability behaviors of the physical systems play a vital role in their performance [37,38]. The confirmation of multistability behavior was first reported experimentally in a Q-switched gas laser [39], since then chaotic systems having multistability behaviors were studied extensively in different systems [40,41]. Recently He et al. [42] examined the complexity of self-reproducing chaotic models with multi-directional infinite number of chaotic attractors. He et al. [43] also studied synchronization and its DSP implementation of fractional-order hyperchaotic systems utilizing the Adomian decomposition method. In recent time Li et al. [44,45] revealed synchronization of distinct chaotic systems and constructed multiwing attractors [46]. However, multistability behavior, as a new research area in the nonlinear physical systems, is still in its infancy. Therefore, the physical systems with multistability characteristics need more research.

Therefore, in the present work, our intension is to report dynamical properties and multistability of IAWs in small amplitude limit in the framework of the modified KP equation in a magnetized TM plasma composing of degenerate cold and hot electrons with mobile ions. A conservative planar dynamical system is formed for the modified KP equation containing three equilibrium points. There are three types of qualitative different trajectories, namely homoclinic trajectory, periodic trajectory and superperiodic trajectories. IA periodic and superperiodic waves are presented corresponding to periodic and superperiodic trajectories of the conservative planer dynamical system. In presence of a external periodic force, multistability of nonlinear IAWs is studied for the perturbed TM plasma system for the first time in the literature. IA coexisting attractors including two different types of quasiperiodic attractors and chaotic attractor are presented for perturbed TM plasma system.

The article is decomposed as follows: In Section 2, basic equations are considered. In Section 3, we derive the modified KP equation. In Section 3, dynamical properties of IAWs are studied considering perturbed and unperturbed dynamical systems. In Section 5, conclusions of the work are given.

2 Basic equations

In our study we considered dense TM magnetoplasma consisting of hot electrons, cold electrons and singly charged ions. An external magnetic field in which the magnetoplasma is confined is $B_0 = B_0\hat{x}$, where $\hat{x}$ denotes a unit vector along the

x-axis and B_0 denotes magnetic field's strength. The nonlinear oblique propagation of the electrostatic IAWs against the external magnetic field in the xy-plane is supervised by the normalized equations of ion continuity and momentum balance:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) + \frac{\partial}{\partial y}(nv) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + (u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y})u + \frac{3}{2}\frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial v}{\partial t} + (u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y})v + \frac{3}{2}\frac{\partial \phi}{\partial y} - w = 0, \tag{3}$$

$$\frac{\partial w}{\partial t} + (u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y})w + v = 0. \tag{4}$$

Equations (1)–(4) are closed by the Poisson equation

$$\Omega\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = \frac{2}{3}\Big[n_c + \alpha n_h - (1+\alpha)n\Big], \tag{5}$$

where $n_c = (1+\phi)^{3/2}$, $n_h = (1+\sigma\phi)^{3/2}$ and $\sigma = \frac{T_{F_c}}{T_{F_h}}$ where T_{F_c} and T_{F_h} respectively are Fermi cold electron temperature and Fermi hot electron temperature. Since plasma a quasineutral collection of charged particles so the quasineutrality condition at equilibrium gives $n_{c_0} + n_{h_0} - n_0 = 0$, from this relation one can obtain $1+\alpha = \frac{n_0}{n_{c_0}}$, $\alpha = \frac{n_{h_0}}{n_{c_0}}$, where n_0, n_{c_0}, n_{h_0} respectively are unperturbed number densities of ions, cold electrons and hot electrons. In equations (1)–(5), n_c, n_h respectively are number densities of cold and hot electrons, n is number density of ions, ϕ is magneto potential, u, v, w are velocity components of ion fluid in the x, y, z directions, respectively. Physical quantities that appeared in equations (1)–(5) are normalized in the following manner: n, n_c and n_h are normalized by (n_0) or n_{c_0} or by n_{h_0}, by Fermi ion-sound speed C_{si} we have normalized velocity of fluid, ϕ by ϵ_F/e, the space and time variables are in units of ion sound gyroradius $\rho_s = C_{si}/\omega_{ci}$ and the ion gyrofrequency ω_{ci} respectively. Here, $\omega_{ci} = eB_0/m_iC$, e is magnitude of electron charge, m_i is ion mass, and C is the speed light in vacuum, $C_{si} = (2\epsilon_F/3m_i)^{1/2}$, $\rho_s = C_{si}/\omega_{ci}$, $\rho_F = 2K_BT_{F_c}$, where K_B is the Boltzmann constant and T_{F_c} is the Fermi cold electron temperature. Furthermore, $\Omega = (\omega_{ci}/\omega_{pi})^2$, $\omega_{pi} = (4\pi e^2 n_0/m_i)^{1/2}$ is ion plasma frequency, $n_0 = (8\pi/3h^3)(P_{F_c})^3$, P_{F_c} is Fermi cold electron momentum, and h is Planck constant.

3 Derivation of the modified KP equation

It is effective to investigate the dynamics of IAWs in small amplitude limit. To study this we are going to use the reductive perturbation technique, in which we will consider the stretching of independent variables in the following way;

$$Y = \varepsilon^2 y, \quad \eta = \varepsilon(x - Vt), \quad \tau = \varepsilon^3 t, \tag{6}$$

where ε is a small parameter that measures the weakness of the dispersion and V is phase velocity of the ion acoustic wave.

We have considered the expression of the dependent variables in the following way:

$$\begin{cases} n = 1 + \varepsilon^2 n_1 + \varepsilon^4 n_2 + \cdots, \\ u = \varepsilon^2 u_1 + \varepsilon^4 u_2 + \cdots, \\ v = \varepsilon^3 v_1 + \varepsilon^5 v_2 + \cdots, \\ w = \varepsilon^3 w_1 + \varepsilon^5 w_2 + \cdots, \\ \phi = \varepsilon^2 \phi_1 + \varepsilon^4 \phi_2 + \cdots. \end{cases} \tag{7}$$

Using equations (6) and (7) into equations (1)–(5) and considering the coefficients of ε^2 and ε^3, we obtain following relations:

$$\begin{cases} n_1 = \frac{1}{V} u_1, \\ u_1 = \frac{3}{2V} \phi_1, \\ n_1 = \frac{3}{2} \left(\frac{1+\sigma\alpha}{1+\alpha} \right) \phi_1, \\ V^2 = \frac{1+\alpha}{1+\sigma\alpha} \cdot \end{cases} \tag{8}$$

Again, considering the coefficients of ε^4, ε^5 and ε^6 we obtain following relations:

$$\frac{\partial n_1}{\partial \tau} - V \frac{\partial n_2}{\partial \eta} + \frac{\partial v_1}{\partial Y} + \frac{\partial}{\partial \eta}(n_1 u_1) + \frac{\partial u_2}{\partial \eta} = 0 \tag{9}$$

$$\frac{\partial u_1}{\partial \tau} - V \frac{\partial u_2}{\partial \eta} + u_1 \frac{\partial u_1}{\partial \eta} = -\frac{3}{2} \frac{\partial \phi_2}{\partial \eta} \tag{10}$$

$$\frac{\partial v_1}{\partial \tau} - V \frac{\partial v_2}{\partial \eta} + u_1 \frac{\partial v_1}{\partial \eta} = -\frac{3}{2} \frac{\partial \phi_2}{\partial Y} \tag{11}$$

$$\frac{\partial w_1}{\partial \tau} - V \frac{\partial w_2}{\partial \eta} + u_1 \frac{\partial w_1}{\partial \eta} = 0 \tag{12}$$

$$\Omega \frac{\partial^2 \phi_1}{\partial \eta^2} = (1 + \sigma\alpha)\phi_2 + \frac{1}{4}(1 + \sigma^2\alpha)\phi_1^2 - \frac{2}{3}(1 + \alpha) n_2. \tag{13}$$

From all these above equations (8)–(12), we have the KP-equation as:

$$\frac{\partial}{\partial \eta} \left[\frac{\partial^3 \phi_1}{\partial \eta^3} + A \phi_1 \frac{\partial \phi_1}{\partial \eta} + B \frac{\partial \phi_1}{\partial \tau} \right] + C \frac{\partial^2 \phi_1}{\partial Y^2} = 0, \tag{14}$$

where $A = \left[\frac{9(1+\sigma\alpha)}{2\Omega V^2} - \frac{(1+\sigma^2\alpha)}{2\Omega} \right]$, $B = \frac{2(1+\sigma\alpha)}{\Omega V}$ and $C = \frac{(1+\sigma\alpha)}{\Omega} \cdot$

Now if $9(1 + \sigma\alpha) = V^2(1 + \sigma^2\alpha)$, i.e., $A \simeq 0$, then the nonlinear term of the KP equation will vanish, as a result we won't be able to study the nonlinear wave features around and at the point where $A \simeq 0$. Because of this fact we can consider more higher order nonlinear equation to study nonlinear wave features around and at the critical points where $A \simeq 0$. We now proceed to find the modified form of KP equation considering same set of stretching of independent variables with

$$\begin{cases} n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 + \cdots, \\ u = \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \cdots, \\ v = \varepsilon^2 v_1 + \varepsilon^3 v_2 + \cdots, \\ w = \varepsilon^2 w_1 + \varepsilon^4 w_2 + \cdots, \\ \phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \cdots. \end{cases} \tag{15}$$

Using equations (6) and (15) into the system of equations (1)–(5) and by considering the coefficients of $\varepsilon, \varepsilon^2, \varepsilon^3$ and ε^4 we obtain following relations:

$$\begin{cases} n_1 = \frac{1}{V} u_1, \\ n_2 = \frac{n_1 u_1}{V} + \frac{u_2}{V}, \\ u_1 = \frac{3}{2V} \phi_1, \\ u_2 = \frac{3}{2V} \phi_2 + \frac{1}{2V} u_1^2, \\ n_1 = \frac{3}{2} \left(\frac{1+\sigma\alpha}{1+\alpha} \right) \phi_1, \\ V^2 = \frac{1+\alpha}{1+\sigma\alpha}, \end{cases} \tag{16}$$

$$\frac{\partial n_1}{\partial \tau} - V \frac{\partial n_3}{\partial \eta} + \frac{\partial v_1}{\partial Y} + \frac{\partial}{\partial \eta}(n_2 u_1) + \frac{\partial}{\partial \eta}(n_1 u_2) + \frac{\partial u_3}{\partial \eta} = 0, \tag{17}$$

$$\frac{\partial u_1}{\partial \tau} - V \frac{\partial u_3}{\partial \eta} + u_1 \frac{\partial u_2}{\partial \eta} + u_2 \frac{\partial u_1}{\partial \eta} = -\frac{3}{2} \frac{\partial \phi_3}{\partial \eta}, \tag{18}$$

$$V \frac{\partial v_1}{\partial \eta} = \frac{3}{2} \frac{\partial \phi_1}{\partial Y}, \tag{19}$$

$$(1+\sigma\alpha)\phi_2 + \frac{1}{4}(1+\sigma^2\alpha)\phi_1^2 - \frac{2}{3}(1+\alpha) n_2 = 0, \tag{20}$$

$$\Omega \frac{\partial^2 \phi_1}{\partial \eta^2} = (1+\sigma\alpha)\phi_3 + \frac{1}{2}(1+\sigma^2\alpha)\phi_1\phi_2 - \frac{2}{3}(1+\alpha) n_3. \tag{21}$$

From all these above equations (16)–(21), we obtain the modified KP-equation as:

$$\frac{\partial}{\partial \eta} \left[\frac{\partial^3 \phi_1}{\partial \eta^3} + D \phi_1^2 \frac{\partial \phi_1}{\partial \eta} + B \frac{\partial \phi_1}{\partial \tau} \right] + C \frac{\partial^2 \phi_1}{\partial Y^2} = 0, \tag{22}$$

where $D = \left[\frac{27(1+\sigma\alpha)}{4\Omega V^4} + \frac{9(1+\sigma^2\alpha)}{8\Omega V^2} \right]$, $B = \frac{2(1+\sigma\alpha)}{\Omega V}$ and $C = \frac{(1+\sigma\alpha)}{\Omega}$.

4 Dynamical properties of IAWs

4.1 Unperturbed system

To have the dynamical system based on the modified KP equation (22), we now consider a traveling wave transformation $\chi = l\eta + mY - c\tau$, $\phi_1(\eta, Y, \tau) = \psi(\chi)$, with boundary conditions $\psi \to 0$, $\frac{d\psi}{d\chi} \to 0$, $\frac{d^2\psi}{d\chi^2} \to 0$ as $\chi \to \pm\infty$, where $l^2 + m^2 = 1$, and c is velocity of the traveling wave in the ηY-plane. Using this transformation in the modified KP equation (22) and integrating twice we obtain,

$$\frac{d^2\psi}{d\chi^2} = P\psi - Q\psi^3, \tag{23}$$

where $P = \frac{lcB - (1-l^2)C}{l^4}$, $Q = \frac{D}{3l^2}$. Now equation (23) can be transformed to dynamical system of the form:

$$\begin{cases} \frac{d\psi}{d\chi} = z, \\ \frac{dz}{d\chi} = P\psi - Q\psi^3. \end{cases} \tag{24}$$

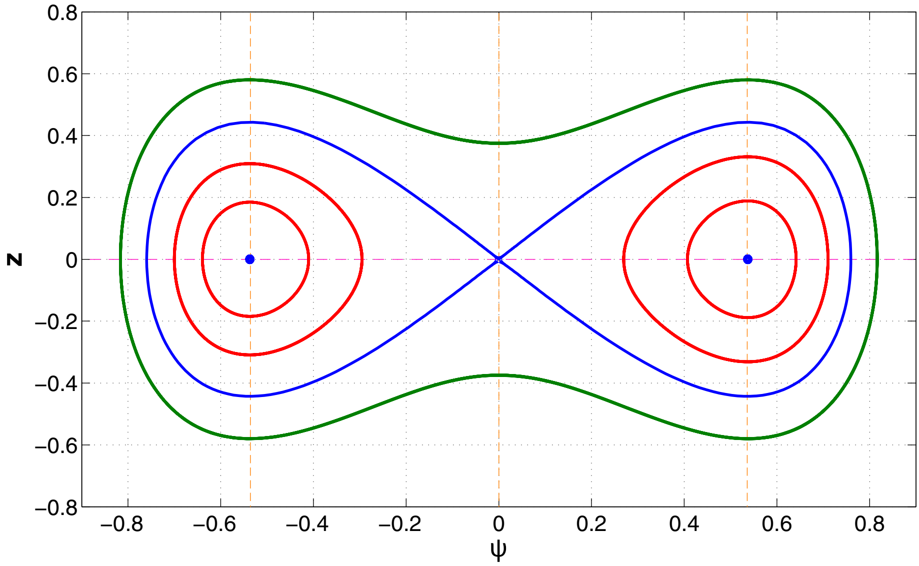

Fig. 1. Phase plot for the dynamical system (24) for $l = 0.6, c = 0.7, \sigma = 0.4$, $\alpha = 0.6$, $\Omega = 0.7$.

If we consider $\overrightarrow{F} = (z, P\psi - Q\psi^3)$ then $\overrightarrow{\nabla}.\overrightarrow{F} = 0$, thus the system (24) is conservative. Here system (24) is a planner Hamiltonian system with Hamiltonian function

$$H(\psi, z) = \frac{z^2}{2} - \frac{P\psi^2}{2} + \frac{Q\psi^4}{4}. \tag{25}$$

Since the phase trajectories defined by the vector field of equation (24) determine all IAWs of equation (22), we instigate the bifurcations of phase plots of equation (24) in the (ψ, z)-plane as $c, l, \Omega, \alpha, \sigma$ are changed. A periodic trajectory (superperiodic trajectory) of system (24) corresponds to a periodic IAWs (superperiodic IAWs) of equation (22). The bifurcation theory of dynamical system plays an important role in our study.

In Figure 1 phase plot of the dynamical system (24) is shown, which consists of three critical points (two centers and one saddle). In this case we find two families of periodic orbits corresponding to the centers, a pair of homoclonic orbits through the saddle point and a family of superperiodic orbits lying outside the pair of homoclinic orbits.

Figure 2 shows effect of parameters Ω, α and σ in the time series plots of the periodic wave solution. Figure 2a shows amplitude of the periodic wave solution decreases slightly if Ω increases. In this case ion gyrofrequency increases compared to ion plasma frequency. Figure 2b shows amplitude of the periodic wave solution decreases drastically if α increases. Here unperturbed number density of hot electrons increases as compared to unperturbed number density of cold electrons. Figure 2c shows amplitude of the periodic wave solution increases proportionally with σ. In this case Fermi temperature of cold electrons increases as compared to Fermi temperature of hot electrons

Figure 3 shows effect of the parameters Ω, α and σ in the time series plot of the superperiodic wave solution. Figure 3a shows there is no significant change in amplitude, when Ω increases. Figure 3b shows amplitude of the superperiodic wave solution decreases slightly if α increases, whereas Figure 3c shows amplitude of the superperiodic wave solution increases proportionally with σ.

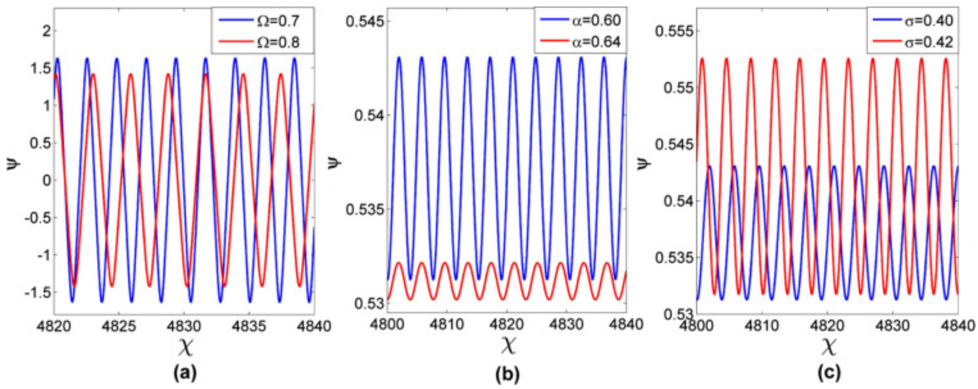

Fig. 2. (a)–(c) represent periodic IAWs of equation (22) for various values of Ω, α and σ respectively with initial condition (0.53, 0).

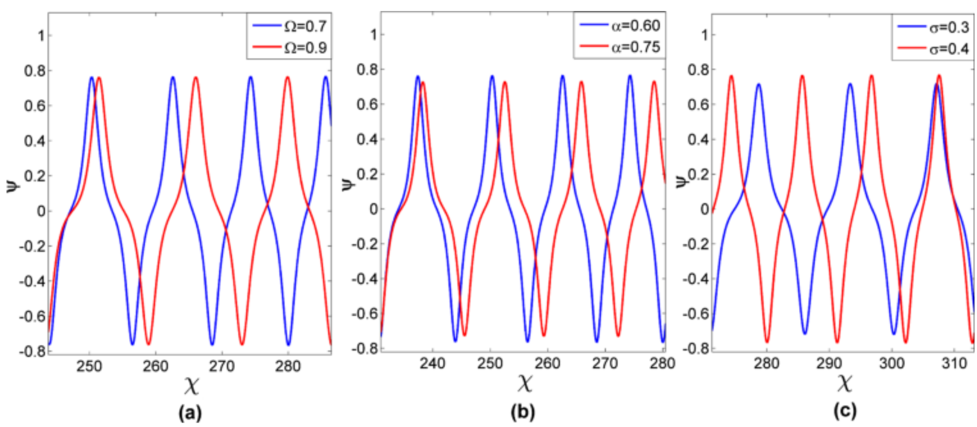

Fig. 3. (a)–(c) represent superperiodic IAWs of equation (22) for various values of Ω, α and σ respectively with initial condition (0.21, 0.09).

4.2 Perturbed system and multistability

In this section, with theoretical analysis and numerical simulations, dynamics of the IAWs in presence of an external periodic force are investigated by means of phase plots and Lyapunov exponents. This type of external periodic force or source term in the form of sine or cosine functions [47] can be present in TM plasma due to the presence of space debris in such plasmas.

We now consider the nonautonomous perturbed dynamical system of the IAWs, with an external periodic force $f_0 \cos(\omega\chi)$ as

$$\begin{cases} \frac{d\psi}{d\chi} = z, \\ \frac{dz}{d\chi} = P\psi - Q\psi^3 + f_0 \cos(\omega\chi), \end{cases} \tag{26}$$

where strength and frequency of the external force are f_0 and ω respectively. The autonomous system corresponding to the nonautonomous system (26) can be

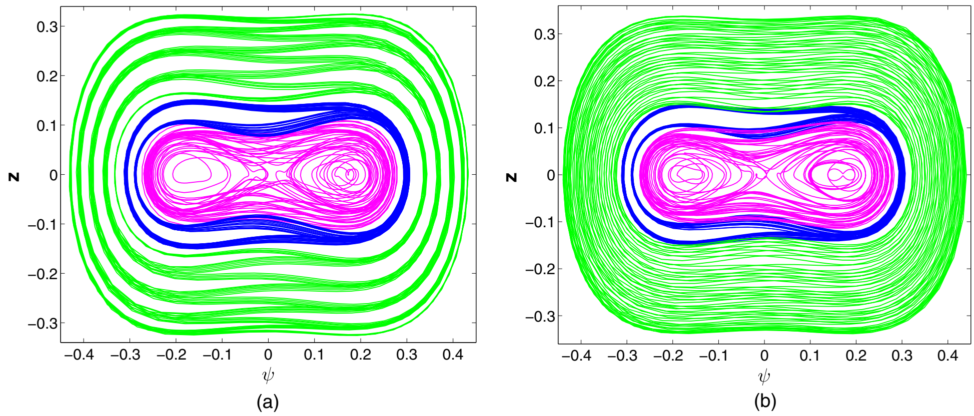

Fig. 4. (a): Coexisting attractors corresponding to IA quasi-periodic motions of two different types (represented by blue and green curves) with initial conditions (0.2209, 0.00048) and (0.15, 0.30) respectively, and IA chaotic motion (represented by magenta curve) with initial condition (0.15, 0.05), for the perturbed dynamical system (26) when $l = 0.7, c = 0.4, \sigma = 0.7, \alpha = 0.6, \Omega = 0.4, \omega = 0.8, f_0 = 0.01$. (b): Coexisting attractors corresponding to IA quasi-periodic motions of two different types (represented by blue and green curves) with initial conditions (0.2253, 0.00048) and (0.15, 0.32) respectively, and IA chaotic motion (represented by magenta curve) with initial condition (0.24001, 0.00048), for the perturbed dynamical system (26) when $l = 0.7, c = 0.4, \sigma = 0.7,\ \alpha = 0.6, \Omega = 0.4, \omega = 0.8, f_0 = 0.01$.

defined as

$$\begin{cases} \frac{d\psi}{d\chi} = z, \\ \frac{dz}{d\chi} = P\psi - Q\psi^3 + f_0\ \cos(\omega\xi), \\ \frac{d\xi}{d\chi} = 1, \end{cases} \tag{27}$$

where ψ, χ and ξ are state variables. It is clear that the system (27) has no equilibrium point, which means that this system exhibits hidden attractors.

In Figure 4, we present different types of coexisting hidden attractors for same parameter values with different initial conditions. Figure 4a shows two types of quasi-periodic attractors (represented by green and blue curves) with initial conditions (0.2207, 0.00048) and (0.15, 0.30) respectively and chaotic attractor (represented by magenta curve) with initial condition (0.15, 0.05). Figure 4b shows two types of quasi-periodic attractors (represented by green and blue curves) with initial conditions (0.2253, 0.00048) and (0.15, 0.32) respectively and chaotic attractor (represented by magenta curve) with initial condition (0.24001, 0.00048).

To show the existence of quasiperiodic and chaotic attractors, the most effective tool is variation of Lyapunov exponents. The dynamics of the system (27) change significantly due to the influence of the external periodic forcing. We investigate the change in the dynamics of system (27) by plotting Lyapunovs with respect to strength (f_0) and frequency (ω) of the external force in Figures 5 and 6 respectively. Figures 5a and 5b show Lyapunovs with respect to f_0 for three different initial conditions represented by blue, green and red curves respectively considering same values of other parameters presented in Figures 4a and 4b respectively. Figures 6a and 6b show Lyapunovs with respect to ω for three different initial conditions represented by blue, green and red curves respectively considering same values of other parameters presented in Figures 4a and 4b respectively. It is interesting to point out that

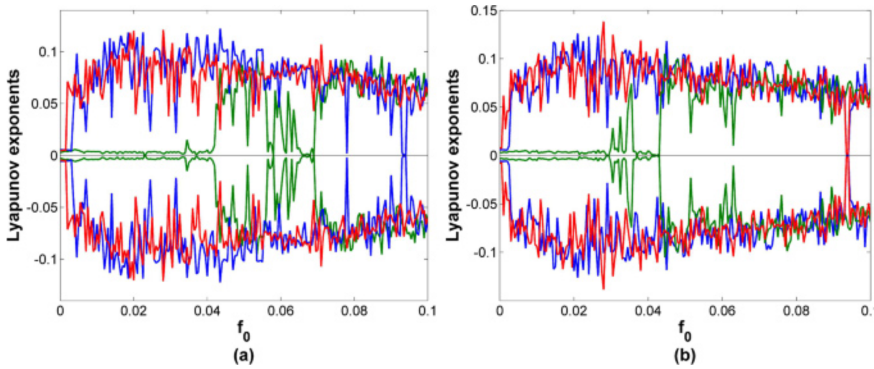

Fig. 5. (a): Lyapunov exponents with respect to f_0 for three different initial conditions (represented by blue, green and red curves respectively) as shown in Figure 4a. (b): Lyapunov exponents with respect to f_0 for three different initial conditions (represented by blue, green and red curves respectively) as shown in Figure 4b.

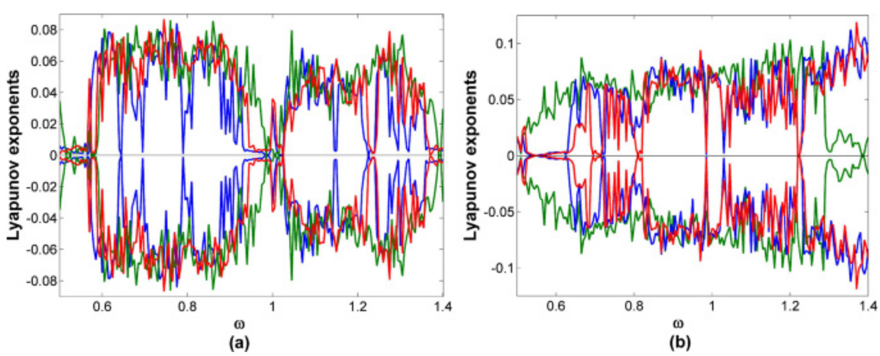

Fig. 6. (a): Lyapunov exponents with respect to ω for three different initial conditions (represented by blue, green and red curves respectively) as shown in Figure 4a. (b): Lyapunov exponents with respect to ω for three different initial conditions (represented by blue, green and red curves respectively) as shown in Figure 4b.

the system (27) still preserves the conservative nature under the external periodic perturbation. The conservative chaos in TM plasma has never been reported before.

5 Conclusions

In this work, dynamical properties of IAWs and its multistability in a dense TM magnetoplasma consisting of hot electrons, cold electrons and singly charged mobile ions have been addressed. The dynamics can produce ion-acoustic periodic and superperiodic waves, which can be quantified by the phase plot and the time series plots. A nonlinear periodic force can affect on various types of dynamical features and existence of lower dimensional chaos. The perturbation can also help as an external forcing, which is the source of multistability and coexisting attractors. This phenomenon is reported for the first time in a dense three-component TM magnetoplasma.

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