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Review

Supersymmetry unification, naturalness, and discovery prospects at HL-LHC and HE-LHC

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Abstract. An overview of recent developments in supersymmetry, supergravity and unification and prospects for supersymmetry discovery at the current and future high energy colliders and elsewhere are discussed. Currently several empirical data point to supersymmetry as an underlying symmetry of particle physics. These include the unification of gauge couplings within supersymmetry, prediction within supergravity unification that the Higgs boson mass lie below 130 GeV supported by the observation of the Higgs boson mass at ∼125 GeV, and vacuum stability up to the Planck scale for the observed value of the Higgs boson mass while the standard model does not do that. Additionally, of course, supersymmetry solves the big hierarchy problem arising from the quadratic divergence to the Higgs boson mass square in the Standard Model, and provides a frame work that allows for extrapolation of physics from the electroweak scale to the grand unification scale consistent with experiment. Currently there is no alternative paradigm that does that. However, the large loop corrections needed to lift the mass of the Higgs boson from its tree value to the experimentally observed values imply that the scale of weak scale supersymmetry lies in the TeV region making the observation of sparticles more challenging. The lightest of the sparticles could still lie with in reach of the High Luminosity (HL)-LHC and High Energy (HE)-LHC operating at an optimal luminosity of 2.5×10^{35} cm^{-2} s^{-1} at a center of mass energy of 27 TeV. Variety of other experiments related to search for dark matter, improved experiments on the measurement of $g_\mu - 2$ and EDMs of elementary particles could lend further support for new physics beyond the standard model and specifically supersymmetry. Supergravity theories may also contain hidden sectors which may interact with the visible sector gravitationally and also via extra-weak or ultra-weak interactions. In this case a variety of new signals might arise in indirect detection and at LHC in the form of long lived charged sparticles which can either decay inside the detector or outside. We note that the discovery of sparticles will establish supersymmetry as a fundamental symmetry of nature, and its confirmation will also lend support for strings.

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1 Introduction

Models in four dimensions formulated using supersymmetry exhibit interesting properties [1–4]. They exhibit much better UV behavior than the standard model [5–11]. Specifically in the standard model one has quadratic divergence in the loop corrections to the Higgs boson mass which is cancelled in the supersymmetry based models. Supersymmetry if it exists would not be an exact symmetry of nature but a broken one. However, spontaneous breaking of global supersymmetry presents a problem because the breaking is not phenomenologically viable. This problem is resolved in supergravity based models, and specifically supergravity grand unified models which provide a framework which allows for the extrapolation of physics from the electroweak scale to the grand unification scale in a phenomenologically viable fashion. In this review we discuss the recent developments in supergravity grand unification and the prospects for the observation of supersymmetry in current and future experiment.

The outline of the rest of the paper is as follows: In Section 2, we discuss the status of supersymmetry in the post Higgs discovery era. In Section 3, we discuss the possibility that the large scale of weak scale supersymmetry and the current limits on the sparticle masses are likely indications that the radiative breaking of the electroweak symmetry originates on the hyperbolic branch and what its implications are for the discovery of supersymmetry. In Section 4, we discuss the status of the supersymmetric electroweak corrections to the muon anomaly in the post Higgs discovery era. In Section 5, we discuss the status of dark matter in the post Higgs boson discovery era. In Section 6 we discuss hidden sector and extended SUGRA models where we consider kinetic mixing and Stueckelberg mass mixing between the Hidden sector and the visible sector. The possibility of a stau being a long lived particle with a lifetime time long enough to leave a track inside an LHC detector is discussed in Section 7. Recent developments in unification in GUTs and strings in discussed in Section 8. The prospects of observation of supersymmetry at HL-LHC and HE-LHC are discussed in Section 9. Conclusions are given in Section 10.

2 SUSY post Higgs

The Higgs boson [12–15] plays a central role in the breaking of the electroweak symmetry. Its discovery in 2012 at a mass of $\sim$125 GeV at the Large Hadron Collider (LHC) [16,17] confirmed the last missing piece in the standard model of electroweak interactions. However, a mass of $\sim$125 GeV is problematic within the standard model. One reason for that is that within the standard model using a renormalization group analysis one finds that vacuum stability holds up to a scale of around 10^{11} GeV [18,19] using the current value of the top mass of $M_t = 173.1 \pm 0.9$ GeV [20]. On the other hand in supergravity grand unification using the minimal sparticle spectrum vacuum stability can be achieved up to the Planck scale. In addition to the above a Higgs mass of $\sim$125 GeV lends further support to supersymmetry and specifically to supergravity grand unification. The reason for that is simple: with in the standard model the Higgs boson mass can acquire values over a very wide range of up to several hundred GeV. In supergravity grand unification [21–23] (for a review of supersymmetry, supergravity and unification see, e.g., [24]), however, the Higgs boson mass is predicted to lie below 130 GeV [25–30] a limit which is respected by experiment. There is a price to be paid, however, in supersymmetry for achieving a mass of $\sim$125 GeV. Thus, in MSSM at the tree level, the lightest CP even Higgs boson has a mass which lies below M_Z [31,32], and a loop correction is needed to lift the Higgs boson mass from below M_Z to its observed value (for a review see [33,34]). The loop correction is sizable and implies that the scale of weak scale SUSY must be in the TeV region [35–40]. The measurement of the Higgs boson mass at ∼125 GeV also sheds light on the mechanism of SUSY breaking. Here one finds that mSUGRA and other supergravity grand unified models easily do the job while some other mechanisms have problems accommodating the large loop correction needed to lift the tree value of the Higgs mass to the experimentally observed value.

The large value of the weak scale supersymmetry implied by the measured Higgs boson mass indicates that the observation of sparticles will be more difficult than previously thought and that turns out to be the case. Further, in supergravity models where the scale of weak scale supersymmetry is large, the lightest neutralino often turns out to be mostly a bino. In this case one needs co-annihilation to achieve the relic density close to the value observed by experiment. However, co-annihilation also implies that the NLSP will lie close to the LSP and thus the decay of the NLPS will lead to soft jets and leptons which makes the detection of supersymmetry more difficult and this may in part explain the lack of observation of supersymmetry thus far.

A large scale of weak scale supersymmetry does have some benefits nonetheless. It help suppress FCNC processes such as $b \to s\gamma$ [41–43] and $B_s \to \mu^+\mu^-$ and explains the absence of any large deviations from the standard model predictions for these processes [44–49]. The large scale of weak scale supersymmetry is also helpful in stabilizing the proton via decays from lepton and baryon number violating dimension five operators. The dressing loop diagrams for these operators involve sfermion exchanges and heavy sfermion masses lead to a suppression of the loop diagrams that lead to proton decay (for a review of proton stability and for tests of unification see [50–52]). Thus there exists a strong correlation between the Higgs boson mass and proton lifetime [53].

3 Natural supersymmetry originates on the hyperbolic branch

One of the very attractive features of supergravity unified models is the breaking of the electroweak symmetry (EWSB) via soft breaking terms induced by gravity mediated breaking (for a review of electroweak symmetry breaking see [54]). However, it was realized some time ago that there exists a new branch of radiative breaking of the electroweak symmetry, i.e., the hyperbolic branch (HB), where the Higgs mixing parameter could be small while the soft parameters get large [55] (for further work see [56–60]). Supersymmetry on the hyperbolic branch is the natural candidate for the discovery at the LHC. The essentials of HB can be understood in terms of the EWSB constraint on the Higgs mixing parameter μ which we can write in the form $\mu^2 + \frac{1}{2}M_Z^2 = m_0^2\ C_1(\tan\beta, ultraM_t, Q) + \Delta(m_{1/2}, A_0, \tan\beta, Q)$. Here m_0 is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, A_0 is the universal trilinear coupling, $\tan\beta = \langle H_2\rangle/\langle H_1\rangle$ where H_2 gives mass to the up quark, and H_1 gives mass to the bottom quark and the lepton, μ is the Higgs mixing parameter which appears in the superpotential in the form $\mu H_1 H_2$, and Q is the renormalization group scale which is chosen to that the two loop correction to the EWSB is minimized. In this case Δ is positive definite and dependent only on $m_{1/2}$, A_0, $\tan\beta$ and Q and not on m_0, while C_1 is a function that depends on $\tan\beta, M_t$ and the renormalization group scale Q but does not depend on soft parameters $m_0, m_{1/2}, A_0$. However, the sign of $C_1(Q)$ depends on $\tan\beta, M_t$ and Q. For the parameter space where the sign of $C_1(Q)$ is positive $|\mu| > m_0$ (up to a small correction from M_Z) and thus a large m_0 requires a $|\mu|$ which is correspondingly large. For the universal scalar mass case, the large loop correction to the Higgs boson mass requires a large m_0 lying in the TeV region, and consequently here $|\mu|$ will be large. On the other hand, if sign$(C_1(Q))$

is negative, one has the possibility that m_0 can be as large as several TeV while μ is relatively small lying in the few hundred GeV region. This latter case, i.e., $\text{sign}(C_1(Q)) = -1$ is the hyperbolic branch. Recently, in several phenomenological analyses, the nomenclature natural supersymmetry has been used (see, e.g., [61–63]). While the exact parametrization of natural supersymmetry varies among various analyses, one common theme is that μ is taken to be relatively small, lying in the few hundred GeV region while the soft parameters could be relatively much larger. The discussion above indicates that this is possible only on the hyperbolic branch of radiative breaking, i.e., the so called natural supersymmetry can only originate on the hyperbolic branch of radiative breaking of the electroweak symmetry and not on the ellipsoidal branch which corresponds to $\text{sign}(C_1(Q)) = +1$. Naturalness also has implications for dark matter (see [64] and the references therein).

4 $g_\mu - 2$ post Higgs

The analysis from the $g_\mu - 2$ experiment [65] indicates that $a_\mu = \frac{1}{2}(g_\mu - 2)$ deviates from the Standard Model prediction [66–69] at the $\sim 3\,\sigma$ level. Thus the most recent analysis of Keshavarzi et al. [70] gives the deviation between experiment and theory $\Delta a_\mu = (28.02 \pm 7.37) \times 10^{-10}$ corresponding to a muon $g - 2$ discrepancy of 3.8σ between the measured value of a_μ and its Standard Model prediction (new data on $g_\mu - 2$ is expected in the near future from experiment currently underway [71,72]). Now it has been known early on that supersymmetric electroweak corrections can be sizable [73–79]. At the one loop level these contributions arise from the exchange of $\chi^\pm - \nu_\mu$ and from the χ^0- smuon in the loops. However, sizable corrections require that the SUSY scale be in the $O(100)$ GeV range. This appears to be on the surface in contradiction with what is indicated by the Higgs boson mass, i.e., a weak scale in the TeV region. However, a possible solution for this can still be achieved within the supergravity unified model. Thus in the $\tilde{g}$SUGRA model [80] one assumes the boundary conditions on soft parameters at the GUT scale [80] for $m_0, m_1, m_2, m_3, A_0, \tan\beta, \text{sign}(\mu)$ so that $(m_0, m_1 = m_2) \ll m_3$, where m_3, m_2, m_1 are soft masses for the gauginos in the $SU(3), SU(2), U(1)$ sectors. In this case the gluino mass is the largest and drives the electroweak symmetry breaking. If we assume that the gluino mass lies in the several TeV region, then the color interactions of the gluino drive the squark masses to the TeV region while the slepton and electroweakino masses remain light. The light slepton and electroweakinos allow one to generate a significant contribution to a_μ. We note here in passing that CP phases from the supersymmetric sector can strongly affect electroweak phenomena and specifically the muon anomaly [81,82]. However, for the case when m_0 is large, the supersymmetric electroweak contributions will be negligible and the observation of a significant persistent muon anomaly would requires contribution from a new sector.

5 Dark matter post Higgs

There is now considerable experimental evidence for the existence of dark matter in the universe. While the standard model does not have a dark matter candidate, there are a variety of dark matter candidates in beyond the standard model physics. Specifically supergravity models provide several possible candidates in the form of LSP which could be a spin zero sfermion, a spin 1/2 neutralino or a spin 3/2 gravitino. Regarding the neutralino it was proposed as a candidate for dark matter soon after the formulation of supergravity grand unified models [83]. This possibility becomes viable since it appears as the LSP over most of the parameter space of models [84] and with R-parity, it becomes a candidate for dark matter. However, experimental measurement of the relic density of dark matter [85] puts a significant constraint on model building. This constraint becomes more stringent for supergravity models taken together with the constraint from the experimental measurement of the Higgs boson mass. As noted earlier, the Higgs boson mass at ∼125 GeV requires a large loop correction within supersymmetry which implies a larger scale of weak scale supersymmetry. In most of the parameter space it leads to a neutralino which is a bino. However, bino-like neutralino cannot annihilate efficiently in the early universe which leads to a relic density far in excess of the current experimental limits. To overcome this problem one needs co-annihilation [86–91] which implies that there are one or more sparticles (NLSPs) lying close to the LSP which implies a compressed spectrum for low lying sparticles. Typically to get an efficient co-annihilation one requires $(m_{NSLP} - m_{LSP})/m_{NLSP}$ to be about ∼1/10 which leads to decays of the NLSP being soft and more difficult to detect. Thus models with sfermion masses in the mass range of 10–100 TeV require co-annihilation which then implies a compressed spectrum making the detection of sparticles more difficult.

Direct detection experiments for the detection of dark matter can provide support for supersymmetry. The recent dark matter experiments (LUX, Panda, XENON100) have reached a sensitivity in the range of 10^{-45}–10^{-46} cm^2 in spin-independent WIMP-nucleon cross-section. A much larger sensitivity up to 10^{-49} cm^2 could be reached by the year 2030 (for a review see [92]). A further increase in sensitivity will be more challenging as after that one would need to deal with neutrino background. There is another possibility in which the relic density constraint can be satisfied and this occurs when the neutralino is higgsino like. Typically in this case the annihilation occurs copiously and the relic density lies below the value measured by experiment. Theoretically this can occur on the hyperbolic branch of radiative breaking of the electroweak-symmetry . Here one may have sfermion heavy along with a relatively small μ and one has the possibility of a higgsino like neutralino (for a recent work see [93]). In this case one would need an additional component to dark matter which could be one of the many possible candidates available, such as an axion [94]. The mass range of possible candidates is enormous. Dark matter particles could be as heavy as the GUT mass and as light as 10^{-21} eV. Specifically an ultralight boson as dark matter has recently been discussed in the context of cosmology at scales less than ∼10 kpc [95]. Thus while the ΛCDM model works quite well for cosmology at large scales, some issues arise at scales smaller than ∼10kpc [95]. One of these issues often called the Cusp-Core problem relates to the fact that N-body simulations show that CDM leads to cuspy dark matter near galaxy cores. One the other hand the observed galaxy rotation have a better fit with constant dark matter density cores. Another issue concerns the fact that CDM predicts too many dwarf galaxies which are not seen. It is claimed that these problems could be resolved by taking into account complex dynamics and baryons along with WIMPs [96]. An alternative possibility is that a proper account of cosmology at small scales may require ultralight dark matter. Such a possibility could be an ultralight axion with mass $\mathcal{O}(10^{-21})$ eV. Such an axion is not a QCD axion but likely a string axion [97] with a decay constraint lying in the range 10^{16} GeV $\leq f \leq 10^{18}$ GeV . Explicit models can be constructed which accommodate a boson as light as $\sim 10^{-21}$ eV. (see, e.g., [98–100]). This particle is a possible candidate for multi-component dark matter along with the neutralino. Other variety of dark matter candidates include sub GeV dark matter, extra-weakly interacting dark matter, self-interacting dark matter, PeV scale dark matter, dynamical dark matter, and dark matter from extra dimensions to name a few (for a few references see [101–106]). Dark matter could be detected at colliders if their production cross-section is large enough and it would be detected as missing energy.

Evidence of dark matter could also emerge in astrophysical observations based on detection of anti-matter in the annihilation of dark matter. Thus two neutralinos can annihilate producing a $f\bar{f}$ where the anti-matter produced could be detected. Specifically an excess of positrons would be an indication of such dark matter and possibly a signal from the hidden sector [107] discussed in the next section. Several satellite experiments [108–110] are exploring the presence of dark matter by these indirect techniques (for a review see [111]).

6 Extended SUGRA models with kinetic and mass mixings with hidden sectors

Supergravity models and strings contain hidden sectors which have extra gauge groups and may also contain matter. Although the hidden sector is a singlet of the visible sector gauge group communication between the visible and the hidden sector can still occur. Aside from gravitational interactions between the two sectors, they can also communicate via kinetic mixing [112,113] and stueckelberg mass mixings [114–124] between a hidden sector $U(1)$ and the $U(1)_Y$ of the visible sector. The kinetic mixing and the Stuckelberg mass mixing lead to different types of interactions between the hidden sector and the visible sector. Thus in kinetic mixing one has mixings between the field strength of the $U(1)$ gauge field in the hidden sector and the field strength of the $U(1)_Y$ hypercharge gauge field of the visible sector. In the diagonalized basis the hidden sector photon couples to both the hidden sector matter as well as the visible sector matter while the visible sector photon couples only to the visible sector matter. In the presence of both kinetic mixing and Stueckelberg mass mixing among the $U(1)$ gauge fields, the hidden sector Z' couples to the visible sector matter and the photon of the visible sector also couples to matter in the hidden sector and these mixings give rise to observable effects. The kinetic energy and Stuckelberg mass mixings of the hidden sector affect the neutralino mass matrix. For the MSSM case, the neutralino mass matrix is 4×4. However, including the mixing effects from the hidden sector, the neutralino mass matrix becomes six by six with four neutralinos residing mostly in the visible sector and the other two in the hidden sector. If the lightest neutralino resides in the hidden sector it will produce some dramatic effects such as the one discussed in the next section. Aside from the kinetic mixing and Stuckelberg mass mixing portals there can be other portals as well such as the Higgs portal [125].

7 Long lived sparticle decays

As discussed above there may be mixing between the visible sector and the hidden gauge groups via kinetic and mass mixings. Because of the mixing the matter sector in the visible sector will have extra weak or super weak interactions with the hidden sector. Next let us suppose that the LSP in the MSSM sector is a stau. Further let us assume that the lightest neutralino in the hidden sector is lighter than the stau. In this case the stau will decay into the hidden sector neutralino. If R parity is conserved, the neutralino in the hidden sector will be stable and a candidate for dark matter. In this case for a range of stau-tau-hidden-sector-neutralino coupling, the stau decay into a tau and a hidden sector neutralino will have a lifetime large enough that it will leave a visible track inside the detector and the decay of the stau into a tau will show a kink providing evidence of dark matter via a missing energy signal. An analysis of this phenomenon has been carried out recently at LHC at 14 TeV and at the prospective future machine HE-LHC at 27 TeV, and the range of the parameter space where such a phenomenon would occur has been identified [126]. (see also the related work [127]). If the interactions are too weak or too feeble, the

NLSP will escape the detector before decay. However, in this case the coupling will be subject to the constrain that the NLSP decay before the BBN. In general the relic density in such processes consists of two parts: one part that arises from out of equilibrium non-thermal-processes where the hidden sector particle is too feeble to be produced by thermal processes but its relic density arises from the decay of other supersymmetric particle. The second component to the relic density arises from the decay of the NLSP using its relic density computed using thermal equilibrium. Another possibility of a long lived particle is a stop which would be the LSP of the MSSM sector and decays into a hidden sector neutralino [128].

8 Unification: GUTs and strings

$SO(10)$ unification contains several interesting properties in that aside from unifying the standard model gauge group, it contains a full generation of quarks and leptons in one irreducible 16-plet representation. Further, the 16-plet also contains a right handed neutrino which is needed for generating a see-saw mass for the neutrinos. However, typically one needs three different representations to break $SO(10)$: one sets of Higgs representations often used consist of a 45 plet of Higgs to break $SO(10)$, a $16 + \overline{16}$ to reduce the rank of the gauge group, and a 10 plets of Higgs to break the electroweak symmetry. There are many available variations of the Higgs representations which accomplish the same thing. Another proposal is the use of $144 + \overline{144}$ multiplets of Higgs [129–133]. This combination can break the $SO(10)$ in one step to the standard model gauge group. Further, by fine tuning one can make one Higgs doublet light which can accomplish electroweak symmetry breaking. In this symmetry breaking pattern, fermion masses arise from quartic couplings $16 \cdot 16 \cdot 144 \cdot 144$ and $16 \cdot 16 \cdot \overline{144} \cdot \overline{144}$. However, much larger third generation masses can arise if one includes additional 45 and 120 matter representations with cubic couplings $16_M \cdot 45_M \cdot \overline{144}$ and $16_M \cdot 120_M \cdot 144$. Inclusion of these couplings allows one to achieve $b - t - \tau$ unification with a low value of $\tan\beta$ [129–134]. This is in contrast to the case where the electroweak symmetry is broken by a 10-plet of Higgs and one needs a $\tan\beta \sim 50$ to achieve $b - t - \tau$ unification [135].

However, grand unified models with 144 multiplet discussed above have the problem of achieving one pair of light Higgs bosons needed for electroweak symmetry breaking in a natural fashion, i.e., without the necessity to fine tune. Thus typically in grand unification the Higgs doublets and triplets belong to a common multiplet and the doublets along with the triplets would tend to get heavy masses of the size the GUT scale and would require a fine tuning of one part in 10^{28} to make one pair of Higgs doublets light. Several proposals have been made to make the doublets light in a natural way that does not involve a high degree of fine tuning. One of these relates to specific choices of heavy and light Higgs representations so that one pair of light doublets arises while all the Higgs triplets and the remaining Higgs doublets are heavy. For $SU(5)$ this is implemented in [136,137] and for the $SO(10)$ model this is implemented in [138,139] (for an application of the $SO(10)$ missing partner model to proton decay see [140]). For example, one may consider the heavy sector to consist of $126+\overline{126}+210$ of fields and the light sector to consist of $2 \times 10 + 120$ Higgs representations which results in all the Higgs triplets becoming heavy leaving just one pair of light Higgs representations (for an early analysis using large representations in $SO(10)$ see [141–144] and for more recent work see [145] and the references therein). Within this framework an analysis of $B - L = -2$ operators was carried out in [146]. The analysis is based on techniques developed in [147–151]. Specifically all allowed dimension five, dimension seven and dimension nine operators arising from matter Higgs interactions were computed. These operators enter in the study of neutrino masses, baryogenesis, proton decay and $n-\bar{n}$ oscillations. Among the exceptional groups only E_6, E_7, E_8 are possible candidates, However, among them only E_6 is acceptable as E_7, E_8 do not have chiral representations. There is an extensive literature on E_6 model building and it is shown that with appropriate symmetry breaking schemes E_6 can produce a low energy theory consistent with data (see, e.g., [152,153]). E_6 as the unification group has also been investigated extensively within string theory. Here E_6 is broken down to the standard model gauge group by a combination of flux breaking and by Higgs fields VEVs. One such possibility is $E_6 \to SO(10)\otimes U(1)_\psi$, $SO(10) \to SU(5)\otimes U(1)_\xi$, $SO(10) \to SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. More recently E_6 unification has also been investigated within F-theory (see e.g., [154]).

However, we note that while GUT theories are consistent effective theories of particles interactions, not all consistent particle physics theories are necessarily UV complete theories of quantum gravity. Typically they are thought to belong to what one may call "swampland" [155] and the probability of such a theory arising as a remnant of a theory of quantum gravity is very small. Thus currently the only viable theory of quantum gravity is viewed as the string theory. Thus it is of interest to derive the general criteria in string model building which on one hand will guarantee unification of gauge couplings [156] and on the other produce a splitting of doublets and triplets in the Higgs sector making one pair of Higgs doublets light and all triplets and remaining doubles superheavy. SUSY/SUGRA also have important implications for cosmology some which are discussed in [157].

9 SUSY prospects at HL-LHC and HE-LHC and elsewhere

The best prospect for the discovery of SUSY in the near future rests with HL-LHC which will operate at 14 TeV and collect as much as 3000 fb^{-1} or more of data. A realistic possibility of a higher energy collider after that is the high energy LHC (HE-LHC), under consideration by the FCC study at CERN, which would use the existing tunnel at CERN with FCC technology magnets to achieve a center-of-mass energy of $\sqrt{s} = 27$ TeV and at a luminosity of 2.5×10^{35} cm^{-2} s^{-1}. Analyses done within supergravity unified models show that model points which might take 5–8 years for discovery at the HL-LHC might be discoverable at HE-LHC in periods ranging from a few weeks to few months [158–160]. Beyond that the parameter space not accessible to HL-LHC can be probed by HE-LHC as shown in [158–160]. A similar situation holds for the case of the discovery of the charged Higgs and CP odd Higgs (for review of the signatures of new physics at HL-LHC and HE-LHC see [161–163]). Machines with even higher energies are also under consideration such as a 100 TeV collider [164,165]). In addition, of course, tests of SUSY can come from the more refined analyses on $g_\mu - 2$, on the EDMs of the electron and the neutron, from a possible detection of dark matter in direct and indirect dark matter experiments, and from a possible detection of proton decay.

10 Conclusion

Supersymmetry and supergravity based models provide a paradigm for the extrapolation of physics from the electroweak scale to the grand unification scale. These models are free of the big gauge hierarchy problem related to the corrections to the Higgs boson mass. Supergravity based models predict a Higgs boson mass to lie below 130 GeV and the observation of the Higgs boson mass at ~ 125 GeV supports the supergravity prediction. As is well known the Higgs boson mass at $\sim$125 GeV requires a large loop correction within supersymmetry/supergravity models which is turn requires the scale of weak scale supersymmetry to lie in the TeV region. A TeV size weak scale makes the observation of sparticles more challenging, first because in this case sparticle masses tend to be large requiring higher collider energies for their observation. Additionally, supergravity models with R parity will have a stable LSP which if neutral could be the dark matter candidate and one needs to include in the theory analyses the relic density constraint. Typically models with a large scale of weak scale supersymmetry lead to a bino like neutralino which requires co-annihilation to satisfy the relic density constraint. Of course co-annihilation leads to the NLSP lying close to the LSP which implies that the decay of the NLSP to LSP will result in soft jets and leptons which consequently makes the detection of supersymmetry more difficult. An alternative possibility for the satisfaction of the relic density is that μ is relatively small, as can happen on the hyperbolic branch, and the neutralino is higgsino-like. A higgsino-like neutralino will undergo a rapid annihilation in the early universe and the neutralino relic density may fall below the experimental value. In this case the relic density deficit will need to be made up from sources other than the neutralino and consequently dark matter will be multi-component.

Of course, the large scale of weak scale supersymmetry although not desirable, since it makes the observation of supersymmetry more challenging, has its benefits. Thus it helps suppress flavor changing neutral currents arising from supersymmetry in processes such as the radiative decay of the b quark to an s quark, and the decay of the B_s meson into muon-anti-muon pair, since deviations from the standard model predictions in these processes are small. Further, a large mass of the sfermions stabilizes the proton from decay via lepton and baryon number violating dimension 5 operators. Because of this there is a strong correlation of the proton lifetime prediction on the Higgs boson mass. We also discussed in this review the communication between hidden sectors and the visible sector which can arise either via kinetic mixing or via Stuckelberg mass mixing and more generally by a mixture of the two. As discussed such a mixing could lead to rather unexpected phenomenon. Thus we considered the possibility of the lightest supersymmetric particle in the MSSM sector being a stau while the hidden sector neutralino lies lower in mass than the stau. In the presence of kinetic and Stuckelberg mass mixing, the stau in the MSSM sector can decay into the hidden sector neutralino. Further, if the mixing between the two sectors is feeble, the stau may be long lived and may leave a track inside the detector before decaying. In this case we will have stau decaying into a tau and missing energy, which is a signal detectable at the LHC. Observation of such an event would constitute discovery of supersymmetry and dark matter.

Finally we discussed prospects for the discovery of supersymmetry at HL-LHC and a prospective future high energy LHC, i.e., HE-LHC. We showed that based on recent analyses one finds that SUGRA models which would take several years to discover at HL-LHC could be discovered in few months at HE-LHC. Further, it was exhibited that model points which are inaccessible at HL-LHC would be accessible at HE-LHC. Thus future machines with energies beyond the LHC are central for the exploration of a larger parameter space of SUGRA models, and further for studying in greater depth the properties of sparticles once they are discovered.

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