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Analytical solitary wave solution of the dust ion acoustic waves for the damped forced modified Korteweg-de Vries equation in q-nonextensive plasmas

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Abstract. Analytical solitary wave solution of the dust ion acoustic waves is studied due to the damped forced modified Korteweg-de Vries equation in an unmagnetized collisional dusty plasma consisting of negatively charged dust grain, positively charged ions, q-nonextensive electrons, and neutral particles in the presence of external periodic force. Using reductive perturbation technique, the damped forced modified Korteweg-de Vries equation is obtained for the dust ion acoustic waves. Momentum consevation law is used to obtain the dust ion acoustic solitary wave solutions in the framework of the damped forced modified Korteweg-de Vries equation. The effects of different physical parameters such as entropic index, dust ion collisional frequency, strength and frequency of the external periodic force, speed of the traveling wave and the parameter which is the ratio between the unperturbed densities of the dust ions and electrons are investigated on the analytical solution of the dust ion acoustic waves. It is observed that those parameters have significant effects on the structures of the damped forced dust ion acoustic solitary waves. The results of the present paper may have applications in laboratory and space plasma environments.

1 Introduction

Dusty plasma is a low temperature plasma consisting of electrons, ions, neutral particles and very massive micrometer-sized solid charged dust grains [1–6]. The interest in studying dusty plasma are gradually increasing because of its tremendous applications in planetary rings, comet tails, asteroids zone, interstellar medium, lower part of the Earth's atmosphere, magnetosphere [7–13], radio frequency discharge [14], plasma crystal [15,16], plasma processing reactors, fusion plasma device etc. The study of the wave phenomena is important because it keeps a nice relation between the theory and

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the experiments. Waves in dusty plasma had been studied exclusively for last three decades or so in different plasma mode like dust acoustic (DA) mode [6], dust ion-acoustic (DIA) mode [17], dust drift mode [18], Shukla-Varma mode [19], dust lattice mode [20], dust cyclotron mode [21], dust Berstain-Green-Krushkal mode [22] etc. Dust ion acoustic waves (DIAWs) involve the motion of massive ions and form region of compression and rarefaction just as in sound waves in the presence of dust grains. Shukla and Silin [32] predicted theoretically DIAWs in dusty plasma consisting of negatively charged static dust grains for the first time and Barken et al. [3] observed the existence of DIAWs experimentally. Linear and nonlinear DIAWs had also been experimentally investigated by Nakamura et al. [33] in a homogeneous unmagnetized dusty plasma. They observed that the phase velocity of the wave increases and the wave endures heavy damping with increasing dust density in the linear regime. Anowar and Mamun [34] investigated the basic features of obliquely propagating dust ion acoustic solitary waves (DIASW) containing adiabatic inertia-less electrons, adiabatic inertial ions, and negatively charged static dust in a hot adiabatic magnetized dusty plasma. Duba and Mamun [35] observed the DIA shock waves in dusty plasma in presence of Boltzmann electrons, mobile ions and charged fluctuating stationary dusts. Ghorui et al. [36] studied the head-on collision of DIAWs in a magnetized quantum dusty plasma. They showed that the quantum parameters (quantum diffraction, ion cyclotron frequency, ratio of densities of electrons to ion) effects the phase shift significantly. Recently, Jharna et al. [37] studied the DIAWs in an unmagnetized collisional nonextensive dusty plasma. They showed that characteristic of the wave effected by the nonextensive parameter and dust ion collisional frequency.

However, in most of the cases, Maxwell distribution had been considered for electrons but the Maxwell distribution is effected to the macroscopic ergodic equilibrium state and it may be insufficient to depict the long range interactions in unmagnetized collisionless plasma having the non-equilibrium stationary state. So, in various physical system this kind of state may exist such as, the presence of external force in natural space plasma environment, turbulence and particle interactions when the Maxwell distribution fails. Non-extensive statistics or Tsallis statistics have proposed for those cases based on the derivation of Boltzmann–Gibbs–Shannon (BGS) entropic measure. Renyi [38] first introduced the nonextensive generalization of BGS entropy for statistical equilibrium. Tsallis [39], suitably extending the standard additivity of the entropies to the nonlinear, nonextensive case where one particular parameter, the entropic index q, characterizes the degree of nonextensivity of the system ($q = 1$ corresponds to the standard, extensive, BGS statistics). Evidence have shown that many astrophysical scenario such as stellar polytopes, solar neutrino problem, and peculiar velocity distribution of galaxy clusters was analyzed by q-entropy. Liyan and Du [44] studied ion acoustic solitary wave (IASWs) in the plasma with power-law q-distribution in non extensive statistics and they suggested that Tsallis [39] statistcs is suitable for the system being the non equilibrium stationary state with inhomogeneous temperature and containing huge supply of the superthermal low velocity particles.

The existence of arbitrary amplitude DIAWs in an unmagnetized plasma had also been investigated by several authors [40–43]. However, all the studies done till today, were based on KdV, Kadomtsev-Petviashvili (KP), Zakharov-Kuznetsov (ZK) or similar type equations, but the effect of external applied force on these equations have not been studied. Very recently Saha et al. [45–47] studied the dynamical behaviour of DIAWs. In the presence of external periodic force, they [45] investigated the periodic and chaotic motion of modified equal width-Burgers (MEW-Burger) equation. Das et al. [46] studied the effect of the dust ion collisional frequency on DIAWs in a magnetized collisional dusty plasma in the frame work of KP equation in the presence of external periodic force. They showed the transition of DIAWs from quasiperiodic behavior to limit cycle oscillation. Considering the external periodic perturbation Saha et al. [47] observed the quasiperiodic, periodic and chaotic structures of DIAWs. We can find the study about nonlinear wave excitation caused by orbiting charged debris object in Sen et al.'s [26] research work. They considered that a charged debris, moving at a speed v_d contributes $S(x - v_d t)$ as the source term in the poisson equation. Ali et al. [27–29] also took the source term in the poisson equation. In their paper, they obtained the analytical solution of KdV and DKdV equation in the presence of external periodic force. These work have motivated us to consider the source term in the Poisson equation. It has known that the value of the parameter set at which the nonlinear term of the KdV equation vanishes is called the critical point. To study solitary wave at critical point, one has to employ modified KdV (MKdV) equation instead of KdV equation. In this paper, we have studied the dust ion acoustic solitary waves (DIASWs) in the presence of externally applied force at the critical point and obtained MKdV equation. Till today no work has been reported to study the solitary wave in the frame work of MKdV equation with an external force.

In the present work, our aim is to derive the analytical DIAWS of the damped force MKdV (DFMKdV) equation in an unmagnetized collisional dusty plasma consisting of negatively charged dust grain, positively charged ions, q-nonextensive electrons, and neutral particles in the presence of external periodic force. Furthermore, the effect of the entropic index (q), dust ion collisional frequency (ν_{id0}), the speed of the traveling wave (M), strength (f_0) and frequency (ω) of the periodic force and the parameter (μ) which is the ratio between the unperturbed densities of the dust ions and electrons are studied on the analytical solution of DIASWs. The rest of the paper is organized as follows: The basic equations are provided in Section 2. In Section 3, we have derived the damped forced modified Korteweg-de Vries (DFMKdV) equation for nonlinear propagation of dust ion acoustic solitary waves. Section 4 presents the effect of the different parameters on analytical solitary wave solution of DFMKdV. Section 5 states the conclusions.

2 Basic equations

In this work, we consider an unmagnetized collisional dusty plasma that contains cold inertial ions, stationary dusts with negative charge and q-nonextensive electrons. The normalized ion fluid equations which include the equation of continuity, equation of momentum balance and Poisson equation, governing the DIAWs, are given by

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x} - \nu_{id} u, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1-\mu)n_e - n + \mu + S(x,t), \tag{3}$$

where n is the number density of ions normalized to its equilibrium value n_0, u is the ion fluid velocity normalized to ion acoustic speed $C_s = \sqrt{\left(\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{m_i}\right)}$, with T_{e} as electron temperature, k_{B} as Boltzmann constant and m_i as mass of ions. The electrostatic wave potential ϕ is normalized to $\frac{k_{\mathrm{B}} T_{\mathrm{e}}}{e}$, with e as magnitude of electron charge. The space variable x is normalized to the Debye length $\lambda_D = \left(\frac{T_{\mathrm{e}}}{4\pi n_{e0} e^2}\right)^{\frac{1}{2}}$ and the time t is normalized to $\omega_{pi}^{-1} = \left(\frac{m_i}{4\pi n_{e0} e^2}\right)^{\frac{1}{2}}$, with ω_{pi} as ion-plasma frequency.

Here ν_{id} is the dust-ion collisional frequency, the term $S(x,t)$ ([26–29]), is a charged density source arising from experimental conditions for a single definite purpose and $\mu = \frac{Z_d n_{d0}}{n_0}$.

In order to describe q-nonextensive electron, we consider the following distribution function [23]

$$f_e(v) = C_q \left\{ 1 + (q-1) \left[\frac{m_e v^2}{2 k_{\mathrm{B}} T_{\mathrm{e}}} - \frac{e\phi}{k_{\mathrm{B}} T_{\mathrm{e}}} \right] \right\}^{\frac{1}{q-1}},$$

where ϕ is the electrostatic potential and other variables or parameter have their usual meaning. It is important to note that this particular distribution function $f_e(v)$ maximizes the Tsallis entropy and, thus, complies the laws of thermodynamics. Then, the constant of normalization is given by

$$C_q = n_{e0} \frac{\Gamma\left(\frac{1}{1-q}\right)}{\Gamma\left(\frac{1}{1-q} - \frac{1}{2}\right)} \sqrt{\frac{m_e(1-q)}{2\pi k_{\mathrm{B}} T_{\mathrm{e}}}} \quad \text{for} \quad -1 < q < 1$$

and

$$C_q = n_{e0} \frac{1+q}{2} \frac{\Gamma\left(\frac{1}{1-q} + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{1-q}\right)} \sqrt{\frac{m_e(1-q)}{2\pi k_{\mathrm{B}} T_{\mathrm{e}}}} \quad for \quad q > 1.$$

Integrating the distribution function $f_e(v)$ after normalization over the velocity space, one can obtain the q-nonextensive electron number density as

$$n_e = n_{e0} \left\{ 1 + (q-1) \frac{e\phi}{k_{\mathrm{B}} T_{\mathrm{e}}} \right\}^{\frac{q+1}{2(q-1)}}.$$

Thus, the normalized q-nonextensive electron number density takes the form as [23]:

$$n_e = n_{e0} \left\{ 1 + (q-1)\phi \right\}^{\frac{q+1}{2(q-1)}}. \tag{4}$$

3 Nonlinear evolution equation and its solution

The reductive perturbation technique (RPT) [24] is used to derive the damped forced KdV (DFKdV) equation in unmagnetized collisional dusty plasma to study the nonlinear wave propagation of DIAWs. The independent variables are stretched as [25]

$$\begin{cases} \xi = \epsilon^{1/2}(x - vt) \\ \tau = \epsilon^{3/2} t \end{cases} \tag{5}$$

where ϵ is the strength of nonlinearity and v be the phase velocity of the DIAWs to be determined from the lowest order of ϵ. The expansions of the dependent variables

$n, u, \phi, \nu, S(x,t)$ are as follows:

$$\begin{cases} n = 1 + \epsilon n_1 + \epsilon^2 n_2 + \cdots, \\ u = 0 + \epsilon u_1 + \epsilon^2 u_2 + \cdots, \\ \phi = 0 + \epsilon \phi_1 + \epsilon^2 \phi_2 + \cdots, \\ \nu_{id} \sim \epsilon^{3/2} \nu_{id0} \\ S \sim \epsilon^2 S_2. \end{cases} \tag{6}$$

Substituting the above expansions (6) along with stretching coordinates (5) into equations (1)–(3) and equating the coefficients of lowest order of ϵ (i.e. coefficient of $\epsilon^{3/2}$ from Eq. (1), coefficient of $\epsilon^{3/2}$ from Eq. (2) and coefficient of ϵ from Eq. (3)), the dispersion relation is obtained as

$$v = \frac{1}{\sqrt{a(1-\mu)}}, \tag{7}$$

with $a = \frac{q+1}{2}$.

Taking the coefficients of next higher order of ϵ (i.e. coefficient of $\epsilon^{5/2}$ from Eq. (1), coefficient of $\epsilon^{5/2}$ from Eq. (2) and coefficient of ϵ^2 from Eq. (3)), we obtain the DFKdV equation

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + C\phi_1 = B\frac{\partial S_2}{\partial \xi}, \tag{8}$$

where $A = \left(\frac{3}{2v} - \frac{bv}{a}\right)$, $B = \frac{v^3}{2}$ and $C = \frac{\nu_{id0}}{2}$, with $b = \frac{(q+1)(3-q)}{8}$.

Now at the certain values, for example $q = 0.6$ and $\mu = 0.5$, there is a critical point at which $A = 0$, which imply the infinite growth of the amplitude of the DIASW solution as nonlinearity goes to zero . Therefore, at the critical point at which $A = 0$ the stretching (5) is not valid. For describing the evolution of the nonlinear system at or near the critical point we introduce the new stretched coordinate as

$$\begin{cases} \xi = \epsilon^1 (x - vt) \\ \tau = \epsilon^3 t \end{cases} \tag{9}$$

and expand the dependent variables as

$$\begin{cases} n = 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \cdots , \\ u = 0 + \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \cdots , \\ \phi = 0 + \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^2 \phi_2 + \cdots , \\ \nu_{id} \sim \epsilon^3 \nu_{id0} \\ S \sim \epsilon^3 S_2. \end{cases} \tag{10}$$

Now substituting equations (9) and (10) into the basic equations (1)–(3) and equating the coefficients of lowest order of ϵ (i.e. coefficient of ϵ^2 from Eq. (1), coefficient of ϵ^2 from Eq. (2) and coefficient of ϵ from Eq. (3)), we obtain the following relations:

$$\begin{cases} n_1 = \frac{1}{v}u_1 \\ u_1 = \frac{1}{v}\phi_1 \\ n_1 = a(1-\mu)\phi_1. \end{cases} \tag{11}$$

Equating the coefficients of next higher order of ϵ (i.e. coefficient of ϵ^3 from Eq. (1), coefficient of ϵ^3 from Eq. (2) and coefficient of ϵ^2 from Eq. (3)), we have

$$\begin{cases} n_2 = \frac{1}{v}(u_2 + n_1 u_1) \\ \frac{\partial u_2}{\partial \xi} = \frac{1}{v}\left(u_1 \frac{\partial u_1}{\partial \xi} + \frac{\partial \phi_2}{\partial \xi}\right) \\ n_2 = (1-\mu)(a\phi_2 + b\phi_1^2). \end{cases} \tag{12}$$

Equating the coefficients of next higher order of ϵ (i.e. coefficient of ϵ^4 from Eq. (1), coefficient of ϵ^4 from Eq. (2) and coefficient of ϵ^3 from Eq. (3)), we have

$$\begin{cases} \frac{\partial n_1}{\partial \tau} - v\frac{\partial n_3}{\partial \xi} + \frac{\partial u_3}{\partial \xi} + \frac{\partial (n_1 u_2)}{\partial \xi} + \frac{\partial (n_2 u_1)}{\partial \xi} = 0 \\ \frac{\partial u_1}{\partial \tau} - v\frac{\partial u_3}{\partial \xi} + \frac{\partial \phi_3}{\partial \xi} + \frac{\partial (u_1 u_2)}{\partial \xi} + \nu_{id0} u_1 = 0 \\ \frac{\partial^2 \phi_1}{\partial \xi^2} = (1-\mu)\left(a\phi_3 + 2b\phi_1\phi_2 + c\phi_1^3\right) - n_3 + S_2, \end{cases} \tag{13}$$

where $a = \frac{(1+q)}{2}$, $b = \frac{(1+q)(3-q)}{8}$ and $c = \frac{(1+q)(3-q)(5-3q)}{48}$.

From equation (11), one can obtain the dispersion relation as $v^2 = \frac{1}{a(1-\mu)}$ and from equations (11)–(13), one can obtain the following nonlinear evaluation equation as:

$$\frac{\partial \phi_1}{\partial \tau} + A_1\phi_1^2\frac{\partial \phi_1}{\partial \xi} + B_1\frac{\partial^3 \phi_1}{\partial \xi^3} + C_1\phi_1 = B_1\frac{\partial S_2}{\partial \xi}, \tag{14}$$

where $A_1 = \frac{15}{4v^3} - \frac{3v^3 c(1-\mu)}{2}$, $B_1 = \frac{v^3}{2}$ and $C_1 = \frac{\nu_{id0}}{2}$.

It has been noticed that the behavior of nonlinear waves changes significantly in the presence of external periodic force and result supports the investigation of [29–31]. It is paramount to note that the source term or forcing term due to the presence of space debris in plasmas may be of different kind, for example, Gaussian forcing term [26], hyperbolic forcing term [26] (in the form of $\text{sech}^2(\xi,\tau)$ and $\text{sech}^4(\xi,\tau)$ functions) and trigonometric forcing term [48] (in the form of $\sin(\xi,\tau)$ and $\cos(\xi,\tau)$ functions). Motivated by these work we assume that S_2 is a linear function of ξ such as $S_2 = f_0\xi\cos(\omega\tau) + P$, where P is some constant and f_0, ω denote the strength and the frequency of the source respectively. Put the expression of S_2 in the equation (14) we get,

$$\frac{\partial \phi_1}{\partial \tau} + A_1\phi_1^2\frac{\partial \phi_1}{\partial \xi} + B_1\frac{\partial^3 \phi_1}{\partial \xi^3} + C_1\phi_1 = B_1 f_0\cos(\omega\tau). \tag{15}$$

Such a form of this source function is observed in experimental situations or conditions for a particular device. Equation (15) is termed as damped force modified Korteweg-de Vries (DFMKdV) equation. Till today no study of waves have been reported in the framework of DFMKdV equation. In absence of C_1 and f_0 i.e., for $C_1 = 0$ and $f_0 = 0$ equation (15) takes the form of well known MKdV equation with the solitary wave solution

$$\phi_1 = \phi_m \mathrm{sech}\left(\frac{\xi - M\tau}{W}\right), \tag{16}$$

where amplitude of the solitary waves $\phi_m = \sqrt{\frac{6M}{A_1}}$ and width of the solitary waves $W = \sqrt{\frac{B_1}{M}}$, with M is the speed of the ion-acoustic solitary waves.

It is well established for the MKdV equation that,

$$I = \int_{-\infty}^{\infty} \phi_1^2 \, d\xi, \tag{17}$$

is a conserved quantity.

For small values of C_1 and f_0, let us assume that amplitude, width and velocity of the ion acoustic waves are dependent on τ and the approximate solution of equation (15) is of the form

$$\phi_1 = \phi_m(\tau) \mathrm{sech}\left(\frac{\xi - M(\tau)\tau}{W(\tau)}\right), \tag{18}$$

where the amplitude $\phi_m(\tau) = \sqrt{\frac{6M(\tau)}{A_1}}$, width $W(\tau) = \sqrt{B_1/M(\tau)}$ and velocity $M(\tau)$ have to be determined.

Differentiating equation (17) with respect to τ and using equation (15), one can obtain

$$\frac{dI}{d\tau} + 2C_1 I = 2B_1 f_0 \cos(\omega\tau) \int_{-\infty}^{\infty} \phi_1 \, d\xi, \tag{19}$$

$$\Rightarrow \frac{dI}{d\tau} + 2C_1 I = \pi f_0 \sqrt{\frac{6B_1^3}{A_1}} \cos(\omega\tau). \tag{20}$$

Again,

$$I = \int_{-\infty}^{\infty} \phi_1^2 \, d\xi, \tag{21}$$

$$I = \int_{-\infty}^{\infty} {\phi_m}^2(\tau) \mathrm{sech}^2\left(\frac{\xi - M(\tau)\tau}{W(\tau)}\right) d\xi. \tag{22}$$

Integrating equation (22), we obtain

$$I = \frac{12\sqrt{B_1 M(\tau)}}{A_1}. \tag{23}$$

Substituting equation (23) into equation (20) with $M(0) = M$, we obtain

$$M(\tau) = \left[\frac{\pi f_0 \sqrt{A_1/6}}{2} e^{-\nu_{id0}\tau} \left(\frac{\omega}{\omega^2 + 4C_1^2}\right) \left\{\sin(\omega\tau) + \frac{2C_1}{\omega}\cos(\omega\tau)\right\} + \left\{\sqrt{M} - \pi f_0 B_1 \sqrt{A_1/24} \left(\frac{2C_1}{\omega^2 + 4C_1^2}\right)\right\}\right]^2. \tag{24}$$

Therefore, the slow time dependence form of the ion acoustic waves solution of the DFMKdV equation (15) is given by,

$$\phi_1 = \phi_m(\tau)\mathrm{sech}\left(\frac{\xi - M(\tau)\tau}{W(\tau)}\right), \tag{25}$$

where $M(\tau)$ is given by equation (24), and the amplitude and width are as follows:

$$\phi_m(\tau) = \frac{\sqrt{6}\left[\frac{\pi f_0\sqrt{A_1/6}}{2}e^{-\nu_{id0}\tau}\left(\frac{\omega}{\omega^2+4C_1^2}\right)\left\{\sin(\omega\tau)+\frac{2C_1}{\omega}\cos(\omega\tau)\right\}+\left\{\sqrt{M}-\pi f_0 B_1\sqrt{A_1/24}\left(\frac{2C_1}{\omega^2+4C_1^2}\right)\right\}\right]}{\sqrt{A}}$$

$$W(\tau) = \frac{\sqrt{B_1}}{\left[\frac{\pi f_0\sqrt{A_1/6}}{2}e^{-\nu_{id0}\tau}\left(\frac{\omega}{\omega^2+4C_1^2}\right)\left\{\sin(\omega\tau)+\frac{2C_1}{\omega}\cos(\omega\tau)\right\}+\left\{\sqrt{M}-\pi f_0 B_1\sqrt{A_1/24}\left(\frac{2C_1}{\omega^2+4C_1^2}\right)\right\}\right]}.$$

4 Effects of parameters

The effects of the parameters f_0, ω, ν_{id0}, q and M on the DIAW solution of the DFMKdV equation (15) have been studied in this section. When $A = 0$, we have the relation between μ, the ratio between the unperturbed densities of the dust ions and electrons and the entropic index q as $q = \frac{3\mu}{4-3\mu}$ or $\mu = \frac{4q}{3(1+q)}$ which gives the critical points. Figures 1a–1c represent the variation of μ with respect to q for the ranges $-1 < q < 0$ in Figure 1a, $0 \leq q < 1$ in Figure 1b and $q > 1$ in Figure 1c. From Figure 1a, it is clear that when $q \to -1$, $\mu \to -\infty$. Since, we have not observed any rarefactive solitary wave solution of the DFMKdV equation (15), this range of q i.e. $-1 < q < 0$ is not important for the present work. In Figure 1b, it is shown that as $q \in (0, 1)$, $\mu \in (0, 0.67)$ and this range of q is highly important to study the effect of the different physical parameters on the DFMKdV solution of equation (15) because it is seen from Figure 1c that when $q > 1$, $\mu \in (0.67, 1.33)$ and if we take any value of μ between 0.75 and 1.33 with corresponding q, the variation of effects of the different parameters on the DIAW solution of DFMKdV equation (15) can not be distinguished properly and thus the effects of the parameters become insignificant.

Figure 2 shows the variation of the dust ion acoustic solitary wave solution ϕ_1 against ξ of the DFMKdV equation (15) for different values of strength f_0 of the externally applied force with other parameters $M = 0.1$, $q = 0.9$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $\omega = 0.4$ and $\tau = 2$. f_0 is taken from the interval $(0, 0.09)$ and to discuss the changes, the figures are drawn for the particular values of $f_0 = 0, 0.03, 0.06$ and 0.09. It is observed that as the strength f_0 of the external periodic force increases the amplitude of the dust ion acoustic wave increases.

In Figure 3, we present the variation of the dust ion acoustic solitary wave solution ϕ_1 against ξ of the DFMKdV equation (15) for different values of frequency ω of the externally applied force with all other parameters $M = 0.1$, $q = 0.9$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $f_0 = 0.06$ and $\tau = 2$. We have taken ω from the interval $(0.1, 1.3)$ and the results are plotted for the fixed values of $\omega = 0.1, 0.5, 0.9$ and 1.3. It is interesting to note that the reverse effect happened as in Figure 1 and it is observed that the amplitude of the dust ion acoustic wave decreases as the frequency ω of the external

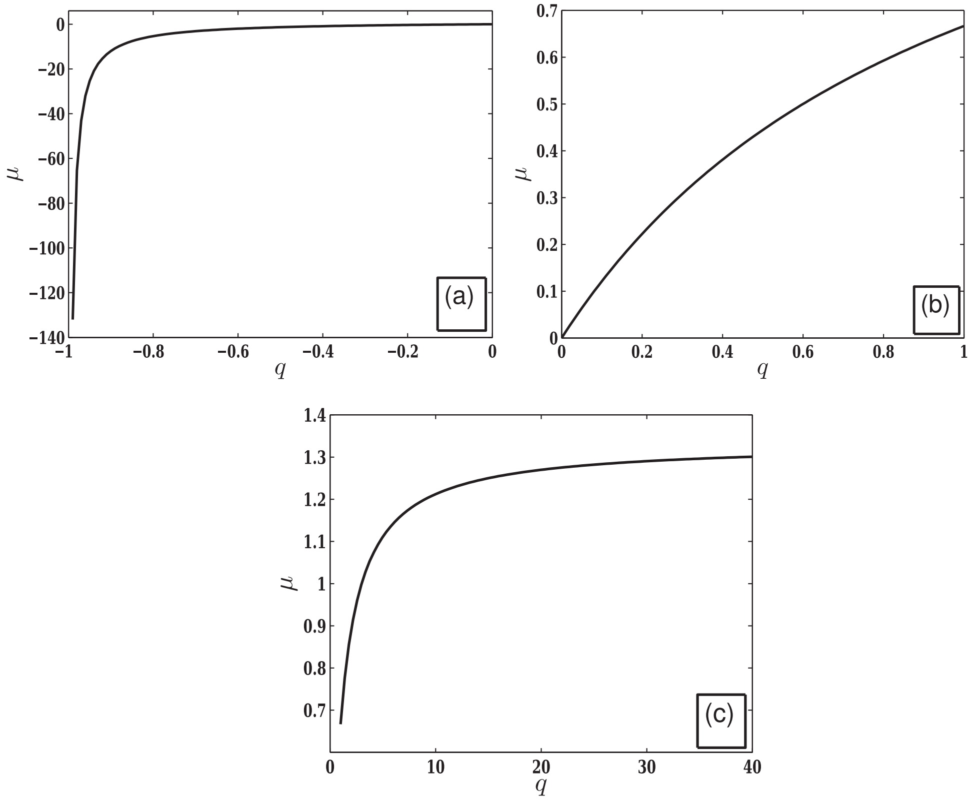

Fig. 1. Variation of μ, the ratio between the unperturbed densities of the dust ions and electrons in the different ranges of the entropic index q.

periodic force increases. It is also seen that when $\omega \in (0.1, 0.5)$ the peaks of the amplitude of the dust ion acoustic wave are almost same.

The variation of the dust ion acoustic solitary wave solution ϕ_1 against ξ of the DFMKdV equation (15) is depicted for different values of ν_{id0} with special values of the other parameters $M = 0.1$, $q = 0.9$, $\mu = 0.63$, $f_0 = 0.06$ and $\tau = 2$ in Figure 4. ν_{id0} is taken from the interval $(0.01, 0.13)$ and to observe the differences the figures are presented for the fixed values of $\nu_{id0} = 0.01, 0.05, 0.09$ and 0.13. It is noticed that the amplitude of the dust ion acoustic wave decreases as ν_{id0} increases but the width of the amplitude remain almost unaltered.

Figure 5 reflects the variation of the dust ion acoustic solitary wave solution ϕ_1 against ξ of the DFMKdV equation (15) for different values of entropic index q with respective values of μ coming from the relation $\mu = \frac{4q}{3(1+q)}$ and other special parameters $M = 0.1$, $\omega = 0.4$, $\nu_{id0} = 0.01$, $f_0 = 0.06$ and $\tau = 2$. Here, $q\ in(0.6, 0.9)$ and corresponding $\mu\ in(0.5, 0.63)$. The figures are plotted for pair of values of ($q = 0.6$, $\mu = 0.5$), ($q = 0.7$, $\mu = 0.55$), ($q = 0.8$, $\mu = 0.59$) and ($q = 0.9$, $\mu = 0.63$). It is observed that as entropic index (q) and μ increases both the amplitude and width of the ion acoustic solitary wave decreases.

Figure 6 presents the variation of the dust ion acoustic solitary wave solution ϕ_1 against ξ of the DFMKdV equation (15) for different values of the speed of the traveling wave (M) with special parameters $q = 0.9$, $\omega = 0.4$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $f_0 = 0.03$ and $\tau = 2$. It is found that as the speed of the traveling wave (M) increases, the peak of the amplitude of the ion acoustic solitary wave increases and the width of

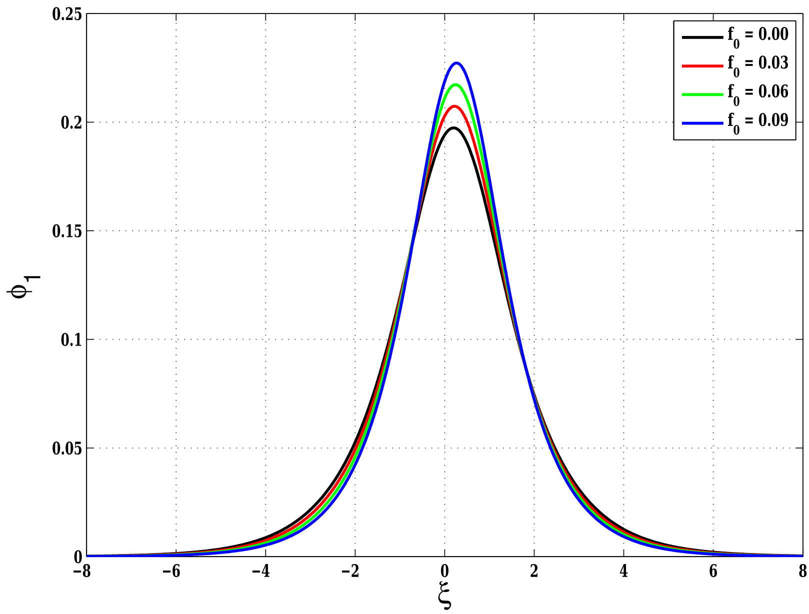

Fig. 2. Variation of the solitary wave solution of the damped forced modified KdV equation (15) for the different values of strength of the periodic force f_0 with other physical parameters $M = 0.1$, $q = 0.9$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $\omega = 0.4$ and $\tau = 2$.

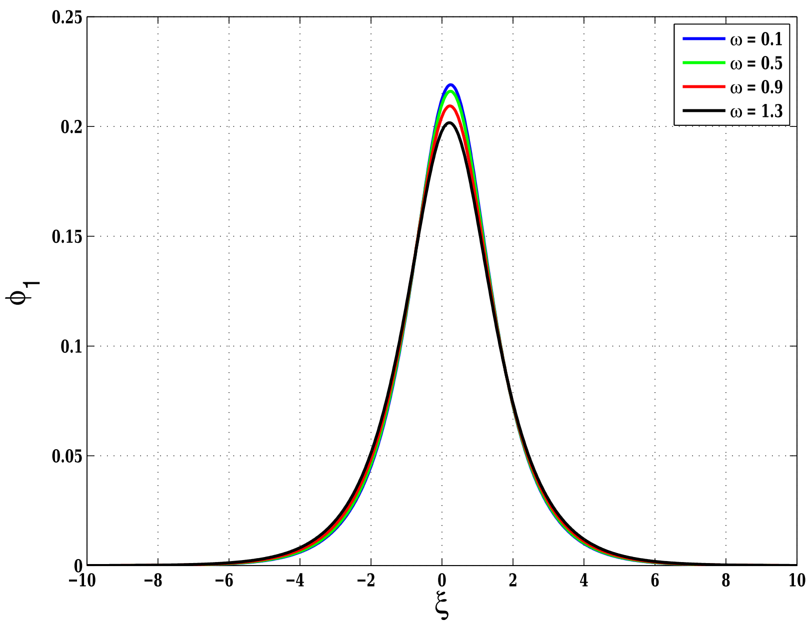

Fig. 3. Variation of the solitary wave solution of the damped forced modified KdV equation (15) for the different values of frequency of the periodic force ω with other special parameters $M = 0.1$, $q = 0.9$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $f_0 = 0.06$ and $\tau = 2$.

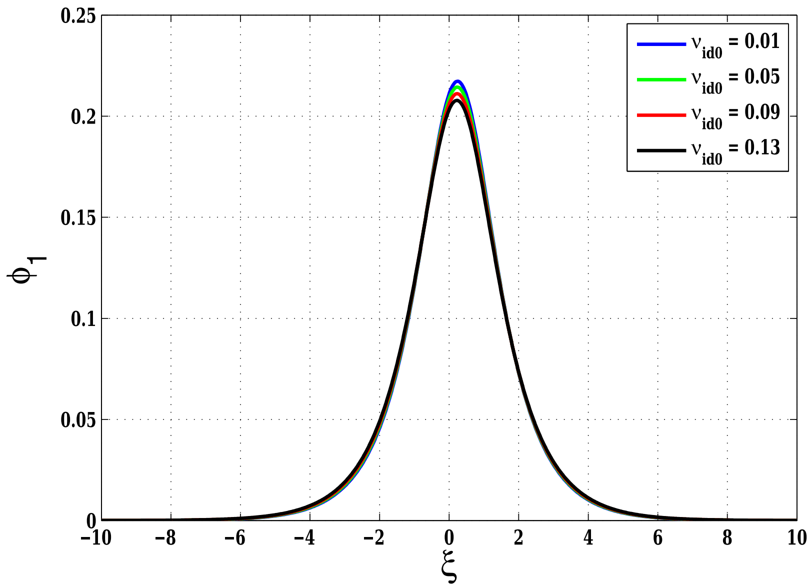

Fig. 4. Variation of the solitary wave solution of the damped forced modified KDV equation (15) for the different values of dust ion collisional frequency ν_{id0} with $M = 0.1$, $q = 0.9$, $\omega = 0.4$, $\mu = 0.63$, $f_0 = 0.06$ and $\tau = 2$.

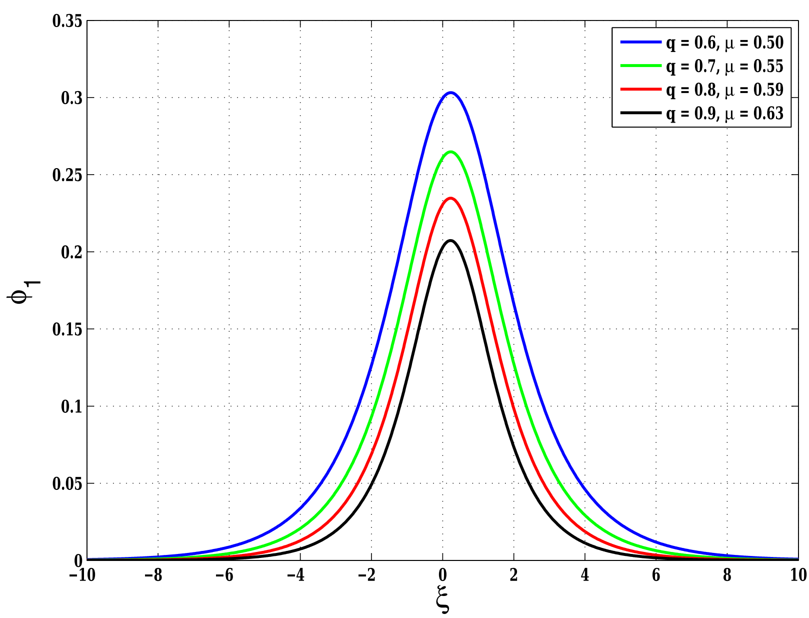

Fig. 5. Variation of the solitary wave solution of the damped forced modified KdV equation (15) for the different values of entropic index q with respective values of μ, the ratio between the unperturbed densities of the dust ions and electrons with $M = 0.1$, $\omega = 0.4$, $\nu_{id0} = 0.01$, $f_0 = 0.06$, $\tau = 2$.

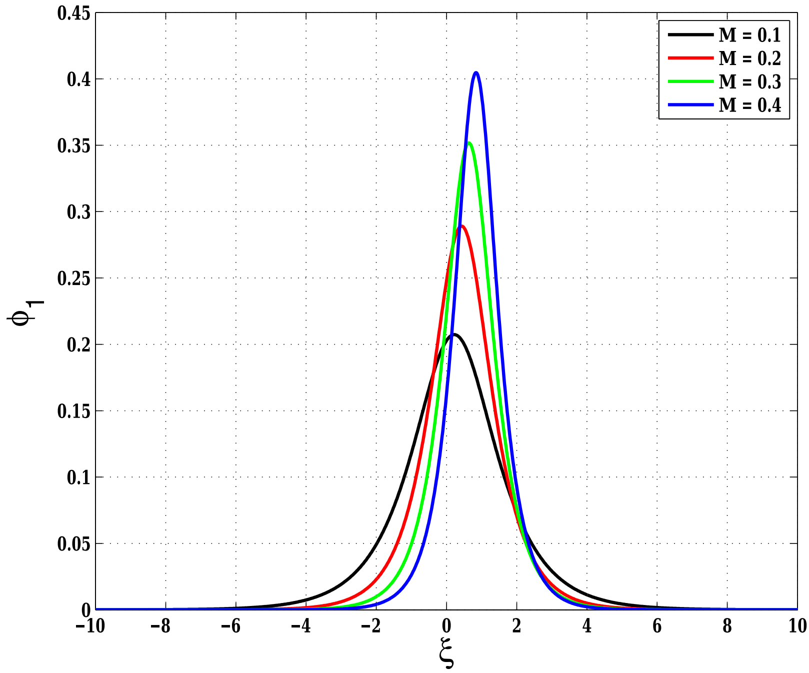

Fig. 6. Variation of the solitary wave solution of the damped forced modified KdV equation (15) for the different values of the speed of the travelling wave M with other physical parameters $q = 0.9$, $\omega = 0.4$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $f_0 = 0.03$ and $\tau = 2$.

the amplitude decreases. A right hand shifting of the dust ion acoustic solitary wave solution is also observed as M increases.

The dependence of the amplitude of the dust ion acoustic solitary wave solution of DFMKdV (15) on the strength (f_0) is presented in Figure 7 for different values of frequency (ω), and all other parameters are same as in Figure 1. The amplitude of the solitary wave increases as the strength of the external periodic force increases. Also at the same time, it is observed that the rate of change of amplitude of the solitary wave decreases when the frequency of the external force increases.

Figure 8 represents the dependence of the amplitude of the dust ion acoustic solitary wave solution of DFMKdV (15) with respect to the strength (f_0) of the external periodic force for different values of entropic index q. It is seen that the amplitude of the solitary wave decreases as the entropic index q increases with the strength (f_0) of the external periodic force.

In Figures 9a and 9b, three dimensional plots of the dust ion acoustic solitary wave solution ϕ_1 are depicted in the plane (ξ, τ) of the damped forced modified KdV equation (15). In Figure 9a special values of the parameters are taken as $M = 0.1$, $q = 0.9$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $\omega = 0.4$, $f_0 = 0.03$ and $\tau = 2$ and in Figure 9b $q = 2.1$, $\mu = 0.9$ with all other parameters are same as in Figure 9a. Both the figures, $\xi \in (-15, 15)$ and $\tau \in (-15, 15)$. In Figure 9a, the maximum amplitude of the dust ion acoustic solitary wave lies between 0.19 and 0.21 and between 0.044 and 0.055 in Figure 9b. Thus, it is observed that as the values of q and corresponding μ increases, the amplitude and width of the dust ion acoustic solitary wave significantly decreases. Also, it is seen that the solitary waves become much sharper in Figure 9b than in Figure 9a.

In Figures 10a and 10b, contour plots of the dust ion acoustic solitary wave solution ϕ_1 are depicted in the plane (ξ, τ) of the DFMKdV equation (15) with other

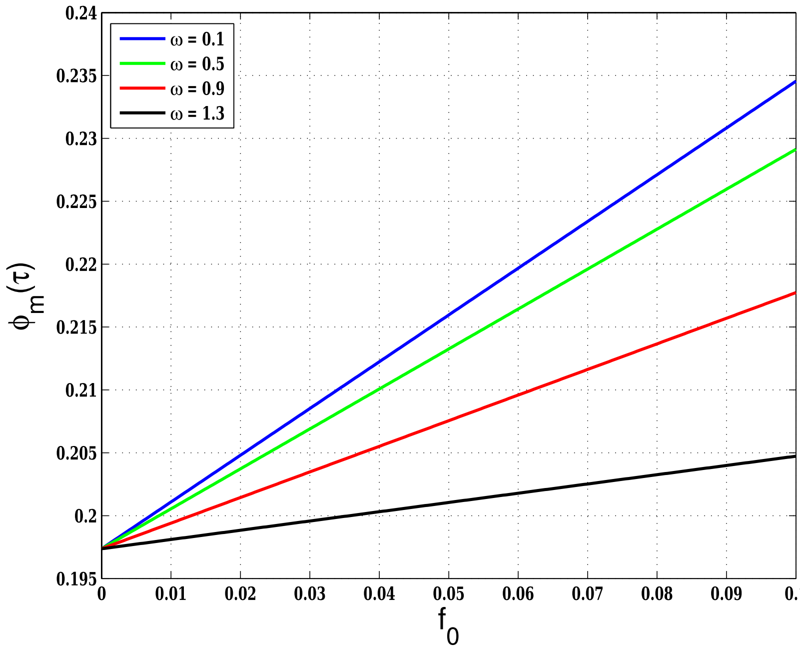

Fig. 7. Variation of the solitary wave amplitude ($\phi_m(\tau)$) with respect to strength of the periodic force f_0 of the damped forced modified KdV equation (15) for the different values frequency ω with all other parameters are same as in Figure 1.

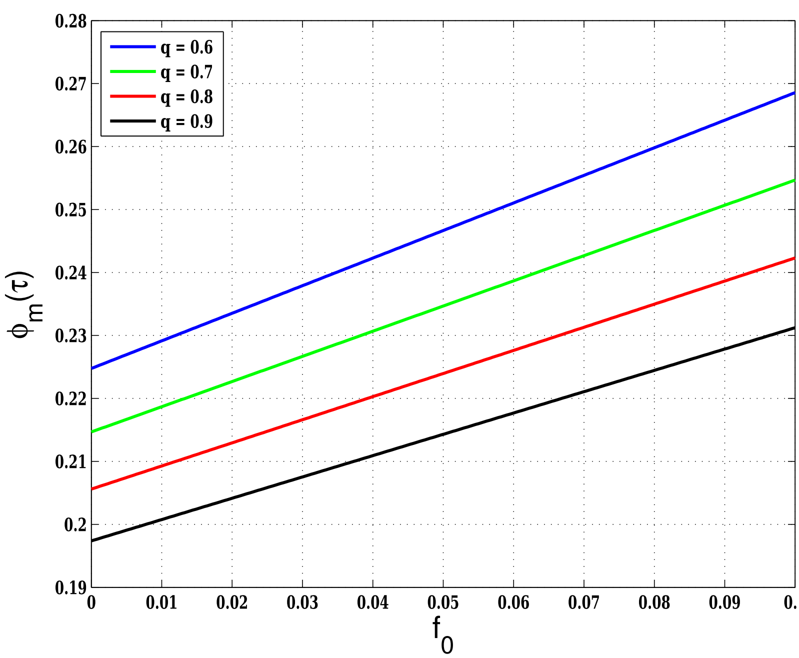

Fig. 8. Variation of solitary wave amplitude ($\phi_m(\tau)$) with respect to strength of the periodic force f_0 of the damped forced modified KdV equation (15) for the different values of entropic index q with all other parameters are same as in Figure 5.

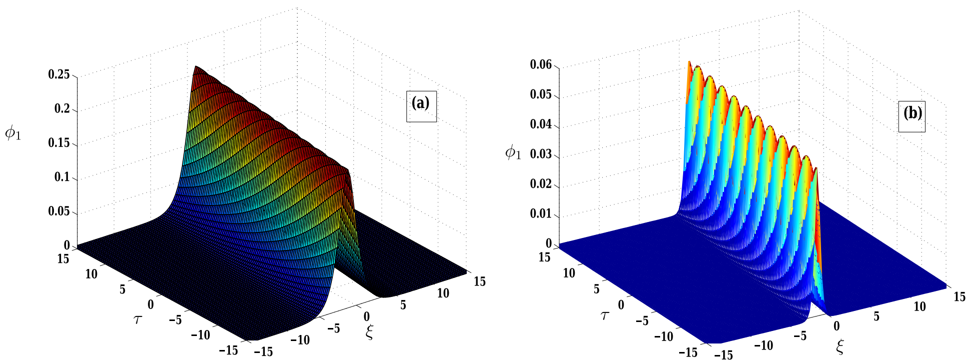

Fig. 9. Three dimensional plots of the solitary wave solution ϕ_1 in the plane (ξ, τ) of the DFMKdV equation (15) with $M = 0.1$, $q = 0.9$, $\mu = 0.63$, $\nu_{id0} = 0.01$, $\omega = 0.4$, $f_0 = 0.03$ and $\tau = 2$ in Figures 9a and 9b, $q = 2.1$ and $\mu = 0.9$ with all other parameters are same as in Figure 9a.

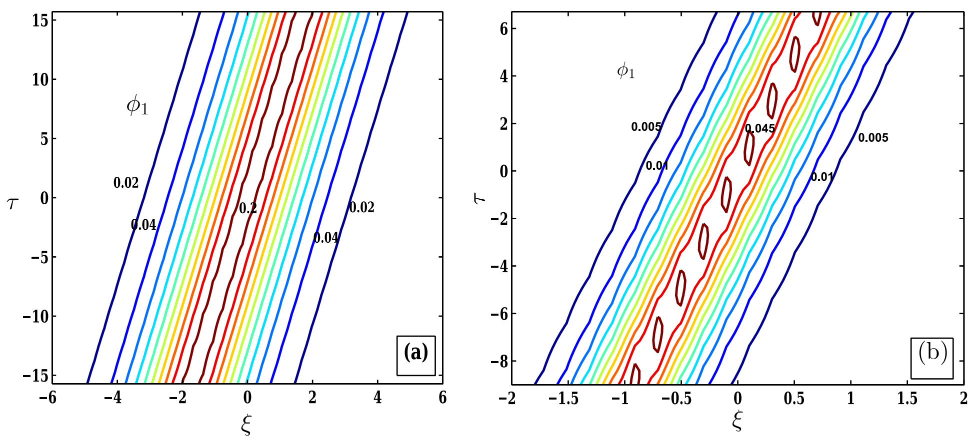

Fig. 10. Contour plot of the solitary wave solution ϕ_1 in the plane (ξ, τ) of the damped forced modified KdV equation (15) with other parameters same as in Figures 9a and 9b.

physical parameters same as in Figures 9a and 9b. Figures represent the equiamplitude solution space of the dust ion acoustic solitary wave solution ϕ_1 and follows a specific pattern in the (ξ, τ) plane with the values of ϕ_1 at the outer most contours 0.02 and 0.005 in Figures 10a and 10b, respectively. From Figures 10a and 10b, it is noticed that the outermost contour has the the same value of ϕ_1 and it increases with the values of the outermost contour towards the centre of the solution space from both sides. This is because we are representing a solitary wave solution. It is found from the contours that the value of the maximum amplitude in Figure 10a is 0.2 and that of in Figure 10b is 0.045.

5 Conclusions

We have studied dust ion acoustic solitary waves in a dusty plasma with negatively charged ions, non-extensive electron and stationary dust particles. The reductive perturbation technique is employed to derive the DFMKdV equation. An analytical solitary wave solution has been derived for DFMKdV equation in the presence of small damped and external applied periodic force. No work have been reported till today on the analytical solution of DFMKdV equations. The effect of the parameters q, f_0, M, ν_{id0} and ω on the dust ion acoustic solitary wave solution with fixed values of other physical parameters μ, τ has been presented. The parameters q, f_0, M, ν_{id0} and ω have played an important role on the nonlinear structure of the DIASW in an collisional dusty plasma.

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Author contribution statement

Contribution of all authors is same.

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