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Review

Bell inequality violation in the framework of a Darwinian approach to quantum mechanics

Carlos Baladrón[1,a] and Andrei Khrennikov[2,3]

[1] Departamento de Física Teórica, Atómica y Óptica, Universidad de Valladolid, 47071 Valladolid, Spain
[2] International Center for Mathematical Modeling in Physics and Cognitive Science, Linnaeus University, 35195 Växjö, Sweden
[3] National Research University of Information Technologies, Mechanics and Optics (ITMO), St. Petersburg 197101, Russia

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Abstract. A fundamental particle in physical space subject to conservation of momentum and energy, and characterized by its average mass and its position is methodologically supplemented with an information processor – a classical Turing machine – and a randomizer both defined on an information space localized on every particle. In this way the particle can be considered a generalized Darwinian system on which natural selection could act steering the evolution on the information space of the algorithms that govern the behaviour of the particles, giving rise plausibly to emergent quantum behaviour from initial randomness. This theory is applied to an EPR-Bohm experiment for electrons in order to analyse Bell inequality violation. A model for the entanglement of two particles has been considered. The model includes shared randomness – each particle stores its own randomizer and that of its partner – and the mutual transfer of their algorithms – sharing programs – that contain their respective anticipation modules. This fact enables every particle to anticipate not only the possible future configurations of its surrounding systems, but also those of the surrounding systems of its entangled partner. Thus, while preserving locality and realism, this theory implies outcome dependence – through shared randomness – and parameter dependence – through shared anticipation – for entangled states and, as a consequence, the violation of the Bell inequality in an EPR-Bohm experiment.

1 Introduction

Bell inequality violation [1] – i.e., non-classical correlations registered (and predicted by quantum mechanical calculations) for certain measurements performed on spatially-separated composite systems in quantum entangled states – constitutes one of the most striking phenomena ever observed in nature. It probably represents better than any other characteristic the hard core of quantum mechanics and its

[a] e-mail: baladron@cpd.uva.es

specificity against classical theories. The work of Bell [1] was central to transforming the metaphysical discussion, usually based on *Gedanken* experiments, that affected the interpretation of the quantum theory into experimental-based debate supported by observations that could be tested and refined in the laboratory.

The first experimental test was performed by Freedman and Clauser [2], reporting the violation of the Bell inequality. Then, among other landmarks [1], Fry and Thompson [3] obtained results that agree with Bell inequality violation, reducing the time needed to gather the experimental data, and Aspect et al. [4] confirmed Bell inequality violations more drastically than in any previous experiment. However, the difficulty of the experiment and the high degree of technological developments that are required to experimentally implement the conditions to test the Bell inequality gave rise to a series of more sophisticated experiments in order to confirm Bell inequality violations when, at the same time, closing the experimental loopholes. This has only been achieved recently [5–7].

The central question that the violation of Bell inequality poses is whether it implies something deep and astounding about nature. A first scrutiny of the explicit assumptions of Bell theorem seems to imply a short list – not exempt of controversy – of possible meanings of Bell inequality violations. Nonlocality [8], contextuality – e.g. counterfactual indefiniteness, multiple Kolmogorov probability spaces, among other several options [9–12] –, the presence of a background field [13–15], negative probabilities [16] are some of the properties of nature or explanatory schemes that might account for the non-classical correlations registered in EPR-Bohm experiments.

Delving into implicit assumptions, a larger list – although by no means exhaustive – of explanations of Bell inequality violations can be enumerated. First, absence – or at least a shortage or necessity of redefinition – of free will in experimenters[1] (this option is explored in the so-called superdeterministic theories [18,19]). Second, a radical extension of the role played by subjectivity in quantum mechanics (e.g. in QBism [20]). Third, enlarging reality by considering that all possible outcomes in an experiment are effectively realized, although in different, mutually unobservable – in principle – worlds (many worlds interpretation of quantum mechanics [21]). Fourth, rejecting the principle of causality and admitting backward in time causation (e.g. in time-symmetric quantum mechanics [22]). In fact, almost every interpretation of quantum mechanics has a different way of looking at the Bell inequality violations. The problem is that nearly every approach on its own way seems to sacrifice one – or at least a part – out of three central pillars of classicality: realism, causality and locality[2] [23,24]. As Jennings and Leifer [25] point out, perhaps this is the message that the Bell theorem and the other interwoven no-go theorems [26] convey about quantum mechanics, namely, that it is not possible to recover classical reality.[3] But is there any unavoidable reason for the world to be quantum? Or “why the quantum?” as Wheeler famously wrote.

In this article, we adopt a constructive standpoint, as advocated by Khrennikov [27], and develop a model of entanglement[4] in the framework of a Darwinian approach

[1]This possibility was explicitly mentioned by Bell [17].

[2]These three basic principles can be briefly defined, following Mückenheim [23], as realism or the possibility of an observer-independent description of nature, causality or the existence of a definite time ordering for cause and effect, and locality (or separability) or the propagation of interactions with a finite speed limit.

[3]The explicit definition for the term “classical” theory adopted by Jennings and Leifer [25] is that such a theory is local, non-contextual and the non-orthogonal pure states, in this kind of theory, are characterized by overlapping statistical distributions on some state space.

[4]This model of entanglement shares with other models the widespread idea that the information travels with the particles. This idea will be further explained in Section 2. See the article by Nieuwenhuizen [12] for a discussion about this idea in other models. In particular, our approach fits very well in the Allahverdyan-Balian-Nieuwenhuizen approach [28,29] to quantum measurement combined with the properties known from EPR quantum measurements.

to quantum mechanics (DAQM) [30–33] that allows us to rebuild, if not a complete classical reality, at least a close sketch of it (a realist, local and quasi-causal theory) that, at the same time, enables to outline an explanation of why the world is the way it is.

The two main non-classical elements that DAQM incorporates to describe a fundamental physical system are defined on an information space associated with every system, assuming that matter is complex and that the state of a particle cannot be completely described by position and momentum.[5] These two non-classical properties are intrinsic randomness and information processing capability. Intrinsic randomness is implemented through the ascription of a random number generator R to every fundamental particle and the information processing capability through a classical Turing machine [34] associated with every fundamental particle on which a program P controls the behaviour of the system. These two characteristics ascribed to matter in DAQM endow physical systems with the capacity of anticipating the configuration of their surrounding systems. This feature applied to entangled systems leads to a new perspective on Bell inequality violations in the framework of a realist and local theory (DAQM).

In Section 2, an EPR-Bohm experiment for electrons is analysed by means of a model of entanglement in the framework of DAQM. A short overview of DAQM is summarized in Section 3. Finally, the conclusion is drawn in Section 4.

We remark that our approach can be considered as a contribution to questioning the fundamental role of physical space in quantum physics. Mathematically it is modelled as $\mathbb{R}^3$, where $\mathbb{R}$ is the real continuum. This space was borrowed from Newtonian mechanics and practically unquestionably incorporated in the structure of quantum theory. We just say that dynamics of quantum systems cannot be described by using solely spatial coordinates. The physical space time has to be extended to include additional degrees of freedom responsible for randomness and anticipation. These degrees of freedom have the informational nature. Thus the physical space $\mathbb{R}^3$ is extended to *fibre bundle* E with the base space $B = \mathbb{R}^3$ and the fibre F which is the *information space*. Although such a fibre bundle can have a complicated geometric structure, this is still a classical state space. Thus in our approach "classicality" is recovered not straightforwardly. By trying to restrict the model to the Newtonian physical space, one confronts nonlocality and variety of other problems and paradoxes.

2 Analysis of an EPR-Bohm experiment for electrons in DAQM

An information-theoretic model for entanglement in the framework of DAQM [30–33] is devised and applied to the study of an EPR-Bohm experiment with electrons.

2.1 Constitution of a fundamental system in DAQM

A fundamental system is characterized in DAQM as a particle in three-dimensional physical space complemented by a methodological classical Turing machine and a random number generator both defined on an information space located on every particle. The magnitudes that identify the particle i in physical space are its average mass m_i and its position $\boldsymbol{X}_i(t)$ at time t. Particles follow continuous trajectories constrained to the conservation of energy and momentum in the processes of absorbing and emitting energy-momentum carriers. These carriers convey

[5] In fact, this is a basic assumption in orthodox quantum mechanics reflected on the characterization of the state of a system by means of a wave function.

information about the position of the emitter. After absorbing a carrier the information is transferred to the classical Turing machine of the receiver defined on its information space.

A key point of the model is the fact that the information space associated with every particle – a different space for every one – is localized at any time on the position occupied by such particle and given that the trajectory followed by any particle in physical space is continuous, then the information (received data and computed data) stored on the information space of every particle travels continuously with the corresponding particle in physical space. Therefore, the data stored on the information space of particle i are located at any moment on the position $\boldsymbol{X}_i(t)$ occupied by such particle i in physical space.

The Turing machine of particle i stores an algorithm P_i that governs the behaviour of the particle by calculating the parameters of the energy-momentum carrier to be emitted after every run of the program. This algorithm includes a module of anticipation A_i whose function is to calculate the possible future positions $\boldsymbol{X}_j$ of the particle's surrounding systems[6] – in fact, as in a radar-like problem, a probability density function $\rho_j(\boldsymbol{X})$ for the position occupied by the particle j. The randomizer[7] R_i completes the software that is stored on the information space of every particle.

Let us assume that P_i, A_i and R_i are able to generate in real time quantum behaviour on the particle i that is controlled by them.[8] Then the model might be envisaged as a kind of generalization of Bohmian mechanics in which every particle is controlled by its own probabilistic classical Turing machine. However, as we are going to show, in this model, in contradistinction to Bohmian mechanics, there is no trace of nonlocality thanks to anticipation.

2.2 An information-theoretic model for entanglement

Two particles in contact at time t in physical space are entangled (see Fig. 1) by sharing and storing their programs (P_1, P_2), anticipation modules (A_1, A_2), randomizers (R_1, R_2) and wave functions (ψ_1, ψ_2) on their respective information spaces. At time $t + \Delta t$, after separation in physical space, each particle conveys its own software $(P_{1(2)}, A_{1(2)}, R_{1(2)})$ and that of its partner $(P_{2(1)}, A_{2(1)}, R_{2(1)})$, in addition to the entangled wave function (ψ) of the bipartite system.

Each particle of the entangled pair is now able to anticipate, at time $t + \Delta t$, the possible locations of its surrounding systems, at time $t + 2\Delta t$, but also that of the

[6]When the surrounding systems of particle i are microscopic systems (particles) the computed – on the anticipation module A_i – future positions of these systems at time $t + \Delta t$ can differ from the actual positions that the systems finally occupy at that time $t + \Delta t$, not only because the programs and data stored on the Turing machines of these surrounding systems are not stored, in general, on the Turing machine of particle i, but mainly because the intrinsic randomness of these microscopic systems represented by the randomizers stored on these particles cannot be anticipated by the particle i. However, the anticipation for the future positions of surrounding macroscopic systems, like meters, is usually straightforward, since they obey classical equations.

[7]The randomizer is the methodological representation of the intrinsic randomness that is postulated in DAQM as a fundamental property of matter. In this sense, it is not necessary to formulate an operational characterization of an underlying mechanism. However, it is possible to give a description of a random generator in terms of classical information processing that could theoretically bring about randomness for all practical purposes. This can be done through a deterministic algorithm $G(t)$ [35] stored in the Turing machine that calculates the next digit of, say, an unknown trascendental number when the algorithm is called by the main program. The starting point at time $t = 0$ in the sequence of digits is unknown. $G(t)$ must be computationally irreducible [36]. These features ensure that the output of $G(t)$ cannot be anticipated even by the own system. See Baladrón [32] and references therein for a deeper discussion.

[8]This key issue will be analyzed in Section 3.

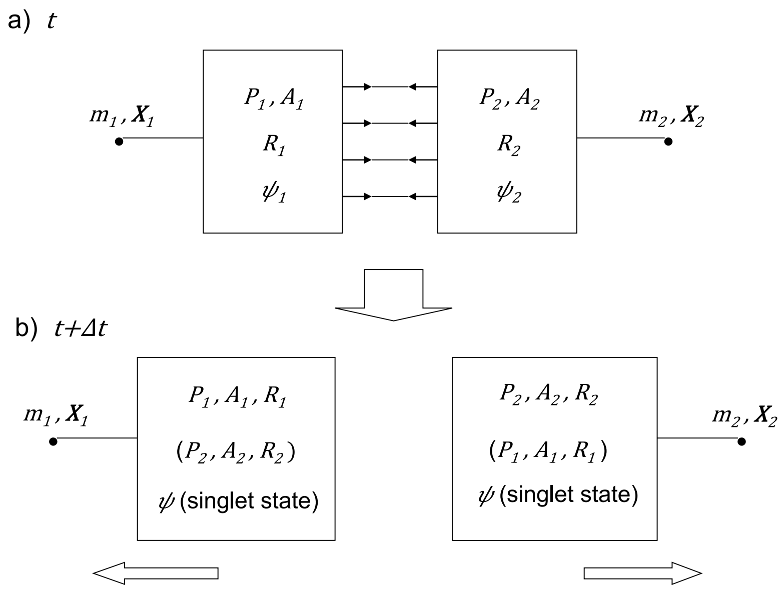

Fig. 1. (a) Two particles of average masses m_1 and m_2 in contact at time t, $\boldsymbol{X}_1(t) = \boldsymbol{X}_2(t)$, are entangled by sharing their programs (P_1, P_2), anticipation modules (A_1, A_2), randomizers (R_1, R_2) and wave functions (ψ_1, ψ_2) on their respective information spaces. (b) At time $t + \Delta t$, after separation in physical space, $\boldsymbol{X}_1(t + \triangle t) \neq \boldsymbol{X}_2(t + \triangle t)$, each particle conveys its own software $(P_{1(2)}, A_{1(2)}, R_{1(2)})$ and that of its partner $(P_{2(1)}, A_{2(1)}, R_{2(1)})$, in addition to the entangled wave function (ψ) of the bipartite system, e.g. the singlet state.

surrounding systems of its partner on its space-separated region. Besides, each particle gets the random number generated by its own randomizer and that of its partner, at the same time $t + \Delta t$ (shared randomness). And, finally, both particles obtain the output of its program – determining the parameters of the energy-momentum carrier to be emitted – and that of its partner at the same time $t + \Delta t$.

Notice that, for an entangled pair $(\boldsymbol{X}_1, \boldsymbol{X}_2)$, present $(t + \Delta t)$ and possible future $(t + 2\Delta t)$ information[9] on events – in physical space: current positions of systems $\boldsymbol{X}_1(t + \triangle t)$, $\boldsymbol{X}_2(t + \triangle t)$ and $\boldsymbol{X}_j(t + \Delta t)$, denoting j surrounding systems to the entangled pair, or emitted energy-momentum carriers; and on information space: possible future positions of systems $\boldsymbol{X}_1(t + 2\triangle t)$, $\boldsymbol{X}_2(t + 2\triangle t)$ and $\boldsymbol{X}_j(t + 2\Delta t)$ or random numbers generated by the randomizers R_1 and R_2 of the entangled pair – that are happening or will possibly happen in a space-separated region is generated by both entangled particles at the same time (instantly) without involving any nonlocal transfer of matter, energy, information or influence. That information on present and possible future events is generated through local computation on the information space that is located at any time on the position occupied by its corresponding particle.

As pointed out by Timpson [37], information, although central to physics as has become patent, is an abstract noun that is not bound to a continuity equation in physical space or to conservation constraints. In our model, information is generated by events in physical space – like the emission of momentum-energy carriers, and therefore it is related to past and current positions and momenta of particles, but it

[9] *Future information* refers to calculated possible future events or properties. Since this data are not about objective, factual events or properties, but potential ones, then this *future information* might also be named as *subjective believes*, taking the term from QBism [20], where it is only applied to rational observers.

is also generated on the information spaces of particles through the execution of the algorithms on the probabilistic classical Turing machine associated to every particle, and therefore it is related to possible future positions and momenta of systems. Information carriers in physical space are constrained by local causality, in the sense of Bell – basically, that these physical carriers transport momentum, energy and information subject to continuity in physical space and the finite transmission rate limit of c (speed of light). However, the information elaborated by the Turing machines' algorithms, in particular by the anticipation modules under the specified conditions for the model of entanglement in DAQM, is not subject to local causality, i.e. these outputs locally calculated at $\boldsymbol{X}_{1(2)}(t)$ may generate at this place information about possible physical events that are happening at the same time in a space-separated region occupied by the entangled partner $\boldsymbol{X}_{2(1)}(t)$, bringing about the appearance of nonlocality. Roughly speaking calculations about possible propagation of a system in the physical space can be performed quicker than real physical propagation, so to say "calculations can be quicker than speed of light".

We also remark that both spaces discussed above, "physical space" and "information space" are just mathematical models serving for representation of physical phenomena, one model is based on the real continuum and another on discrete binary representation of information. In particular, one has not to forget that the real Cartesian or Minkowsky spaces are just mathematical constructions.

2.3 Theoretical analysis of an EPR-Bohm experiment for electrons

Let us apply the information-theoretic model of entanglement in DAQM to explain an EPR-Bohm experiment with electrons. A schematic representation of such an experiment is sketched in Figure 2. A source of entangled electron pairs is located between two analysers – Stern-Gerlach apparatuses where their characteristic axes, $\hat{a}$ (left) and $\hat{b}$ (right), represent the directions of their respective magnetic fields along which the spin component of the impinging electron is measured – on the left and right wing of the setup. The pair of electrons that travel in opposite directions from the source towards the analysers are generated in the singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle\right). \tag{1}$$

A Lorentz frame is assumed for the analysis of the experiment in which the electron 2 reaches the right analyser ($\hat{b}$-axis) at time t, whereas the electron 1 travelling towards the left analyser ($\hat{a}$-axis) arrives at time $t + \triangle t$.

Let us first consider a configuration of the setup in which the axes of the analysers are parallel, i.e. the axes form an angle $\theta = (\theta_1 - \theta_2) = 0°$ (see Fig. 2). At time t on the right side of the setup, the random number generated by R_2, the randomizer on the probabilistic Turing machine of electron 2, has determined whether the spin component[10] of electron 2 along the axis $\hat{b}$ is $+1$ ($\uparrow$) or -1 ($\downarrow$) with 50% of the values[11] yielding the output $+1$ and the other 50% producing -1, in accordance with the fact that the program P_2 that controls electron 2 self-interactions induces

[10] The eigenvalues of the spin component operator are written taking $\hbar/2$ as unity, where $\hbar$ is the Planck constant divided by 2π.

[11] For simplicity, it can be assumed that the randomizer yields a random digit within the range 0–9 at every run. See Footnote 7 for details.

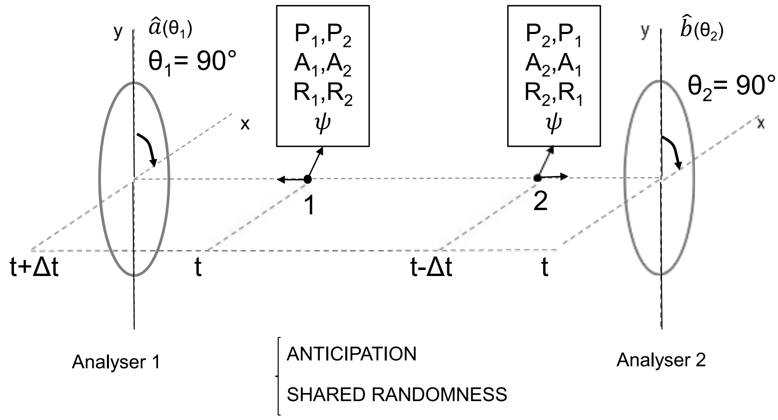

Fig. 2. An EPR-Bohm experiment is schematically represented. A source of entangled electrons is located between two analysers (Stern-Gerlach apparatuses) whose axes $\hat{a}$ (left wing) and $\hat{b}$ (right wing) are parallel. The angles formed by these axes with respect to the x-axes of reference are $\theta_1 = \theta_2 = 90°$. The electron 2 that travels towards the right analyser (analyser 2) reaches the analyser at time t, whereas the electron 1 travels in opposite direction towards the left analyser (analyser 1) and arrives at time $t + \triangle t$. Both entangled electrons share its software $(P_{1(2)}, A_{1(2)}, R_{1(2)})$ and that of its partner $(P_{2(1)}, A_{2(1)}, R_{2(1)})$. Shared randomness and anticipation are the key elements of the model of entanglement.

quantum behaviour on electron 2 as previously assumed.[12] Let us suppose that the output of P_2 is $-1(P_2 \to\ \downarrow_2)$ along axis $\hat{b}$ at time t.

At the same time t on the left side, the electron 1 has not yet reached the analyser 1. However, the Turing machine of electron 1 not only stores and executes P_1, A_1 and R_1, but as a consequence of the entanglement between both electrons (see Sect. 2.2), the electron 1 also stores and executes P_2, A_2 and R_2. Therefore, the output of P_2 $(P_2 \to\ \downarrow_2)$ along axis $\hat{b}$ at time t has also been calculated on the Turing machine of electron 1 whose location is $\boldsymbol{X}_1(t)$ close to the analyser 1 on the left side of the setup. This implies *parameter dependence* – i.e. the orientation of the analyser's axis $\hat{b}$ on the right wing of the setup is known at time t and position $\boldsymbol{X}_1(t)$ on the Turing machine of electron 1. In addition, the output of R_2 at time t is also at disposal on the Turing machine of electron 1 located at $\boldsymbol{X}_1(t)$. This shared randomness (every entangled particle stores its own randomizer and that of its partner) implies *outcome dependence*.[13]

Therefore at time $t + \Delta t$, the program P_1 drives electron 1 to the outcome $+1$ $(P_1 \to\ \uparrow_1)$ for the spin component value, provided that the output of P_2 was -1 $(P_2 \to\ \downarrow_2)$ along axis $\hat{b}$ at time t, yielding perfect anticorrelation for the outcomes on both sides of the setup for this configuration in which the axes of the analysers are parallel $(\theta = 0°)$ in accordance with the standard quantum mechanical calculation.

[12]Technically, the description of the measurement of the spin component of an electron in DAQM would be similar to the description in Bohmian mechanics [38,39], more precisely, to the Bohm-Vigier version [40] of Bohmian mechanics in which a random term is incorporated to the guiding equation. In DAQM, the random term is generated using the randomizer stored on the information space of the particle. As in Bohmian mechanics, the result of the measurement could be described in terms of the coupling of the system to the meter and its final configuration, i.e. the positions of the particle and the Stern-Gerlach apparatus.

[13]See, for example, Shimony [1], Vervoort [15] and the text below for a general characterization of the concepts of *parameter independence* and *outcome independence*.

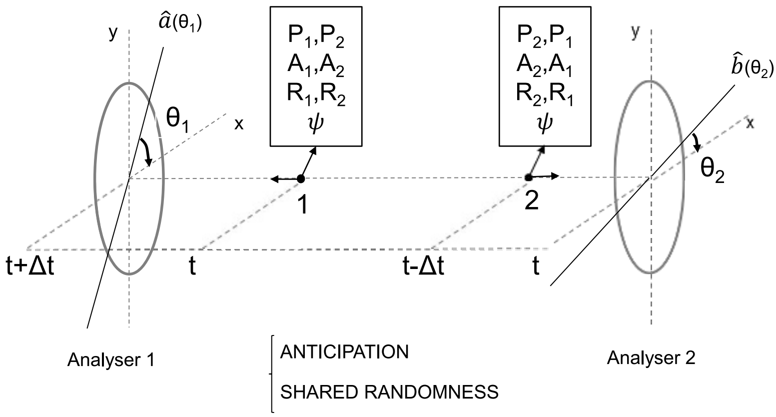

Fig. 3. An EPR-Bohm setup as described in Figure 2, but now the axes of the analysers are not parallel ($\theta_1 \neq \theta_2$).

Let us consider now a generic configuration of the setup in which the axes of the analysers are not parallel and therefore form an angle $\theta = (\theta_1 - \theta_2) \neq 0°$ (see Fig. 3). The analysis proceeds in a quite similar way as for the case $\theta = 0°$. As *outcome dependence* and *parameter dependence* are ensured in this model of entanglement in DAQM, then the correlations C for the outcomes on both sides of the setup in DAQM in the general case of a configuration for which the angle θ formed by the analysers' axes takes any value also coincide with the ordinary quantum mechanical calculation $C = -\cos\theta$.

Let us check that result. At time t (see Fig. 3) on the right wing of the experiment, the randomizer R_2 yields 50% of occasions an output $+1$ for the spin component of electron 2 along axis $\hat{b}(\theta_2)$ and 50% an output -1, according to the program P_2 that reproduces quantum behaviour on electron 2. Let us assume that the result has been -1 ($P_2 \rightarrow \downarrow_2$) along the axis $\hat{b}$ forming an angle θ_2 with the reference x-axis. As both electrons are entangled, according to our model this output has also been computed on the Turing machine of electron 1 located at $\boldsymbol{X}_1(t)$ on the left wing of the experiment at the same time t, very close to analyser 1. This implies *parameter dependence*, since electron 1 has registered the orientation (θ_2) of axis $\hat{b}$ at time t by means of its copy of P_2 and A_2, and *outcome dependence*, since electron 1 has also obtained the output of R_2 at time t and, therefore, has calculated as well by means of its copy of P_2 the result of the measurement performed on electron 2 by the analyser on the right side of the setup at time t. Now, at time $t + \Delta t$, the program P_1 can calculate the orientation (θ_1) of axis $\hat{a}$ and the angle $\theta = (\theta_1 - \theta_2) \neq 0°$ between both axes. Then, applying the expression for the outcomes correlations[14] $C = -\cos\theta$, along with the result of the measurement performed on electron 2 at time t (stored on the information space of electron 1) and the output of the randomizer R_1 at time $t + \Delta t$, the program P_1 drives electron 1 to the particular outcome that will contribute to reproduce the orthodox quantum statistics for the experiment when applied to a sequence of entangled pairs.

Finally, let us consider the experimental arrangement of Figure 3, but now with the axes of the analysers ($\hat{a}$ and $\hat{b}$) rotating, i.e. the parameters θ_1 and θ_2 are function of time. These functions $\theta_1(t)$ and $\theta_2(t)$ may be selected by the experimenter, therefore, reflecting their free will or may be controlled by a random – for all practical purposes –

[14]Notice that it has been assumed that the programs stored on the Turing machines of the particles generate orthodox quantum behaviour.

number generator. In any case, it is assumed that $\theta_1(t)$ and $\theta_2(t)$ are continuous functions and their variation is smooth. Under these conditions, the values of $\theta_1(t)$ and $\theta_2(t)$ at the moment in which the entangled pair is generated at the source are different from those encountered by the particles when arriving at the analysers. As a consequence, the computation by the anticipation module A_2, that is also stored on electron 1, of the particular value of $\theta_2(t)$ when the electron 2 arrives at the right wing analyser (analyser 2) must be updated all along the way from the source to the left wing analyser (analyser 1) for the electron 1, by means of the information carriers emitted by the analyser 2 and the entangled partner (electron 2). Notice that the *violation of parameter independence* is caused not by any nonlocal process, but through anticipation by on-site computation on the Turing machine of the particles. A process that is completely local.

The on-site computation of the parameter $\theta_2(t)$ by the copy of P_2 on the Turing machine of electron 1 at the location occupied by this electron 1 (left wing of the setup) at time t in which the electron 2 arrives at the analyser 2 (right wing of the setup) may be performed, in spite of the random selection of the function $\theta_2(t)$, thanks to the fact that the analysers are macroscopic systems, therefore obeying classical equations of motion that can be straightforwardly solved by the anticipation modules (A_2 in this particular case), and the additional plausible conditions of continuity and smooth[15] variation imposed on the functions $\theta_1(t)$ and $\theta_2(t)$.

The theoretical analysis of an EPR-Bohm experiment for electrons in the framework of DAQM can be summarized in terms of the mathematical characterization of *outcome independence* (OI) and *parameter independence* (PI).

Let us denote by $p\left(\sigma_1 \left|\hat{a},\ \lambda\right.\right)$ the probability for finding the outcome σ_1 (+1 or −1) (in this example the outcome for the measurement of the spin component of electron 1 on the left wing of the experiment) given the conditions $\hat{a}$ (in this example the orientation for the analyser axis on the left wing as previously defined) and λ (in this example the hidden variables of the model) and so forth. Then, *outcome independence* and *parameter independence* are characterized by the following expressions:

$$p\left(\sigma_1 \left|\sigma_2, \hat{a}, \hat{b}, \lambda\right.\right) = p\left(\sigma_1 \left|\hat{a}, \hat{b}, \lambda\right.\right) \quad \text{(OI)} \tag{2}$$

$$p\left(\sigma_1 \left|\hat{b}, \hat{a}, \lambda\right.\right) = p\left(\sigma_1 \left|\hat{a}, \lambda\right.\right) \text{ and equivalently for } \sigma_2 \text{ (PI).} \tag{3}$$

The conjunction of both properties defines local causality (or the so-called locality condition) as considered by Bell [15]. DAQM satisfies OI and PI when these probabilities are calculated averaging on the information space variables of every electron, i.e. when only physical space time degrees of freedom are considered. However, when the information spaces of every particle are included in the description, then *outcome independence* and *parameter independence* are violated in DAQM. Let us incorporate the information space degrees of freedom to the calculation of probabilities, enclosing the information space entities (randomizers, R, and anticipation modules, A) between

[15] Since the analysers are macroscopic objects, the angles $\theta_1(t)$ and $\theta_2(t)$ should vary in time several orders of magnitude slower than both the speed of light and the speed of the entangled electrons travelling from the source to their respective analysers in an hypothetical EPR-Bohm experiment with electrons. Although it is assumed that the functions may vary randomly, the variation cannot be instantaneous, but continuous and smooth in comparison with the variation rate of the parameters of the electrons in physical space and the calculation time on the information space of the electrons. Therefore, the position that a rotating analyser axis is going to occupy in a future instant of time (in particular, at the arrival time of the entangled pair at their respective analysers) can be extrapolated from the anticipation modules of the programs and the information carriers that reach the electrons and continuously allow them to update and improve the estimation.

square brackets. Now, OI and PI are no longer satisfied:

$$p(\sigma_1 \left|\sigma_2 \left[A_2^1, R_2^1\right], \hat{a}, \hat{b}, \lambda) \neq p(\sigma_1|\hat{a}, \hat{b}, \lambda) \quad \text{(OD)} \tag{4}$$

where A_2^1 represents the anticipation module for electron 2 and its environment that is stored on the Turing machine of electron 1. Notice that $A_2^1 = A_2^2$ (the superscript just indicates the Turing machine at which the anticipation module for the electron identified by the subscript is located) due to the entanglement between both particles. The same terminological description is equivalently valid for the randomizers (R_2^1).

The presence of A_2^1 and R_2^1 (copies of A_2^2 and R_2^2) on the Turing machine of electron 1 implies that the outcome σ_2 measured on the right wing of the experiment (electron 2) is instantaneously at disposal of the program P_1^1 that controls electron 1 and is stored on the Turing machine of the same electron 1 located on the left wing of the experiment. As a consequence, *outcome independence* is not satisfied.

A similar discussion explains *parameter independence* violation in DAQM when the information space elements are considered. The fact that the anticipation module for electron 2 and its environment is stored not only on the information space of electron 2 (A_2^2) but also on the Turing machine of electron 1 (A_2^1) enables to extrapolate on the information space of electron 1 the orientation for the axis of the analyser on the right wing of the experiment ($\hat{b}$) at the moment in which the electron 2 arrives at the right wing. In this case, the condition that indicates the presence of *parameter dependence* (PD) in DAQM is:

$$p(\sigma_1 \left|\hat{b} \left[A_2^1\right], \hat{a}, \lambda) \neq p(\sigma_1 \left|\hat{a}, \lambda) \quad \text{(PD)}. \tag{5}$$

As a final point, let us remark that DAQM is compatible with the free will of the experimenter for selecting the orientations of the analysers axes (this property is usually named *measurement independence*, e.g. see Hall [41] and Vervoort [15]). The particles self-interactions that are determined by the outputs of the programs stored on the information spaces of the particles do not depend exclusively on past events (events in the past light cones of the particles) and the outputs of their respective randomizers. The actions of the particles also involve the analysis of the possible future configurations of the surrounding systems (through the anticipation modules) and, as it will be further discussed in Section 3, the assessment of the different potential options with respect to the stability expectations for the particle. Thus, in DAQM, there are *free choice* elements for the microscopic systems that come from the future, but not travelling backward in time in physical space (so that the classical principle of causality is preserved). The freedom of choice resides in a possible future that is computed on the particle information space. Therefore, the key component is again the anticipation module stored on the information space of every particle. DAQM presents an explicit mechanism that might explain the central role that potentiality seems to play in quantum mechanics. This microscopic model of free choice for fundamental physical systems[16] in the framework of DAQM could establish the basis for free will in complex biological systems.

3 Overview of DAQM

As analysed in Section 2, the model of entanglement developed in the framework of DAQM is able to explain Bell inequality violations preserving locality and realism

[16] See Baladrón [32] for a deeper discussion of this microscopic free-choice model in DAQM.

by introducing randomness and information processing capability as intrinsic properties of matter, provided that the program stored on every particle induces quantum behaviour on the particle.

The central tenet of DAQM is that assuming intrinsic randomness and the capability of processing information as fundamental properties in nature, then quantum mechanics would emerge from an otherwise classical scheme as a consequence of Darwinian evolution under natural selection acting on physical systems that have become generalized Darwinian systems precisely through the incorporation to every system of an information space endowed with a randomizer and a classical Turing machine. The possibility that physical and biological systems admit a unified description in a generalized information-theoretic Darwinian framework is explored [31,33].

The antecedents of this theory can be traced back to the works of Lotka [42] – who considered the possible key role played by natural selection in physics, Whitehead [43] – who studied the possibility of the evolution of physical laws and the importance of the concept of anticipation in the physical world, and Wheeler [44] – whose well-known aphorisms (e.g. *law without law*, *it from bit* or *why the quantum?*) summarize his deep influence in the recent development of quantum information and the foundations of quantum mechanics. In addition, Darwinism has been previously applied to study several fundamental problems in physics (e.g. Smolin [45] and Zurek [46]).

There are not universal laws in DAQM. Every physical system at time $t = 0$ is controlled by its randomizer R. As time increases the information conveyed by the randomly emitted energy-momentum carriers reaches the absorbers, and through a process of variation, selection and retention, characteristic of Darwinian systems, those systems that develop the fittest programs (P) will survive. In this information-theoretic Darwinian scenario, it is assumed that the optimization, against the system stability, of the information flows – past, present and anticipated information – would lead to the emergence of quantum behaviour. This process is studied by DAQM [30–33]. Darwinian natural selection would act as a meta-law that would determine the way in which the physical laws or regularities – the programs that control the behaviour of the systems – would evolve.

The general principle of optimization of the information flows in physical systems would shape the regulating principles that would determine the structure, the dynamics and the interaction for physical systems.

Principle 1 (structure): The complexity of a system is optimized (maximized).

Principle 2 (dynamics): The outwards information flow of a system is optimized (minimized).

Principle 3 (interaction): The interaction established between two subsystems optimizes (maximizes) the complexity of the composite system.

There are several definitions of complexity[17] and information. In DAQM, a contextual definition of complexity [33] seems well adapted to conceptually study the possible emergence of quantum behaviour from the enunciated regulating principles. The complexity of a system is then defined as the capability of the system to anticipate the positions that its surrounding systems will occupy after a certain increment of time. This definition is suited for a qualitative analysis.

As for information in the Principle 2, it refers to the Fisher information measure [47,48] of the probability distribution function of the system's position. Roughly speaking, it quantifies the sharpness of the function.

DAQM aims to deduce the postulates of quantum mechanics from these three regulating principles. This is a work in progress, but several interesting results have

[17] See Baladrón and Khrennikov [31] for a brief discussion on the adequacy of several definitions of complexity from the point of view of DAQM.

been already obtained. Applying the Principle 1, taking into account the particular definition of complexity that has been adopted, then the maximization of complexity directly implies the maximization of the system's predictive power. Several studies [49–53] analyse the way in which optimal statistical inference capability would induce the complex Hilbert space structure for the space of states of a system. The Principle 2 is basically a rephrasing of the minimum Fisher information or maximum Cramer-Rao bound [47]. Frieden [47] deduces the Schrödinger equation applying the minimum Fisher information principle on the probability distribution function for the position of the system. Therefore, the Principle 2 implies the Schrödinger equation for the dynamics of a system. The Principle 3 should lead to the appearance of entanglement as a basic natural phenomenon in the formation of composite systems as a consequence of the tendency to increase the complexity.

From this analysis, it stands out that the two most characteristic quantum properties, contextuality and entanglement, would be naturally suited in DAQM as features that increase the stability of systems endowed with the capability of processing information. Contextuality in DAQM reflects the fact that a system in physical space has not identity parameters, apart from its location, that the values of magnitudes are computed on the information space depending on the context in which the system is immersed and aiming to maximize the stability of the system. Entanglement is a resource for improving the anticipation or predictive power of a composite system that increases the stability of the system.

Some studies support the interest of analysing the implications of DAQM. First, the experimental observation of certain quantum-like properties for a macroscopic liquid drop that is coupled with the surface waves produced by itself when bouncing on a vibrating bath [54,55]. Second, some computer simulations [56] demonstrate that evolution by means of certain mechanisms might perform exponential-time tasks, when accomplished by most evolutionary processes, in polynomial-time. This is considered a distinctive quantum characteristic. Third, certain cosmological tests proposed to check Bohm-like theories [57] could also be adapted in the future to experimentally investigate DAQM. Fourth, DAQM could constitute a physical basis for quantum information biology (QIB) [58,59] by explaining the crucial role played by quantum information for certain bio-systems at the macroscopic level, and QIB in turn could supply a macroscopic testing ground to DAQM.

4 Conclusion

Bell inequality violation has been analysed in the framework of DAQM in which every fundamental particle is supplemented with a randomizer and a classical Turing machine on an information space locally associated with every particle. A natural model of entanglement in this theory, based on shared randomness and anticipation, explains Bell inequality violations while preserving locality and realism. DAQM clarifies in a constructive manner the reason why randomness, objective indefiniteness, and the relationship between potentiality and actuality are fundamental elements in the quantum mechanical description of nature.

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Author contribution statement

The authors have equally contributed to the realization of this article.

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