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Vertical vibration effect on the Rayleigh-Benard-Marangoni instability in a two-layer system of fluids with deformable interface

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Abstract. The effect of vertical vibrations on the Rayleigh-Benard-Marangoni instability of a two-layer system of immiscible incompressible viscous fluids subjected to a constant vertical heat flux at the external boundaries is studied in the framework of the generalized Boussinesq approximation taking into account the interface deformations. The study is performed using the averaging approach under the assumption that the vibration period is small in comparison with the hydrodynamical time scales and the product of the vibration amplitude and the Boussinesq parameter is small in comparison with the layer thickness. It has been found that the long-wave instability is not affected by vibrations of small and moderate intensity. It turned out that vibrations have a stabilizing effect on the finite-wavelength perturbations in a wide range of parameters.

1 Introduction

It is known (see, for example [1]), that in the case of small density inhomogeneities caused by nonisothermal conditions the deviation of the fluid-fluid interface from a flat shape is proportional to the Boussinesq parameter. This is the reason why in the framework of the Boussinesq approximation the interface deformations are usually neglected. As shown in [2], taking account of the interface deformations in the framework of conventional Boussinesq approximation can produce physically incorrect results. At the same time, in some physical situations, the non-deformable interface approximation has proved to be insufficient. For example, if the difference in densities is of the same order of magnitude as the density inhomogeneities caused by nonisothermal conditions, the gravity force is unable to keep the interface flat at finite values of the Grashof number. In this case the interface deformations can be large and should be taken into account.

The generalized Boussinesq approximation, allowing us to take into account the interface deformations in a proper way when both the relative density difference and the Boussinesq parameter are small, was formulated in [3,4]. In [3], the onset of

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thermal buoyancy convection in a two-layer system of immiscible fluids with deformable interface was studied in the framework of this approximation for the case when the thicknesses of layers and all fluid parameters are identical, except for the fluid densities. The monotonic and oscillatory finite-wavelength instability modes were found. The same problem was studied in [4] for fluids of different properties. A monotonic long-wave instability mode, associated with the interface deformations (this mode is absent in the case considered in [3]) was discovered. It was shown, that the long-wave monotonic perturbations are most dangerous over a wide range of parameters.

In reference [5], the case of a two-layer system of immiscible fluids with deformable interface and fixed vertical heat flux at the external boundaries was studied in the framework of generalized approximation developed in [3,4]. It was shown, that in this case, in addition to the long-wave monotonic instability mode related to the interface deformations, there exists an oscillatory long-wave instability mode. The boundary of stability to the monotonic long-wave perturbations in the parameter plane Rayleigh number – modified Galileo number consists of two branches. The critical perturbations at one of these branches represent a convective flow through the entire system and large interface deformations. The second branch corresponds to perturbations of a two-floor structure and small interface deformations. The boundary of the oscillatory long-wave instability is a straight line connecting two branches of the monotonic long-wave instability boundary. The structure of critical perturbations changes from the one-floor to the two-floor structure with the displacement along the oscillatory instability boundary.

In references [6–8], a weakly nonlinear analysis of large-scale convective flows in a two-layer system with deformable interface and perfectly heat conducting external boundaries was performed. The nonlinear amplitude equation was formulated and investigated in [6] for layers of equal thicknesses. The case of fluid layers of different thicknesses was considered in [7]. Here the amplitude equation was constructed to describe the large-scale convective flows accompanied by the interface deformation. The work [8] deals with the investigation of periodical regimes of the large-scale convection in a two-layer system with deformable interface. The amplitude equation describing the interface deformation was obtained in the limiting case of large surface tension. Two-dimensional periodical flow regimes were investigated.

The influence of the Marangoni effect on the onset of the Rayleigh-Benard convection in a two-layer system with deformable interface was studied in [9]. It was found that in the case of heating from below the thermocapillarity produces a stabilizing effect on the long-wave perturbations, and at some values of the parameters the state of the system, in which a denser fluid is located above a less dense fluid, becomes stable. However, the analysis of finite wave-length perturbations in the presence of thermocapillary effect shows that in the case of heating from below the Rayleigh-Taylor instability is not suppressed. For any values of the parameters the perturbations with finite wavelength have proved to be more dangerous.

It is important to find the ways of controlling the stability of conductive states in hydrodynamic systems. Vibrations were found to be one of the simplest and efficient means of control. The effect of high-frequency vibrations is studied through the application of the averaging procedure, in which all hydrodynamic fields are decomposed into two parts – a pulsation part and an average one. The averaging method was first applied for studying thermal convection in [10]. It has been found that high-frequency vibrations have a stabilizing effect on the stability of conductive state of the fluid layer with perfectly conducting external boundaries. The mechanisms of vibrational control of flows and heat and mass transfer in crystal growth are discussed in [11,12]. A comprehensive review of the main results obtained in the field of thermal vibrational convection can be found in [13].

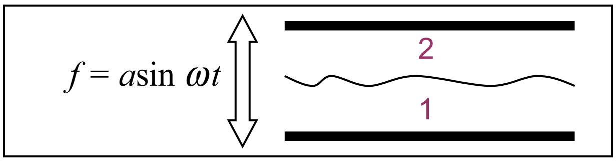


Fig. 1. Problem configuration.

The present work is devoted to the study of vertical high-frequency vibration effect on the stability of the conductive state in a two-layer system of immiscible fluids with close densities and deformable interface.

2 Problem statement. Governing equations

Let us consider a system of two superposed horizontal immiscible fluid layers with a deformable interface. The layer thicknesses are assumed to be the same and equal to h. A constant vertical heat flux is applied to the external rigid boundaries. Different directions of vertical heat flux, corresponding to heating from below and from above, are considered. The layer boundaries oscillate vertically with the amplitude $\mathrm{Ma} = -100$ and frequency ω according to the law $f = a \sin \omega t$ (Fig. 1).

In the reference frame of layer boundaries the vibrations lead to a redefinition of the acceleration of gravity:

$$\mathbf{g_{eff}} = -g\gamma - a\omega^2 f\gamma.$$

Here $\mathrm{Ra_v} = 100$ is the acceleration of gravity, γ is the unit vector directed vertically upwards.

As it has been already mentioned, the conventional Boussinesq approximation proves to be inadequate to the case when the interface deformations are taken into account. In the present work we study fluids of close densities. In this case we can take into account the interface deformation in a proper way by solving the problem in the framework of the generalized Boussinesq approach suggested by Busse and Lyubimov in [3,4]. In this approach, along with the limiting transition $\mathrm{Ga}^* \to \infty$ ($\mathrm{Ga}^* = gh^3/\nu^2$), not only the relative temperature heterogeneity $\beta_*\theta$ but also the relative difference in fluid densities $\delta = (\rho_{02} - \rho_{01})/(\rho_{02} + \rho_{01})$ is assumed to be small. Here ν is the kinematic viscosity of the fluid, β_* is the reference value of the thermal expansion coefficient, θ is the representative temperature difference, ρ_{01}, ρ_{02} are the densities of the lower and upper fluids at a certain temperature taken as a reference value.

The system of equations and boundary conditions describing thermal buoyancy convection in a two-layer system of fluids in the presence of vertical vibrations is written in the framework of the Busse-Lyubimov model as

$$\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \nabla)\,\mathbf{v}_j = -\frac{1}{\rho_{0j}} \nabla p_j + \nu_j \Delta \mathbf{v}_j + \mathbf{g}_{eff}\,\beta_j T_j \tag{1}$$

$$\frac{\partial T_j}{\partial t} + (\mathbf{v}_j \nabla) T_j = \chi_j \Delta T_j \tag{2}$$

$$\nabla \cdot \mathbf{v}_j = 0 \tag{3}$$

$$z = \pm h: \quad \mathbf{v}_j = 0, \quad \frac{\partial T_j}{\partial z} = -A_j \tag{4}$$

$$\begin{gathered} z = \zeta: \quad [\mathbf{v}] = 0, \quad [p] + [\sigma_{nn}] = -\alpha K, \quad [\sigma_{n\tau}] = \frac{\partial \alpha}{\partial T} \cdot \nabla T \\ [T] = 0, \quad \left[\kappa \frac{\partial T}{\partial n}\right] = 0, \quad \frac{\partial \zeta}{\partial t} + (\mathbf{v}\nabla)\,\zeta = (\mathbf{v}\,\gamma)\,. \end{gathered} \tag{5}$$

Here $j = 1, 2$, and the quantities related to the lower fluid are marked by subscript 1, while the quantities related to the upper fluid by subscript 2, χ_j is thermal diffusivity of the jth fluid, the square brackets $\mathrm{Ra_v} = 100$ stand for a jump of the value f across the interface, $\mathbf{n}$ is the vector normal to the interface between media, $\bar{\bar{\sigma}}$ is the viscous stress tensor, K is the interface curvature. The interface between the fluids is defined by the equation $z = \zeta\,(x, y, t)$, where x,y are the horizontal coordinates and z is the vertical coordinate.

The problem formulated above admits a stationary solution corresponding to the conductive state of the fluids ($\mathbf{v}_j = 0$) with a flat horizontal interface $\zeta = 0$, wherein the temperature and pressure distributions in the layers take the following form:

$$\frac{\partial \tilde{\mathbf{v}}_j}{\partial t} = -\frac{1}{\rho_{0j}} \nabla \tilde{p}_j + \nu_j \Delta \tilde{\mathbf{v}}_j + \mathbf{g}_{eff}\; \beta_j \tilde{T}_j. \tag{6}$$

Let us examine the stability of the conductive state. Since the problem is uniform in the horizontal plane(x, y), we restrict ourselves to two-dimensional perturbations.

Representing the velocity, temperature and pressure fields as the sums of the basic state (conductive state) fields and small perturbation fields, substituting these sums into equations and boundary conditions (1)–(5), using the equations of the base state and linearizing them, we obtain the problem for small perturbations $\tilde{\mathbf{v}}$, $\tilde{p}$, $\tilde{T}$ of the conductive state written as

$$\frac{\partial \tilde{\mathbf{v}}_j}{\partial t} = -\frac{1}{\rho_{0j}} \nabla \tilde{p}_j + \nu_j \Delta \tilde{\mathbf{v}}_j + \mathbf{g}_{eff}\; \beta_j \tilde{T}_j \tag{7}$$

$$\frac{\partial \tilde{T}_j}{\partial t} + (\tilde{\mathbf{v}}_j \nabla)\tilde{T}_{0j} = \chi_j \Delta \tilde{T}_j \tag{8}$$

$$\nabla \cdot \tilde{\mathbf{v}}_j = 0 \tag{9}$$

$$z = \pm h: \quad \tilde{\mathbf{v}}_j = 0, \quad \frac{\partial \tilde{T}_j}{\partial z} = 0 \tag{10}$$

$$\begin{gathered} z = \zeta: \quad [\tilde{\mathbf{v}}] = 0, \quad [\tilde{p}] + \left[\frac{\partial p_0}{\partial z}\right]\zeta + [\sigma_{nn}] = -\alpha K, \quad [\sigma_{n\tau}] = \frac{\partial \alpha}{\partial T} \cdot \nabla \tilde{T} \\ \left[\tilde{T}\right] = 0, \quad \left[\kappa \frac{\partial \tilde{T}}{\partial n}\right] = 0, \quad \frac{\partial \zeta}{\partial t} = (\tilde{\mathbf{v}}\,\gamma)\,\cdot \end{gathered} \tag{11}$$

Let us exclude the pressure and horizontal velocity components by applying the double curl procedure to equation (7) and projecting the obtained equation on the z-axis.

After introducing the perturbations, which are periodic along the x-axis ($\exp(ikx)$), we get

$$\frac{\partial}{\partial t}\Delta w_j = \nu_j \Delta^2 w_j - \beta_j k^2 \left(g + a\omega^2 f''\right) \vartheta_j$$
$$\frac{\partial}{\partial t}\vartheta_j = \chi_j \Delta \vartheta_j + A_j w_j \tag{12}$$

$$z = \pm 1: \quad w_j = 0, \quad w_j' = 0, \quad \vartheta_j' = 0 \tag{13}$$

$$\begin{aligned} z = 0: \quad [\mathrm{w}] = 0, \quad [\mathrm{w}'] = 0, \quad [\eta w'''] + 2k^2[\eta w'] - [\rho_0]\, g_{eff}\zeta = -\alpha k^2 \zeta, \\ \left[\eta w'' + \eta k^2 w\right] = \frac{\partial \alpha}{\partial T}\vartheta', \quad [\vartheta] = \zeta\,[A]\,, \quad [\kappa\vartheta'] = 0, \quad \frac{\partial \zeta}{\partial t} = w. \end{aligned} \tag{14}$$

Here w is the amplitude of the vertical component for velocity perturbations, ϑ is the amplitude of temperature perturbations, the prime denotes differentiating with respect to z, the Laplace operator $\Delta = \partial^2/\partial z^2 - k^2$.

We assume that the vibration period is small compared to the viscous and thermal time scales, so that for the frequency the following conditions are fulfilled:

$$\omega \gg \nu/h^2, \quad \omega \gg \chi/h^2. \tag{15}$$

At the same time the vibrations are supposed to be non-acoustic assuming that the sound wavelength at the imposed vibration frequency $\lambda = 2\pi c/\omega$ is large in comparison with the characteristic size $\lambda \gg h$.

The vibration amplitude is assumed to be small [13]:

$$a\beta\theta \ll h. \tag{16}$$

Decomposing all fields into rapidly changing (pulsation) components and slowly changing (time-average) components, applying the procedure of averaging over the vibration period and introducing perturbations of the type $\exp(-\lambda t)$, depending exponentially on slow time, we obtain the equations and boundary conditions for average and pulsation components (due to restrictions (15, 16) the nonlinear and viscous terms in the equations for pulsations can be neglected):

$$\begin{aligned} &\frac{\lambda}{\mathrm{Pr}}\Delta w_j = \nu_j \Delta^2 w_j - \mathrm{Ra}\, k^2 \beta_j \vartheta_j - \mathrm{Ra_v}\, k^2 \nu_j \beta_j A_j W_j^2 \\ &\lambda \vartheta_j = \chi_j \Delta \vartheta_j + w_j A_j, \quad \Delta W_j = -\beta_j k^2 \vartheta_j \end{aligned} \tag{17}$$

$$\begin{gathered} z = \pm 1: \ w_j = 0, \quad w_j' = 0, \quad \vartheta_j' = 0, \quad W_j = 0 \\ z = 0: \quad [w] = 0, \quad [w'] = 0, \quad [\kappa\vartheta'] = 0, \\ [\vartheta] = \zeta\,[A]\,, \quad \lambda\zeta = w \\ [\eta w'''] - k^2 \mathrm{Ga}\zeta + k^2 \mathrm{Ga_v} W + 2k^2\,[\eta w'] = -\left(Ca\zeta + \mathrm{Ma}\vartheta\right) k^4 \\ \left[\eta w'' + \eta k^2 w\right] = \mathrm{Ma}\vartheta', \quad [W] = 0, \quad [W'] = -k^2 \frac{\mathrm{Ga_v}}{\mathrm{Ra_v}}\zeta\,. \end{gathered} \tag{18}$$

Here w is the perturbation amplitude of the vertical component of the average velocity, ϑ is the perturbation amplitude of the average temperature, W is the perturbation amplitude of the vertical component of the pulsation velocity $w_p = a\,\omega\,W\frac{\partial f}{\partial t}$ (where symbol p stands for the pulsation component).

The system of equations and boundary conditions (17, 18) is written in the non-dimensional form. The quantities h^2/χ_*, h, χ_*/h, A_*h, $\rho\nu_*\chi_*/h^2$, $a\omega$ are used as the scales for slow time, length, average velocity, temperature, pressure and pulsation velocity, respectively. Here $\nu_*, \kappa_*, \beta_*, \chi_*, A_*$ are the arithmetical means of the viscosity, thermal conductivity, thermal expansion, thermal diffusivity coefficients and temperature gradients in the base state, which are taken as the scales of the corresponding quantities, for which the following relations hold true

$$\nu_1 + \nu_2 = 2, \quad \kappa_1 + \kappa_2 = 2, \quad \beta_1 + \beta_2 = 2, \quad \chi_1 + \chi_2 = 2, \quad A_1 + A_2 = 2.$$

The problem (17)–(18) is governed by the following non-dimensional parameters: the Prandtl number Pr, the Rayleigh number Ra, the Galileo number Ga, the capillarity parameter Ca, the vibrational Rayleigh number $\mathrm{Ra_v}$, the Marangoni number Ma:

$$\mathrm{Pr} = \frac{\upsilon_*}{\chi_*}, \quad Ra = \frac{g\beta_* A_* h^4}{\upsilon_*\chi_*}, \quad \mathrm{Ca} = \frac{\alpha h}{\upsilon_*\chi_*\rho_{0*}}, \quad Ga = \frac{(\rho_{01}-\rho_{02})}{\rho_{0*}}\frac{gh^3}{\eta_*\chi_*},$$
$$\mathrm{Ra_v} = \frac{1}{2}\frac{\left(a\omega\beta_* A_* h^2\right)^2}{\upsilon_*\chi_*}, \quad \mathrm{Ga_v} = \frac{1}{2}\frac{(\rho_{01}-\rho_{02})}{\rho_{0*}}\frac{a^2\omega^2\beta_* A_* h^3}{\upsilon_*\chi_*}, \quad \mathrm{Ma} = \left(\frac{\partial\alpha}{\partial T}\right)\frac{h^2 A_*}{\chi_*\,\eta_*}.$$

Here, $\eta_* = \rho_0\nu_*$ is the average value of dynamic viscosity. Note that, for the majority of fluids $\partial\alpha/\partial T$ is negative, and therefore the positive values of the Marangoni number correspond to heating from above, negative – to heating from the bottom. Due to the relation $\mathrm{Ga_v} = \mathrm{Ga\,Ra_v/Ra}$, the vibrational Rayleigh number is not an independent parameter of the problem.

3 Effect of vibrations on the long-wave instability of conductive state

The problem (17)–(18) does not admit an analytical solution for an arbitrary set of parameters. However, in the case of long-wave perturbations it is possible to derive a complete analytical solution.

At $k = 0$ the problem (16)–(17) admits a two-parameter family of solutions, which corresponds to neutral perturbations:

$$\begin{aligned} &w_j^{(0)} = 0, \quad \zeta^{(0)} = C_1, \quad W_j^{(0)} = 0, \\ &\vartheta_1^{(0)} = C_2 + \frac{1}{2}(A_1 - A_2)\,C_1, \quad \vartheta_2^{(0)} = C_2 - \frac{1}{2}(A_1 - A_2)\,C_1 \end{aligned} \tag{19}$$

where C_1 and C_2 are the integration constants.

The perturbations with $C_1 = 0$ correspond to the problem invariance with respect to a shift of the reference temperature. Such a mode of neutral perturbations is typical for the problems with a fixed vertical heat flux at the boundaries. The perturbations with $C_2 = 0$ correspond to a shift of the interface as a whole. Thus, there is a doubly degenerate level with $\lambda = 0$ at $k = 0$. At $k \neq 0$ this level will either split into two real levels or give rise to a pair of complex-conjugate levels. In this case the real part of an increment can be either positive or negative, i.e. the long-wave perturbations will grow or decay depending on the parameters of the problem. To answer the question of which of the scenarios is realized, we will seek for a solution to the spectral problem (8)–(9) and the rate of exponential decay in the form of a series expansion with respect to the wavenumber:

$$\lambda = k\lambda^{(1)} + k^2\lambda^{(2)} + \ldots, \quad w_j = kw_j^{(1)} + k^2 w_j^{(2)} + \ldots, \quad W_j = kW_j^{(1)} + k^2 W_j^{(2)} + \ldots$$
$$\vartheta_j = \vartheta_j^{(0)} + k\vartheta_j^{(1)} + \ldots, \quad \zeta = \zeta^{(0)} + k\zeta^{(1)} + \ldots \tag{20}$$

The problem to the first order in k has the form, which is almost identical to that of the zeroth order problem, since the equations and boundary conditions contain only terms with even degrees of the wave number. The second order expansion with respect to the wave number provides two solvability conditions, which allow us to determine the stability boundaries in the Rayleigh – Galileo number plane. The first condition is derived from the kinematic condition $w_1^{(2)} = \lambda^{(2)}\zeta^{(0)}$ at $z = 0$, (the perturbation amplitude of the vertical component of the velocity $w_j^{(2)}$ is determined from the continuity equations in the second-order expansion $iu_j^{(1)} + w_j^{(2)'} = 0$). The second condition follows from the energy equations. The energy equations for the lower and upper fluids can be written as

$$\frac{\partial^2\theta_1^{(2)}}{\partial z^2} = f_1(z), \quad f_1(z) = \frac{\theta_1^{(0)}\left(\lambda^{(2)} + 1\right) - w_1^{(2)}A_1}{\chi_1};$$
$$\frac{\partial^2\theta_2^{(2)}}{\partial z^2} = f_2(z), \quad f_2(z) = \frac{\theta_2^{(0)}\left(\lambda^{(2)} + 1\right) - w_2^{(2)}A_2}{\chi_2}.$$

Integrating the first of these equations with respect to the vertical coordinate from -1 to 0 and the second equation from 0 to 1 and taking into account the boundary conditions for temperature perturbations in the second order expansion

$$\left.\frac{\partial\theta_1^{(2)}}{\partial z}\right|_{(z=-1)} = \left.\frac{\partial\theta_2^{(2)}}{\partial z}\right|_{(z=1)} = 0, \quad \kappa_1\left.\frac{\partial\theta_1^{(2)}}{\partial z}\right|_{(z=0)} = \kappa_2\left.\frac{\partial\theta_2^{(2)}}{\partial z}\right|_{(z=0)}$$

we obtain the second solvability condition.

As one can see from (19), the amplitude of the vertical component of pulsation velocity in the zeroth order expansion is zero $W_j^{(0)} = 0$, and therefore the vibrational term drops out from the second-order momentum equation at finite values of the vibrational Rayleigh and Galileo numbers, i.e. vibrations produce no effect on the long-wave instability of the conductive state up to the terms with the wave number k of second order.

Let us consider the case of large vibrational Rayleigh and Galileo numbers. We introduce the modified Rayleigh and Galileo numbers $\mathrm{Ra}_\mathrm{v}^*, \mathrm{Ga}_\mathrm{v}^*$ by relations

$$\mathrm{Ra}_\mathrm{v} = \varepsilon^{-2}\mathrm{Ra}_\mathrm{v}^*, \quad \mathrm{Ga}_\mathrm{v} = \varepsilon^{-2}\mathrm{Ga}_\mathrm{v}^*.$$

Here ε is a small parameter related to the spatial scale of perturbations as $k = k_1\varepsilon$, where k_1 (index 1 will be omitted below), $\mathrm{Ra}_\mathrm{v}^*, \mathrm{Ga}_\mathrm{v}^*$ are finite quantities. We derive the quadratic equation for λ_2 from the solvability conditions for the algebraic equations obtained in the second order series expansion with respect to ε

$$\lambda_2^2 + B\lambda_2 + C = 0, \tag{21}$$

where B and C are the functions of Ra, Ga, Ra_v^* k^2.

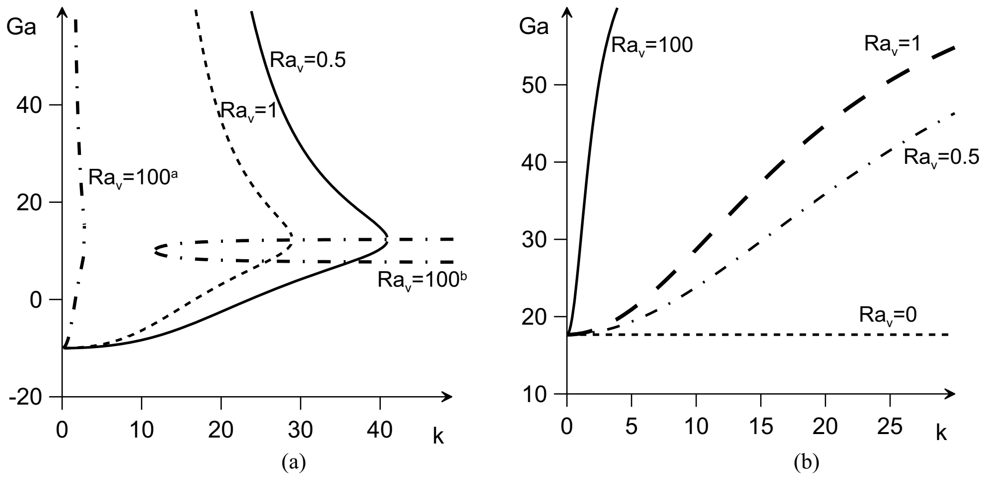


Fig. 2. Neutral curves Ga(k) at Ma = −10, Ra = 200: (a) neutral curves for monotonic instability mode, (b) for oscillatory instability mode. Instability domains are located above the curves.

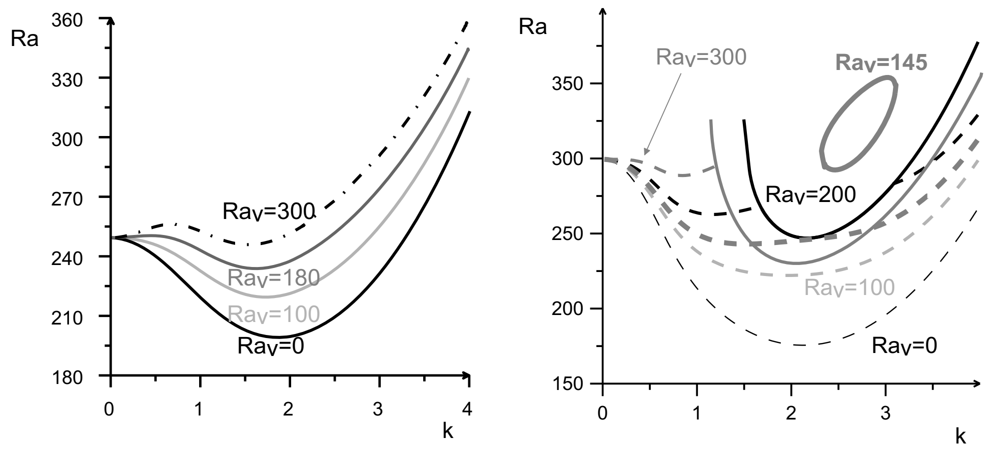
The analysis of relation (21) shows that as in the absence of thermocapillary effect [14], the vertical high-frequency vibrations of high intensity as well as the vibrations of moderate intensity do not affect the threshold of the oscillatory long-wave instability.

The neutral curves of monotonic and oscillatory instabilities at Ma = −10 and large values of the vibrational Rayleigh number are presented in Figure 2. As is seen from the figure, the vibrations lead to the appearance of a new finite-wavelength monotonic instability mode (Fig. 2a). The oscillatory instability mode is not affected by vibrations.

4 Vibration effect on the instability to perturbations of a finite wavelength

The vibration effect on the stability of the conductive state to the finite-wavelength perturbations was investigated numerically by the differential sweep method applied for the first time to the hydrodynamic stability problem in [15] and by the shooting method. The calculations were performed for a two-layer system containing formic acid and transformer oil. The stability of conductive state in this system in the absence of vibrations was studied in [5,16]. We carried out computations for zero value of the capillary parameter Ca. Since in the problem under consideration the capillary parameter appears only in the dynamic boundary condition in a combination Ga − k^2Ca, the threshold of the long-wave instability does not change with a change of Ca. The analysis of the capillarity effect on the finite-wavelength perturbations performed in [5] for the formic acid – transformer oil system in the absence of vibrations has shown that the growth of the capillary parameter leads to a decrease in the threshold of instability with respect to monotonic perturbations and extension of the parameter range, in which the long wave oscillatory perturbations are more dangerous than the finite-wavelength oscillatory perturbations.

In Figure 3a the neutral Ra(k)-curves are plotted for Ma = −100, Ga = − 250 and different values of the vibrational Rayleigh number. From the results of Section 3 it



(a)
Ga = –250

(b)
Ga = –60

Fig. 3. Neutral curves Ra(k) for Ma $= -100$, Ga $= -250$ (a) and Ga $= -60$ (b) and different vibrational Rayleigh number values. Instability domains are located above the curves. Solid lines neutral curves for monotonic perturbations, dashed lines for oscillatory perturbations. Instability domains are located above the curves and inside the close loop.

follows that the vertical high-frequency vibrations of low and moderate intensity do not affect the stability of the conductive state to the long-wave perturbations. The numerical results presented in Figure 3a support this conclusion: the values of the critical Rayleigh number obtained at $k \to 0$ are the same for all values of $\mathrm{Ra_v}$.

As one can see from Figure 3a, in the absence of vibrations ($\mathrm{Ra_v} = 0$) at Ga $= -250$ the monotonic finite-wavelength perturbations are more dangerous than the long-wave ones. The vibrations produce a stabilizing effect on these perturbations: with increase of the vibrational Rayleigh number the minima on the neutral curves are shifted to the region of larger values of Ra. Since the long-wave monotonic instability mode is not affected by vibrations of low and moderate intensity, at a certain value of the vibrational Rayleigh number the long-wave perturbations become most dangerous (as seen from Fig. 3a, this change of instability mode occurs at $\mathrm{Ra_v} \approx 290$).

Figure 3b shows the effect of vibrations for Ma $= -100$, Ga $= -60$ and different values of $\mathrm{Ra_v}$. Again we can see that the values of the critical Rayleigh number obtained at $k \to 0$ are the same for all values of $\mathrm{Ra_v}$, i.e. the obtained numerical results support the conclusion that the vertical high-frequency vibrations of low and moderate intensity do not affect the stability of the conductive state against the long-wave perturbations. In the absence of vibrations ($\mathrm{Ra_v} = 0$) at Ga $= -60$ the finite-wavelength oscillatory perturbations are most dangerous. As one can see from Figure 3b, the vibrations have a stabilizing effect on these perturbations: with increase of the vibrational Rayleigh number the minima on the neutral curves are shifted to the region of larger values of $\mathrm{Ra_v}$ (see neutral curves for $\mathrm{Ra_v} = 0$ and $\mathrm{Ra_v} = 100$ for comparison). Since the long-wave oscillatory instability mode is not affected by vibrations of low and moderate intensity, at a certain value of the vibrational Rayleigh number the long-wave oscillatory perturbations become more dangerous than the finite-wavelength ones (as seen from Fig. 3b, this change of the instability mode occurs at $\mathrm{Ra_v}$ slightly larger than 300). However, in this case the growth of the vibration intensity also leads to the appearance of a new, monotonic finite-wavelength instability mode. This mode appears at $\mathrm{Ra_v} \approx 142$ and initially the domain of this

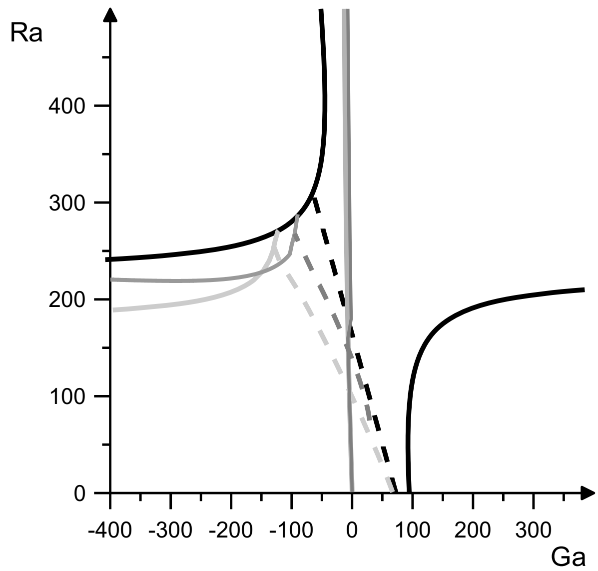


Fig. 4. Stability map for $\mathrm{Ma} = -100$ and two values of vibrational Rayleigh number: $\mathrm{Ra_v} = 0$ and $\mathrm{Ra_v} = 100$: Solid lines – monotonic instability, dashed lines – oscillatory instability. Black lines – longwave instability, light grey lines – finite-wavelength instability at $\mathrm{Ra_v}= 0$, dark grey lines – finite-wavelength instability at $\mathrm{Ra_v}= 100$.

instability has the form of a close loop (see, Fig. 3b, solid line for $\mathrm{Ra_v} = 145$) located above the neutral curve of oscillatory finite-wavelength perturbations; the instability domain is located inside the loop. However, the vibrations exert a destabilizing effect on this mode, and already at $\mathrm{Ra_v} = 200$ becomes more dangerous than the oscillatory finite-wavelength instability mode.

Full stability maps for $\mathrm{Ma} = -100$ in the absence and in the presence of vibrations with $\mathrm{Ra_v} = 100$ are shown in Figure 4. As one can see, except for a narrow range $\mathrm{Ga} \approx [-142, -92]$, the vibrations at $\mathrm{Ra_v} = 100$ have a stabilizing effect.

As shown in [9], in the case when vibrations are absent, with increase of the Marangoni number the shape of the curves changes qualitatively: the recoupling of the branches of monotonic longwave instability boundaries takes place at $\mathrm{Ma} \approx -187.1$. We analyzed the vibration effect at $\mathrm{Ma} = -200$. The neutral curves at $\mathrm{Ma} = -200$ and different vibrational Rayleigh numbers are presented in Figures 5a and 5b for $\mathrm{Ga} = -250$ when the monotonic finite-wavelength perturbations are most dangerous and $\mathrm{Ga} = -60$ when the oscillatory finite-wavelength perturbations are responsible for instability.

As one can see, the behavior is qualitatively similar to the case when $\mathrm{Ma} = -100$ (see Figs. 3 and 5 for comparison). This similarity is related to the fact that unlike the long-wave instability boundaries, the boundaries of the finite-wavelength instability, which is most dangerous at $\mathrm{Ma} = -200$, do not change their shape.

Full stability maps for $\mathrm{Ma} = -200$ in the absence of vibrations and in the presence of vibrations with $\mathrm{Ra_v} = 100$ are presented in Figure 6. As in the case of $\mathrm{Ma} = -100$, except for a narrow range $\mathrm{Ga} \approx [-161, \ -129]$, the vibrations at $\mathrm{Ra_v} = 100$ have a stabilizing effect.

In the case of heating from above ($\mathrm{Ra} < 0$, $\mathrm{Ma} > 0$) the monotonic finite-wavelength perturbations are responsible for the onset of instability (Fig. 7). The vibrations produce a destabilizing effect on these perturbations: the instability boundaries are shifted to the range of negative Rayleigh number larger in modulus.

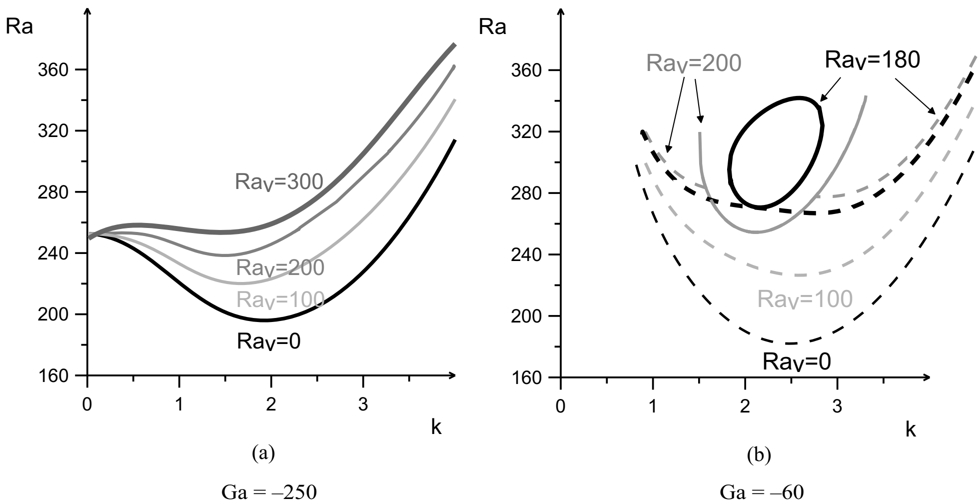


Fig. 5. Neutral curves $\mathrm{Ra}\,(k)$ at $\mathrm{Ma} = -200$ and different vibrational Rayleigh numbers. Solid lines – neutral curves for monotonic perturbations, dashed lines – for oscillatory perturbations. Instability regions are located above the curves and inside the close loop.

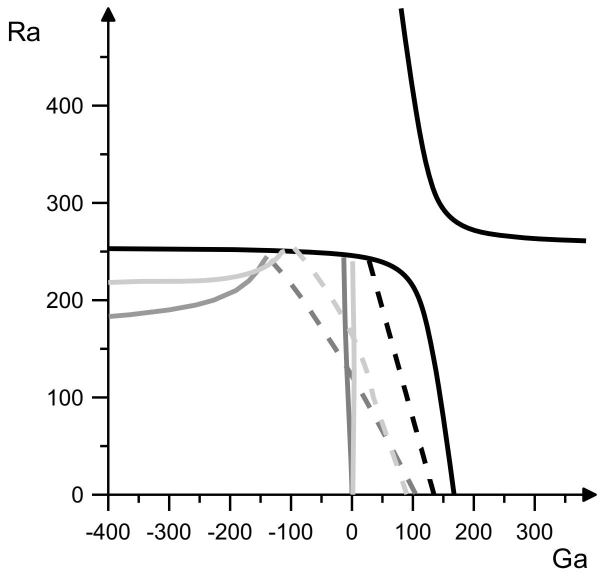


Fig. 6. Stability map for $\mathrm{Ma} = -200$ and two values of vibrational Rayleigh number: $\mathrm{Ra_v} = 0$ and $\mathrm{Ra_v} = 100$. Solid lines – monotonic instability boundaries, dashed lines – oscillatory instability boundaries. Black lines – longwave instability boundaries, dark grey lines – finite-wavelength instability boundaries for $\mathrm{Ra_v} = 0$, light grey lines – finite-wavelength instability boundaries for $\mathrm{Ra_v} = 100$.

5 Conclusions

The effect of vertical vibrations on the Rayleigh-Benard-Marangoni instability in a two-layer system of immiscible incompressible viscous fluids of close densities has been studied using the generalized Boussinesq approximation, which allows us to take into account the interface deformations in a proper way. A constant vertical heat flux is

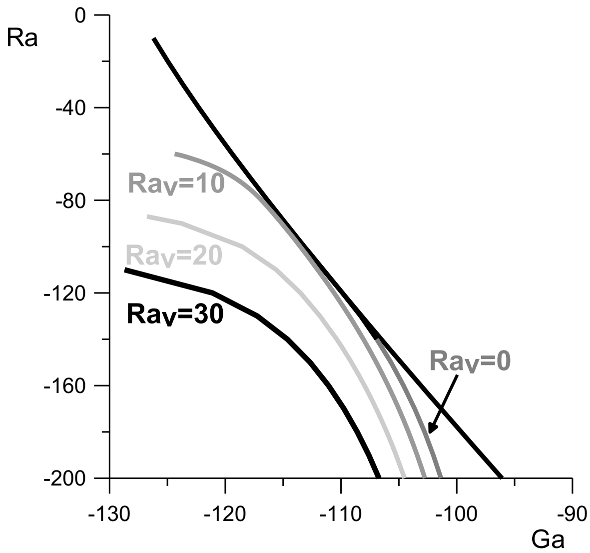


Fig. 7. Stability map for Ma = 100 and different values of vibrational Rayleigh number. Black line – longwave monotonic instability boundary, grey and bold black lines – finite-wavelength monotonic instability boundaries.

prescribed at the external rigid boundaries. The study has been performed using the averaging approach based on the assumption that the vibration period is small in comparison with the characteristic time scales (viscous and thermal) and the product of the vibration amplitude and the Boussinesq parameter is small in comparison with the layer thickness. It has been shown, that, as in the case when thermocapillary effect is absent, vibrations of low and moderate intensity do not influence the long-wave instability threshold and the high-intensity vibrations lead to the appearance of a new finite-wavelength monotonic instability mode characterized by small wave numbers, which, however, does not become the most dangerous at any values of parameters.

The investigation of instability with respect to finite-wavelength monotonic perturbations shows that the vibrations have a stabilizing effect on these perturbations. Since the long-wave monotonic instability mode is not affected by vibrations of low and moderate intensity, at a certain value of the vibrational Rayleigh number the long-wave monotonic perturbations become more dangerous than the finite-wavelength ones.

The effect of vibrations on oscillatory finite-wavelength perturbations is also stabilizing. Since the long-wave oscillatory instability mode is not affected by vibrations too, at certain value of the vibrational Rayleigh number the long-wave oscillatory perturbations become more dangerous than the finite-wavelength ones. However, the growth of vibration intensity also leads to the appearance of new, monotonic finite-wavelength instability mode. Initially, the domain f of this instability mode represents a close loop located above the neutral curve for oscillatory finite-wavelength perturbations. However, the vibrations exert a destabilizing effect on this mode, and with a further growth of Ra_v becomes more dangerous than the oscillatory finite-wavelength instability mode.

In the case of heating from above the monotonic finite-wavelength perturbations are responsible for the onset of instability. Vibrations produce a destabilizing effect on these perturbations.

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