



A practical observer for state and sensor fault reconstruction of a class of fractional-order nonlinear systems

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Abstract Practical observers have been widely used in the literature for the purpose of estimating states for integer-order disturbed and/or uncertain systems. However, much less research has been done to propose practical observers for fractional-order disturbed and/or uncertain systems. As for sensor fault estimation, and to the best of the authors' knowledge, no previous works have developed an adaptive practical observer to address the problem. In this paper, the authors propose, for the first time, an adaptive practical observer to reconstruct simultaneously the states and sensor faults for Lipschitz fractional systems. Another merit of this work, is that the class of nonlinear systems, used here, is some kind of disturbed Lipschitz system. That is to say, the Lipschitz assumption here is a modified version of the classical Lipschitz condition. And this modified version has been newly introduced in the literature, in a recent article. To validate the developed theoretical results, a numerical example is studied in the final section.

1 Introduction

In the control theory, it is important to access the system states for various causes. Sometimes, it is technologically impossible to access some states through sensors. In such situations, we can use observers, as software sensors. Generally speaking, it is possible to reconstruct either the entire state vector (full-order observers), or a part of the state vector (reduced-order observers) [1]. Apart from reconstructing states, researchers have extensively exploited observers in other contexts, such as control problems [2]. Note that diverse kinds of observers have been introduced in the literature. Kalman was the first scientist who introduced observers, and this was for stochastic linear systems [3]. And then, Luenberger has defined the well-known Luenberger theory for observing deterministic linear systems [4]. After that, various types of observers have been introduced by researchers. Among these types, we can cite the unknown input observers [5].

The real dynamics of several real physical processes contain non-integer derivatives. That's why they can't be modelled using integer-order differential equations.

In the goal of describing these systems with accuracy, researchers have exploited the fractional differential equations. In this case, when the derivative order α satisfies $0 < \alpha \leq 1$, the investigated class of systems is a generalization of the classical integer-order systems ($\alpha = 1$). In the last years, extensive works have been established to analyze fractional systems and phenomena. For example, in [6], the authors have suggested a fractional memristor map, in the presence of hidden chaotic attractors. The fractional calculus has been used to model diverse real systems. In the field of biological diseases and infections, more and more research works are modelling such diseases using fractional derivatives. This was the case for the Hand–Foot–Mouth Disease [7], the zoonotic infection [8], the dengue infection [9], and the COVID19 pandemic [10]. On the other hand, in [11], it was a question of modelling the propagation of classical optical solitons. In [12], the authors have tackled fractional modelling of a heat transfer system. Note that, in the last decades, the fractional calculus have been remarkably used in the stability theory [13, 14].

Synthesizing observers for fractional systems is becoming an attractive axis of research. Several works have been recently done in this context, for linear systems as well as for nonlinear systems. Talking about

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linear systems, the authors in [15] have recently synthesized an observer for polytopic perturbed fractional linear systems. Another work [16] has designed and used an observer for the control of fractional linear systems with interval variable delay. In [17], the authors have used an observer for the control of fractional linear multi-agents systems. Dealing with nonlinear systems, the authors in [18] have synthesized a Luenberger-like observer for a class of fractional nonlinear systems. In another paper [19], it was a question of designing a non-fragile H-infinity observer for a class of fractional nonlinear systems. The subject of another research work [20] was the interval observer query for a fractional multi-agent system in the presence of nonlinear dynamics.

In diagnosis and fault-tolerant control, it's important to reconstruct possible faults that might appear. Generally speaking, these faults include component faults, sensor faults, and actuator faults. In a recent doctoral dissertation [21], the author has estimated these three types of faults for doubly-fed induction generators. In another research work [22], it was a question of reconstructing actuator faults for three physically-linked 2WD mobile robots. In [23], the authors have designed and exploited a sliding mode observer, in the goal of estimating actuator faults. Talking about sensor faults, they denote any malfunctioning that may affect the process sensors. In the present paper, we investigate constant sensor faults. Estimating sensor faults for fractional systems is an interesting area of research. The reason is that, only some papers have been published, in this context. For instance, in [24], the authors have reconstructed sensor faults for one-sided Lipschitz descriptor fractional systems. And in another sister-paper [25], it was a question of estimating sensor faults for fractional systems, in the presence of monotone nonlinearities.

Practical Stability is a particular stability concept introduced in the literature in the 60 s. And there has been diverse works published about it. For instance, the authors in [26] have tackled this query with respect to some variables, for stochastic differential equations. In [27], the authors have tackled the practical stability for time-varying positive systems, in the presence of delay. This type of stability has been even used for artificial neural networks, and readers can refer to the research paper [28], in this context. The use of practical stability, in the context of studying observers, has risen in the last years. For instance, the author in [29] has designed an observer and demonstrated its practical stability for some class of nonlinear systems, with unknown delay. In another work [30], it was a question of using an unknown input observer that converges practically, to estimate faults for some class of nonlinear systems. Talking about fractional systems, some researchers have recently published noteworthy results, in relation to practical stability. In [31], the authors have investigated the practical stability for fractional artificial neural nets. Another work [32] has focused on designing a stabilization law for some class of fractional uncertain systems, and the practical stability concept

has been used therein. The readers can also refer to [33] as an interesting work, where it was a question of merging both concepts of practical stability and finite-time stability to control neural networks for incommensurate fractional nonlinear systems.

Based on all the above, the present research work investigates the design of a practical observer, in the purpose of estimating simultaneously states and sensor faults, for some specific class of nonlinear fractional-order systems. The concept of Caputo fractional derivative order is used. For the nonlinear part of the system, the authors take advantage of a newly introduced condition in the literature [34], which is a more general version of the classical Lipschitz condition (see Assumption 1, in the next section). And that's been said, the contribution of this work comes from the merging of the following aspects:

- To the authors' knowledge, no research work exists for sensor fault estimation using a practical observer in the literature. The already published paper [30], mentioned above, investigates actuator faults, not sensor faults.
- The used class of systems is wider than the classical Lipschitz systems. Such a class has only been introduced in the new literature work [34], and after that, no works have been published using this condition.
- As mentioned above, fractional systems with $0 < \alpha \leq 1$ are an extension of integer-order systems ($\alpha = 1$).

What's left in this paper has the following organization. In the next section, the authors present the mathematical class of systems to be treated, as well as some relevant definitions and useful lemmas. In Sect. 3, the developments and result of the paper are exposed. And in the final section, a simulation example is provided and used, to show the efficiency of the paper's results.

2 Preliminaries and system description

Various fractional derivative concepts have already been defined by researchers [35, 36]. For this research work, the Caputo definition is chosen.

Definition 1 The Riemann–Liouville fractional integral of order $\alpha > 0$ is:

$$I_{t_0}^{\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t - \tau)^{\alpha-1} x(\tau) d\tau,$$

$\Gamma(\alpha) = \int_0^{+\infty} e^{-t} t^{\alpha-1} dt$ denotes the Gamma function.

Definition 2 If n is the integer part of $\alpha + 1$, then the Caputo fractional derivative has the following expression:

$${}^C D_{t_0,t}^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t (t-\tau)^{n-\alpha-1} \frac{d^n}{d\tau^n} x(\tau) d\tau, \quad (n-1 < \alpha < n)$$

and if $0 < \alpha < 1$, then this Caputo derivative for an absolutely continuous function $x(t)$ becomes:

$${}^C D_{t_0,t}^\alpha x(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^t (t-\tau)^{-\alpha} \frac{d}{d\tau} x(\tau) d\tau. \quad (1)$$

Definition 3 The following function is called the Mittag-Leffler function:

$$E_{\alpha,\gamma}(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma(k\alpha + \gamma)},$$

where $\alpha > 0, \gamma > 0, z \in \mathbb{C}$. When $\gamma = 1$, one has $E_\alpha(z) = E_{\alpha,1}(z)$, furthermore, $E_{1,1}(z) = e^z$.

If one considers:

$$\begin{aligned} {}^C D_{t_0,t}^\alpha X(t) &= G(t, X) \\ X(t_0) &= X_0, \end{aligned}$$

where $t_0 \in R_+$ and $G : R_+ \times R^n \rightarrow R^n$ is supposed to be continuous, then the following definition can be given.

Definition 4 [37] The above system is practically Mittag-Leffler stable, if one can find positive scalars ρ, λ and $\sigma, s.t.$:

$$\|X(t)\| \leq [\omega(\|X_0\|) E_\alpha(-\lambda(t-t_0)^\alpha)]^\sigma + \rho \quad \forall t \geq t_0,$$

where ω is a locally Lipschitz function with $\omega(0) = 0$.

Lemma 1 [38] Define $Y : [t_0, +\infty) \rightarrow R$ that satisfies:

$${}^C D_{t_0,t}^\alpha Y(t) \leq -\lambda Y(t) + h(t), \quad \forall t \geq t_0$$

with $h(t)$ is continuous nonnegative function that satisfies:

$$\begin{aligned} t \rightarrow \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(-\lambda(t-s)^\alpha) \\ \times h(s) ds \text{ is a bounded function.} \end{aligned} \quad (2)$$

Then there exists $K > 0$ such that

$$Y(t) \leq E_\alpha(-\lambda(t-t_0)^\alpha) + K, \quad \forall t \geq t_0.$$

For this work, the authors investigate the following fractional system (3):

$$\begin{aligned} {}^C D_{t_0,t}^\alpha x(t) &= Ax + Bu + g(t, x), \quad t \geq t_0 \\ y(t) &= Cx(t) + Ff, \end{aligned} \quad (3)$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$ denotes the input, $y(t) \in \mathbb{R}^q$ denotes the output, $f \in \mathbb{R}^p$ denotes the constant sensor fault. $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{q \times n}, F \in \mathbb{R}^{q \times p}$, are well known constant matrices. The nonlinear part $g : R_+ \times R^n \rightarrow R^n$ which satisfies the following assumption:

Assumption 1 [34] g is supposed to be continuous, and

$$\|g(t, x) - g(t, y)\| \leq b\|x - y\| + \varphi(t),$$

$$\forall t \in R_+, \forall (x, y) \in R^n,$$

$$\text{where } b > 0 \text{ and } \varphi \in C(R_+, R_+).$$

Lemma 2 [39] Let $\alpha \in (0, 1)$ and $P \in \mathbb{R}^{n \times n}$ a constant symmetric and positive definite matrix. Then, the following relationship holds

$$\frac{1}{2} {}^C D_{t_0,t}^\alpha (x^T(t) P x(t)) \leq x^T(t) P {}^C D_{t_0,t}^\alpha x(t), \quad \forall t \geq t_0$$

3 Estimation of states and sensor faults

We define the following adaptive Luenberger-like observer:

$$\begin{aligned} {}^C D_{t_0,t}^\alpha \hat{x}(t) &= A\hat{x}(t) + Bu(t) + g(t, \hat{x}) - L(\hat{y}(t) - y(t)), \\ \hat{y}(t) &= C\hat{x}(t) + F\hat{f}(t), \end{aligned} \quad (4)$$

where $\hat{x}(t)$ and $\hat{f}(t)$ are the state vector estimate and the fault vector estimate, and L is the observer gain to be designed.

Theorem 1 Consider the fractional system (3), the observer (4), Assumption 1 and the adaptation law (5). If one can find matrices X, G and $P = P^T > 0$, as well as scalars $\varepsilon > 0$ and $\eta > 0$, such that the inequality (6) is feasible under condition (7), this gives that system (4) is a practical observer that makes it possible to reconstruct the states and sensor faults, for system (3).

$${}^C D_{t_0,t}^\alpha \hat{f}(t) = G(y(t) - \hat{y}(t)), \quad t \geq t_0 \quad (5)$$

$$\Gamma = \begin{bmatrix} PA + A^T P - XC - C^T X^T + 2b^2 \varepsilon I + \frac{1}{\varepsilon} PP & XF + \frac{1}{\eta} (GC)^T \\ (XF)^T + \frac{1}{\eta} GC & -\frac{2}{\eta} GF \end{bmatrix} < 0 \quad (6)$$

$$\begin{aligned} t \rightarrow \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(-\lambda(t-s)^\alpha) \varphi^2(s) \\ \times ds \text{ is a bounded function,} \end{aligned} \quad (7)$$

where $X = PL$ and $\lambda = -\frac{\lambda_{max}(\Gamma)}{\lambda_{max}(S)}$ with $S = \begin{bmatrix} P & 0 \\ 0 & \frac{1}{\eta} \end{bmatrix}$.

Proof Consider the following estimation errors: $e(t) = \hat{x}(t) - x(t)$ and $\tilde{f}(t) = f - \hat{f}(t)$. Then, one has:

$${}^C D_{t_0,t}^\alpha e(t) = (A - LC)e(t) + \Delta g(t) + LF\tilde{f}(t). \tag{8}$$

where $\Delta g(t) = g(t, \hat{x}) - g(t, x)$.

Consider now the Lyapunov function $V(e, \tilde{f}) = e^T P e + \frac{1}{\eta} \tilde{f}^T \tilde{f}$, where η is a real design scalar.

Using Lemma 2, one can have:

$$\begin{aligned} & {}^C D_{t_0,t}^\alpha V(t, e, \tilde{f}) \\ & \leq 2e^T P {}^C D_{t_0,t}^\alpha e + \frac{2}{\eta} \tilde{f}^T {}^C D_{t_0,t}^\alpha \tilde{f} \\ & \leq 2e^T P \left((A - LC)e(t) + \Delta g(t) + LF\tilde{f}(t) \right) \\ & \quad - \frac{2}{\eta} \tilde{f}^T {}^C D_{t_0,t}^\alpha \tilde{f} \leq 2e^T P(A - LC)e \\ & \quad + 2e^T P \Delta g(t) + 2e^T P L F \tilde{f} - \frac{2}{\eta} \tilde{f}^T G(y(t) - \hat{y}(t)) \\ & \leq 2e^T P(A - LC)e + 2e^T P \Delta g(t) \\ & \quad + 2e^T P L F \tilde{f} - \frac{2}{\eta} \tilde{f}^T G F \tilde{f} + \frac{2}{\eta} \tilde{f}^T G C e. \end{aligned}$$

On the other hand, one has:

$$2e^T P \Delta g(t) \leq \varepsilon \|\Delta g(t)\|^2 + \frac{1}{\varepsilon} \|P e\|^2.$$

Using Assumption 1, it follows that:

$$\begin{aligned} 2e^T P \Delta g(t) & \leq \varepsilon (2b^2 \|e\|^2 + 2\varphi^2(t)) + \frac{1}{\varepsilon} \|P e\|^2 \\ & \leq e^T \left(2b^2 \varepsilon I + \frac{1}{\varepsilon} P P \right) e + 2\varepsilon \varphi^2(t). \end{aligned}$$

Thus,

$$\begin{aligned} & {}^C D_{t_0,t}^\alpha V(t, e, \tilde{f}) \leq \begin{bmatrix} e \\ \tilde{f} \end{bmatrix}^T \\ & \quad \begin{bmatrix} P(A - LC) + (A - LC)^T & P L F + \frac{1}{\eta} (G C)^T \\ P + 2b^2 \varepsilon I + \frac{1}{\varepsilon} P P & P L F + \frac{1}{\eta} (G C)^T \\ (P L F)^T + \frac{1}{\eta} G C & -\frac{2}{\eta} G F \end{bmatrix} \\ & \quad \begin{bmatrix} e \\ \tilde{f} \end{bmatrix} + 2\varepsilon \varphi^2(t). \end{aligned}$$

Now considering (6), one can write:

$${}^C D_{t_0,t}^\alpha V(t, e, \tilde{f}) \leq -\lambda V(t, e, \tilde{f}) + 2\varepsilon \varphi^2(t). \tag{9}$$

And both conditions (7) and (9) make it possible to exploit Lemma 1. So we can find:

$$\begin{aligned} V(t, e, \tilde{f}) & \leq E_\alpha(-\lambda(t - t_0)^\alpha) V(t_0, e_0, \tilde{f}_0) + K, \\ & \quad \forall t \geq t_0 \end{aligned}$$

Then, there exist $m > 0$ and $r > 0$, such that

$$\|X(t)\| \leq [m E_\alpha(-\lambda(t - t_0)^\alpha) \|X(t_0)\|^2]^{1/2} + r, \forall t \geq t_0$$

where $X(t) = (e(t), \tilde{f}(t))$.

Finally, using Definition 4, we can conclude that (4) is a practical observer that makes it possible to reconstruct the states and sensor faults for system (3). This ends the proof.

Remark Inequality (6) can be easily transformed into linear matrix inequality by applying Schur Complement.

We obtain the following LMI:

$$\begin{bmatrix} P A + A^T P - X C - C^T X^T + 2b^2 \varepsilon I & X F + (W C)^T & P \\ (X F)^T + W C & -2W F & 0 \\ P & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{10}$$

where $W = \frac{1}{\eta} G$.

4 Numerical study

Consider the following fractional system:

$$\begin{cases} {}^C D_{t_0,t}^\alpha x(t) = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) \\ \quad + \begin{pmatrix} 0.01 \cos(t^2 x_1) \\ 0.1 \sin(x_2) \end{pmatrix}, t \geq t_0 \\ y(t) = \begin{pmatrix} 0.2 & 0.1 \\ -0.1 & 0.2 \end{pmatrix} x + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \end{cases}$$

f_1 and f_2 represent two constant sensor faults to be estimated. Let's assign the fractional-order derivative to: $\alpha = 0.7$. It's clear that Assumption 1 ($\|g(t, x) - g(t, y)\| \leq b \|x - y\| + \varphi(t)$) is satisfied for this numerical example, where:

$$g(t, x) = \begin{pmatrix} 0.01 \cos(t^2 x_1) \\ 0.1 \sin(x_2) \end{pmatrix}, b = 0.1, \varphi(t) = 0.02,$$

And $\varphi(t) = 0.02$ satisfies condition (7).

That's been said, the observer (4) together with the adaptation law (5) can be exploited, and Theorem 1

Fig. 1 State estimation errors

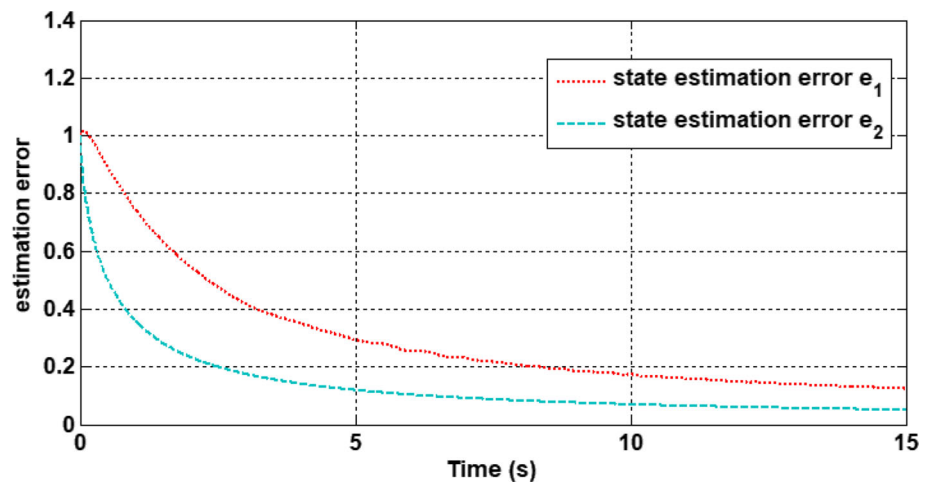
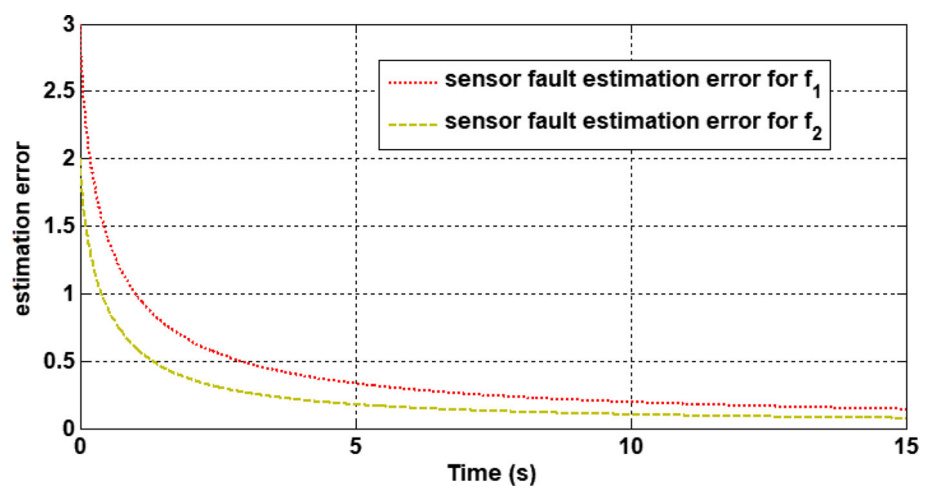


Fig. 2 Sensor fault estimation errors



is applied. The LMI (10) is feasible, and it has these solutions: $\varepsilon = 33.8936$, $\eta = 1$ and:

$$P = \begin{pmatrix} 12.8693 & 2.3782 \\ 2.3782 & 16.3915 \end{pmatrix},$$

$$G = \begin{pmatrix} 1.4666 & 0.0392 \\ -0.0551 & 1.4953 \end{pmatrix}, X = \begin{pmatrix} -0.6131 & 2.3260 \\ 0.4784 & -2.5471 \end{pmatrix}$$

So the observer gain matrix is computed: $L = P^{-1}X = \begin{pmatrix} -0.0545 & 0.2152 \\ 0.0371 & -0.1866 \end{pmatrix}$. And then, the simulation results are ready to be depicted. The initial conditions are chosen as follows: $x_0 = [-3 \ -1]^T$, $e_0 = [1 \ 1]^T$, $\tilde{f}_0 = [3 \ 2]^T$. In Fig. 1, the state estimation errors are illustrated, while in Fig. 2, the sensor fault reconstruction errors are illustrated. Both figures insure the practical stability of the state estimation and fault estimation errors, as demonstrated in the previous section.

5 Conclusion

In the present paper, the authors have designed an adaptive Luenberger-like observer, in the goal of reconstructing the states and sensor faults for some specific class of nonlinear fractional systems. As the nonlinear part of the system is a special Lipschitz condition, it was necessary to investigate and to use the practical Mittag-Leffler stability definition. The Caputo fractional derivative definition was used in this work. The synthesized observer was found to converge practically if a matrix inequality condition is satisfied. To further consolidate the paper’s results, a simulation example has been investigated, and numerical results have shown the practical stability of such systems.

This research work has focused on constant sensor faults. As a future paper, the authors plan to extend the present work to estimating time-varying sensor faults.

Data availability No data was associated with the manuscript.

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