Regular Article

Relationship between water injection volume and liquid injection pressure under the condition of pipeline deformation

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Abstract Water injection in a closed pipe at one end causes force deformation of the pipe. The expansion changes of the pipeline under different liquid injection pressures are studied combined with theoretical and experimental results. A new relationship between liquid injection pressure and water injection volume of the pipeline is proposed based on elastic mechanics, which is analyzed and verified. The experimental results show that at the early stage, the variation of water injection amount and pipeline deformation show nonlinear change, and the change rate liquid injection volume is constantly decreasing after the liquid injection pressure is increased to some value. Subsequently, there is a linear relationship between water injection and water injection pressure. The inner diameter and length of pipeline have significance influences on the deformation of the pipeline. With the increase of inner diameter and length of the pipeline, the rising rate of water injection volume is accelerated. The proposed equation can well express the relationship between the water injection change volume and the pipeline deformation under water injection pressures. The theory could provide useful theoretical reference for the study of pipeline blockage in the future.

1 Introduction

Due to the influence of external media and internal sediments, the pipeline may be blocked, which is inevitable in all research fields [7, 19]. In the study of pipeline, different conditions may cause blockage of pipes such as freezing of liquid in pipes under cold condition, deposition of internal sediments and closure of valves [14, 17], leading to a change in the pressure at the blockage of the pipeline and endangering the stability of the pipes.. Some scholars have studied the change of formation stress by hydraulic fracturing [16, 18, 20, 21]. Similarly, blockage of blood vessels can lead to disease in medicine [3, 8].The mechanism of pipeline expansion under water pressure will be helpful for the pipeline safety or similar fluid–solid coupling phenomena.

The damage of underground pipeline will cause a great loss of energy and material resources. However, the change of the environment outside the pipe and the impact of the liquid inside the pipe will have influences on the pipeline. These uncontrollable factors make pipeline maintenance become a challenge. Similarly, the characteristics of closed fractures in the rock filed provide a research idea for the study of hydraulic fracturing. Scholars in various fields pay more attention to the influence of water injection and pressure on the research object under the condition of blockage [4, 11, 13]. Through the study of hydraulic fracturing, Schultz et al. [18] found that the most important operating parameter to control induced earthquakes was the amount of injection, which was linearly related to the total number of earthquakes. Duan et al. [5, 6] studied that the blockage of the pipeline may lead to the redistribution of the fluid flow network and the change of the flow and pressure distribution at the clogged place, which may lead to the leakage or destruction of the pipeline. Watanabe et al. [22] carried out hydraulic fracturing experiments by injecting water into cylindrical granite samples with holes, resulting in the extension of fractures in rock samples under high pressure. Fracture mechanism was used to study the exploitability of thermal energy under deep strata. However, scholars pay attention to the energy loss caused by the blockage of the channel and the change of pressure it may produce a series of adverse consequences. But there is a lack of theoretical analysis of the blockage impact and there is no quantitative analysis of the impact of injection and pressure on the blocked pipeline. The influence of pipe length and diameter on pipe safety during water injection has not been noticed. For solving these problems, we propose the strain test pipe deformation

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method under the internal water pressure to study the change of the water injection volume, and analyze the influence of the pipe length and scale on the above relationship.

The blockage of the pipeline will increase the pressure of the pipeline and cause the pipeline to expand during water injection. The pressure change and pipeline deformation caused by this kind of water injection are rarely studied. Therefore, we choose pipes with different lengths and different diameters to carry out the experiment. This experiment studies the relationship among the liquid injection pressure, the amount of water injected and the deformation of the pipeline under the condition of pipeline blockage. On the basis of the theoretical model, this paper puts forward the theoretical relationship between the amount of water injection volume and the formed water pressure when the pipeline is deformed by water pressure. Combined with the experiment, the relationship between water injection volume and liquid injection pressure under different lengths and diameters is obtained. The theoretical and experimental results have been a good agreement, providing mechanical reference for channel monitoring and protection.

2 Methodology

2.1 Testing equipment

Type III polypropylene (PPR) is widely used in building water supply and drainage, industrial fluid transmission, agricultural irrigation and agricultural fields. Therefore, in this experiment, to be close to the engineering significance, PPR pipeline is selected as the research sample material (Fig. 1).

The instruments involved in the experiment include strain gauge, pump and strain system (Fig. 1). The strain gauge has the characteristics of strong anti-interference and good stability [10, 12, 23]. The strain gauge can measure the expansion change of the pipe due to water injection. The pump can provide the water injection pressure needed in the experiment, and can provide a stable flow rate. The strain system is DH3816N static strain test and analysis system. During the experiment, the strain gauge is connected with the instrument through the corresponding signal line, so that the strain data can be transmitted to the instrument after processing.

2.2 Specification selection of PPR pipe

Pipes with different diameters and lengths are selected for experiments to study the relationship between water injection and liquid injection pressure in the case of blockage (Table 1).

Five strain gauges are pasted on every pipe to measure strain under different liquid injection pressures. For every pipe paste one strain gauge is pasted at a location

away from the pipe water inlet 100 mm. The distance between the pipe strain gauges of length 1000 mm is 200 mm; the distance between the pipe strain gauges of length 1500 mm is 325 mm; and the distance between the pipe strain gauges of length 2000 mm is 400 mm. The regular distribution of strain gauges can effectively measure the strain changes under different liquid injection pressure.

2.3 Testing methods

The specific steps of the experiments are as follows (Fig. 1).

The inlet of pipe and the instrument pipe are cemented by epoxy resin to prevent the outflow of water during the experiment (Fig. 2).

Air is removed by filling the pipe with water. When the water is full, the outlet of the pipe is filled with epoxy resin to mimic the blockage of the pipe (Fig. 2). It also allows the pipe and the instrument to form a closed system to provide stable liquid injection pressure.

The strain gauges are pasted to the surface of the pipe and connected to the strain system.

Liquid injection pressure is provided stable, the change of strain and water injection volume are recorded.

3 Theoretical analysis between water injection volume and liquid injection pressure

In practical engineering application, because of the complexity of the surrounding environment of the pipeline, whether the pipeline is blocked or not only can be judged according to the change of frequency [9, 13, 15]. A completely clogged pipe expands when the volume of the injected liquid exceeds its own volume, which increases the volume of the pipe to accommodate more liquid (Fig. 3a). The pressure distribution under the blocked pipeline is the largest in the middle, which gradually decreases from the middle to both sides. As a result, it can be inferred that the deformation of the pipeline should be the largest in the middle, which gradually decreases from the middle to both sides. Water injection volume will produce pipeline pressure and pipeline expansion, and these changing elements are related to each other. Therefore, studying the deformation of the pipeline under water injection pressure will provide more means for the monitoring of pipeline blockage.

The pipeline expands under the influence of liquid injection pressure (Fig. 3b). The wall thickness of pipe is h, the length of pipe is $2\ l$ and liquid injection pressure is p. Because the liquid injection pressure in the pipeline is the same everywhere, the corresponding function form can be established. According to the mechanics of materials, bending stress is σ_x; extrusion stress is σ_y; and tangential stress is τ_{xy}. Therefore, the
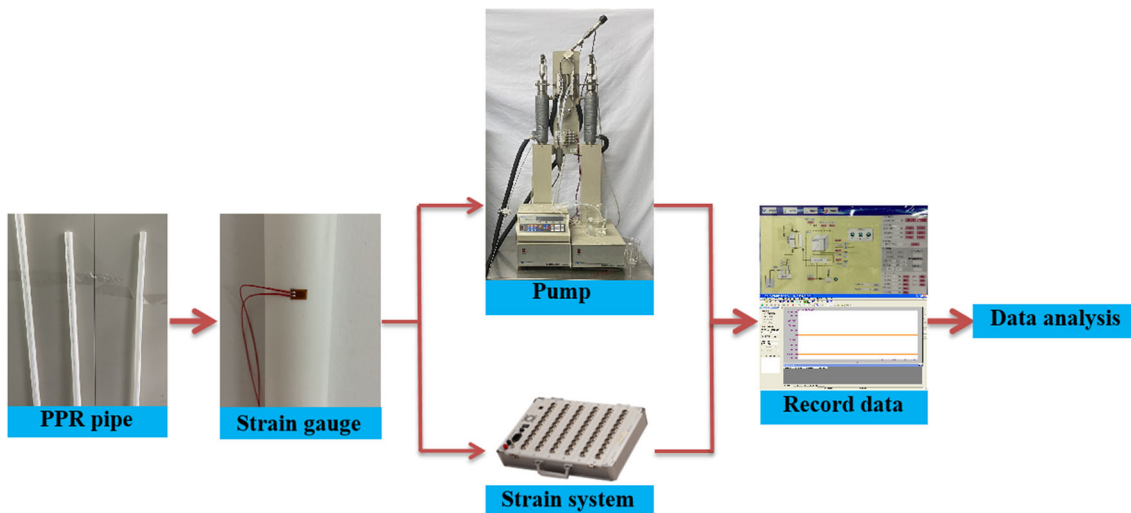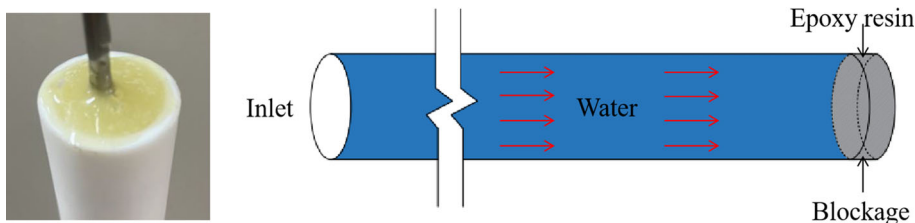
Fig. 1 Experiment process

Table 1 Specification information of PPR pipeline

Pipe	Material	Length (mm)	External diameter (mm)	Inner diameter (mm)	Length/Inner diameter
PPR	Type III polypropylene	1000	20	15.4	65:1
			25	19.4	52:1
			32	24.8	40:1
		1500	20	15.4	97:1
			25	19.4	77:1
			32	24.8	60:1
		2000	20	15.4	130:1
			25	19.4	103:1
			32	24.8	80:1

Fig. 2 Schematic of the blockage

liquid injection pressure does not change with x, so extrusion stress (σ_y) does not change with x. σ_y is only a function of y:
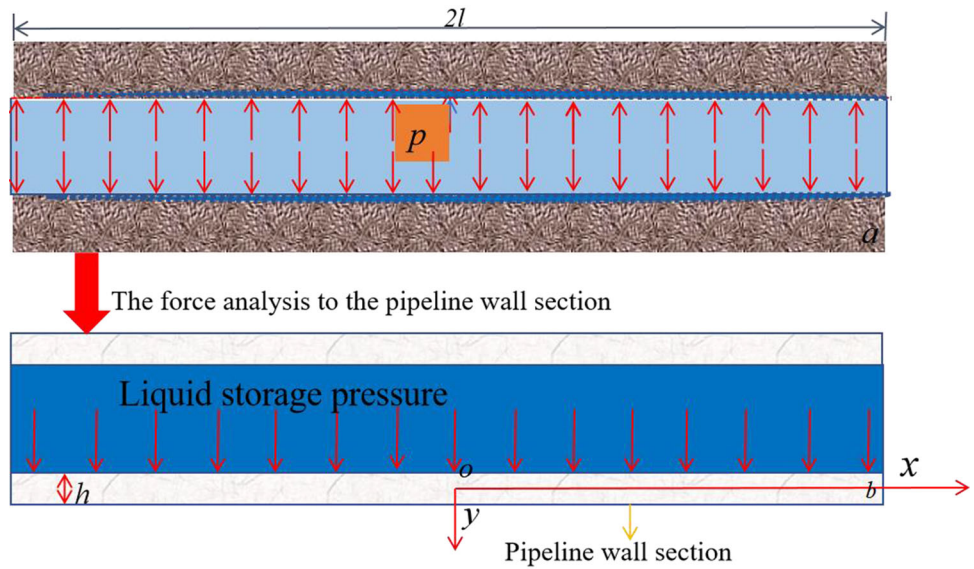
$$\sigma_y = f(y) = \frac{\partial^2 \varphi}{\partial x^2}. \tag{1}$$

φ is called the stress function of the plane problem, also known as the Airy stress function.

When the body force is zero, Airy stress function could be written:

$$\begin{cases} \sigma_x = \dfrac{\partial^2 \varphi}{\partial y^2} \\[2mm] \sigma_y = \dfrac{\partial^2 \varphi}{\partial x^2} \\[2mm] \tau_{xy} = -\dfrac{\partial^2 \varphi}{\partial x \partial y}. \end{cases} \tag{2}$$

Fig. 3 Blockage of pipeline deformation caused by water injection in soil

By integrating x: Eq. (1) can be

$$\varphi = \frac{1}{2}x^2 f(y) + x f_1(y) + f_2 y), \qquad (3)$$

where $f(y), f_1(y)$ and $f_2(y)$ are undetermined functions of y. Replacing the stress equation into the compatibility equation:$\nabla^4 \phi = 0$ [2], the specific equation of the compatibility equation is:

$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0. \qquad (4)$$

To make the stress function satisfy the compatibility equation, by substituting Eqs. (3) to (4), we can get:

$$\frac{1}{2}\frac{d^4 f(y)}{dy^4}x^2 + \frac{d^4 f_1(y)}{dy^4}x + \frac{d^4 f_2(y)}{dy^4} + 2\frac{d^2 f(y)}{dy^2} = 0. \qquad (5)$$

The stress function in pipe must satisfy the compatibility equation, and Eq. (5) is the quadratic equation of x. It can be known that its coefficient and free term must be equal to 0. The corresponding equation can be obtained:

$$\begin{cases} \dfrac{d^4 f(y)}{dy^4} = 0 \\ \dfrac{d^4 f_1(y)}{dy^4} = 0 \\ \dfrac{d^4 f_2(y)}{dy^4} + 2\dfrac{d^2 f(y)}{dy^2} = 0. \end{cases} \qquad (6)$$

Thus, the relation of $f(y)$, $f_1(y)$, $f_2(y)$ can be obtained:

$$\begin{cases} f(y) = Ay^3 + By^2 + Cy + D \\ f_1(y) = Ey^3 + Fy^2 + Gy \\ f_2(y) = -\dfrac{A}{10}y^5 - \dfrac{B}{6}y^4 + Hy^3 + Ky^2, \end{cases} \qquad (7)$$

where $A, B, C \ldots K$ are undetermined constant.

The stress function is brought into Airy stress function:

$$\begin{cases} \sigma_x = \dfrac{x^2}{2}(6Ay + 2B) + x(6Ey + 2F) \\ -2Ay^3 - 2By^2 + 6Hy + 2K \\ \sigma_y = Ay^3 + By^2 + Cy + D \\ \tau_{xy} = -x(3Ay^2 + 2By + C) - (3Ey^2 + 2Fy + G). \end{cases} \qquad (8)$$

According to the symmetry of the problem, σ_x and σ_y are even functions of x, and τ_{xy} is odd function of x.

$$E = F = G = 0.$$

The pipeline is mainly affected by the liquid injection pressure, so the boundary conditions of the pipeline can be written:

$$(\sigma_y)_{y=-h/2} = -p, \ (\sigma_y)_{y=h/2} = 0. \qquad (9)$$

Bringing the boundary condition into Eq. (8), the result is:

$$A = -\frac{2p}{h^3}, \quad B = 0, \quad C = \frac{3p}{2h}, \quad D = -\frac{p}{2}.$$

Because of the symmetry of the pipe, we can get the following conditions [1]:

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x)_{x=l} dy = 0, \tag{10}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x)_{x=l} y dy = 0, \tag{11}$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} (\tau_{xy})_{x=l} dy = -pl. \tag{12}$$

Substituting Eqs. (10)–(12) into Eq. (2), we can get the answer:

$$\begin{cases} \sigma_x = \dfrac{6p}{h^3}(l^2 - x^2)y + p\dfrac{y}{h}\left(4\dfrac{y^2}{h^2} - \dfrac{3}{5}\right) \\[2mm] \sigma_y = -\dfrac{p}{2}\left(1 + \dfrac{y}{h}\right)\left(1 - \dfrac{2y}{h}\right)^2 \\[2mm] \tau_{xy} = -\dfrac{6p}{h^3}x\left(\dfrac{h^2}{4} - y^2\right). \end{cases} \tag{13}$$

In Eq. (13), σ_x and σ_y are even functions of x, following the law of weakening from the middle to both sides.

The vertical displacement of the pipeline is caused by the expansion of water injection. Then, the physical equation must be considered.

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \sigma_x) = \frac{\partial v}{\partial y}, \tag{14}$$

where E is elastic modulus of pipeline, v is vertical displacement of pipeline. The vertical displacement can be written:

$$v = \frac{1}{E}\left(\left(-\frac{p}{2}y + \frac{21py^2}{20h} - \frac{3py^4}{h^3} - \frac{3py^2}{h^3}(l^2 - x^2)\right) + g(x)\right). \tag{15}$$

According to the symmetry, the displacement v in the vertical direction is an even function of x. Two ends of the pipe can be regarded as the fixed end. Therefore, the displacement boundary condition of the fixed end is generally approximated to that the midpoint of the boundary is fixed (the point cannot be moved and horizontal line cannot be rotated). Since the symmetry is taken into account, only the displacement boundary condition on the right side needs to be considered. That is:

$$(v)_{x=l,\, y=0} = 0. \tag{16}$$

It can be deduced that the unknown term($g(x)$) is equal to 0. However, Eq. (15) only represents the displacement of one surface, and the three-dimensional

characteristics of the pipeline should be taken into account. The volume change of pipeline expansion should be a collection of countless surfaces. The volume change of the pipe is independent of the direction of the coordinate system as:

$$V_p = \int_0^{2\pi} \left(\frac{1}{E}\int_{-l}^{l}\left(\frac{p}{2}y - \frac{21py^2}{20h} + \frac{3py^4}{h^3} + \frac{3py^2}{h^3}(l^2 - x^2)\right)dx\right)d\theta. \tag{17}$$

The volume of pipeline deformation is equivalent to the volume change of pipeline water injection. Further, it can be concluded that:

$$V = \frac{2\pi}{E}\left(pyl - \frac{21py^2}{10h}l + \frac{6py^4}{h^3}l + \frac{4py^2l^3}{h^3}\right), \tag{18}$$

where V_p is the initial volume of the pipe, and V is water injection volume. Equation (18) expresses the relationship between water injection volume and liquid injection pressure.

4 Results and discussion

The experiment explores the relationship between liquid injection pressure and water injection volume. The water injection pressure is set, and under the condition of changing the length and diameter of the pipeline. The change of water injection volume (water injection volume is the volume of water after the expansion of the pipe minus the volume of the initial state of the pipe), under different water injection pressure is explored. The strain change collected by the strain gauge on the surface of the pipeline is observed at the same time.

4.1 The relationship between water injection pressure and water injection volume

Under the condition of changing the water injection pressure, the strain and water injection of the pipeline change are shown in Fig. 4.

Pipes with different lengths and diameters are selected as the research objects. By setting different liquid injection pressure, the change of pipeline surface strain and the influence of liquid injection pressure on pipeline expansion are studied (Fig. 4).

Although the strain change rates are different at different locations, the relationship between liquid injection pressure and strain is linear. This shows that the expansion of the pipeline under every pressure gradient is same. Because the change of the volume of the pipeline represents the change of the amount of water rejected. The volume change of the pipeline under every pressure gradients is same, which means that the change of the amount of water injected under every pressure gradients is same. The relationship between the amount of water injection and liquid injection pressure is linear.
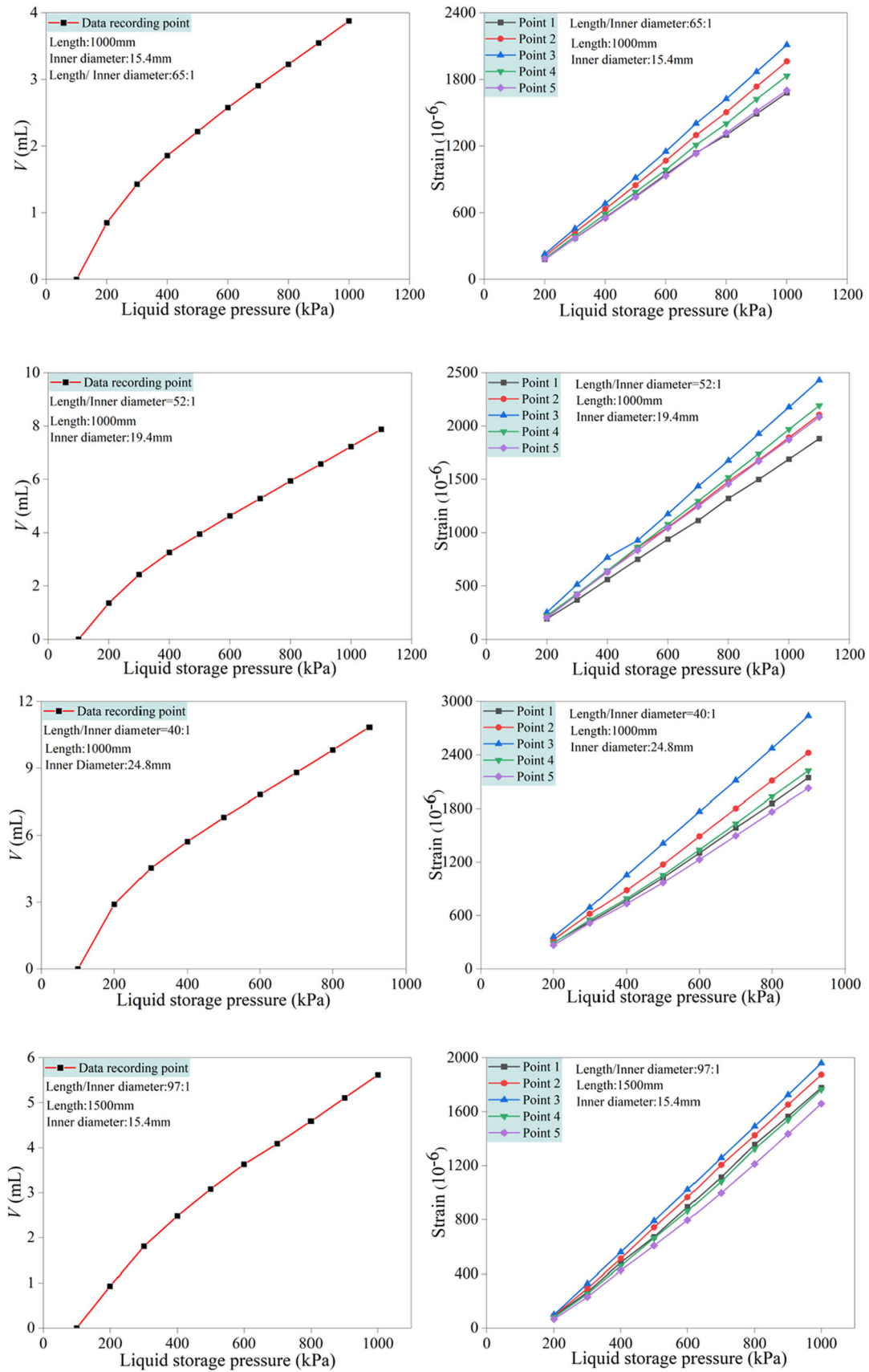
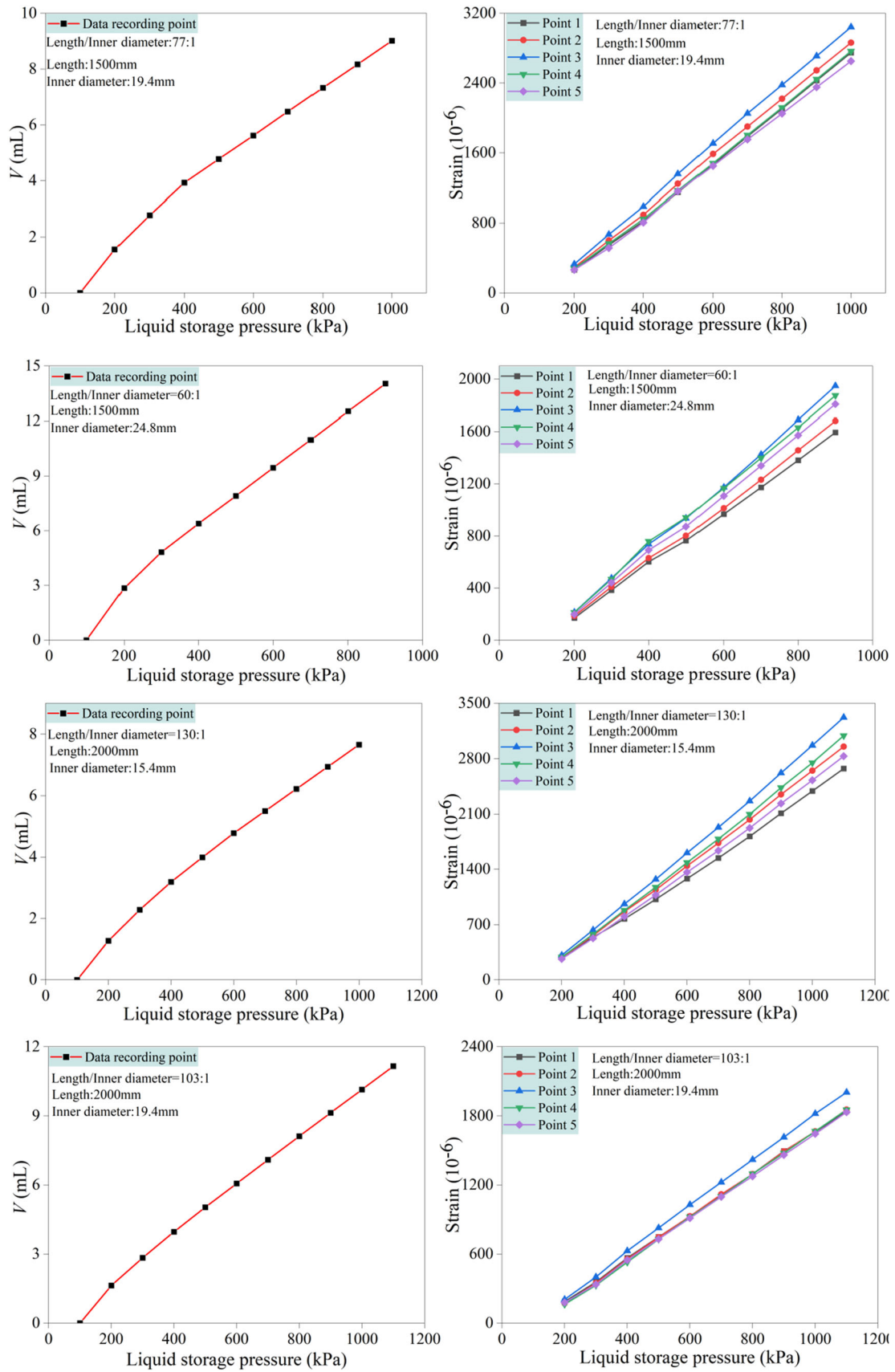**Fig. 4** Variation of strain and water injection under different water storage pressure
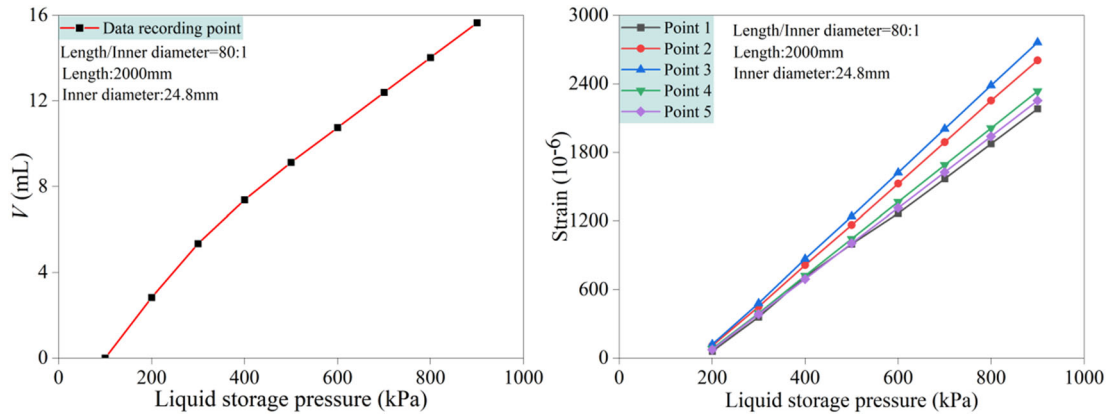
Fig. 4 continued

Fig. 4 continued

However, with the increase of liquid injection pressure, the water injection volume can be divided in to stages: curve stage and linear stage. Under the curve stage, the slope of the curve decreases gradually until it is the same as the linear stage. The curve relationship between water injection volume and liquid injection pressure is caused by the microcompressibility of water or closure of micropores. Under the bondage of the pipeline, with the injection of water, the molecular force between water molecules becomes lager. It is
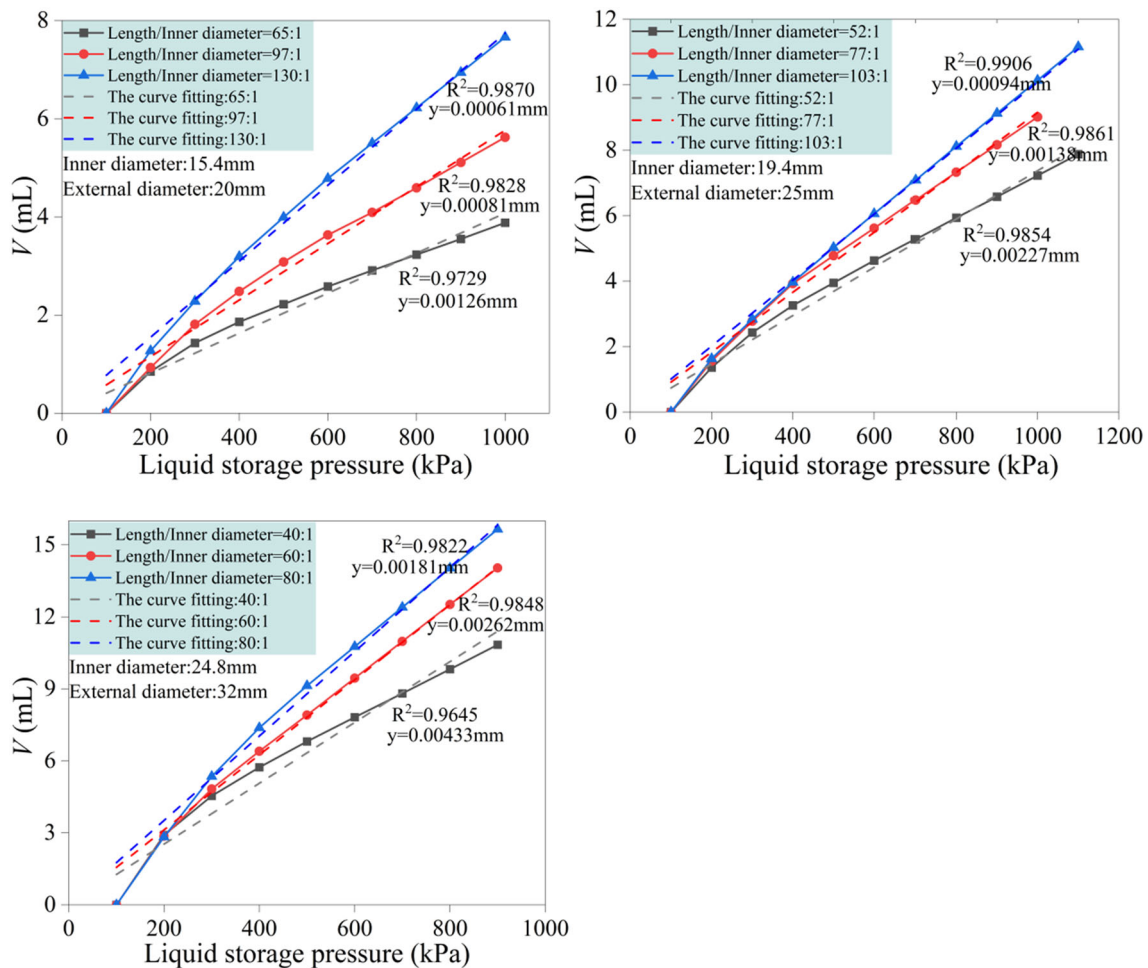

Fig. 5 Comparison of water injection volume of pipes with the same diameter and different length
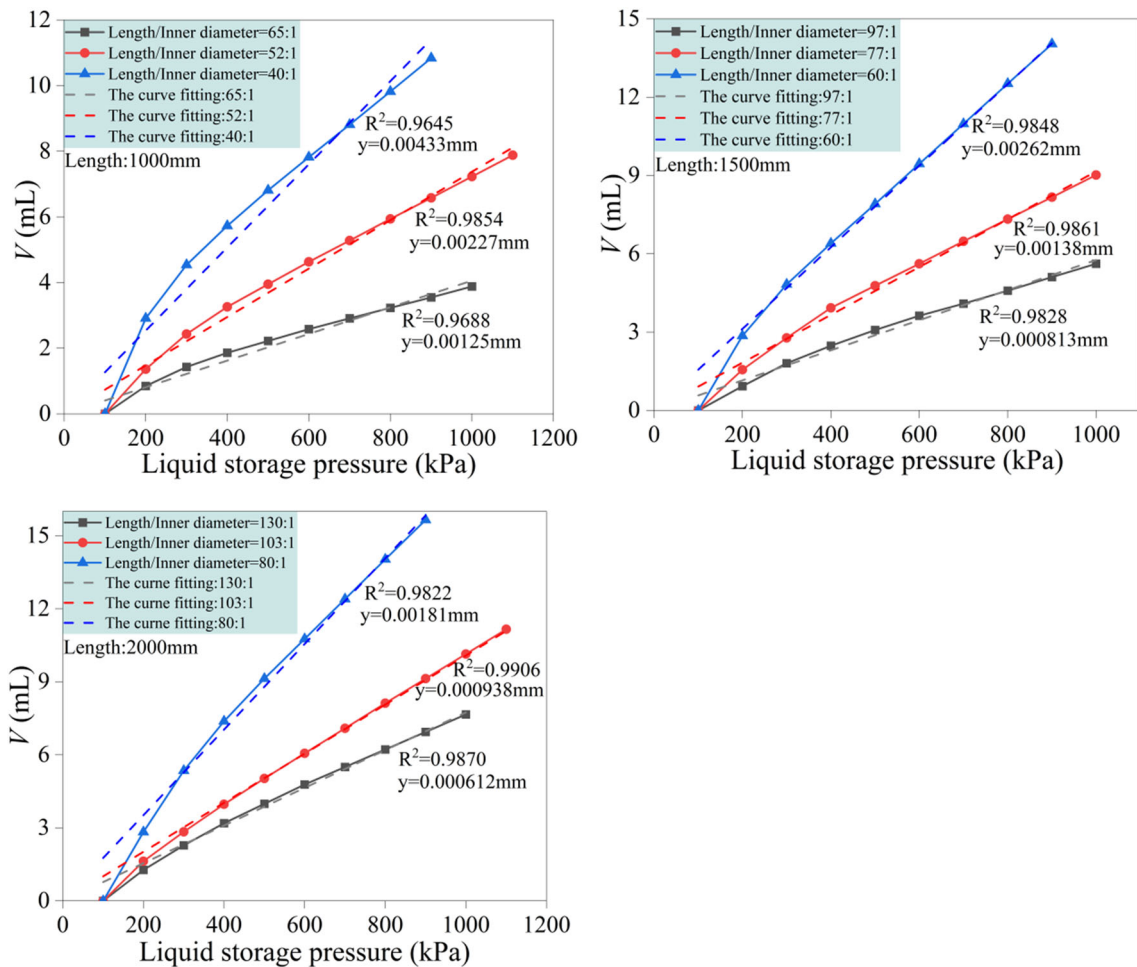
Fig. 6 Comparison of water injection volume of pipes with the same length and different inner diameter

more difficult to compress water until it is incompressible. The relationship between water injection volume and liquid injection pressure changes from curve to linear.

Although the measured strain change indicates that the pipeline is in the elastic range, the relationship between water injection volume and liquid injection pressure is gradually from nonlinear to linear.

4.2 Effect of pipe length on water injection volume

According to Eq. (18), there is a linear relationship between water injection volume and liquid injection pressure. The experimental results are the transition from curve to straight line. Therefore, we fit the results to determine the rationality of the derived equation (Figs. 5, 6).

The solid line is the actual measured curve, and the dotted line is the curve fitted by Eq. (18). From the correlation coefficient (R^2) of fitting, the fitting effect is better, and it shows that the proposed equation (Eq. 18) has good adaptability in pipeline research. With the increase of pipe length, the average bending degree (y) of pipe wall decreases, and the water injection volume

increases. The rate of rise of water injection volume also increases. In large span pipe engineering projects, the pipeline deformation caused by blockage is not easy to detect, but the danger increases with the increase of water injection volume.

4.3 Effect of pipe inner diameter on water injection volume

In this experiment, the influence of pipe diameter on water injection volume is considered through different pipe inner diameter (Fig. 6).

It can be seen from Eq. (18) that the thickness of the pipe wall is inversely proportional to the water injection volume. The greater the thickness of the pipeline wall is, the more difficult it is for the pipe to deform. The water injection volume of the pipeline will be relatively reduced. In the selected pipeline, the larger diameter, the greater the wall thickness. From the experiments results, with the increase of the inner diameter of pipeline, the deformation of the pipeline become larger, which means that the inner diameter plays a leading role in the competition between the inner diameter and the wall thickness of the pipe. With the increase of pipe

inner diameter, the average bending degree(y) of pipe wall increases, the water injection volume increases, and the rising rate of water injection volume increases.

5 Conclusions

Through the study of PPR pipeline, we discuss the relationship between water injection volume and liquid injection pressure, and study the influence of pipeline length and inner diameter on pipeline deformation under different liquid injection pressures. According to the experiment results, the following conclusions are obtained:

1. The relationship between water injection volume and liquid injection pressure could be divided into two stages: the nonlinear stage and the linear stage.
2. Under the same diameter pipeline, the change of water injection volume is related to the length, and the rising rate of water injection volume increases with the increase of length.
3. Under the same length pipeline, the change of water injection volume is related to the inner diameter, and the rising rate of water injection volume increases with the increase of inner diameter.
4. According to the experimental results, the proposed equation about volume change of pipeline water injection has a good fitting effect and has a good reference significance for the study of pipeline blockage.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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