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Review

Pions: the original Nambu–Goldstone bosons

An introduction and precision pion physics

B. Ananthanarayan[a]

Centre for High Energy Physics, Indian Institute of Science, Bangalore 560012, India

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Abstract In this special issue being brought in the centenary year of the birth of Yoichiro Nambu, we exemplify on his discovery of spontaneous symmetry breaking in elementary particle physics, and review precision pion physics in the present era. The notion of spontaneous symmetry breaking in elementary particle physics was introduced by Nambu, and found a realization in the strong interaction sector. It allows one to view the pions as the approximate Nambu–Goldstone bosons of spontaneously broken axial-vector symmetries associated with the (near) masslessness of quarks. Inspired by the phenomenon of superconductivity of condensed matter physics, Nambu found this application in a remarkable *tour de force.* Nambu's work in collaboration with G. Jona-Lasinio gave a dynamical model where such pions may arise. Pions today play the role of being sensitive probes of the ground state of quantum chromodynamics, the Lagrangian field theory of the strong interactions with (confined) quark and gluon degrees of freedom, and whose ground state spontaneously breaks the approximate chiral symmetry. The presence of non-zero quark masses renders the symmetries approximate, and yet the properties of the low-energy sector can both be described and measured at high precision both in experiment and on the lattice. Notable physical quantities include the neutral pion lifetime and pion scattering lengths. An important role of pions is their contribution to the hadronic radiative corrections to the anomalous magnetic moment of the muon, which is being measured at high precision at Fermilab. We review some of the important aspects of the state of the art. We also say some words about the outstanding contributions of the recently departed Murray Gell-Mann who was a pioneer in the field initiated by Nambu.

1 Introduction

It was a great honour for me to have spoken at the Yoichiro Nambu Centenary Conference. I am particularly indebted to Prof. Bindu A. Bambah for thinking of me in this regard.[1] In this contribution I recall some of the main features of the work of Yoichiro Nambu in the context of spontaneous symmetry breaking and his discovery that pions could be viewed as approximate, what are now known as Nambu–Goldstone bosons of spontaneously broken axial-vector symmetries associated with the near masslessness of quarks, the basic building blocks of the strong interaction Lagrangian. A collection of important papers of Nambu may be found in Ref. [1].

Among the topics I have worked on in over three decades of research in elementary particle physics, pion physics is a topic that has occupied an important place. Their existence had been predicted by Hideki Yukawa to account for the inter-nucleon forces. They come in three types and weigh a little over 135 MeV/c^2 in the case of the neutral pion and nearly 140 MeV/c^2 in case of the charged pion, in units where the proton and neutron are approximately 939 MeV/c^2. They were subsequently discovered in 1947 in cosmic rays by C. Powell and G. Occhialini.[2] A recent history of pion physics has been made available, see Ref. [3].

We use this opportunity to recall some properties of the strong interaction Lagrangian and its parameters, and how pions play an important role in our understanding of the ground state of the strong interactions. We also briefly discuss the importance of $\pi\pi$ scattering and the measurement of the lifetime of the neutral pion, an experimental challenge, and the status of corrections to the prediction of the lifetime from the anomaly. We

[1] A review prepared for EPJ ST Special Issue: Symmetry, Dynamics and Strings: A Centennial Issue in Honor of Yoichiro Nambu, Bindu A. Bambah, ed.

[a] e-mail: anant@iisc.ac.in (corresponding author)

[2] Recent historical research shows that D. M. Bose and Bibha Chowdhury had been conducting research in India. They may have been on the track of this discovery even earlier (cf. "The woman who could have won a Nobel: Despite being a pioneer in the study of cosmic rays in India, Bibha Chowdhuri remains practically unknown" by Amitabha Bhattacharya, The Telegraph, November 25, 2018). See also Ref. [2].

also discuss the role of virtual pions in radiative corrections to the anomalous magnetic moment of the muon, which is now being measured at high precision at Fermilab. Recent measurements based on Run I of the experiment confirm the results from the Brookhaven Laboratory experiment, and when combined differ from the Standard Model value at 4.2 σ. The result was announced at a live streaming press conference on April 7, 2021 with talks given by Aida X. El-Khadra giving the theoretical talk and Chris Polly for the experimental result. Furthermore, an early pioneer who worked on these topics was Murray Gell-Mann who passed away recently and we also pay tribute to his contributions. We provide a biographical note recalling his important contributions and also quote from various notices appearing after his demise, as well as notes about his life and achievements. In addition, we provide a small note on each of Jeffrey Goldstone and Gianni Jona-Lasinio whose names are inter-twined with that of Nambu in the context of spontaneous symmetry breaking.

1.1 Pions and Nambu–Goldstone bosons

Pions are particles which are some of the earliest realization of what we now call Nambu–Goldstone bosons. They are associated with the spontaneous symmetry breaking of the axial-vector symmetries of the strong interaction Lagrangian and are pseudo-scalar bosons.

Nambu was awarded one half of the Nobel Prize in 2008 for which the citation read "...for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics." The other half was shared by Makoto Kobayashi and Toshihide Maskawa. Since Nambu was too elderly to travel, it is my recollection that his prize was collected by his early collaborator Gianni Jona-Lasinio. At the same time there was the story that made the headlines that Maskawa was ruing the fact that he now had to get a passport.

The ingenuity of Nambu was to realize that although pions are massive, they are extremely light on the hadronic scale. Nambu realized that the Bardeen-Cooper-Schrieffer theory of superconductivity that suggested the existence of massless modes must also have a parallel in elementary particle physics. The latter is a manifestly relativistic theory. His proposal wherein he studied the role of gauge invariance and breakdown of a continuous symmetry and the existence of a massless mode to cure apparent paradoxes as given in Refs. [4, 5].

The symmetries we talk about are realized in the limit of the quark masses being zero in the strong interaction Lagrangian. Although the quarks do have a mass, they are very small in magnitude, and the approximate symmetry happens to be an excellent approximation to an ideal world where the u- and d-quarks are exactly massless. The approximation that the s-quark mass is also zero is not as good but is a useful approximation. In the real world there are also the electromagnetic interactions, and it was Richard Feynman who showed that in the presence of these and if the u- and d-quark masses were equal the proton would be heavier than the neutron, which in reality is not the case. Thus, we know from such elementary arguments that the d-quark must be heavier than the u-quark. The neutron decays into a proton and an electron and its anti-neutrino precisely because it is heavier than the proton, and it is due to the weak interactions. The pions themselves are also unstable. Whereas the neutral pion decays in a pair of photons, the charged pions decay primarily into muons and their neutrinos and less often to electrons and their neutrinos, and there are also rare decays where the charged pion can decay to a neutral pion, the so-called pion beta decay as well as with the emission of a photon.

In the 1950s with the discovery of what we now call strange or s- quarks in cosmic rays, and interpreted by Abraham Pais and Gell-Mann as thus, we have a larger picture of the physics of the strong interactions that involve these three light quarks. Heavier quarks, namely the charm, b- and top-quarks came later in the 1970s and eventually in the 1990s in laboratory experiments, thereby completing the quark picture. The sector containing three light quarks offer a sensitive laboratory to test the properties of the strong interaction and its ground state.

2 The standard model

Everything has been said so far is the crystallization of a century of information and experience, where we have a clear picture of the whole constellation of phenomena and data, and we have a clear picture of what has come to be known as the Standard Model: a theory of quarks and leptons, interacting strongly via the exchange of gluons, amongst each other through the exchange of intermediate vector bosons, the $W\pm$, Z and photons. The W and the Z bosons become massive through the Higgs phenomenon and become short range. The electromagnetic interactions remain long ranged and the photon remains massless. This is the subject of textbooks, see, e.g. [6].

The strong interactions describe particles that are made up of the fundamental constituents known as quarks and gluons. This is a gauge theory of interactions known as Quantum Chromodynamics (QCD), which was formulated in the 1970s, see Refs. [7–10]. The quarks carry 3 'colors' and the gluons of which there are 8 carry both color and anti-color, and these are the force carriers. The quarks come in six varieties, of which the lightest two, the u and d quarks, are the constituents of stable matter, as the others decay due to the weak interactions.

The quarks get trapped inside baryons such as protons and neutrons and their heavier counterparts, and mesons such as the pions. Underlying these are symmetries, some manifest, some broken, some broken explicitly, others broken spontaneously. The underlying quantum field theories enriched and embellished by over seven or eight decades of experience have been worked into a highly sophisticated science. Today we also refer to the realization of symmetries in nature as the 'Wigner-Weyl' realization in contrast to the 'Goldstone' realization of a symmetry. Let us also note that many of the notions are those that are born in classical field theory.

We now know all about running coupling constants, renormalization group flow, the Coulomb phase, the Higgs phase, the confining phase. The idea that there must be conserved charges and currents dates back to Emmy Noether, and yet they must survive in quantum field theories, to all orders in the loop-expansion. These are all today proved in the renormalization programme. These profound ideas first found in the context of quantum electrodynamics is due to the work of Erst Carl Gerlach Stuckelberg and André Petermann, and of Gell-Mann and Francis Low, and of Bogoliubov and Dmitri V.Shirkov.

It is ironical that one of the proponents and champions of this theory Martinus Veltman who passed away recently was never satisfied with having a massive scalar as he considered it to be 'unnatural'. And yet nature has proved otherwise. Like it or not there is a Higgs boson. We have to deal with the naturalness question. We are lucky that we have so many excellent books today that explain all this to us that give the impression that all these notions and ideas have evolved in one linear manner. Nothing could be further than the truth. A recent representative textbook is, e.g. Ref. [11].

2.1 Pre-standard model and Y. Nambu

Underlying many of the concepts mentioned above are notions and ideas many of which go back to Y. Nambu. I will discuss only a few of them. Writing in "From BCS to NJL—An Old Story Retold" Nambu [12] talks about his own background about being a young man in the days of the war and working in Japan and how he came in contact with the schools of Hideki Yukawa and Sun-Itiro Tomonaga. The world of elementary particle physics was just born. There was so much confusion. The picture of strong interactions, for instance, did not exist. It was nuclear forces in those days. Today we call them the residue of the strong interactions, which is a gauge theory of quarks and gluons, which is asymptotically free and is confining in the infra-red. In the spectrum of this theory lie the pions and protons and neutrons. It was the ingenuity of Nambu to realize that the smallness of the mass of the pion signalled something. By linking it to the notions of superconductivity, where spontaneous symmetry breaking required the existence of a massless mode, Nambu thought that the vacuum itself could be like the BCS groundstate where such massless modes exist and propagate.

Writing in the CERN Courier of 21 January 2008, in article entitled 'From BCS to the LHC' [13], Steven Weinberg says "Though spontaneous symmetry breaking was not emphasized in the BCS paper, the recognition of this phenomenon produced a revolution in elementary-particle physics. The reason is that (with certain qualification, to which I will return), whenever a symmetry is spontaneously broken, there must exist excitations of the system with a frequency that vanishes in the limit of large wavelength. In elementary-particle physics, this means a particle of zero mass."

We are lucky that the pion decays and that by his reckoning the axial-vector current generator could interpolate between the one-pion state and the vacuum the magnitude of which is measured by the pion decay constant, F_π. The clear distinction between weak and strong interactions was not known at the time of his work.

Furthermore, although the notion of a continuous symmetry was known to Werner Heisenberg in the context of protons and neutrons, and one could also group the pions into an iso-triplet, it was astonishing that the latter could turn out to be the Nambu–Goldstone bosons of the strong interaction Lagrangian, at a time one did not even know if there was a Lagrangian! and what its symmetries would be. This was the profound leap of insight that was made by Nambu. In his essay he talks about his reservation and relates that things were cleared when he learned about the work of Nikolai Bogoliubov and John G. Valantin in the BCS theory. And yet, he notes that although he was aware of the word of Fritz London and that he knew that a gauge boson could behave like it was massive, it did not occur to him that he could take this forward.

In this regard, Weinberg [13] says, "The first clue to this general result was a remark in a 1960 paper by Yoichiro Nambu, that just such collective excitations in superconductors play a crucial role in reconciling the apparent failure of gauge invariance in a superconductor with the exact gauge invariance of the underlying theory governing matter and electromagnetism. Nambu speculated that these collective excitations are a necessary consequence of this exact gauge invariance."

Prodded on by these investigations, Nambu needed a model which had symmetries which needed to be broken. By introducing nucleons and pions into the same model and with 4 fermion interactions included and interactions amongst the nucleons and pions, he was able to produce, along with Gianni Jona-Lasinio a model that exhibited all the required properties [14,15].

Weinberg [13] then says, "In a subsequent paper with Giovanni Jona-Lasinio, Nambu presented an illustrative theory in which, with some drastic approximations, a suitable chiral symmetry was found to be spontaneously broken, and in consequence the light pion appeared as a bound state of a nucleon and an antinucleon." Furthermore, other relations such as those between the axial-vector coupling of the neutron and the f_π the famous Goldberger-Treiman relationship and current algebra slowly began to appear.[3]

All the above concern what we call global symmetries. An interesting version of symmetries could manifest themselves as gauge symmetries, electromagnetism

[3] Jona-Lasnio himself in Ref. [16] says, "Yoichiro somehow legitimized my inclinations. I learned from him two important principles: never be afraid of thinking unconventionally; and analogies are a powerful source of ideas."

being the prototype of these. Just as global symmetries can break spontaneously, so can gauge symmetries when a scalar field acquires a vacuum expectation value in a Lagrangian model. This model was considered by Higgs and it turned the massless force carrier of the symmetry into a massive one and left behind a particle called the Higgs particle. Legend has it that in the revised version of the famous paper of Peter Higgs [17] the sentence regarding such a particle was suggested by the referee, who is said to be none other than Nambu.

2.2 Other significant results

Along with Chew et al. [18], Nambu explored dispersion relations in the context of pion-nucleon scattering. Other famous relations of those days include the Goldberger-Miyazawa-Oehme sum rule. All these tools and ideas even today form the basis of analysis of data and information and probes of the strong interaction sector. Other notions introduced by Nambu were with Han, the so-called Han-Nambu model. Nambu also constructed an interesting monopole which is named after him. Other contributions in string theory and Nambu mechanics will be covered elsewhere. An important recapitulation of his achievements may be found in the obituary written by Sumit R. Das and Spenta Wadia in the CERN Courier [19].

2.3 Spontaneous symmetry breaking and Murray Gell-Mann

A colleague of Nambu at the University of Chicago was Murray Gell-Mann a prodigy who had come to the faculty and contributed a lot to the understanding of the strong interaction spectrum, and one who clarified the picture of spontaneous symmetry breaking and its implications to phenomenology of particles containing what we now know to be the u-, d- and s-quarks. The early relations pioneered by Murray Gell-Mann who came to the University of Chicago soon after his Ph. D. where he must have undoubtedly benefited from the knowledge of Nambu, include the Gell-Mann–Oakes–Renner relation between the pion mass and the quark condensate of QCD which today we know breaks the symmetry spontaneously, the Gell-Mann–Okubo mass relations between various states of the strong interaction spectrum, the Gell-Mann–Nishijima relation amongst the generators of the $SU(3)$ algebra. In the biographical notes section we include some more detailed discussion on these results.

3 Strong interactions in the low-energy sector today

Fast forward to the present, and today we use many tools to test the picture of the strong interaction and weak interaction physics. There is extensive work on the lattice, the computational approach to solving the strong interactions on the computer, to compute and test the picture of spontaneous symmetry breaking.

3.1 Effective theory of the strong interaction

Some decades ago, the effective theory of strong interactions at low energy, due first to Roger Dashen, and Marvin Weinstein, and then Heinz Pagels, and turned into a really systematic science by Juerg Gasser and Heinrich Leutwyler [20,21]. Nambu himself mentions that part of his ideas would be what is now called the loop-expansion, in other words as precursor to this modern theory. This was extended by Jan Stern [22] to ask whether it is really true that the quark condensate breaks the vacuum? Stern had named this framework generalized chiral perturbation theory.

Let us recall some basic facts. The effective theory at hand is one in which we have an expansion in powers of the masses of the quarks and of the momenta, which is suitable for a theory with Nambu–Goldstone bosons which have only derivative couplings. This is ideal for the meson sector and can also be extended to include baryons. The chiral symmetry that is broken is the approximate chiral symmetry due to the near masslessness of the quarks. For a theory with N_f-light quarks, the symmetry which would be independent left- and right-chiral transformations, of which the axial-vector part, a combination of the chiral projects is spontaneously broken by the vacuum expectation value of the quark condensate, a standard assumption. This would make accurate predictions for several low-energy measurable quantities after some parameters are fitted to certain experiments. Relaxing certain assumptions about the condensate leads to an extended formalism. In the framework of generalized chiral perturbation theory, the pion scattering lengths would be quite different from that predicted by standard chiral perturbation theory. Today high precision experiments have ruled out the generalized picture. The original notions of Nambu and of Gell-Mann and others stand vindicated.

We have, with our collaborators and our group carried out an extensive analytical study of two-loop integrals that appear in the effective theory. Our work adds to the picture of turning the picture of Nambu–Goldstone bosons into a precision science. Work goes on now in the pion-nucleon sector as well.

3.2 Light quarks at high precision on the lattice

Light quark masses are three of the fundamental parameters of the SM. In the past, the best estimates came from the use of sum rules and the use of chiral perturbation theory. Today, they are extracted from lattice simulations at an unprecedented level of accuracy. A recent review is Ref. [23].

In Fig. 1 we give a plot of various determinations of the masses of the lightest quarks and that of the $s-$quark. The main lesson compared to some years ago is that the light quarks are now known to be lighter

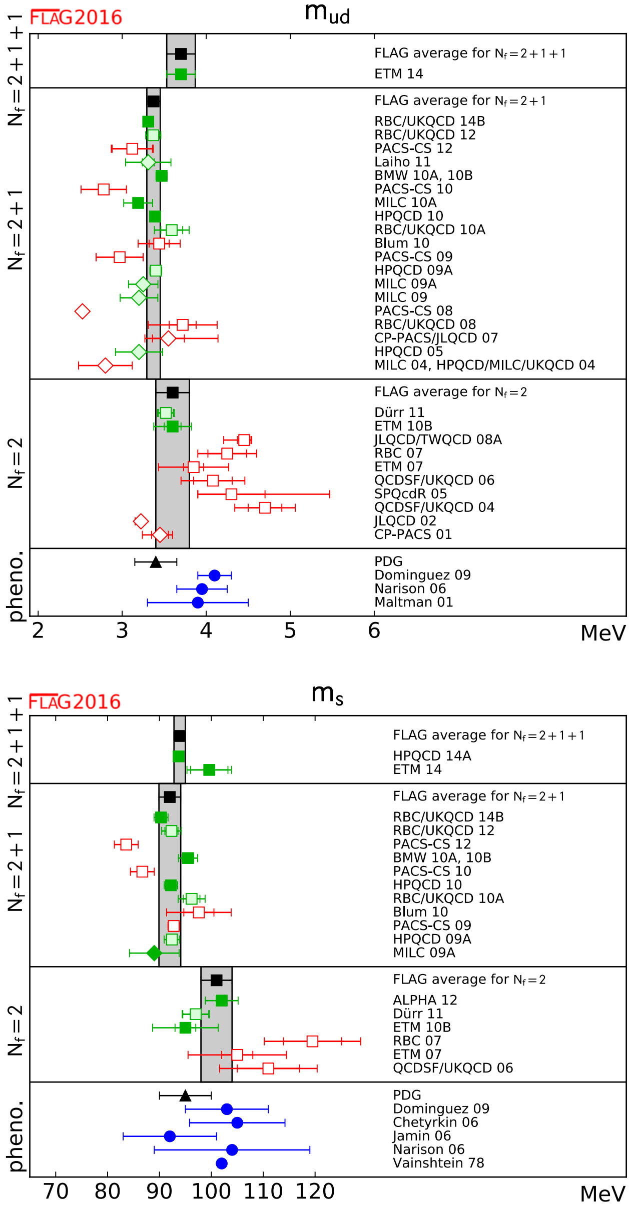

Fig. 1 Some mass determinations from FLAG [23] of the average values of the 2 lightest quark masses, the $s-$ quark mass. These are the mass parameters quoted at $\mu = 2$ GeV

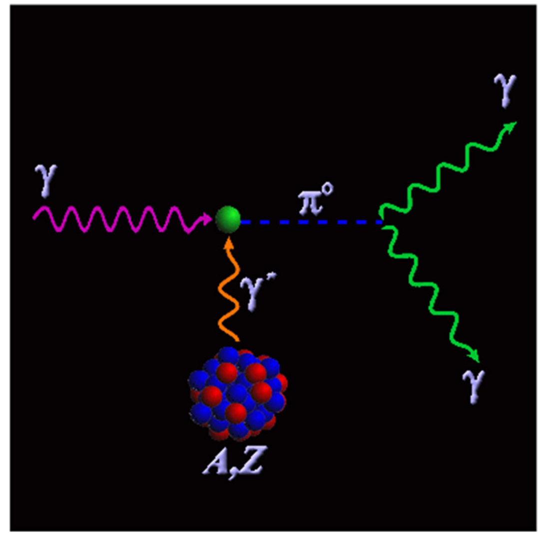

Fig. 2 The Primakoff effect (from JLab web-site)

than concluded from initial determinations. An accessible introduction is Ref. [24].

A sensible approach requires us to match chiral perturbation theory in a controlled manner with the lattice information in three flavoured chiral perturbation theory. Pion, kaon and eta masses and decay constants were evaluated nearly two decades ago up to two-loop order. Nevertheless, some of the so-called sunset diagrams which are the simplest two-loop self-energy diagrams cannot all be evaluated in terms of known functions and had to be evaluated only numerically. In a series of recent publications [25, 26], we have advanced a suitable Mellin-Barnes technique to obtain double series expansions in ratios of the masses of the three pseudoscalar mesons, which allows a controlled comparison to be performed.

3.3 π^0 lifetime

Recall that the pions themselves are unstable particles. The charged pions decay via the weak interactions mainly into muons and their neutrinos, and with a small branching ratio to electrons and their neutrinos, as a result of helicity suppression, due to the parity violating nature of the weak interactions. The neutral pion on the other hand decays to two photons with a very short lifetime, τ. An important quantity is the chiral anomaly of the light flavour sector which primarily determines the neutral pion lifetime. This is fixed almost entirely by the F_π the charged pion decay constant, the neutral pion mass M_{π^0}, and α and $\hbar$. Experimental measurements of the lifetime, coming from the Primakoff process shown in Fig. 2 of collisions of X-rays with nuclear targets.

The anomaly prediction reads:

$$\begin{aligned}\Gamma(\pi^0 \to 2\gamma) &= \left(\frac{M_{\pi^0}}{4\pi}\right)^2 \left(\frac{\alpha}{F_\pi}\right)^2 \\ &= 7.760\,\text{eV}\,[\tau \equiv 1/\Gamma = 8.38 \times 10^{-17}\,\text{s}].\end{aligned}$$

A recent high precision experiment at JLab, the Primex experiment has measured the lifetime to the desired precision which brings theory into agreement with experiment at an unprecedented level. This reads $\Gamma = 7.82 \pm 0.14(\text{stat.}) \pm 0.17(\text{syst.})$ eV. In the 2018 Reviews of Particle Properties [27] is $\tau = (8.52 \pm 0.18) \times 10^{-17}$ s. Figure 3 gives a recent representation of theory, Refs. [28–32] and experimental determinations. These are given as representative numbers with updates from time to time. The most recent impressive measurement with a further reduction of error superseding Ref. [33] is Ref.

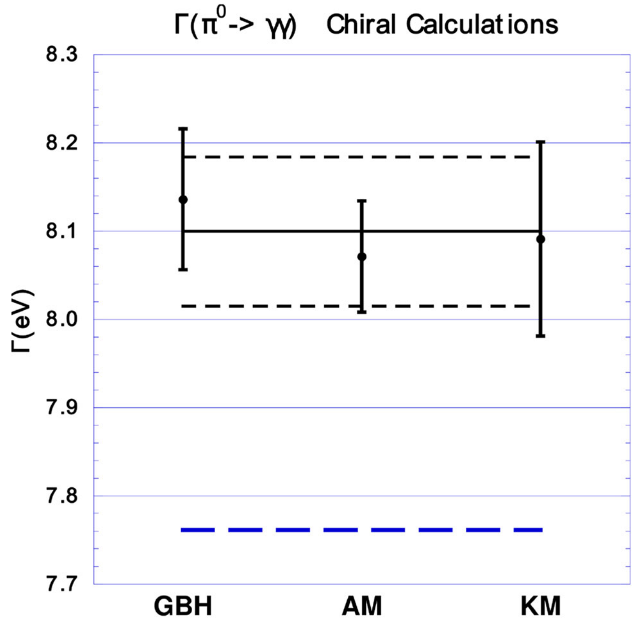

Fig. 3 The anomaly prediction (large dashed line), also shown are the predictions from Refs. [28,29,31], and the upper solid line is the average of these and 1% error is the band. Summary of chiral corrections from Ref. [32]

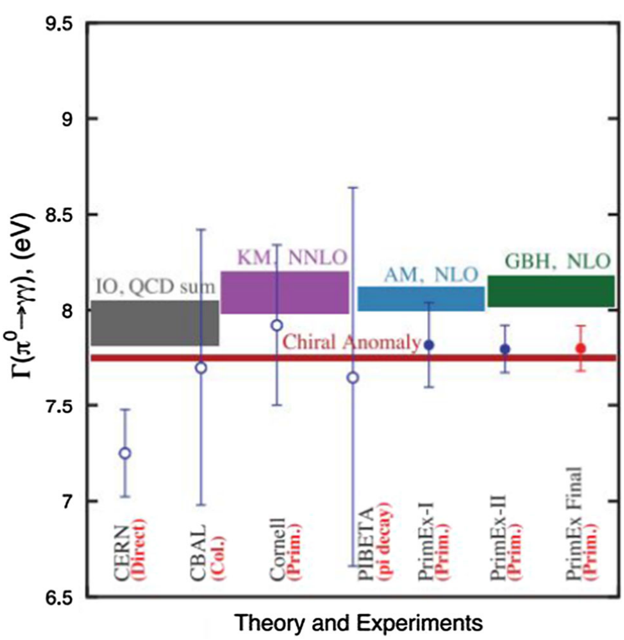

Fig. 4 Summary of measurements of several experiments and theory predictions including from Refs. [28–31] reproduced from Ref. [34]

[34]. This is shown in Fig. 4. This seems to suggest that estimates coming from large N_c arguments may not be accurate enough for the desired level of precision. A clarification could be the subject of future research.

3.4 Pion scattering lengths and partial waves

$\pi\pi$ scattering (for a review, see Ref. [35]) provides a paradise for theoreticians due to the simplicity of the process, and for testing consequences of general principles. It also led to the rise of dual resonance theory, the Veneziano amplitude and its interpretation in terms of a bosonic string, paving the path to the development of string theory. The interpretation of these amplitudes arose from the Nambu–Goto action. Furthermore, dispersion relations, relations which follow from the principle of causality in field theory are also of great use. Dispersion relations arise from the application of Cauchy's theorem of complex variable theory to scattering amplitudes. Other important principles are 'crossing symmetry' and unitarity. In the context of $\pi\pi$ scattering, a system of dispersion relations were established that entailed the presence of certain unknown functions of the momentum transfer which limited the power of the dispersion relations.

In 1971, S. M. Roy [36] established a system of dispersion relations allowing one to solve for the low-lying waves. Partial knowledge of the low-lying waves and assumptions on the higher waves could be used to produce a determination of pion scattering lengths. An important comprehensive study taking into account the needs of effective theories is Ref. [37].

A series of experiments based on different principles such as the rescattering of pions from a kaon decay to two pions and a lepton pair conducted at Brookhaven National Laboratory with small errors have brought theory and experiment into agreement [38]. Furthermore, the standard picture of spontaneous symmetry breaking has been confirmed by these measurements, ruling out the generalization by Stern. See also Ref. [39] for a detailed discussion on the subject. It is worthy of note that today there are measurements of scattering amplitudes on the lattice, for recent important papers, see Refs. [40,41]

Furthermore, the phase shift analysis that went into the above has found a new use in the determination of the hadronic contributions to the anomalous magnetic moment of the muon which is the subject of the next sections.

3.5 Form factors

All the above said, we note here that the pion interaction offers not just a probe of the properties of the pion, but is also a crucial ingredient for evaluating the contributions of vacuum fluctuations to low energy observables, which are sensitive probes of the Standard Model, and of interactions beyond it. For instance, the pion phase shift information is crucial today for evaluating the low-energy contributions of hadrons propagating in loops to the muon $g-2$, also known as the anomalous magnetic moment of the muon, which has been measured at high precision and is again being measured at Fermilab at high precision. In this regard, one approach that has been used to improve the phenomenology here

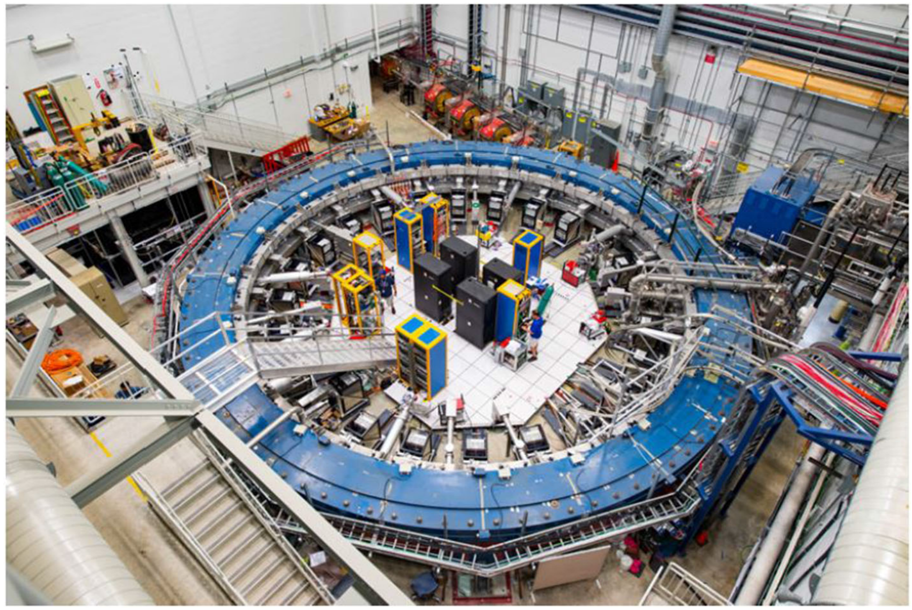

Fig. 5 Fermilab ring

is the method of unitarity bounds and other functional methods inspired by this approach. Of great importance is the Fermi-Watson theorem which relates the phase of the form factor to the $I = 1$ $p-$wave phase, which is known from Roy equation studies.

4 The anomalous magnetic moment of the muon and hadronic corrections

The anomalous magnetic moment of the muon is a very sensitive laboratory for testing the predictions of the standard model and also a place to look for deviations from it, see for important overviews, Refs. [42,43]. The last important experiment was conducted in Brookhaven National Laboratory [44] which indicated a discrepancy between the SM and the experiment at a little over 3σ. Thus, it was important to carry out the measurement at higher accuracy and simultaneously improve the theoretical effort to evaluate the standard model contributions. A recent authoritative survey is Ref. [45].

On April 7, 2021 in a much awaited press conference, the spokesman Chris Polly of the E821 Fermilab experiment representing The Muon g-2 Collaboration, a multi-national, multi-institutional collaboration announced the results coming from the Run I of the experiment, after a theory talk given by Aida El-Khadra. The principle is similar to that of the precursor, namely the Brookhaven National Laboratory experiment, with improvements in purity of the beam and detector components. The new result confirmed the prior measurement, although the new central value is somewhat smaller than the previous one.

This experiment uses a 14-m diameter electromagnet that was already used in Brookhaven and moved to Fermilab in 2013. It met a new milestone with reassembly in summer 2014. In Fig. 5 we display a picture of the ring after it was refitted.

The principle of the experiment is to have a highly uniform, essentially pure, 1.45 T dipole field throughout the circumference. The muons have to meet a condition known as the magic energy condition, to cancel certain terms in the spin equations so that the dependence on the less precisely known electric quadrupole fields used for focusing is reduced. 3.1 GeV muons produced from pion-decay enter the magnetic field along a nearly field-free path. The $(g-2)_\mu$ measurement is based on polarized muons being injected, and the precession is measured, with parity violating weak decay being the spin analyser. Since the electrons from the decay have less energy than the muon, they curl into the storage ring and detected by an array of counters. Their arrival time is measured as a function of time after injection. The oscillation modulating an exponentially falling rate gives the precession frequency and finally correlation is made between the anomalous magnetic moment and the modulation in the so-called 'wiggle plot'. Several excellent articles are available on the internet describing the fascinating principles.

The results of Run I of the Fermilab experiment while being in agreement with the Brookhaven result have a smaller central value and closer to the band predicted by the Standard Model. Nevertheless since the two evaluations are in agreement, it is possible to combine them to get the 4.2 σ discrepancy. These results are now available in a series of publications, Refs. [46–49].

In Fig. 6 we present this result from the Fermilab web-pages.

The Standard Model computation comes from a variety of sources. The largest is the computation in quantum electrodynamics at five loops but known at very high precision. There is a minor change when the value of the Sommerfeld fine structure constants is taken from the caesium compared to that from rubidium. These

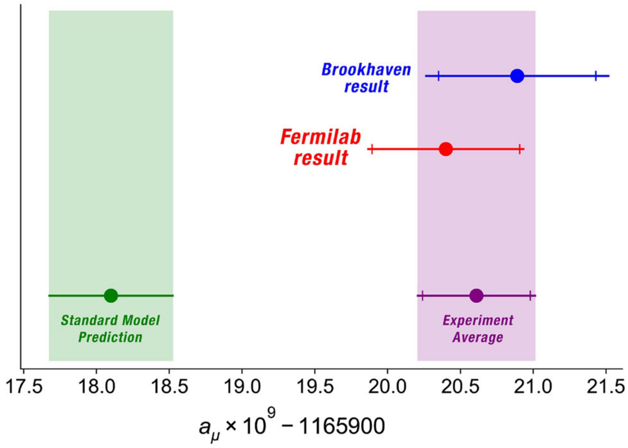

Fig. 6 Latest results from the Fermilab web-site as of April 2021

two determinations are themselves in disagreement but that is a different issue.

The main discrepancy comes from the so-called hadronic vacuum polarization contributions and another some what less numerically important but with relatively larger uncertainties known as the hadronic light by light scattering contributions. These latter require a controlled computation and a vast theoretical effort has gone into compiling these are collected. Our own contribution to understanding the constraints from unitarity and analyticity and its impact on these determinations is Ref. [50]. An excellent summary of all these discussions was recently presented, see slides of [51].

The determination of these contributions poses a major challenge because it has to be obtained from low-energy scattering data. As a representative example, we display a slide in Fig. 7 from a recent talk, Ref. [51] which offers a detailed anatomy of these determinations. Cited are the contributions evaluated by different groups using different methods.

When all the dust settles, the contribution has a significantly smaller central value and controlled errors, which when compared with the experimental measurement suggests a discrepancy at the level of 4.2 σ. In the meantime, there has been excitement because a computation on the lattice due to the Budapest-Marseille-Wuppertal collaboration [52] reports a value that is closer to the experiment and with uncertainties which would imply less than 2 σ deviation. In Fig. 8 we display a slide from Ref. [51] which summarizes the situation.

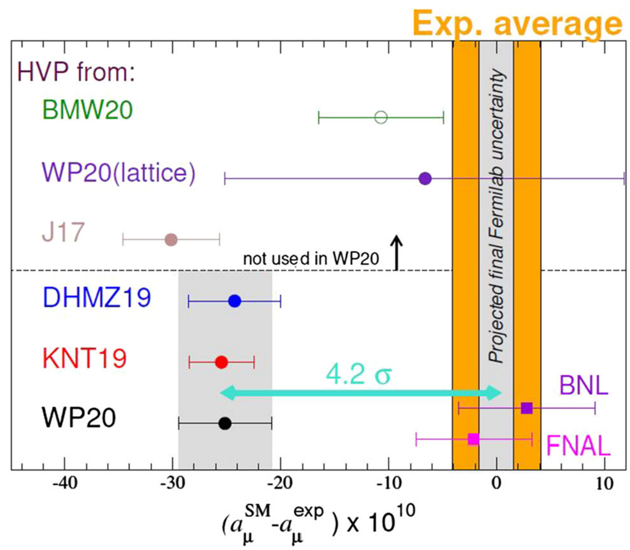

Fig. 8 Experiment vs. theory from Ref. [51]

5 Summary

In this article we have had the honour of recounting the contributions of Yoichiro Nambu, the pioneer of spontaneous symmetry breaking in global symmetries, and its implications to strong interaction phenomenology. At a time before the theory of strong interactions was even known, Nambu figured out that the near masslessness of pions may be related to their being associated with the breakdown of an approximate symmetry. The

Fig. 7 2π contributions from [51]. The columns indicate evaluations by different groups

Energy range	ACD18	CHS18	DHMZ19	KNT19
$\leq$ 0.6 GeV		110.1(9)	110.4(4)(5)	108.7(9)
$\leq$ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
$\leq$ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
$\leq$ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
$\leq$ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.3(1)
$\leq$ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

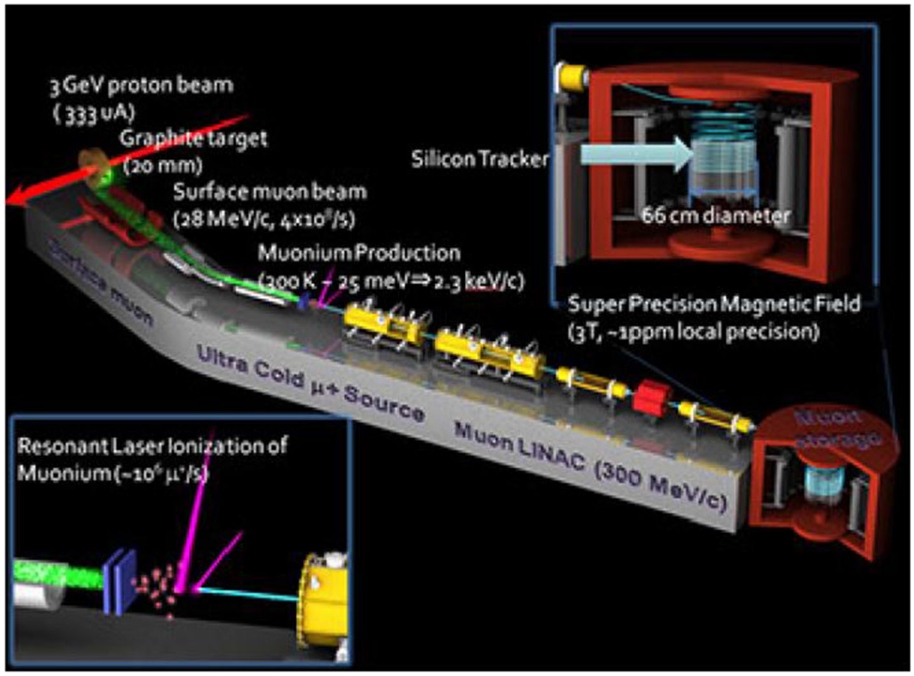

Fig. 9 Jparc experiment (from Jparc web-site)

confirmation of this picture has been validated over six decades of research.

The property of pions learnt from effective theories and their applications to strong interaction phenomenology in theory and in experiment, and in terms of comparison with properties derived from lattice gauge theories has led to a wealth of precision studies. Besides the confirmation of predicted scattering lengths and lifetimes, their contributions to the radiative corrections to the muon anomalous magnetic moment are an important subject and of implication to ongoing experiments.

While high precision determination will become possible as the Fermilab experiment gathers data in its Runs II and III, there is also a proposal to carry out a measurement at JParc in Japan. The scheme of the proposed experiment which uses a different principle is given in Fig. 9.

It has been our privilege to work on these subjects and many of the mathematical questions that have ensued from these investigations. For instance, an important subject we have tackled has arisen in a class of diagrams associated with the $g-2$ of leptons, as it produced analytical results for such three-loop contributions in QED, see, Ref. [53]. Spurred on by the methods we have developed in that context, we have recently made spectacular progress in solving a nearly 100 year old problem of finding series representations for Mellin-Barnes integrals spurred by these investigations, and have applied them, see Refs. [54–56].

Biographical notes

In this we give short biographical notes on Yoichiro Nambu (longer ones will be available elsewhere in this collection) and of Jeffrey Goldstone and of Gianni Jona-Lasinio whose names are inter-twined with his. We also provide a longer note on Murray Gell-Mann to honour his memory at his recent demise.

Yoichiro Nambu

Yoichiro Nambu was born in 1921 in Japan and after his studies at the University of Tokyo, he joined the group at Osaka. His early researches were in nuclear physics. After moving to the United States of America, Nambu spent time at the Institute for Advanced Study, Princeton and later moved to the University of Chicago where he spent the rest of his career. Besides his work on spontaneous symmetry breaking in 1960 which was to earn him the Nobel Prize in physics, he also worked on dispersion relations in his early years. His other celebrated work was the discovery of the Nambu–Goto action which laid the foundation to string theory. Nambu also worked on mechanics. Other contributions to this issue cover these topics.

Nambu is remembered for his scholarship and his acumen, and many have commented on his ability to see several years ahead of his contemporaries. Some of his Ph. D. students include Bindu A. Bambah, Sumit Das, Madhushree Mukherjee, and Savas Dimopoulos among others.

Jeffrey Goldstone

Jeffrey Goldstone is a British born physicist who worked at the Massachusetts Institute of Technology and is irrevocably associated with the name of Nambu due to the discovery of spontaneous symmetry breaking. Together with Abdus Salam and Steven Weinberg he proved the validity of the Goldstone theorem in relativistic quantum field theory from first principles. He is a recipient of the Dirac Medal of the Abdus Salam International Centre for Physics, Trieste, Italy.

Gianni Jona-Lasinio

Gianni Jona-Lasinio is an Italian physicist who spent a majority of his working life at the Sapienza University of Roma. His noted work is in statistical mechanics besides his work with Nambu. Joan-Lasinio travelled to Stockholm to collect the Nobel Prize on behalf of Nambu who was too elderly to travel by the time he became a Laureate.

Murray Gell-Mann

Late Murray Gell-Mann, one of the most influential physicists of the twentieth century and considered the sharpest mind during his hey day was born on 15 September 1929 and passed away recently, on 24 May 2019. Murray Gell-Mann was born in New York City in 1929, the year of the Great Depression. His parents were Arthur Isidore Gell-Mann and Pauline Reichstein Gell-Mann. His father was a language teacher. His parents and their personalities played a decisive role in shaping him. Gell-Mann studied at the Columbia Grammar & Preparatory School, Yale University (1948) for his undergraduate education, and then at MIT (1951) and obtained his Ph. D. under the supervision of Victor Weisskopf. Gell-Mann held various positions at the Institute for Advanced Study, Princeton, University of Illinois at Urbana-Champaign, Columbia University, University of Chicago and finally California Institute of Technology 1955-1993. He then moved to the Santa Fe Institute in New Mexico. Some of his most noted Ph. D. students were Kenneth G. Wilson and Sidney Coleman and James Hartle.

The Nobel Prize in Physics 1969 was awarded to Murray Gell-Mann of which he was the sole winner and the citation said "for his contributions and discoveries concerning the classification of elementary particles and their interactions."

Murray Gell-Mann produced a large number of scientific works that are named after him, although by his own admission he suffered from a life-long writer's block which affected him greatly. Among the most famous scientific pieces of work are

1. Gell-Mann—Low renormlization group equation with Francis Low, with similar work done at the same time by E. C. G. Stuckelberg and A. Petermann, and by N. Bogoliubov and D. Shirkov
2. Gell-Mann—Low theorem in quantum field theory and the ground state of interacting field theories,
3. Associated production of strange particles in cosmic rays with Abraham Pais,
4. Neutral kaon mixing foundations with Abraham Pais,
5. Gell-Mann and Bruckner theory of Many Body Physics, with Kenneth A. Bruckner,
6. V-A theory along with Richard Feynman, a little after E. C. George Sudarshan and Robert Marshak, as well as the conserved vector current hypothesis,
7. Dispersion relations in field theory with Marvin Goldberger and Walter Thirring, at roughly the same time as Hans-Joachim Bremmermann, Reinhard Oehme and John G. Taylor,
8. Crossing symmetry in quantum field theory with Marvin Golberger, at roughly the same time at Jacques Bros, Henri Epstein and Vladimir J Glaser,
9. Extension of iso-spin symmetry of Werner Heisenberg to the inclusion of strangeness,
10. Introduction of SU(3) into particle physics and classification of particles,
11. SU(3) and the 8 fold way (reported in the preprint numbered TID-12608; CTSL-20) [discussed at more or less the same time byYuval Ne'eman (and Abdus Salam)],
12. Quark Model, also George Zweig who called them 'aces' [at more or less the same time discussed also by Shoichi Sakata, and by Oscar Greenberg]. In the process, the following came to be named after him. or associated entirely with him:
    (a) Gell-Mann matrices λ_i, $i = 1, \ldots 8$,
    (b) Gell-Mann–Nishijima formula, with Kazuhiko Nishijima, which relates electric charge to the baryon number (B), hyper-charge (strangeness (S)) and the third component of isospin (T_3).

$$Q = I_3 + \frac{1}{2}(B + S)$$

    (c) Gell-Mann—Okubo mass formula, with Susumu Okubo,
        i For the baryon octet with members being the nucleon (N), the Σ and the cascade Ξ.

$$\frac{m_N + m_\Xi}{2} = \frac{3m_\Lambda + m_\Sigma}{4}$$

        ii. For the baryon decuplet * denoting the 3/2-spin partner of the spin 1/2 octet, and the Ω being the made of 3 s- quarks,

$$m_\Delta - m_{\Sigma^*} = m_{\Sigma^*} - m_{\Xi^*} = m_{\Xi^*} - m_\Omega$$

        iii. For the meson octet, with the members being the pions, kaons and the η,

$$m_K^2 = \frac{3m_\eta^2 + m_\pi^2}{4}$$

(d) Prediction of Ω^- and its mass for which he was awarded the Nobel Prize
(e) Gell-Mann–Oakes–Renner formula, with Robert Oakes and B. Renner,
(f) Gell-Mann–Lévy sigma model, with Maurice Lévy,
(g) Current algebra and charge algebra.

13. The 'Cabibbo angle' was already presented in a footnote of a paper of Gell-Mann and Lévy Nuovo Cimento 1960,
14. Quantum chromodynamics with Harald Fritzsch and Heinrich Leutwyler
15. The see-saw mechanism for neutrino masses with Pierre Ramond and Richard Slansky, preceded by the work of Peter Minkowski, and followed by the work of Yanagida, and Mohapatra and Senjanović

By the 1980s, Gell-Mann's interests had shifted to complexity theory and he became a member of the Santa Fe Institute in New Mexico. Gell-Mann wrote a popular book entitled 'The Quark and the Jaguar' in this regard.

Among numerous awards and honors, Gell-Mann was the recipient of the Albert Einstein Medal of the Albert Einstein Society (2005), the Ernest Orlando Lawrence Memorial Award of the Atomic Energy Commission (1996), the Franklin Medal of the Franklin Institute (1967), and the John J. Carty Award of the National Academy of Sciences (1968). In 1988, Gell-Mann was listed on the United Nations Environment Programme's Roll of Honour for Environmental Achievement. He also shared the 1989 Ettore Majorana "Science for Peace" prize.

Caltech web-site says:

In an oral history for the Caltech Archives, Gell-Mann noted that his mother "always had dreams for me, of doing great things," and that her dreams were realized when he tested into a private grammar school with a full scholarship. He ended up skipping several grades.

Gell-Mann described his broad interests in his oral history: "My principal interests were all in subjects involving individuality, diversity, evolution. History, archeology, linguistics, natural history of various kinds-birds, butterflies, trees, herbaceous flowering plants, and so on-those are the things that I loved. Plus mathematics. Plus all sorts of other things-art, for example, and music," he said.

Personal recollections

My own personal connections with many of the *dramatis personae* of this narrative are limited. I was in ICTP, Trieste at the time Jeffrey Goldstone received the Dirac Medal. I have seen G. Jona-Lasinio once at the University of Lausanne where I was a post-doctoral fellow, when he came to give a seminar. I met M. Sogame couple of times at Singapore who told me that he was the last student of Yukawa. His ideas, he told me, had been considered by B. V. R. Tata and Ajay Sood. He told me he liked my talk. That my English was good. The mathematics was good. So I was wondering what the catch was. Then he concluded by saying, you Indians are good at maths but not good at physics! I thought I would leave you with that.

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Author contribution statement

I am the sole author of this contribution.

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