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Review

Review on Goldstone dark matter

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Abstract We review the theory and phenomenology of models in which the dark matter is made of composite pseudo-Nambu Goldstone bosons. We focus predominantly on models in which the Higgs is also composite and dark matter is a singlet and heavier than the Standard Model fields. Then we discuss a variety of departures from this main setup, including: electroweak charged dark matter, lighter dark matter, issues related to quantum anomalies, ultraviolet completions or composite dark matter not related to the hierarchy problem.

1 Composite dark matter

1.1 Goldstone dark matter

As we discussed previously, one of the most popular and successful hypothesis for the experimental evidence of DM is that it consists of Weakly Interacting Massive Particles (WIMPs). In fact it has been shown that a WIMP of mass around the electroweak (EW) scale can explain the experimentally inferred DM abundance via a simple freeze-out mechanism [1], thus indicating an appealing link between the Higgs boson and DM. This result is popularly known as the *WIMP miracle*. This observation, together with the success of Composite Higgs Models (CHMs) in explaining a variety of problems,[1] suggests that both the Higgs and the DM could be pseudo Nambu–Goldstone Bosons (pNGBs) of the same symmetry-breaking pattern. This proposal was first suggested in Ref. [2] and it has received much attention is the last years. It is further supported by the fact that, with the exception of the minimal CHM, based on the coset $\mathcal{G}/\mathcal{H} = \mathrm{SO}(5)/\mathrm{SO}(4)$ [3], all (custodial-symmetry preserving) CHMs contain pNGBs in addition to the Higgs degrees of freedom, often singlets of the SM gauge group and therefore electrically neutral and colourless. Moreover, as we will discuss below, pNGB DM can evade strong constraints from direct detection experiments, given that its interaction with nuclei is velocity suppressed.

The simplest such CHMs (i.e. with smaller number of pNGBs) include the cosets SO(6)/SO(5) [2], SO(5) × SO(2)/SO(4) [4], SO(7)/SO(6) [5–8], SO(7)/G_2 [9,10], SO(6)/SO(4) [11,12], SO(7)/SO(5) [12] among others. (Note that we do not write explicitly the unbroken SU(3) group of colour.) Details about these cosets are given in Table 1.

As for any CHM, the relevant effective Lagrangian of each of the models of composite DM above can be split into two pieces: (i) The non-linear sigma model [13,14] L_σ parametrising the pNGB self-interactions; it is fully determined by $\mathcal{G}/\mathcal{H}$ and by the scale (or scales) of compositeness f (if the pNGBs transform in reducible representations of $\mathcal{H}$). (ii) The Yukawa Lagrangian L_Y and the scalar potential $\mathcal{V}$, which reflect respectively the explicit breaking of $\mathcal{G}$ at tree and loop level. The stability of the DM particle is in general not guaranteed; vertices involving the DM and e.g. two SM fields can arise. The stability of the DM must be instead enforced. The simplest option is assuming that the strong sector respects a $\mathbb{Z}_2$ symmetry under which the DM is odd while all SM fields are even. Or equivalently assuming that the global SO(n) group is uplifted to the corresponding $O(n)$, e.g. SO(6)/SO(5) $\mapsto O(6)/O(5)$. One must also assume that this symmetry is not broken at the quantum level; this holds automatically in non-anomalous cosets such as SO(7)/SO(6), while it needs justification in the UV completion for models based on SO(6). We revisit this issue later on this section.

The proof that this symmetry is compatible with the shift symmetry of the pNGBs is however non trivial,[2] and it goes as follows [10]. Let us denote by T^i and

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[1] As we have seen in previous sections, CHMs can shed light on the gauge hierarchy problem as well as on the flavour problem. We also highlight again in this section that they can be much more predictive than other models.

[2] The following toy example should illustrate this conflict. Consider the function $f(x, y) = A(x - y)^2$. This function is trivially invariant under the shift $x \to x + \alpha$, $y \to y + \alpha$. However, this symmetry is broken if $x \to -x$ is enforced, unless $A = 0$. A similar conflict in the composite DM model could require $L_\sigma = 0$ and therefore no DM propagating field.

Table 1 Some of the minimal cosets $\mathcal{G}/\mathcal{H}$ that provide a custodial symmetric Higgs doublet and at least one singlet

coset	# pNGBs	Rep of $\mathcal{H}$	Rep of $SU(2)_L \times SU(2)_R$
$\mathrm{SO}(6)/SO(5)$	5	$\mathbf{5}$	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$
$\mathrm{SO}(5)\times\mathrm{SO}(2)/SO(4)$	5	$\mathbf{4}+\mathbf{1}$	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$
$\mathrm{SO}(7)/SO(6)$	6	$\mathbf{6}$	$(\mathbf{2},\mathbf{2})+2\times(\mathbf{1},\mathbf{1})$
$\mathrm{SO}(7)/G_2$	7	$\mathbf{7}$	$(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$
$\mathrm{SO}(6)/\mathrm{SO}(4)$	9	$2\times\mathbf{4}+\mathbf{1}$	$2\times(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$
$\mathrm{SO}(7)/\mathrm{SO}(5)$	11	$2\times\mathbf{5}+\mathbf{1}$	$2\times(\mathbf{2},\mathbf{2})+3\times(\mathbf{1},\mathbf{1})$

We ignore cosets of the form $\mathrm{SO}(n+1)/\mathrm{SO}(n)$ with $n>6$, which give always rise to a Higgs doublet and $n-5$ singlets. Note also the difference in the branching rule of SO(7)/SO(5) with respect to that given in Refs. [17,18]

X^a the unbroken and the coset generators, respectively. We also define $\Pi = \Pi_a X^a$, with Π_a running over the pNGB fields. The leading order non-linear sigma model Lagrangian,

$$L_\sigma = \frac{1}{2} f^2 \mathrm{Tr}(d_\mu d^\mu), \tag{1}$$

is built upon the d symbol entering the Maurer–Cartan one form,

$$\omega_\mu = -iU^{-1}\partial_\mu U = d^a_\mu X^a + E^i_\mu T^i\,, \tag{2}$$

where the Goldstone U matrix reads $U = \exp(i\Pi/f)$. The d symbol can be written as

$$\begin{aligned} d_\mu &= \sum_{m=0}^{\infty} \frac{(-i)^m}{f^{m+1}(m+1)!} \mathrm{ad}^m_\Pi(\partial_\mu \Pi)_X \\ &= \frac{1}{f}\partial_\mu\Pi - \frac{i}{2f^2}[\Pi,\partial_\mu\Pi]_X - \frac{1}{6f^3}[\Pi,[\Pi,\partial_\mu\Pi]]_X \\ &\quad + \frac{1}{24f^4}[\Pi,[\Pi,[\Pi,\partial_\mu\Pi]]]_X + \cdots \end{aligned} \tag{3}$$

We have defined $\mathrm{ad}_A(B) = [A,B]$ and the sub-index X stands for projection into the broken generators. The crucial observation is that (almost) all cosets mentioned above are symmetric spaces. This means that the commutator of two broken generators belongs to the Lie algebra of the unbroken group, i.e. $[X^a, X^b] = ic^{abj}T^j$ for some constants c^{abj}. Consequently, all terms involving even powers of f in Eq. (3) vanish. This in turn implies that L_σ only involves even powers of $1/f$, and therefore only terms with even number of fields. The $\mathcal{H}$ symmetry can be subsequently used to demonstrate that only terms with even number of DM fields are allowed. For example, in SO(6)/SO(5), which gives a doublet and a singlet, gauge invariance requires that only even number of doublets are in place, and therefore only even number of singlets.

The only coset above that is not symmetric is $\mathrm{SO}(7)/G_2$. Still, the leading order non linear sigma model contains only even powers of $1/f$. This is so because, in this case [10]:

$$\begin{aligned} d_\mu &= \frac{1}{f}\partial_\mu\Pi + g_1(\Pi)\hat{\Pi}^2[\Pi,\partial_\mu\Pi]_X \\ &\quad + g_2(\Pi)\hat{\Pi}^3[\Pi,[\Pi,\partial_\mu\Pi]]_X, \end{aligned} \tag{4}$$

with $\hat{\Pi} = \sqrt{\Pi^a\Pi^a}$. The functions g_1 and g_2 consist of only even and odd powers of $1/f$, respectively. Moreover, it can be shown that both $\mathrm{Tr}(\partial^\mu\Pi[\Pi,\partial_\mu\Pi]_X)$ and $\mathrm{Tr}([\Pi,\partial^\mu\Pi]_X[\Pi,[\Pi,\partial_\mu\Pi]]_X)$ vanish. Accordingly, L_σ contains only even powers of f. Then $\mathrm{SU}(2)_L \times U(1)_Y$ invariance requires that the DM singlet appears always in pairs; being then compatible with the parity symmetry.

Thus, in all the aforementioned cosets one can (and must) force the stability of the singlet DM. Let us note that this is by no means different to what occurs within SUSY models of DM. Although it is widely believed that the symmetries—external to SUSY—required to avoid proton decay, such as R-parity, make the neutralino automatically a good DM candidate, this is not really true. R-parity is one of the options *chosen* to make the neutralino stable. Other ways of avoiding too fast proton decay include baryon parity and lepton parity among many others. None of them is fundamentally more appealing than the rest [15]. Thus, DM stability within minimal SUSY is also set *ad-hoc*. In this respect, DM within CHMs is on an equal footing to SUSY scenarios; and in both scenarios the connection between the DM and the (protected) EW scale is apparent.

In comparison to models of elementary scalar DM, CHMs are also much more predictive. To see why, consider the simplest example of SO(6)/SO(5) with q_L and t_R embedded in the representations **20** and **1** of $SO(6)$; hereafter denoted by $q_L \sim \mathbf{20}$ and $t_R \sim \mathbf{1}$. (Obviously, the embedding must respect the $\mathbb{Z}_2$ symmetry [17].) The right-handed top does not break the global symmetry, only q_L does. Therefore, given that under $SO(5)$ we find $\mathbf{20} = \mathbf{1}+\mathbf{5}+\mathbf{14}$, the leading order pNGB scalar potential depends on only two couplings parameterising the only two independent invariants that can be built to first order in the number of spurion insertions. It reads [12]

$$\begin{aligned} \mathcal{V} = c_1\left(2f^2|H|^2 - \frac{16}{3}|H|^4 - \frac{8}{3}S^2|H|^2\right) + c_2\left(-\frac{7}{2}f^2|H|^2\right. \\ \left. + \frac{19}{3}|H|^4 - 2S^2 + \frac{23}{6}S^2|H|^2\right). \end{aligned} \tag{5}$$

The two unknowns, c_1 and c_2, can be in turn traded for the Higgs quartic coupling λ and the mass term

$\mu^2 = -\lambda v^2$ (with v being the Higgs VEV), obtaining

$$\mathcal{V} = \mu^2|H|^2 + \lambda|H|^4 + \frac{1}{3}f^2\lambda S^2 + \frac{5}{18}\lambda S^2|H|^2 + \mathcal{O}\left(\frac{v^2}{f^2}\right). \tag{6}$$

Thus, the mass of S and the portal coupling depend solely on f. Moreover, the enforced $\mathbb{Z}_2$ symmetry is *predicted* not to be broken after EWSB, given that the S mass squared term is strictly positive. (Something similar happens also in other CHMs, e.g. in SO(7)/SO(6) with $q_L \sim \mathbf{27}$ and $t_R \sim \mathbf{1}$ [8] as well as in SO(7)/G_2 with $q_L \sim \mathbf{35}$, $t_R \sim \mathbf{1}$ [10].)

Evidently the full Lagrangian depends also on L_σ, but, as we stressed before, this is fully determined by f provided the pNGBs transform in irreducible representations of the unbroken group.[3] In this particular case it reads

$$L_\sigma = |D_\mu H|^2\left[1 - \frac{S^2}{3f^2}\right] + \frac{1}{2}(\partial_\mu S)^2\left[1 - 2\frac{|H|^2}{3f^2}\right] + \frac{1}{3f^2}\partial_\mu|H|^2(S\partial_\mu S) + \cdots \tag{7}$$

The scale f can be fixed by requiring that S accounts for the whole DM abundance. Interestingly, in this case it leads to $f \sim 3$ TeV, which in turns implies $m_S \sim 900$ GeV. At this point the phenomenology of the model is fully determined.

The singlet DM phenomenology of the CHMs in Table 1 has been mostly studied in the regime in which the DM is the lightest pNGB. (The prominent exception is the study of Ref. [8], that we discuss later on this section.) The phenomenology of all these cases can be therefore conveniently captured by the following Lagrangian [12]:

$$\begin{aligned} L = {} & |D_\mu H|^2\left(1 - a_1\frac{S^2}{f^2}\right) + \frac{a_2}{f^2}\partial_\mu|H|^2(S\partial_\mu S) \\ & + \frac{1}{2}(\partial_\mu S)^2\left(1 - 2a_3\frac{|H|^2}{f^2}\right) \\ & - m_\rho^2 f^2\frac{N_c y_t^2}{(4\pi)^2}\left(-\alpha\frac{|H|^2}{f^2} + \beta\frac{|H|^4}{f^4} + \gamma\frac{S^2}{f^2} + \delta\frac{S^2|H|^2}{f^4}\right) \\ & + \left(i\epsilon\frac{y_t}{f^2}S^2\overline{q_L}Ht_R + \text{h.c.}\right). \end{aligned} \tag{8}$$

In this equation, S represents the DM singlet and H stands for the Higgs doublet. The sigma model parameters a_1, a_2, a_3 as well α, β, γ and δ in the scalar potential and ϵ in the Yukawa Lagrangian are expected to be at most $\mathcal{O}(1)$. y_t stands for the top Yukawa coupling, $N_c = 3$ is the number of QCD colours and $m_\rho = g_\rho f$, with g_ρ being the typical coupling among composite resonances. As an example, by comparing Eqs. (6) and (7) to Eq. (8), we find that $a_1 = a_2 = a_3 = 1/3$, $\gamma = 1/4$ and $\delta = 1/5$ in SO(6)/SO(5) with $q_L + t_R \sim \mathbf{20} + \mathbf{1}$. The values of these couplings in other models are shown in Table 2.

This parametrisation is not only simple yet generic enough to capture the structure of the several CHMs; it also reflects the power counting in CHMs [16]. We warn however that not all the parameters are independent. For example, f could be absorbed into the dimensionless couplings. Also, only particular combinations of these parameters enter the relevant observables such as the DM annihilation cross section.

The annihilation cross section is triggered by S^2H^2 interactions, at both zero momentum (proportional to δ) and at order $\mathcal{O}(p^2)$ (driven by the derivative couplings a_1, a_2 and a_3). This cross section is dominated by the final states hh, ZZ and W^+W^-. (By virtue of the Goldstone equivalence theorem they contribute in the proportion $1:1:2$ in the limit of large m_S.) The Feynman rule for the corresponding vertex V reads

$$V \sim \frac{2iN_c m_\rho^2}{(4\pi)^2 f^2}[2(2a_1 + 2a_2 + a_3)\gamma - \delta]. \tag{9}$$

For the spin independent DM-nucleon cross section we have [12]

$$\sigma \sim \frac{9}{256\pi^5} m_N^4 f_N^2 \delta^2 \frac{g_\rho^4}{m_h^4 m_S^2}, \tag{10}$$

with $f_N \sim 0.3$ [19,20] and $m_N \sim 1$ GeV.

In light of these expressions, there are two important lessons to extract from DM within CHMs, which differ from elementary models of singlet scalar DM in the derivative interactions. (i) It is likely that the derivative interactions dominate the annihilation cross section; see the values of a_1, a_2, a_3 and γ, δ in Table 2. In particular, in the aforementioned fully predictive model SO(6)/SO(5) with $q_L + t_R \sim \mathbf{20} + \mathbf{1}$ we have that $(10/3)\gamma > \delta$ (~ 0.8 versus 0.2). If the derivative interactions dominate, CHMs with DM become extremely predictive as they depend only on the coset structure, the constant f and the DM mass [2]. (ii) Consequently, in the regime in which $\delta \to 0$, DM annihilation can still be efficient enough to accommodate the observed relic abundance $\Omega h^2_{\text{obs}} \sim 0.11$ [21] while completely evading constraints from direct detection experiments. In fact, note that Eq. (10) depends only on δ, not on the derivative interactions, because the DM-nucleon scattering occurs at very low momentum. This observation was first pointed out in Ref. [2]. (See also Ref. [22] for a later discussion in the context of elementary pNGB DM.)

One would expect $\delta \sim 0$ if the DM shift symmetry is exact. However, this limit is forbidden as the DM is massive. Sources of explicit symmetry breaking are unavoidable to generate the DM mass and the Higgs parameters as well as the Yukawa couplings, and in general they produce a non-negligible δ. When and how δ is very suppressed has been recently studied in Ref. [23]. (See also Ref. [24] for an insightful discussion on this

[3] While SO(6)/SO(5) fulfils this condition, SO(5) $\times$ $U(1)$/SO(4) must be parametrised by two scales f and f_S, unless $f_S \gg f$ as we often assume for simplicity. Similar considerations apply to SO(6)/SO(4) and SO(7)/SO(5).

Table 2 Value of the dimensionless parameters in Eq. (8) in different CHMs with singlet pNGB DM, for different fermionic representations

$\mathcal{G}/\mathcal{H}$	$q_L + t_R$	a_1	a_2	a_3	γ	δ
SO(6)/SO(5)	$\mathbf{6}+\mathbf{1}$	1/3	1/3	1/3	–	–
	$\mathbf{6}+\mathbf{15}$				$\ll 1$	$\ll 1$
	$\mathbf{15}+\mathbf{15}$				$\ll 1$	$\ll 1$
	$\mathbf{20}+\mathbf{1}$				1/4	1/5
SO(5) $\times$ $U(1)$/SO(4)	$\mathbf{5}+\mathbf{5}$	0	0	0	$\ll 1$	$\ll 1$
SO(7)/SO(6)	$\mathbf{7}+\mathbf{1}$	1/3	1/3	1/3	–	–
	$\mathbf{7}+\mathbf{7}$				–	–
	$\mathbf{27}+\mathbf{1}$				$\leq 1/4$	$\leq 1/5$
SO(7)/G_2	$\mathbf{8}+\mathbf{8}$	1/3	1/3	1/3	–	–
	$\mathbf{35}+\mathbf{1}$				1/4	1/5
SO(6)/SO(4)	$\mathbf{6}+\mathbf{6}$	0	1/6	1/3	–	–
SO(7)/SO(5)	$\mathbf{7}+\mathbf{7}$	$< 1/3$	$< 1/3$	1/3	–	–

More details can be found in Ref. [12]

topic.) One possibility, previously mentioned in Ref. [2] consists in breaking the S shift symmetry only in the b sector, which works *e.g.* for $q_L, b_R \sim \mathbf{6}$ and $t_R \sim \mathbf{15}$. Still, Ref. [23] demonstrates that in generic models most of the viable parameter space is already being tested by Xenon1T [25] or will be tested by LZ [26], as we also discuss below. Likewise, there is also a limit in which the two terms within the brackets of Eq. (9) can cancel each other. While this allows large values of the portal coupling to be compatible with the measured relic density for a mass range larger than in elementary models, it is disfavoured by direct detection experiments.

The complementarity between different DM searches for exploring the parameter space of Eq. (8) has been discussed at length in Ref. [12]. In the first panel of Fig. 1, we see that the whole parameter space region of the CHM $SO(6)/SO(5)$ with $q_L + t_R \sim \mathbf{20}+\mathbf{1}$, which leads to sizable values of δ and γ (see Table 2), could be probed by the direct detection experiments,[4] in particular the future LZ experiment. This is not necessarily so for models with smaller δ and/or larger γ.

The regions enclosed by the solid and dashed red lines correspond to searches for vector-like quarks (VLQs) in CHMs with DM. As it was pointed out in Refs. [10, 12,32], the assumption that DM within CHMs is the only thermal relic, sets an upper bound on f (above which DM is over-abundant) and therefore on the scale of new resonances, much more robust than naturalness arguments. In turn, searches for VLQs complement DM searches. In Fig. 1 we see that while current LHC data can only constrain a small region of the different parameter spaces, a future 100 TeV collider could complement DM searches so that these models are almost entirely probed.

Other collider tests of CHMs with singlet pNGB DM include traditional measurements of modified Higgs couplings [33] and Higgs invisible decays [34] as well as other LHC searches such as mono-jet and mono-photon analyses [35,36].

None of them provide insight into the DM itself, though. The most adequate probes in this respect are tests of the DM-Higgs interactions, which for DM masses above the Higgs threshold can be tested in e.g. vector boson fusion (VBF) at hadron colliders: $pp \to SSjj$. Recently, Ref. [37] has shown that for marginal portal couplings of $\mathcal{O}(1)$, DM masses of only about 200 GeV could be probed at a future 100 TeV collider; see Ref. [38] for an earlier study with similar findings in this respect. Moreover, this coupling does not shed light on the composite nature of the DM. The derivative DM-Higgs coupling instead does, and because its contribution to VBF increases with the energy, it is shown [37] that DM mass closer to the TeV could be probed in this channel.

Other collider tests of the DM scenarios under consideration could rely on rare top decays $t \to j$SS triggered by the last term in Eq. (8), in consonance with studies aimed to other CHMs [39,40]. To the best of our knowledge no analysis has been yet developed in this respect.

Going beyond the minimal CHM with DM, the coset SO(7)/SO(6) is particularly interesting because, contrary to the models based on SO(6) or SO(5), it is anomaly free [4]. That is, there cannot be topological terms destabilizing the DM, such as $\sim \mathrm{SF}^{\mu\nu} F_{\mu\nu}$. Because of this, the phenomenology of this model has been also studied in different works, particularly in regimes of the parameter space in which the DM is not just the lightest singlet scalar.

[4] Despite being presumably negligible for pNGBs, it is worth to mention that interpreting experimental data in the context of composite DM might be more subtle than in the elementary case for several reasons. The limits reported by experimental collaborations usually assume that the DM-nucleon cross section is independent of the recoil energy; i.e. it depends only on the nucleon form factor. This is however not true for composite DM [28,29]. In turn, theoretical estimations of the DM form factors are challenging and rely typically on the lattice [30]. Likewise, the DM velocity distribution and profile in models of composite DM could be appreciably different from that assumed by the experimental collaborations [31].

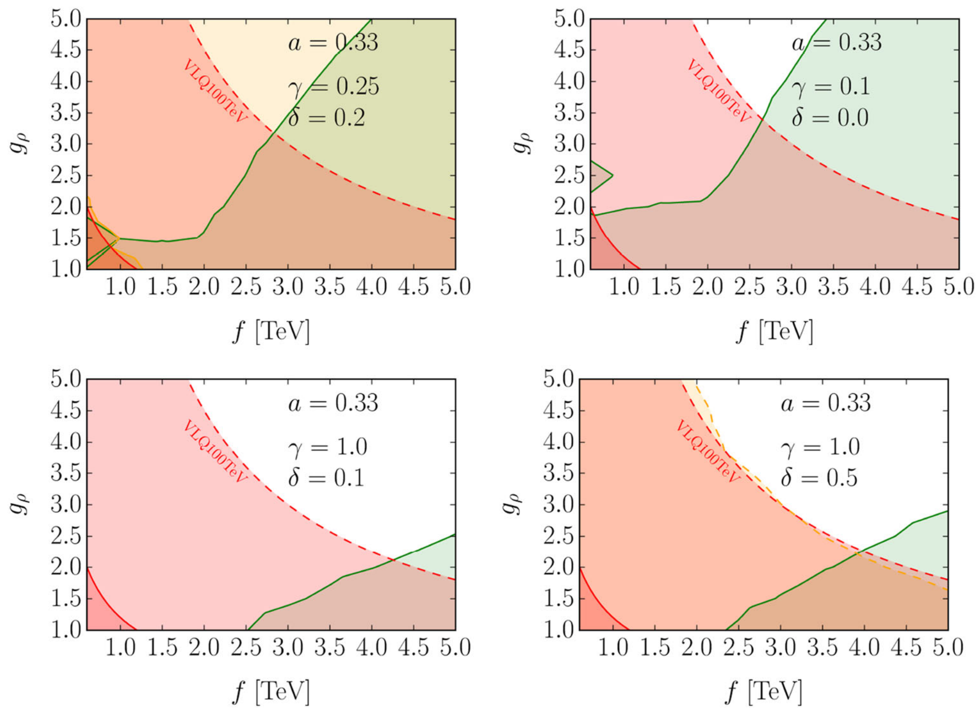

Fig. 1 Reach of DM searches for different choices of $a \equiv (2a_1 + 2a_2 + a_3)/5$, γ and δ. In the green area, enclosed by the green solid line, the DM is over-abundant. The LUX experiment [27] excludes the orange area enclosed by the solid orange line. The area enclosed by the dashed orange line could be tested by LZ [26]. The area enclosed by the solid red line is excluded by the LHC searches [12]. The area enclosed by the dashed red curve could be tested by a 100 TeV collider [12]. This figure is directly taken from Ref. [12], which did not consider Xenon1T data. Xenon1T bounds on the DM direct detection cross section for this range of masses are only about a factor of two stronger than those from LUX, though

One such important regime is that in which $U(1) \subset$ SO(6) is not explicitly broken by the SM interactions, so that the two singlets in SO(7)/SO(6) are degenerate. In this case the DM is a complex scalar χ; it has been studied in depth in Refs. [6,23].[5] If the new $U(1)$ is gauged, the DM mass can arise radiatively in a similar way to how photon loops provide a mass to the QCD pions. The fermion interactions can therefore preserve completely the shift symmetry; this happens for example if $q_L \sim \mathbf{7}$ and $t_R, b_R \sim \mathbf{21}$. The marginal portal coupling is not even generated at one loop, because the Higgs is not charged under the new $U(1)$. As such, current and future direct detection constraints can be fully avoided.

In any case, indirect detection constraints still apply. Recently, Ref. [8] has shown that they can be avoided in a different regime of this coset SO(7)/SO(6), namely when the second scalar singlet is lighter than the DM and it dominates the DM annihilation. Ref. [8] shows that this occurs in non negligible regions of the parameter space. The lightest scalar, κ, can be well leptophilic if its shift symmetry is only broken in the lepton sector. In particular, Ref. [8] shows that if κ is muonphilic, the indirect detection bounds from Fermi-LAT are very weak. We emphasize that, in doing so, Ref. [8] does not rely on the bounds reported by the experimental collaboration, which only provide constraints on DM annihilating directly into SM final states. Instead, Ref. [8] computes from scratch the theoretical gamma ray flux from DM annihilation $SS \to \kappa\kappa, \kappa \to \mu^+\mu^-$ (and also other final states) and compares that to the Fermi-LAT data. Given the potential interest for other DM models, composite or not, we reproduce these results in Fig. 2 for $m_S = 2m_\kappa$.

In this case though, the smallness of the portal coupling needed to avoid direct detection constraints can not be derived from symmetry principles. To the best of our knowledge, the existence of a natural mechanism to avoid both direct and indirect detection constraints within CHMs with DM is still an open question.

A different CHM with singlet pNGB DM has been considered in Ref. [41]. It it based on the coset SO(7)/SO(5) $\times$ $U(1)$, thus triggering two Higgs doublets and two singlets, the lightest of which is the DM. The authors identify a viable parameter space for DM masses in the range $\sim$ 130–160 GeV, corresponding to $f \gtrsim 0.8$ TeV and thus well in agreement with naturalness arguments. The viability of this region of DM mass lies mostly in a cancellation between the

[5] In Ref. [12] it was shown that the DM phenomenology of this model can be still captured by the parametrisation in Eq. (8) with $a_1, a_2, a_3 = 31/(75\sqrt{2}) \sim 0.3$ and γ and δ being, as usual, dependent on the fermionic couplings.

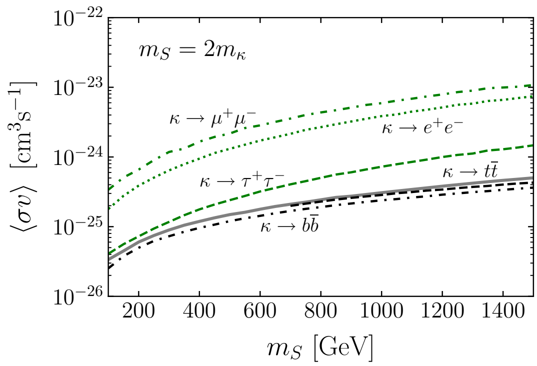

Fig. 2 Indirect detection bounds on the DM annihilation cross section. The solid grey line stands for $hh + W^+W^- + ZZ$ channel. The dashed and dotted curves represent the annihilation into the lighter scalar singlet, each for a specific SM decay of the latter. See Ref. [8] for more details

s-channel contributions of both Higgs doublets to the DM-nucleon coupling, which enhances the relic abundance while suppressing the direct detection cross section. The authors point out that this result, in turn, is tightly connected to the 2HDM-like structure of the model at low energies.

The authors also show that the heavier pNGB singlet might decay to the DM after freezing out in the early Universe, thus providing an extra non-thermal contribution to the DM abundance. No other model in the CHM literature has been shown to provide this possibility. Collider and indirect detection implications of this model have not yet been studied in depth.

DM within CHMs can be instead EW charged. This is the case, for example, of SO(6)/SO(4) $\times\, U(1)$, which develops an inert doublet [17,36]. Likewise, if the SM SU(2) gauged group in SO(7)/G_2 is the $SU(2)_R$, then this coset generates an inert triplet; see Table 1. The main consequence of DM being EW charged is the large DM annihilation cross section, mostly into gauge bosons. As a result, the DM mass and in turn f must be very large, implying large fine tuning on the Higgs mass. For example, in Ref. [10] it was shown that the DM abundance can be only explained within SO(7)/G_2 for $f \sim 8$ TeV. Moreover, the DM annihilation cross section at the low velocities relevant for indirect detection experiments is greatly enhanced by Sommerfeld effects [42]; being in tension with current data. In fact, the EW triplet cannot provide more than $\sim 80\%$ of the DM abundance if it is a thermal relic. Likewise, DM-nucleon cross sections are at the reach of experiments [10,36] such as Xenon 1T and definitely future facilities such as LZ.

A drawback common to almost all the models discussed so far is that they can not be UV completed in four dimensions. Needless to say, explicit QCD-like confining theories are *a priori* more appealing than non UV-completable models. Moreover, the mass spectrum and other observables in models with UV completions can be accurately predicted e.g. by using lattice field theory [43]; see also Refs. [44,45,45,46]. The only model that does admit a UV completion is SO(6)/SO(5), or equivalently SU(4)/Sp(4). One example of such an underlying theory is SU(2) with two Dirac flavours transforming in the fundamental representation [47]. This theory is however not consistent because the topological terms, associated to quantum anomalies, break the DM stability inducing DM decays into SM gauge bosons.

For the very same reason, the next to minimal UV completed CHM of DM, based on SU(5)/SO(5) [48], is also ruled out. Some minimal viable choices involve instead [49]:

1. Four Dirac fermions transforming in the fundamental of SU(N). The symmetry breaking pattern in this case is SU(4) $\times$ SU(4)/SU(4). It develops 15 pNGBs, which can be split into $\mathbb{Z}_2$ even and $\mathbb{Z}_2$ odd (or *dark*) sectors. In the first one we find pNGBs transforming as $(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$ under the custodial symmetry group. In the dark sector one has instead $(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1})$, the lightest neutral state of which plays the role of the DM.
2. Three Dirac fermions transforming in the pseudo-real fundamental representation of the confining gauge group SU(2). Up to an anomalous $U(1)$, the global symmetry of the strong sector is SU(6). If the masses of all fundamental fermions ψ are equal, then the condensate $\langle\overline{\psi}\psi\rangle$ breaks the global symmetry down to Sp(6). The 14 pNGBs of SU(6)/Sp(6) transform as $2 \times (\mathbf{2},\mathbf{1}) + 2 \times (\mathbf{1},\mathbf{2})$ (dark) and $(\mathbf{2},\mathbf{2}) + 2 \times (\mathbf{1},\mathbf{1})$.
3. Six Weyl fermions ψ transforming in a real representation of either SO(7) or SO(9). A condensate $\langle\overline{\psi}\psi\rangle$ can thus break SU(6) $\to$ SO(6). The 20 pNGBs, labelled by their quantum numbers under the custodial symmetry group, are: $(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$ (dark pNGBs) and $(\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{3}) + 2 \times (\mathbf{1},\mathbf{1})$.

In all cases, the EW symmetry is embedded by assigning non trivial EW quantum numbers to the fundamental fermions. The DM phenomenology of the first of the models above was first considered in Ref. [50], and thoroughly studied in Ref. [51]. It was shown that the DM carries a small component of gauge charged neutral scalars. Due to the several dark particles around the DM mass, it was also shown that co-annihilations play a very important role in the DM phenomenology. Overall, the DM abundance is well reproduced for DM masses of 0.5–2 TeV if DM is thermal.

The DM in the SU(6)/Sp(6) model has been studied within the broader context of vacuum misalignment [52]. It was shown that in the absence of CP violation, if the Yukawas are aligned in SU(6) space and two SU(2) technifermions are degenerated, there is a residual unbroken $U(1)$ symmetry that prevents some of the pNGBs from decay. The DM phenomenology has not yet been explored in detail, though.

The third of the aforementioned models, based on SU(6)/SO(6), was introduced in Ref. [53]. It was shown that top interactions required by partial compositeness tend to induce a tadpole for the triplet, thus breaking the custodial symmetry. To avoid this issue, the authors propose to embed the SM top fields in the adjoint representation of the global SU(6) (in a similar fashion to what was previously proposed for SU(5)/SO(5) [54]), while lighter fermion Yukawas are generated through four fermion operators [47]. Consequently, custodial symmetry is only broken at order $\mathcal{O}(m_b^2/m_h^2)$, thereby not conflicting with experimental data [55].

The DM candidates within this model can therefore be either the singlet pseudo-scalar or the neutral pseudo-scalar in the second Higgs doublet. The DM phenomenology of the singlet has been studied recently in Ref. [55]; the authors find that despite being tightly constrained, there is a viable parameter space for DM masses in the range $\sim$ 400–1000 GeV. The $t\bar{t}$ channel dominates the DM annihilation. The phenomenology of the DM candidate in the second doublet, despite being *a priori* similar to that of other inert doublet models [42,56], has not been fully explored yet.

PNGB DM from strongly interacting dynamics has been also considered in contexts different from CHMs. One close scenario is the Little Higgs Model (LHM). One of the simplest realizations of this model is based on SU(5)/SO(5) [57]. The main difference with respect to CHMs is that two copies of $\mathrm{SU}(2)\times U(1) \subset \mathrm{SU}(5)$ are gauged; they are spontaneously broken to the diagonal subgroup upon confinement. The SU(5)/SO(5) coset of the LHM can be endowed with a $\mathbb{Z}_2$ parity, also known as T-parity, that exchanges both $\mathrm{SU}(2) \times U(1)$ groups thereby avoiding large corrections to EW observables from heavy resonances, as these can only mediate through loops. The lightest T-odd particle can therefore be a DM candidate.

In the last years it has been shown that embedding the SM matter content in this LHM is however problematic [58,59]. More recently, following Refs. [58,60] shows that a possible way out consists in extending the coset to $\mathrm{SU}(5)\times SU(2)\times U(1)/[\mathrm{SU}(2)\times U(1)]_{1+2+3}$. In addition to the 14 pNGBs of SU(5)/SO(5), this coset provides 10 pNGBs transforming as $\mathbf{1}_0+\mathbf{3}_0+\mathbf{2}_{1/2}+\mathbf{1}_{1/2}$ under the SM. It was shown that the singlet can be DM in regions of the parameter space where the marginal Higgs portal coupling is $\mathcal{O}(0.01)$.

A more drastically different paradigm of composite pNGB DM was introduced in Ref. [61]. In this case, it is assumed that the whole SM, including the Higgs, is elementary, and it is extended with a new strongly interacting sector. As in CHMs, the latter is assumed to break a global symmetry $\mathcal{G}$ down to $\mathcal{H}$ at the confinement scale f_D. No linear mixing between elementary and composite fermions is required. The explicit breaking of $\mathcal{G}$, that must be at place to generate the mass of the DM, is only induced by gauging the SM group within $\mathcal{H}$. The VEV of the Coleman-Weinberg potential induced in this way is known to be aligned in the direction that preserves the global symmetry [62]. Thus, provided $\mathcal{G}/\mathcal{H}$ is symmetric, the DM field is stable [61].

It can be easily seen that the smallest viable cosets within this framework are the products $[\mathrm{SU}(2)^2 \times U(1)]/[\mathrm{SU}(2)\times U(1)]$ and $\mathrm{SU}(3)/[SU(2)\times U(1)]$.[6] In the first case there are three pNGBs that transform as a hyperchargeless triplet of the EW gauge group. In the second case four pNGBs emerge transforming as a complex doublet Φ with $Y = 1/2$. So they provide natural realizations of the inert triplet and doublet models, respectively. It must be stressed, though, that in the latter case there is a global $U(1)$ symmetry that makes the charged and neutral components of the doublet exactly degenerate [61], being the model *a priori* in tension with direct detection constraints due to Z exchange [63]. (Quantum corrections do not split the two components significantly to avoid these constraints.) Therefore, it must be assumed that the aforementioned $U(1)$ symmetry is broken at a higher scale and manifests in *e.g.* operators of the form $\lambda_{H\Phi}[(H^\dagger\Phi)^2 + \mathrm{h.c.}]$ which, after EWSB, split the components of Φ by $\sim \lambda v^2$.

Because DM within this framework is charged under the EW group, it must be quite massive to account for the relic abundance. This translates into a value of the compositeness scale f_D much larger than the usual "natural" value assumed in CHMs. In turn, this weakens the stringent bounds from the LHC and other experiments. In Ref. [61] these models were described using a dual description of the composite sector in extra dimensions with modified boundary conditions; all observables depending on solely f_D and a dimensionless coupling g_D in the strong sector. The mass of the DM reproducing the whole relic abundance is of order $\sim$ 1 TeV (500 GeV) in the triplet (doublet) model. In both cases, $g_D f_D \gtrsim 10$ TeV.

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[6] In the simpler coset $[\mathrm{SU}(2)\times U(1)^2]/[\mathrm{SU}(2)\times U(1)]$, the unbroken $U(1)$ commutes with the whole SM gauge group and therefore the only pNGB is exactly massless. Likewise, in $\mathrm{SU}(2)^2/[\mathrm{SU}(2)\times U(1)]$ there are only two *electrically charged* pNGBs.

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