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Charmonium mass shifts in an unquenched quark model

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Abstract In this paper, we performed a coupled-channel calculation and evaluated the mass shifts for all $1S$, $2S$, $1P$, $2P$ and $1D$ charmonium valence states below 4 GeV, by incorporating the four-quark components (D, D^*, D_s and D_s^* meson pairs) into the quark model. The valence-continuum coupling is provided by the $^3P_0$ quark-pair creation model. The induced mass shifts appear to be large and negative with the original transition operator in $^3P_0$ model, which raised up challenges for the valence quark model. More QCD-motivated models should be employed for the quark-pair creation Hamiltonian. So herein, we recalculated the mass shifts with the improved $^3P_0$ transition operator introduced in our previous work and the mass shifts are reduced by 75% averagely. Besides, as a exercise, we adjust the confinement parameter Δ and recalculate the spectrum of the charmonium states. The masses of some charmonium states are reproduced well.

1 Introduction

The discovery of many hidden charm states, the so-called X, Y, Z mesons [1] and many bottomonium states, such as $\eta_b(1S)$ [2], $\Upsilon(^3D_J)$ [3] has created challenges for the conventional quenched quark model and given great impetus to study on heavy quarkonium spectroscopy recent years, because some members of them have unexpected properties. Now, it may be a good time to develop the unquenched quark model, in which, the effects of hadron loops (also called coupled-channel effects) were also considered. In recent years, the coupled-channel effects in the charmonium spectrum have been further studied [4–11] and provided important information on the identifications of the newly reported states.

Godfrey and Isgur gave the predictions of the mass spectra of charmed and charmed-strange states in the nonrelativistic potential model [12]. However, the observed masses are generally lower than the predicted ones, such as the narrow charm-strange mesons $D_{s0}^*(2317)^+$ [13] and $D_{s_1}(2460)^+$ [14], which also raised the special concern in both experiment and theory. The coupling to mesonic channels may be responsible for these anomalously low masses [15–17]. $X(3872)$ is the most widely discussed state in the charmonium states. As the state sits just at the $D\bar{D}^*$ threshold, it might be a $D\bar{D}^*$ molecule bound state. The study [18] indicated that it may be a mixture of a $D\bar{D}^*$ molecule and the $\chi_{c_1}(2P)$ ($c\bar{c}$) considering the effects of coupled-channel using the $^3P_0$ quark-pair creation model. B. Q. Li et al. supported the assignment of the $X(3872)$ as $\chi_{c_1}(2P)$-dominate charmonium state in two different models: the coupled-channel model and the screened potential model [7]. Recent study by Zheng Cao and Qiang Zhao investigated the effects of S-wave thresholds $D_{s_1}\bar{D}_s+c.c.$ and $D_{s_0}\bar{D}_s^*+c.c.$ on vector charmonium spectrum and found that it can lead to formation of exotic states Z_{cs} in the decay of $\psi(4415) \to J/\psi K\bar{K}$ [11]. There are many other studies which presented good descriptions of the charmonium states when considering the mass shifts induced by the intermediate hadron loops [5, 19, 20].

The $^3P_0$ model [21] is the simplest model for light-quark pair creation which is widely applied in the effects of hadron loops in the most of the above-mentioned papers. It assumes that the pair is created in the vacuum with the $^3P_0$ quantum numbers uniformly in space. The application of this model to the coupled-channels calculations has a long history. For example, T. Barnes has first reported results for hadronic mass shifts of lower charmonium due to mixing with D, D^*, D_s and D_s^* meson pairs, calculated within $^3P_0$ model and the shifts appear to be alarmingly large [22]. Refs. [4, 23, 24] also arrived at the same conclusion that $q\bar{q}$ ($q = u$, d) pairs were found to induce very large mass shifts in the $^3P_0$ model. In our previous work [24], we compute the masses of ground state for the light mesons, incorporating hadron loops in a chiral quark model using the $^3P_0$ model to describe the pair creation, and explored the impact of physically motivated modifications of the associated operator. For the light-quark system, the coupling between $n\bar{n}$ ($n = u,\ d$) and meson pairs component is weakened, producing mass shifts that are around 10%∼20% of the hadron

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bare masses. In our present work, we will keep exploring the effect of modified operator of the $^3P_0$ model for the heavy-quark charmonium system and trying to understand the properties of the newly found charmonium states.

In this paper, the effects of coupled-channels for charmonia levels including all $1S$, $2S$, $1P$, $2P$ and $1D$ valence states are presented. We calculated the mass shifts of these charmonium states based on the nonrelativistic chiral quark model and solved the quantum mechanics problem using the Gaussian expansion method (GEM) [25] instead of the simple harmonic oscillator (SHO) ones [4, 6, 26]. In Sect. 2, the chiral quark model and the GEM are outlined. Sect. 3 introduces the $^3P_0$ model briefly. And Sect. 4 is devoted to a discussion of the results. In Sect. 5, the paper ends with a short summary.

2 Chiral quark model

In the nonrelativistic quark model, we obtained the meson spectrum by solving a Schrödinger equation:

$$H\Psi^{IJ}_{M_I M_J}(1,2) = E^{IJ}\Psi^{IJ}_{M_I M_J}(1,2), \tag{1}$$

where 1, 2 represents the quark and antiquark labels. $\Psi^{IJ}_{M_I M_J}(1, 2)$ is the wave function of a meson composed of a quark and a antiquark with quantum numbers IJ^{PC} and reads,

$$\Psi^{IJ}_{M_I M_J}(1,2) = \sum_{\alpha} C_\alpha[\psi_l(\mathbf{r})\chi_s(1,2)]^J_{M_J}\omega^c(1,2)\phi^I_{M_I}(1,2), \tag{2}$$

where $\psi_l(\mathbf{r})$, $\chi_s(1, 2)$, $\omega^c(1, 2)$, $\phi^I(1, 2)$ are orbit, spin, color and flavor wave functions, respectively. α denotes the intermediate quantum numbers, l, s and possible flavor indices. In our calculations, the orbital wave functions is expanded using a series of Gaussians,

$$\psi_{lm}(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n\psi^G_{nlm}(\mathbf{r}), \tag{3a}$$

$$\psi^G_{nlm}(\mathbf{r}) = N_{nl}r^l e^{-\nu_n r^2}Y_{lm}(\hat{\mathbf{r}}), \tag{3b}$$

with the Gaussian size parameters chosen according to the following geometric progression

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{\max}}}{r_1}\right)^{\frac{1}{n_{\max}-1}}. \tag{4}$$

This procedure enables optimization of the ranges using just a small number of Gaussians.

At this point, the wave function in Eq. (2) is expressed as follows:

$$\Psi^{IJ}_{M_I M_J}(1,2) = \sum_{n\alpha} C_\alpha c_n\Big[\psi^G_{nl}(\mathbf{r})\chi_s(1,2)\Big]^J_{M_J}\omega^c(1,2)\phi^I_{M_I}(1,2). \tag{5}$$

We employ the Rayleigh–Ritz variational principle for solving the Schrödinger equation due to the non-orthogonality of Gaussians, which leads to a generalized eigenvalue problem

$$\sum_{n',\alpha'}(H^{IJ}_{n\alpha,n'\alpha'} - E^{IJ}N^{IJ}_{n\alpha,n'\alpha'})C^{IJ}_{n'\alpha'} = 0, \tag{6a}$$

$$H^{IJ}_{n\alpha,n'\alpha'} = \langle\Phi^{IJ}_{M_I M_J,n\alpha}|H|\Phi^{IJ}_{M_I M_J,n'\alpha'}\rangle, \tag{6b}$$

$$N^{IJ}_{n\alpha,n'\alpha'} = \langle\Phi^{IJ}_{M_I M_J,n\alpha}|1|\Phi^{IJ}_{M_I M_J,n'\alpha'}\rangle, \tag{6c}$$

with $\Phi^{IJ}_{M_I M_J,n\alpha} = [\psi^G_{nl}(\mathbf{r})\chi_s(1, 2)]^J_{M_J}\omega^c(1, 2)\phi^I_{M_I}(1, 2)$, $C^{IJ}_{n\alpha} = C_\alpha c_n$.

We get the mass of the four-quark system also by solving a Schrödinger equation:

$$H\,\Psi^{4IJ}_{M_I M_J} = E^{IJ}\Psi^{4IJ}_{M_I M_J}, \tag{7}$$

where $\Psi^{4IJ}_{M_I M_J}$ is the wave function of the four-quark system, which can be constructed as follows. In our calculations, we only consider the meson-meson picture with the color singlet for the four-quark system in coupled-channel effects. First, we write down the wave functions of two meson clusters,

$$\Psi^{I_1 J_1}_{M_{I_1} M_{J_1}}(1,2) = \sum_{\alpha_1 n_1} C^{\alpha_1}_{n_1} \times \left[\psi^G_{n_1 l_1}(\mathbf{r}_{12})\chi_{s_1}(1,2)\right]^{J_1}_{M_{J_1}}\omega^{c_1}(1,2)\phi^{I_1}_{M_{I_1}}(1,2), \tag{8a}$$

$$\Psi^{I_2 J_2}_{M_{I_2} M_{J_2}}(3,4) = \sum_{\alpha_2 n_2} \mathcal{C}^{\alpha_2}_{n_2} \times \left[\psi^G_{n_2 l_2}(\mathbf{r}_{34})\chi_{s_2}(3,4)\right]^{J_2}_{M_{J_2}} \omega^{c_2}(3,4)\phi^{I_2}_{M_{I_2}}(3,4), \tag{8b}$$

then the total wave function of the four-quark state is:

$$\begin{aligned}
\Psi^{4\,IJ}_{M_I M_J} &= \mathcal{A}\sum_{L_r}\left[\Psi^{I_1 J_1}(1,2)\Psi^{I_2 J_2}(3,4)\psi_{L_r}(\mathbf{r}_{1234})\right]^{IJ}_{M_I M_J} \\
&= \sum_{\alpha_1\,\alpha_2\,n_1\,n_2\,L_r} \mathcal{C}^{\alpha_1}_{n_1}\mathcal{C}^{\alpha_2}_{n_2}\Bigg[\left[\psi^G_{n_1 l_1}(\mathbf{r}_{12})\chi_{s_1}(1,2)\right]^{J_1} \\
&\quad \times \left[\psi^G_{n_2 l_2}(\mathbf{r}_{34})\chi_{s_2}(3,4)\right]^{J_2}\psi_{L_r}(\mathbf{r}_{1234})\Bigg]^J_{M_J} \\
&\quad \times \left[\omega^{c_1}(1,2)\omega^{c_2}(3,4)\right]^{[1]}\left[\phi^{I_1}(1,2)\phi^{I_2}(3,4)\right]^I_{M_I},
\end{aligned} \tag{9}$$

Here, $\mathcal{A}$ is the antisymmetrization operator: if all quarks (antiquarks) are taken as identical particles, then

$$\mathcal{A} = \frac{1}{2}(1 - P_{13} - P_{24} + P_{13}P_{24}). \tag{10}$$

$\psi_{L_r}(\mathbf{r}_{1234})$ is the two-cluster relative wave function which is also expanded in a series of Gaussians. L_r describes the relative cluster orbital angular momentum. Need to be noted that, in our calculations, the angular momentum for the two mesons l_1 and l_2 equals zero. So for the $1S$, $2S$ and $1D$ states, the relative angular momentum L_r equals 1 (P wave); for the $1P$ and $2P$ states, we only consider the $L_r = 0$ with S wave between the two clusters, and $L_r = 2$ with D wave is not considered herein, which is our future work. For the quark model introduction, we take four-quark system as an example. (The two-quark system is relative simple, here we will omit it). The Hamiltonian of the chiral quark model for the four-quark system consists of three parts: quark rest mass, kinetic energy, potential energy:

$$\begin{aligned}
H &= \sum_{i=1}^{4} m_i + \frac{p_{12}^2}{2\mu_{12}} + \frac{p_{34}^2}{2\mu_{34}} + \frac{p_r^2}{2\mu_r} \\
&\quad + \sum_{i<j=1}^{4}\left(V^C_{\mathrm{CON}}(\boldsymbol{r}_{ij}) + V^C_{\mathrm{OGE}}(\boldsymbol{r}_{ij}) + V^{\mathrm{SO}}_{\mathrm{CON}}(\boldsymbol{r}_{ij}) + V^{\mathrm{SO}}_{\mathrm{OGE}}(\boldsymbol{r}_{ij}) + \sum_{\chi=\pi,K,\eta} V^\chi_{ij} + V^\sigma_{ij}\right).
\end{aligned} \tag{11}$$

where m_i is the constituent mass of ith quark (antiquark). $\frac{\mathbf{p_{ij}^2}}{2\mu_{ij}}(ij = 12; 34)$ and $\frac{\mathbf{p_r^2}}{2\mu_r}$ represents the inner kinetic of two-cluster and the relative motion kinetic between two clusters, respectively, with

$$\mathbf{p}_{12} = \frac{m_2\mathbf{p}_1 - m_1\mathbf{p}_2}{m_1 + m_2}, \tag{12a}$$

$$\mathbf{p}_{34} = \frac{m_4\mathbf{p}_3 - m_3\mathbf{p}_4}{m_3 + m_4}, \tag{12b}$$

$$\mathbf{p}_r = \frac{(m_3 + m_4)\mathbf{p}_{12} - (m_1 + m_2)\mathbf{p}_{34}}{m_1 + m_2 + m_3 + m_4}, \tag{12c}$$

$$\mu_{ij} = \frac{m_i m_j}{m_i + m_j}, \tag{12d}$$

$$\mu_r = \frac{(m_1 + m_2)(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}. \tag{12e}$$

V^C_{CON} and V^C_{OGE} is the central part of the confinement and central part of one-gluon-exchange. $V^{\mathrm{SO}}_{\mathrm{CON}}$ and $V^{\mathrm{SO}}_{\mathrm{OGE}}$ is the noncentral potential energy. In our calculations, a quadratic confining potential is adopted. For the mesons, the distance between q and $\bar{q}$ is relatively small, so the difference between the linear potential and the quadratic potential is very small by adjusting the confinement strengths. Both of them can conform to the linear Regge trajectories for $q\bar{q}$ mesons.

$V^{\chi=\pi,K,\eta}_{ij}$, and σ exchange represents the one Goldstone boson exchange. Chiral symmetry suggests dividing the quarks into two different sectors: light quarks (u, d and s) where the chiral symmetry is spontaneously broken and heavy quarks (c and b) where the symmetry is explicitly broken. The origin of the constituent quark mass can be traced back to the spontaneous breaking of chiral symmetry, and consequently, constituent quarks should interact through the exchange of Goldstone bosons. Beginning with the chiral Lagrangian $L = \bar{\Psi}(i\gamma^\mu\partial_\mu - MU^{\gamma_5})\Psi$, through the method of field theory, we can get the potentials of Goldstone bosons.

The nonrelativistic reduction in this potential has been performed for the study of nuclear forces. The detailed derivation process can be found in several theoretical works [27, 28]. In this manuscript, for saving space we will not show the particular steps.

The forms of the potentials are [29]:

$$V^{C}_{\mathrm{CON}}(\boldsymbol{r}_{ij}) = (-a_c r^2_{ij} - \Delta)\boldsymbol{\lambda}^c_i \cdot \boldsymbol{\lambda}^c_j, \tag{13a}$$

$$\begin{aligned} V^{\mathrm{SO}}_{\mathrm{CON}}(\boldsymbol{r}_{ij}) = \boldsymbol{\lambda}^c_i \cdot \boldsymbol{\lambda}^c_j \cdot \frac{-a_c}{2m_i^2 m_j^2}\Bigg\{\bigg((m_i^2+m_j^2)(1-2a_s) \\ +4m_i m_j(1-a_s)\bigg)(\boldsymbol{S}_+\cdot\boldsymbol{L}) + (m_j^2-m_i^2) \\ (1-2a_s)(\boldsymbol{S}_-\cdot\boldsymbol{L})\Bigg\}, \end{aligned} \tag{13b}$$

$$V^{C}_{\mathrm{OGE}}(\boldsymbol{r}_{ij}) = \frac{\alpha_s}{4}\boldsymbol{\lambda}^c_i \cdot \boldsymbol{\lambda}^c_j\left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\delta(\boldsymbol{r}_{ij})\right], \tag{13c}$$

$$\begin{aligned} V^{\mathrm{SO}}_{\mathrm{OGE}}(\boldsymbol{r}_{ij}) = -\frac{1}{16}\cdot\frac{\alpha_s}{m_i^2 m_j^2}\boldsymbol{\lambda}^c_i \cdot \boldsymbol{\lambda}^c_j\{\frac{1}{r^3_{ij}} - \frac{e^{-r_{ij}/r_g(\mu)}}{r^3_{ij}}\cdot \\ (1+\frac{r_{ij}}{r_g(\mu)})\}\times\Bigg\{\bigg((m_i+m_j)^2+2m_i m_j\bigg) \\ (\boldsymbol{S}_+\cdot\boldsymbol{L}) + (m_j^2-m_i^2)(\boldsymbol{S}_-\cdot\boldsymbol{L})\Bigg\}, \end{aligned} \tag{13d}$$

$$\delta(\boldsymbol{r}_{ij}) = \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{4\pi r_{ij} r_0^2(\mu_{ij})}, \mathbf{S}_{\pm} = \mathbf{S}_1 \pm \mathbf{S}_2, \tag{13e}$$

$$V_{\pi}(\boldsymbol{r}_{ij}) = \frac{g^2_{ch}}{4\pi}\frac{m^2_{\pi}}{12m_i m_j}\frac{\Lambda^2_{\pi}}{\Lambda^2_{\pi}-m^2_{\pi}}m_{\pi}v^{\pi}_{ij}\sum_{a=1}^{3}\lambda^a_i\lambda^a_j, \tag{13f}$$

$$V_{K}(\boldsymbol{r}_{ij}) = \frac{g^2_{ch}}{4\pi}\frac{m^2_{K}}{12m_i m_j}\frac{\Lambda^2_{K}}{\Lambda^2_{K}-m^2_{K}}m_{K}v^{K}_{ij}\sum_{a=4}^{7}\lambda^a_i\lambda^a_j, \tag{13g}$$

$$\begin{aligned} V_{\eta}(\boldsymbol{r}_{ij}) = \frac{g^2_{ch}}{4\pi}\frac{m^2_{\eta}}{12m_i m_j}\frac{\Lambda^2_{\eta}}{\Lambda^2_{\eta}-m^2_{\eta}}m_{\eta}v^{\eta}_{ij} \\ \times\left[\lambda^8_i\lambda^8_j\cos\theta_P - \lambda^0_i\lambda^0_j\sin\theta_P\right], \end{aligned} \tag{13h}$$

$$v^{\chi}_{ij}(\boldsymbol{r}_{ij}) = \left[Y(m_{\chi}r_{ij}) - \frac{\Lambda^3_{\chi}}{m^3_{\chi}}Y(\Lambda_{\chi}r_{ij})\right]\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j, \tag{13i}$$

$$\begin{aligned} V_{\sigma}(\boldsymbol{r}_{ij}) = -\frac{g^2_{ch}}{4\pi}\frac{\Lambda^2_{\sigma}}{\Lambda^2_{\sigma}-m^2_{\sigma}}m_{\sigma} \\ \times\left[Y(m_{\sigma}r_{ij}) - \frac{\Lambda_{\sigma}}{m_{\sigma}}Y(\Lambda_{\sigma}r_{ij})\right], \end{aligned} \tag{13j}$$

where $\mathbf{S}_1$ and $\mathbf{S}_2$ is the spin of the two meson clusters. $Y(x) = e^{-x}/x$; $r_0(\mu_{ij}) = s_0/\mu_{ij}$; $\boldsymbol{\sigma}$ are the $SU(2)$ Pauli matrices; $\boldsymbol{\lambda}$, $\boldsymbol{\lambda}^c$ are $SU(3)$ flavor, color Gell-Mann matrices, respectively. For the parameter Λ_i $(i = \pi, K, \eta, \sigma)$, it is a cut-off parameter, which is equivalent to the form factor. The aim is to remove the short-range contribution of Goldstone bosons exchanges. Besides, Λ_i is dependent on the mass of Goldstone bosons, so a unique parameter is not used. $g^2_{ch}/4\pi$ is the chiral coupling constant, determined from the π-nucleon coupling; and α_s is an effective scale-dependent running coupling [29],

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln\left[(\mu^2_{ij}+\mu^2_0)/\Lambda^2_0\right]}. \tag{14}$$

In our calculations, for the two-quark system, besides the central potential energy, the noncentral potential energy is also included. But in the four-quark system calculations, we find that the influence of the noncentral potential energy on the mass shift of the state is tiny.

Lastly, we show the model parameters [30] in Table 1. Need to be noted that, in the reference [30], the confinement item takes the form $V^C_{ij} = (-a_c(1-e^{-\mu_c r_{ij}})+\Delta)(\boldsymbol{\lambda}^c_i\cdot\boldsymbol{\lambda}^c_j)$. And in our present calculations, the usual quadratic confinement $V^C_{ij} = (-a_c r^2_{ij} - \Delta)\boldsymbol{\lambda}^c_i\cdot\boldsymbol{\lambda}^c_j$ is employed, so some parameters are different such as quark mass, a_c and Δ.

Table 1 Model parameters, determined by fitting the meson spectrum, leaving room for unquenching contributions in the case of light-quark systems

Quark masses	$m_u = m_d$	313
(MeV)	m_s	536
	m_c	1728
	m_b	5112
Goldstone bosons	m_π	0.70
($\mathrm{fm}^{-1} \sim 200\,\mathrm{MeV}$)	m_σ	3.42
	m_η	2.77
	m_K	2.51
	$\Lambda_\pi = \Lambda_\sigma$	4.2
	$\Lambda_\eta = \Lambda_K$	5.2
	$g_{ch}^2/(4\pi)$	−0.54
	$\theta_p(°)$	−15
Confinement	a_c (MeV fm^{-2})	101
	Δ (MeV)	−78.3
OGE	α_0	3.67
	$\Lambda_0(\mathrm{fm}^{-1})$	0.033
	μ_0(MeV)	36.98
	s_0(MeV)	28.17

Using the model parameters, we calculated the masses of some mesons from light to heavy, especially the relevant charmonium $c\bar{c}$ mesons η_c, J/ψ, $\chi_{cJ}(J = 0, 1, 2)$, h_c in the chiral quark model, which are demonstrated in Table 2. In order to obtain the stable masses, we take the Gaussian size parameters $r_1 = 0.01$, $r_n = 2$, $n = 16$ in Eq. (4). From the table, we can find that the quark model achieves great success on describing the hadron spectra, especially for the ground-state mesons such as most light mesons and heavy mesons $\eta_c(1S)$, $J/\psi(1S)$. But it still be faced some challenges on the charmonium excited states such as $\eta_c(2S)$, $\psi(2S)$, $\chi_{cJ}(1P)$, $\chi_{cJ}(2P)$ and $1D$ states since more higher charmonium states have been observed experimentally. For $b\bar{b}$ system, the masses of the ground-state $\eta_b(1S)$ and $\Upsilon(1S)$ are not so satisfactory, but for the excited states, the masses are well consistent with the experimental values unexpectedly such as $\Upsilon(2S)$, $\chi_{bJ}(1P)$ and $\chi_{bJ}(2P)$.

3 $^3P_0$ model

The $^3P_0$ quark-pair creation model [21, 32, 33] has been widely applied to OZI rule allowed two-body strong decays of hadrons [34–39]. If the quark and antiquark in the source meson are labeled by 1, 2, and the quark and antiquark ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$) generated in the vacuum are numbered as 3, 4, the operator of the $^3P_0$ model reads:

$$T_0 = -3\gamma \sum_m \langle 1m1(-m)|00\rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \times \mathcal{Y}_1^m\left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2}\right)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34} b_2^\dagger(\mathbf{p}_3) d_3^\dagger(\mathbf{p}_4), \tag{15}$$

where γ describes the probability for creating a quark-antiquark pair with momenta $\mathbf{p}_3$ and $\mathbf{p}_4$ from the 0^{++} vacuum. It is normally determined by fitting an array of hadron strong decays. This yields $\gamma = 6.95$ for $u\bar{u}$ and $d\bar{d}$ pair creation, and $\gamma = 6.95/\sqrt{3}$ for $s\bar{s}$ pair creation [40]. ω_0^{34} and ϕ_0^{34} are the color and flavor wave function components, respectively.

4 Numerical results

By incorporating the four-quark components ($c\bar{c}q\bar{q}$ ($q = u, d, s$)) into the charmonium $c\bar{c}$ mesons, we can get the eigenvalues of the $c\bar{c} + c\bar{c}q\bar{q}$ system by solving the Schrödinger equation,

$$H\Psi = E\Psi, \tag{16}$$

where Ψ and H is the wave function and the Hamiltonian of the system, it takes,

$$\Psi = c_1\Psi_{2q} + c_2\Psi_{4q}, \tag{17}$$

$$H = H_{2q} + H_{4q} + T_0. \tag{18}$$

Table 2 The mass spectrum in the chiral quark model, in comparison with the experimental data [31] (in unit of MeV)

Name	$J^{P(C)}$	Mass	PDG [31]
π	0^-	134.9	135.0
K	0^-	489.4	493.7
ρ	1^{--}	772.3	775.3
K^*	1^-	913.6	892.0
ω	1^{--}	701.6	782.7
η	0^{-+}	669.2	547.9
$\phi(1020)$	1^{--}	1015.9	1019.5
D^0	0^-	1861.9	1864.8
D^{*0}	1^-	1980.6	2006.9
D_s^+	0^-	1950.1	1968.4
D_s^{*+}	1^-	2079.9	2112.2
B^-	0^-	5280.7	5279.3
B^*	1^-	5319.6	5324.7
B_s^0	0^-	5367.4	5366.9
B_s^*	1^-	5410.2	5415.4
$\eta_c(1S)$	0^{-+}	2964.4	2983.9
$\eta_c(2S)$	0^{-+}	3507.8	3637.5
J/ψ	1^{--}	3096.4	3096.0
$\psi(2S)$	1^{--}	3605.0	3686.1
$\chi_{c_0}(1P)$	0^{++}	3362.8	3414.7
$\chi_{c_0}(2P)$	0^{++}	3814.7	$\chi_{c_0}(3915)$?
$\chi_{c_1}(1P)$	1^{++}	3393.9	3510.7
$\chi_{c_1}(2P)$	1^{++}	3851.9	$\chi_{c_1}(3872)$?
$\chi_{c_2}(1P)$	2^{++}	3435.8	3556.2
$\chi_{c_2}(2P)$	2^{++}	3901.1	$\chi_{c_2}(3930)$?
$h_c(1P)$	1^{+-}	3416.1	3525.4
$h_c(2P)$	1^{+-}	3877.4	$Z_c(3900)$?
$\eta_b(1S)$	0^{-+}	9561.5	9398.7
$\Upsilon(1S)$	1^{--}	9647.8	9460.3
$\Upsilon(2S)$	1^{--}	10016.7	10023.3
$\chi_{b_0}(1P)$	0^{++}	9916.8	9859.4
$\chi_{b_0}(2P)$	0^{++}	10198.4	10232.5
$\chi_{b_1}(1P)$	1^{++}	9925.4	9892.8
$\chi_{b_1}(2P)$	1^{++}	10208.2	10255.5
$\chi_{b_2}(1P)$	2^{++}	9938.9	9912.2
$\chi_{b_2}(2P)$	2^{++}	10223.2	10268.7
$h_b(1P)$	1^{+-}	9932.4	9899.3
$h_b(2P)$	1^{+-}	10216.1	10259.8
$\eta_{c_2}(1D)$	2^{-+}	3675.1	?
$\psi(1D)$	1^{--}	3653.3	$\psi(3770)$?
$\psi_2(1D)$	2^{--}	3668.3	$\psi_2(3823)$?
$\psi_3(1D)$	3^{--}	3688.1	$\psi_3(3842)$?

Because the number of particles is conserved in the nonrelativistic quark model, the H_{2q} only acts on the wave function of two-quark $c\bar{c}$ system, Ψ_{2q}, and the H_{4q} only acts on the wave function of four-quark system, Ψ_{4q}. The transition operator T_0 (Eq. (15)) in the $^3P_0$ model is responsible for the coupling of the two- and four-quark system.

In this way, we can get the matrix elements of the Hamiltonian,

$$\begin{aligned}\langle\Psi|H|\Psi\rangle &= \langle c_1\Psi_{2q} + c_2\Psi_{4q}|H|c_1\Psi_{2q} + c_2\Psi_{4q}\rangle \\ &= c_1^2\langle\Psi_{2q}|H_{2q}|\Psi_{2q}\rangle + c_2^2\langle\Psi_{4q}|H_{4q}|\Psi_{4q}\rangle\end{aligned}$$

$$+c_1c_2^*\langle\Psi_{4q}|T_0|\Psi_{2q}\rangle + c_1^*c_2\langle\Psi_{2q}|T_0^\dagger|\Psi_{4q}\rangle, \tag{19}$$

and the block-matrix structure for the Hamiltonian and overlap takes,

$$(H)=\begin{bmatrix}(H_{2q}) & (H_{24})\\ (H_{42}) & (H_{4q})\end{bmatrix},\quad (N)=\begin{bmatrix}(N_{2q}) & (0)\\ (0) & (N_{4q})\end{bmatrix}, \tag{20}$$

with

$$(H_{2q})=\langle\Psi_{2q}|H_{2q}|\Psi_{2q}\rangle, \tag{21a}$$

$$(H_{24})=\langle\Psi_{4q}|T_0|\Psi_{2q}\rangle, \tag{21b}$$

$$(H_{4q})=\langle\Psi_{4q}|H_{4q}|\Psi_{4q}\rangle, \tag{21c}$$

$$(N_{2q})=\langle\Psi_{2q}|1|\Psi_{2q}\rangle, \tag{21d}$$

$$(N_{4q})=\langle\Psi_{4q}|1|\Psi_{4q}\rangle. \tag{21e}$$

where (H_{2q}) and (H_{4q}) is the matrix for the pure two-quark $c\bar{c}$ system and pure four-quark system, respectively. (H_{24}) is the coupling matrix of two-quark system and four-quark system.

Finally, the eigenvalues (E_n) and eigenvectors (C_n) of the system are obtained by solving the diagonalization problem,

$$\Big[(H)-E_n(N)\Big]\Big[C_n\Big]=0. \tag{22}$$

In our calculations, a convergence factor $e^{-f^2\mathbf{p}^2}$ was inserted into the operator T_0 in Eq. (15) in order to be Fourier transformed, because the two- and four-quark system are solved in coordinate space. The Fourier transformed factor is written as,

$$T_1=-i3\gamma\sum_m\langle 1m1(-m)|00\rangle\int \mathrm{d}\mathbf{r_3}\mathrm{d}\mathbf{r_4}(\frac{1}{2\pi})^{\frac{3}{2}}2^{-\frac{5}{2}}f^{-5}$$
$$rY_{1m}(\hat{\mathbf{r}})\mathrm{e}^{-\frac{\mathbf{r}^2}{4f^2}}\chi^{34}_{1-m}\omega^{34}_0\phi^{34}_0 b_3^\dagger(\mathbf{r_3})d_4^\dagger(\mathbf{r_4}). \tag{23}$$

There is one more parameter f in the transition operator T_1. When f takes the limit to zero, the original form of the ${}^3P_0$ quark model is recovered. By the way, $\mathbf{r}$ in Eq. (23) is the relative distance between the quark pair in the vacuum, $\mathbf{r}=\mathbf{r}_3-\mathbf{r}_4$.

By solving Eq. (22) with the transition operator in Eq. (23) and in the limit $f\to 0$, $\gamma=6.95$, we obtained the mass shifts for $\eta_c(1S)$, $\eta_c(2S)$, $J/\psi(1S)$, $J/\psi(2S)$, $\chi_{c_0}(1P)$, $\chi_{c_1}(1P)$, $\chi_{c_2}(1P)$, $h_c(1P)$ charmonium valence states, as well as the higher charmonium $2P$ and $1D$ states, by incorporating the four-quark components (D, D^*, D_s and D_s^* meson pairs) into the two-quark $c\bar{c}$ system. The results are shown in Table 3. In order to get the stable mass shifts of the states, we take the Gaussian size parameters $r_1=0.01$, $r_n=2$, $n=16$ in Eq. (4) for the two-quark charmonium system. For the four-quark system, we take $r_1=0.1$, $r_n=2$, $n=8$ for inner two meson pairs, and the relative Gaussian size parameters between the two meson pairs take $r_1=0.1$, $r_n=6$, $n=9$.

There exist three open channels in our calculations, $\chi_{c_0}(2P)\to D\bar{D}$, $\chi_{c_1}(2P)\to D\bar{D}^*$, $h_c(2P)\to D\bar{D}^*$. For these open channels, the mass shifts of the states will change with the Gaussian distribution. Especially, the mass shifts will change with the increasing of spatial volume periodically. In our calculations, we picked up the biggest mass shifts as the contributions of this open channel by varying the Gaussian size parameter r_n between the two meson pairs. Let us take the channel $\chi_{c_0}(2P)\to D\bar{D}$ as an example. For $D\bar{D}$ state, it has the discrete energy levels which will change with the varying Gaussian distribution in the theoretical calculations even if it is a scattering state. When considering the coupling of the $D\bar{D}$ and $\chi_{c_0}(2P)$, the strength of coupling will be increased as the energy of $D\bar{D}$ state is close to that of $\chi_{c_0}(2P)$, and the induced mass shift will become bigger. We take the biggest one as the mass shift of the state $\chi_{c_0}(2P)$ to the $D\bar{D}$ state. Besides, if we expand the space further with higher r_n values, the same biggest mass shift will be repeated. From the table, we can also find that for the open channels, the mass shifts are always larger than the close channels.

In Table 3, the bare mass of the states is obtained in the quenched quark model, viz. solved with only the $c\bar{c}$ component. When considering the coupled-channel effects, we get the large negative mass shifts. Such large shifts invalidate the traditional quenched quark model. In our previous work [24], when we investigate the hadron loop effects of the $n\bar{n}$ ($n=u$, d) states, similarly, large mass shifts are obtained with the original operator T_1 in Eq. (23) in the limit $f\to 0$. In order to develop a more realistic unquenching procedure, in the work [24], we gave some modifications of the operator T_1. It reads,

$$T_2=-3\gamma\sum_m\langle 1m1(-m)|00\rangle\int d\mathbf{r_3}d\mathbf{r_4}(\frac{1}{2\pi})^{\frac{3}{2}}ir2^{-\frac{5}{2}}f^{-5}$$
$$Y_{1m}(\hat{\mathbf{r}})\mathrm{e}^{-\frac{\mathbf{r}^2}{4f^2}}\mathrm{e}^{-\frac{R_{AV}^2}{R_0^2}}\chi^{34}_{1-m}\phi^{34}_0\omega^{34}_0 b_3^\dagger(\mathbf{r_3})d_4^\dagger(\mathbf{r_4}), \tag{24}$$

compared with Eq. (23), the factor $e^{-\frac{R_{AV}^2}{R_0^2}}$ is introduced because the creation of quark-antiquark pairs should become less likely as the distance from the bare-hadron source is increased. R_{AV} is the relative distance between the source particle and quark-antiquark pair in the vacuum. In Eq. (24), there are three parameters need to identify, γ, f and R_0. According to our previous work [24], we find

$$\gamma = 32.2, \quad f = 0.5 \text{ fm}, \quad R_0 = 1 \text{ fm}. \tag{25}$$

In the present work, we also apply the transition operator in Eq. (24) with improvements and remain the values of parameters γ, f and R_0 in Eq. (25). The newly mass shifts of the charmonium valence states are demonstrated in Table 4. From the table, we found that the mass shifts are reduced greatly by 75% averagely, compared with the results in Table 3. Plainly, our modified $^3P_0$ pair-creation model generates modest unquenching corrections, with mass renormalizations just $1\% - 4\%$ of a given meson's bare mass. So the effects of including the new term $e^{-\frac{R_{AV}^2}{R_0^2}}$ on the mass shifts are relatively stable. On the other hand, we calculated the decay width of mesons using the transition amplitude T_2 in Eq. (24), and we find that the results are similar with the ones using the original transition amplitude T_1 with $f \to 0$ in Eq. (23).

In Table 2, we obtained the masses of the charmonium mesons in the quenched quark model, and the masses of $1S$, $2S$, $1P$, $2P$ and $1D$ are all smaller than the experimental values from PDG [31]. In the unquenched quark model, the coupled-channel effects result in the negative mass shifts, which leads to the smaller unquenched masses for the states. Notably, although the mass shifts reported in Table 4 are sensible, they destroy agreement with the empirical masses. This is because the model parameters in Table 1 were determined by fitting the meson spectrum from light to heavy, without considering the coupled-channel effects. As a exercise, we choose to illustrate a remedy. We adjust the confinement parameter Δ in order to increase the quenched masses for only $c\bar{c}$ mesons such that unquenching delivers the empirical masses, an outcome achieved with

$$\Delta = -62 \text{ MeV}. \tag{26}$$

The results are listed in Table 5. We can find that the mass shifts are not very sensitive to the parameter Δ. Having made our point, we leave for the future a complete refit of the parameters in Table 1 in order to arrive finally at a fully unquenched quark model.

Now, let us focus on the numerical analysis on the results in Table 5. Firstly, the mass shift of the each single coupled-channel $D_s^{(*)}D_s^{(*)}$ is smaller than and is about $\frac{1}{3}$ of the $D^{(*)}D^{(*)}$, due to the γ values. For open channels, the mass shifts are also larger than the close channels. For example, for $\chi_{c_0}(2P) \to D\bar{D}$, the mass shift is about 60 MeV. For $\chi_{c_1}(2P) \to D\bar{D}^* + D^*\bar{D}$ and $h_c(2P) \to D\bar{D}^* + D^*\bar{D}$, the mass shift is about 58 MeV and 36 MeV, respectively, and they are all much larger than the other close channels. Because the coupling of all channels is not so significant, as an approximation, the "Total" column represents the total mass shifts of the state, which is obtained by summing the mass shifts of the each coupled-channel simply.

Secondly, the $J/\psi - \eta_c$ and $\psi(2S) - \eta_c(2S)$ loop-induced mass splitting has been discussed previously by Eichten et al. [41]. The authors find a small loop-induced $J/\psi - \eta_c$ mass splitting of -3.7 MeV and a $\psi(2S) - \eta_c(2S)$ splitting of -20.9 MeV, bringing their model into good agreement with the experimental $\psi(2S) - \eta_c$ mass difference. Table 5 shows that we find a numerically similar $\psi(2S) - \eta_c(2S)$ splitting of -18.4 MeV, but the ground state $J/\psi - \eta_c$ mass difference is -13.4 MeV. In Ref. [4], Barnes and Swanson get the $\psi(2S) - \eta_c(2S)$ splitting of -24 MeV, which is well consistent with our results. But the $J/\psi - \eta_c$ mass difference is -34 MeV which is larger than ours.

Thirdly, let us compare our "Unquenched mass" (the last column in the table) with the experiment values. By simple correction of model confinement parameter Δ in Table 1, the bare masses of $c\bar{c}$ states are increased. After considering the coupled-channel effects, the unquenched masses of the states, $\eta_c(1S)$, J/ψ, $\chi_{c0}(1P)$, $\chi_{c_1}(2P)$, $\chi_{c_2}(2P)$ and $h_c(2P)$ are well consistent with the experimental values. In our future work, we will adjust the model parameters related to the charm quark in Table 1 and keep the light meson sector unchanged as much as possible.

What's more, for comparisons, we show some theoretical results about the mass shifts of charmonium mesons in Table 6. In the table, the mass shifts have minus sign overall. The second column ΔM_1 are the mass shifts with the original transition operator of the $^3P_0$ model. For $1S$ and $2S$ states, the mass shifts are larger than the other theoretical works in Table 6. For $1P$ states, our results are basically consistent with the references [5–8]. Therein, the spherical harmonic oscillator (SHO) wave function is applied to describe the meson dynamics, and the relative motion between two mesons is described by plane-wave functions. Besides, the mass shifts are dependent on the parameter β in SHO and γ in the $^3P_0$ model. The systematic errors due to the approximations are unpredictable for the bound-state calculation, although they are not a bad approximation for the decay width calculation. Our results ΔM_2 in the third column, obtained with the improved $^3P_0$ model, are comparable to each other, and some of them are much smaller than the other theoretical results in 4-8 columns.

5 Summary

In the present work, the spectrum of $1S$, $2S$, $1P$, $2P$ and $1D$ charmonium states below 4 GeV is calculated taking into account coupling to the pairs of lowest D and D_s pairs. To minimize the error from the calculation, a powerful method for dealing with

Table 3 Mass shifts computed for $c\bar{c}$ charmonium mesons using the transition matrix constructed from T_1 in Eq. (23) with $f \to 0$, $\gamma = 6.95$. (Units of MeV)

Bare $c\bar{c}$ state			Mass shifts by channels									$c\bar{c} + qq\bar{q}\bar{q}$
State$(n^{2S+1}L_J)$	Bare mass	Exp	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$	$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$	Total	Unquenched mass
$\eta_c(1S)(1^1S_0)$	2964.4	2983.9	–	−197.6	−197.6	−369.4	–	−80.7	−80.7	−155.8	−1081.8	1882.6
$\eta_c(2S)(2^1S_0)$	3507.8	3637.5	–	−127.8	−127.8	−228.7	–	−46.9	−46.9	−89.0	−667.1	2840.6
$J/\psi(1S)(1^3S_1)$	3096.4	3096.0	−60.6	−111.6	−111.6	−370.5	−22.1	−41.6	−41.6	−139.6	−899.2	2197.2
$\psi(2S)(2^3S_1)$	3605.0	3686.1	−48.4	−83.7	−83.7	−259.5	−15.5	−29.1	−29.1	−96.2	−645.3	2959.7
$\chi_{c0}(1P)(1^3P_0)$	3362.8	3414.7	−111.5	–	–	−30.4	−36.0	–	–	−10.5	−188.5	3174.3
$\chi_{c1}(1P)(1^3P_1)$	3393.9	3510.7	–	−65.3	−65.3	–	–	−20.7	−20.7	–	−172.2	3221.7
$\chi_{c2}(1P)(1^3P_2)$	3435.8	3556.2	–	–	–	−113.8	–	–	–	−35.6	−149.5	3286.3
$h_c(1P)(1^1P_1)$	3416.1	3525.4	–	−32.5	−32.5	−57.3	–	−9.9	−9.9	−18.6	−160.7	3255.4
$\chi_{c0}(2P)(2^3P_0)$	3814.7	$\chi_{c0}(3915)$?	−130.8	–	–	−29.8	−33.7	–	–	−9.8	−204.1	3610.6
$\chi_{c1}(2P)(2^3P_1)$	3851.9	$\chi_{c1}(3872)$?	–	−74.2	−74.2	–	–	−19.8	−19.8	–	−188.0	3663.9
$\chi_{c2}(2P)(2^3P_2)$	3901.1	$\chi_{c2}(3930)$?	–	–	–	−111.2	–	–	–	−34.8	−145.9	3755.1
$h_c(2P)(2^1P_1)$	3877.4	$Z_c(3930)$?	–	−49.8	−49.8	−57.8	–	−9.7	−9.7	−18.0	−194.8	3682.6
$\eta_{c2}(1D)(1^1D_2)$	3675.1	?	–	−32.2	−32.2	−53.8	–	−8.7	−8.7	−16.2	−151.8	3523.4
$\psi(1D)(1^3D_1)$	3653.3	$\psi(3770)$?	−62.8	−27.0	−27.0	−18.5	−16.3	−7.5	−7.5	−5.6	−172.1	3481.2
$\psi_2(1D)(1^3D_2)$	3668.3	$\psi_2(3823)$?	–	−47.4	−47.4	−28.5	–	−13.1	−13.1	−8.2	−157.8	3510.5
$\psi_3(1D)(1^3D_3)$	3688.1	$\psi_3(3842)$?	–	–	–	−106.5	–	–	–	−31.6	−138.2	3549.9

Table 4 Mass shifts computed for $c\bar{c}$ charmonium mesons using the transition matrix constructed from T_2 in Eq. (24) with $f = 0.5$ fm, $\gamma = 32.2$, $R_0 = 1$ fm. (Units of MeV)

Bare $c\bar{c}$ state			Mass shifts by channels									$c\bar{c} + qq\bar{q}\bar{q}$
State$(n^{2S+1}L_J)$	Bare mass	Exp	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$	$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$	Total	Unquenched mass
$\eta_c(1S)(1^1S_0)$	2964.4	2983.9	–	−14.5	−14.5	−27.1	–	−3.2	−3.2	−6.4	−68.9	2895.4
$\eta_c(2S)(2^1S_0)$	3507.8	3637.5	–	−31.6	−31.6	−53.1	–	−4.9	−4.9	−9.1	−135.3	3372.5
$J/\psi(1S)(1^3S_1)$	3096.4	3096.0	−6.4	−11.8	−11.8	−38.9	−1.4	−2.6	−2.6	−8.7	−84.2	3012.2
$\psi(2S)(2^3S_1)$	3605.0	3686.1	−16.5	−25.2	−25.2	−71.2	−2.1	−3.7	−3.7	−11.9	−159.5	3445.5
$\chi_{c0}(1P)(1^3P_0)$	3362.8	3414.7	−20.5	–	–	−5.2	−3.4	–	–	−1.0	−30.1	3332.7
$\chi_{c1}(1P)(1^3P_1)$	3393.9	3510.7	–	−12.9	−12.9	–	–	−2.2	−2.2	–	−30.3	3363.5
$\chi_{c2}(1P)(1^3P_2)$	3435.8	3556.2	–	–	–	−25.1	–	–	–	−4.5	−29.6	3406.1
$h_c(1P)(1^1P_1)$	3416.1	3525.4	–	−6.9	−6.9	−11.7	–	−1.2	−1.2	−2.2	−30.1	3386.0
$\chi_{c0}(2P)(2^3P_0)$	3814.7	$\chi_{c0}(3915)$?	−106.5	–	–	−11.7	−6.6	–	–	−1.3	−126.1	3688.6
$\chi_{c1}(2P)(2^3P_1)$	3851.9	$\chi_{c1}(3872)$?	–	−49.7	−49.7	–	–	−3.4	−3.4	–	−106.2	3745.7
$\chi_{c2}(2P)(2^3P_2)$	3901.1	$\chi_{c2}(3930)$?	–	–	–	−56.3	–	–	–	−6.0	−62.3	3838.8
$h_c(2P)(2^1P_1)$	3877.4	$Z_c(3930)$?	–	−44.8	−44.8	−28.2	–	−1.8	−1.8	−2.9	−124.3	3753.1
$\eta_{c2}(1D)(1^1D_2)$	3675.1	?	–	−12.1	−12.1	−18.3	–	−1.6	−1.6	−2.8	−48.4	3626.8
$\psi(1D)(1^3D_1)$	3653.3	$\psi(3770)$?	−25.4	−9.3	−9.3	−5.8	−2.8	−1.2	−1.2	−0.9	−55.9	3597.3
$\psi_2(1D)(1^3D_2)$	3668.3	$\psi_2(3823)$?	–	−17.4	−17.4	−9.5	–	−2.3	−2.3	−1.4	−50.2	3618.0
$\psi_3(1D)(1^3D_3)$	3688.1	$\psi_3(3842)$?	–	–	–	−38.3	–	–	–	−5.8	−44.1	3644.1

Table 5 Mass shifts computed for $c\bar{c}$ charmonium mesons using the transition matrix constructed from T_2 in Eq. (24) with $f = 0.5$ fm, $\gamma = 32.2$, $R_0 = 1$ fm and $\Delta = 62$ MeV. (Units of MeV)

Bare $c\bar{c}$ state			Mass shifts by channels									$c\bar{c} + qq\bar{q}\bar{q}$
State($n^{2S+1}L_J$)	Bare mass	Exp	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$	$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$	Total	Unquenched mass
$\eta_c(1S)(1^1S_0)$	3051.3	2983.9	–	−13.7	−13.7	−25.7	–	−3.1	−3.1	−6.1	−65.4	2985.9
$\eta_c(2S)(2^1S_0)$	3594.7	3637.5	–	−27.9	−27.9	−48.2	–	−4.5	−4.5	−8.4	−121.3	3473.4
$J/\psi(1S)(1^3S_1)$	3183.3	3096.0	−5.9	−11.0	−11.0	−36.3	−1.3	−2.4	−2.4	−8.3	−78.8	3104.6
$\psi(2S)(2^3S_1)$	3691.9	3686.1	−13.2	−21.6	−21.6	−63.9	−1.8	−3.4	−3.4	−10.9	−139.7	3552.3
$\chi_{c_0}(1P)(1^3P_0)$	3449.7	3414.7	−18.1	–	–	−4.7	−3.1	–	–	−1.0	−26.9	3422.8
$\chi_{c_1}(1P)(1^3P_1)$	3480.8	3510.7	–	−11.5	−11.5	–	–	−2.1	−2.1	–	−27.2	3453.5
$\chi_{c_2}(1P)(1^3P_2)$	3522.7	3556.2	–	–	–	−22.7	–	–	–	−4.2	−26.9	3495.8
$h_c(1P)(1^1P_1)$	3502.9	3525.4	–	−6.1	−6.1	−10.7	–	−1.1	−1.1	−2.0	−27.1	3475.9
$\chi_{c_0}(2P)(2^3P_0)$	3901.7	$\chi_{c0}(3915)$?	−60.5	–	–	−9.2	−4.8	–	–	−1.2	−75.8	3825.9
$\chi_{c_1}(2P)(2^3P_1)$	3938.8	$\chi_{c1}(3872)$?	–	−28.6	−28.6	–	–	−2.8	−2.8	–	−62.9	3875.9
$\chi_{c_2}(2P)(2^3P_2)$	3988.0	$\chi_{c2}(3930)$?	–	–	–	−42.1	–	–	–	−5.2	−47.3	3940.7
$h_c(2P)(2^1P_1)$	3964.3	$Z_c(3930)$?	–	−17.8	−17.8	−20.7	–	−1.5	−1.5	−2.5	−61.8	3902.5
$\eta_{c_2}(1D)(1^1D_2)$	3762.1	?	–	−9.8	−9.8	−15.9	–	−1.4	−1.4	−2.5	−41.1	3721.0
$\psi(1D)(1^3D_1)$	3740.2	$\psi(3770)$?	−19.2	−7.6	−7.6	−5.1	−2.5	−1.1	−1.1	−0.8	−45.3	3694.9
$\psi_2(1D)(1^3D_2)$	3755.2	$\psi_2(3823)$?	–	−14.3	−14.3	−8.2	–	−2.1	−2.1	−1.3	−42.3	3712.8
$\psi_3(1D)(1^3D_3)$	3775.0	$\psi_3(3842)$?	–	–	–	−33.3	–	–	–	−5.3	−38.6	3736.4

Table 6 The comparison of the mass shifts of the charmonium mesons between several theoretical works. The second and third column denote our results corresponding to Table 3 and Table 4. (Units of MeV)

States	ΔM_1	ΔM_2	[7]	[5]	[8]	[6]	[4]
1^1S_0	−1081.8	−68.9	−148	−148	−208	−165	−423
2^1S_0	−667.1	−135.3	−208	−158	−84	−200	−416
1^3S_1	−899.2	−84.2	−159	−148	−238	−177	−457
2^3S_1	−645.3	−159.5	−228	−157	−99	−216	−440
1^3P_0	−188.5	−30.1	−181	−157	−141	−198	−459
1^3P_1	−172.2	−30.3	−195	−173	−191	−215	−496
1^3P_2	−149.5	−29.6	−210	−154	−218	−228	−521
1^1P_1	−160.7	−30.1	−201	−150	−189	−219	−504
2^3P_0	−204.1	−126.1	−179	−218	−38	–	–
2^3P_1	−188.0	−106.2	−300	−214	−58	–	–
2^3P_2	−145.9	−62.3	−268	−203	−64	–	–
2^1P_1	−194.8	−124.3	−230	−153	−51	–	–
1^1D_2	−151.8	−48.4	−226	–	−112	–	–
1^3D_1	−172.1	−55.9	−233	−188	−125	–	–
1^3D_2	−157.8	−50.2	−226	–	−121	–	–
1^3D_3	−138.2	−44.1	−230	–	−116	–	–

few-body systems (GEM) was used. In our work, the angular momentum of the two mesons takes zero, and the relative motion between the two mesons denotes to P wave for $1S$, $2S$ and $1D$ states. For $1P$ and $2P$ states, we only consider the relative motion to be S wave for the preliminary work, and D wave-related calculations will be our future work.

The transition operator of the $^3P_0$ model is required to relate the valence part to the four-quark components. We demonstrated the mass shifts of the charmonium states with the original transition operator of the $^3P_0$ model, as well as with the modified version of the transition operator. By contrast, the masses shifts are reduced greatly by 75% averagely within the modified $^3P_0$ model. Plainly, our modified $^3P_0$ pair-creation model generates modest unquenching corrections, with mass renormalizations just 1% − 4% of a given meson's bare mass.

As a preliminary work, we only fine-tune the model confinement parameter Δ, and we obtained unquenched masses for the charmonium states. We find that the masses of the charmonium states, $\eta_c(1S)$, J/ψ, $\chi_{c0}(1P)$, $\chi_{c1}(2P)$, $\chi_{c2}(2P)$ and $h_c(2P)$ are well consistent with the experimental values. We leave for the future a complete refit of the model parameters in order to arrive finally at a fully unquenched quark model. More experimental data in the future can help us better understand the spectrum of the charmonium states.

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