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Impact of the generalized uncertainty principle on the thermodynamic characteristics of Schwarzschild black hole veiled with quintessence matter

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Abstract We conduct an analysis to examine the thermodynamic character of a Schwarzschild black hole surrounded by quintessence matter under a generalized Heisenberg algebra with a quadratic and linear terms in momentum. We compute the temperature, heat capacity, and entropy associated with this black hole using this modified algebra in the presence of quintessence matter surrounding the black hole. When computing the thermodynamical variables for this black hole using this deformed algebra, we also examine the behavior of the isotherms for this black hole. We compare each computed result with the standard result of the Schwarzschild black hole. As we can see, a remnant of the black hole is indicated by this modified algebra, which is seen to be a plausible solution to the information loss problem.

1 Introduction

A fascinating prediction of the general theory of relativity is the possibility of the existence of black holes. Astronomical observation has now made it clear that black holes might exist in nature. Strong evidence for the existence of black holes includes the observation of gravitational waves, the success in capturing the shadow of the $M87*$ supermassive black hole, and the subsequent success in capturing the shadow of the $SgrA*$. Naturally, the study of different aspects of black holes has gained renewed interest. It is generally agreed that black holes are thermal bodies, and the laws of thermodynamics can be used to describe their characteristics. This proposal was put forward a little more than half a century ago by Hawking and Bekenstein [1–3]. According to the information obtained from these papers [1–3], a black hole is a thermodynamic system that emits radiation with a characteristic temperature closely related to its surface gravity. Moreover, the entropy of these objects is linearly proportional to the area covered by the event horizon in Planck units. With these innovative proposals, they tested several models of black holes by computing Hawking temperature and mass in the framework of statistical mechanics and reached the conclusion that the basic laws of thermodynamics are no longer violated [4].

According to the most recent observational astronomical data, our Universe is expanding at an accelerating rate [5–7]. To explain this acceleration, the existence of a distinctly different form of matter having negative pressure has been postulated. This dark energy content is expected to be around seventy percent of the total energy density of the Universe. Although the use of the concept of the cosmological constant could have been an effective technique to characterize this acceleration, its experimental value is found to be significantly smaller than what is theoretically anticipated [8]. Different alternative models were consequently suggested. These compelling models [9–14] differ from one another in terms of the parameters relating to the ratio of pressure to the density of dark energy. These models are essentially based on the dynamics of specific scalar fields. Black hole thermodynamics is thought to be significantly influenced by the dark energy surrounding them. The literature of recent times contains a wealth of studies on this subject. We suggest readers to the review [15] and the references therein for an in-depth discussion and evaluation of those models. The quintessence matter, which is notable among the dark energy contenders, was taken into consideration by Kiselev in the foundational work, and a useful study has been conducted. The literature of recent times contains a wealth of studies on this subject [16]. A few years later, Chen et al. investigated the Hawking radiation of any D-dimensional, spherically symmetric, static black hole, where the very nature of matter is thought to be a potential candidate for dark matter [17]. The study of the thermodynamic characteristics of black holes, inspired by the quintessence, attracted more and more attention throughout time. Therefore, exploration of thermodynamics in the presence of quintessence matter for the Reissner–Nordstrom (NR) black hollow [18], the NR black hollow in de-Sitter spacetime [19], Narai-kind black holes [20, 21] was carried out. Bardeen-type regular black holes were presented later in due course [22]. Shahjalal recently compared the thermodynamics of this black hollow in the presence of the quintessence matter

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[23], taking into account quantum correction to the Schwarzschild black hole. Thermodynamic parameters of a rotating nonlinear magnetic-charged black hole embedded with the quintessence matter were recently estimated by Ndogmo et al. [24].

It is well known that the famous Heisenberg algebra plays a pivotal role in the quantization of a theory. As a matter of fact, the role of gravitational interaction, which is a phenomenon associated with the quantization of gravity, is believed to be the most challenging problem in physics. Unfortunately, the entire wing has not yet been developed. Therefore, it has recently been the focus of numerous theoretical investigations to address the issues that arise when one tries to combine general relativity and quantum mechanics to obtain a comprehensive quantum gravity theory [25–30]. Since the quantum influence of gravity cannot be disregarded at a very large energy scale, a number of techniques, including string theory, loop quantum gravity, noncommutative geometry, etc. [31–36], have surfaced. All of these admirable and decent attempts, incidentally, suggest that there is a minimum measurable length of the order of the Planck length.

In order to render a faithful and decent account for the quantum gravity effect, GUP has been given substantial consideration. Therefore, the amendment of quantum correction due to gravity motivated through GUP is considered seriously and accepted as an adequate tool for describing black holes and their thermodynamic properties to capture quantum gravity correction. The prominence of the GUP can be attributed to the fact that it was first proposed as a sequel to perturbative string theory with addition an additional quadratic term in momentum along with conventional Heidelberg algebra [29, 30, 37–39]. Although it is has been found instrumental to capture the effect of quantum gravity, it is worth mentioning that such an extension is not unique. Hence, different GUP proposals appeared in the literature. A series of papers were perused adding only a quadratic term in momentum which was proposed initially as a modification of standard Heisenberg algebra. Later on, by inserting an extra linear term with the quadratic term in momentum further modification of Heisenberg algebra was pursued. Using the exponential term in momentum is the most general extension that has also has come in the literature. In the papers [40–52], the authors used several types of deformed algebra consistent with a minimum measurable length or a minimum measurable length with a maximum measurable momentum proposition. Consequently, a number of phenomenological methods were implemented in an effort to constrain the parameter associated with the GUP [53–56] in due course.

In the paper [57], the authors used the deformed algebra with a quadratic term in the momentum to study the black hole thermodynamics of a Schwarzschild black hole immersed in quintessence matter within the GUP framework. This allowed them to study quantum corrected thermodynamic variables inspired by GUP, such as temperature, entropy, heat capacity, energy density, etc. The generalization that includes both linear and quadratic terms in momenta together is often called linear and quadratic GUP (LQGUP). This GUP proposal was simulated by making the uncertainty principle compatible with Doubly Special Relativity (DSR) [58, 59]. Each of the non-equivalent algebraic deformations results in different new characteristics. The minimum measurable minimal length scenario occurred when the deformation is associated with only the quadratic term in the momentum [60]. The minimum measurable length along with maximum measurable momentum follows from the deformed algebra containing both the linear and quadratic terms (LQGUP) [58, 59]. Using different modified Heidelberg algebra, several studies have been carried out which have appeared in a series of publications [61, 61–67]. The LQGUP framework is exploited in [68] in order to make the Thomas–Fermi model consistent with the Planck scale.

In this work, we explore the thermodynamic properties of the Schwarzschild black hole within the LQGUP framework, which carries the coexistence of both linear and quadratic terms in momentum in the deformed Heisenberg algebra. The purpose of this project is to examine how novel properties like the existence of a minimal measurable length and a maximal observable momentum afforded by LQGUP contribute to our knowledge of the thermodynamics of Schwarzschild black holes surrounded by quintessential matter.

The paper is organized as follows. In Sect. 2, we introduce the metric taking into consideration the black hollow surrounded by the quintessence matter. In Sect. 3, we describe LQUP framework associated with the deformation of Heisenberg algebra through a linear and quadratic term in momentum. Section 4 is devoted to study the LQGUP inspired thermodynamical variables like temperature, pressure, heat capacity, and entropy. After having those thermodynamic features, in Sect. 5 with the precise admissible values of the quintessence state parameter, we study the GUP-corrected density and equation of state. Finally, in Sect. 6, we round up with a conclusion and discussion.

2 Description of generalization of Heisenberg algebra

The Heisenberg algebra is given by

$$[x, p] = i\hbar, \tag{1}$$

which corresponds to the celebrated Heisenberg principle

$$(\Delta x)(\Delta p) \geq \frac{1}{2}\hbar. \tag{2}$$

It is stated in the introduction that GUP has drawn a lot of attention since it offers a credible correction of quantum gravity. The significance of GUP can be traced to the fact that it is an outcome of perturbative string theory, and it can be formulated from standard Heisenberg algebra by adding a quadratic term in momentum [29, 30, 37–39]. The following is a generalization of Eq. (1) with that quadratic term in the momentum

$$(\Delta x)(\Delta p) \geq \frac{\hbar}{2}\big(1+\alpha(\Delta p)^2\big), \tag{3}$$

which correspond to the extended Heisenberg algebra [60]

$$[x, p] = \hbar\big(1+\alpha(\Delta P)^2\big), \tag{4}$$

where α is a parameter proportional to the Planck length that leads to a minimum uncertainty in position $(\Delta x)_{\min} = \sqrt{\alpha}$. While it has been demonstrated that this modification (3) is helpful in capturing the influence of quantum gravity, it is important to note that this extension is not the only one that can render this service. Later, Heisenberg algebra was further modified by adding a second linear term over quadratic term of the momentum. It was found in [59] that the simulation for this new extension came from double special relativity (DSR). It is supplied by

$$(\Delta x)(\Delta p) \geq \frac{1}{2}\big(1-2\eta(\Delta p)+4\eta^2(\Delta p)^2\big), \tag{5}$$

where η is called the GUP parameter; it is non-negative and proportional to the Planck length. The generalized algebra which follows from the LQGUP in Eq. (5) is

$$[x, p] = i(1-\eta p+2\eta^2 p^2). \tag{6}$$

This deformed algebra leads to a minimum uncertainty in the position as well as maximum uncertainty in momentum:

$$(\Delta x)_{\min} \approx \eta, \tag{7}$$

$$(\Delta p)_{\max} \approx \frac{1}{\eta}. \tag{8}$$

Here $\eta = \frac{\eta_0}{m_p c} = \eta_0 \frac{l_p}{\hbar}$, where $m_p c^2 = 10^{19}$ GeV, and the Planck length $l_p = 10^{-35}$ m. Unlike the quadratic generalization, the concept of maximum momentum along with the minimum length is admissible here and here lies the fundamental difference between these two. The minimum length and maximum momentum admissible to this deformed algebra, respectively, are

$$\Delta x \geq \Delta x_{\min} \approx \eta_0 l_p,\ \Delta p \leq \Delta p_{\max} \approx \frac{m_p c}{\eta_0}. \tag{9}$$

Note that Eq. (6) is satisfied by the following representation of position and momentum, respectively.

$$\begin{aligned} x_i &= x_{0i}, \\ p_i &= p_{0i}(1-\eta p+2\eta^2 p^2). \end{aligned} \tag{10}$$

The invariant phase space volume in D dimensions for this deformation needs the following modification

$$[D\mu] = \frac{d^D x d^D p}{J}, \tag{11}$$

where J is the jacobin for transformation which is given by [33, 61]

$$J = \left[1-\eta p+\left(\frac{2}{D+1}+\frac{1}{2}\right)\eta^2 p^2\right]^{D+1}. \tag{12}$$

3 Description of the metric

We take into account a static, spherically symmetric spacetime that is connected to a black hole encircled by quintessential matter. It was derived in the paper [16]. The basis for the derivation was that the vacuum was replaced in the exact solution of Einstein's equations by a slowly fluctuating quantum field with negative pressure, which simulated the accelerating expansion of the Universe.

$$\mathrm{d}s^2 = -F(r)\mathrm{d}t^2 + \frac{1}{F(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2, \tag{13}$$

where, for the Schwarzschild black hole, $N(r)$ can be taken as

$$F(r) = 1 - \frac{2M}{r} - \frac{\xi_{\tau_q}}{r^{(3\tau_q+1)}}, \tag{14}$$

Here τ_q is the quintessential state parameter and τ_{τ_q} is the positive normalization factor that depends on the density of quintessence matter and the mass of the black hole is specified by$M$. It is a remarkably popular model. A special feature of this metric is that it can explain the accelerated expansion of the universe when the consideration of the quintessence state parameter lies in the range $-1 < \tau_q < -1/3$. One of the other reasons for its popularity is its generic character: $\xi_{\tau_q} = 0$ corresponds to Schwarzschild metric, $\tau_q = 1/3$ corresponds to Reissner–Nordström metric, and $\tau_q = -1$ refers to the Schwarzschild–(anti)-de Sitter (Kottler) metric. Another intriguing reason is that it includes the effect of the anisotropic surrounding medium. The non-vanishing components of the stress–energy–momentum tensor of the quintessence matter fluid for the Kiselev black hole solution are given by

$$T_t^t = T_r^r = -\rho, \tag{15}$$

$$T_\theta^\theta = T_\phi^\phi = \frac{\rho}{2}(3\tau_q + 1). \tag{16}$$

The matter–energy density ρ_q has the form

$$\rho_q = -\frac{3\xi\tau_q}{2r^{3(\tau_q+1)}}. \tag{17}$$

, and the pressure is expressed as

$$P_q = \tau_q \rho_q. \tag{18}$$

Therefore, the matter–energy density takes only positive values; however, the pressure of the quintessence matter takes only negative values if the quintessence state parameter lies in the range $-1 < \tau_q < -1/3$.

With this settings, let us see the position of the event horizon. It is known that the solution of the equation

$$F(r)|_{r=r_H} = 1 - \frac{2M}{r} - \frac{\xi}{r^{(3\tau_q+1)}}|_{r=r_H} = 0, \tag{19}$$

gives the description of the horizon corresponding to this black hole. A careful look reveals that the solutions of Eq. (19) can be classified in three distinct cases for different values of τ_q: we find two horizons with radii, namely inner horizon (r_-) and outer horizon (r_+), for $\tau_q = -2/3$:

$$r_- = \frac{1}{2\xi}\left(1 - \sqrt{1 - 8M\xi}\right), \tag{20}$$

$$r_+ = \frac{1}{2\xi}\left(1 + \sqrt{1 - 8M\xi}\right). \tag{21}$$

However, for $\tau_q = -1/3$, only one horizon is found and the radius of the horizon is given by

$$r_H = \frac{2M}{1-\xi}. \tag{22}$$

A completely new situation arises for $\tau_q = -1$. A de-Sitter–Schwarzschild solution is obtained, if $\xi = \frac{\Lambda}{3}$ is chosen. Here Λ refers to the cosmological constant.

4 LQGUP-inspired thermodynamical features

In the vicinity of the Planck scale, quantum correction to gravity should be taken into account. However, mature formulation of quantum gravity is not in hand right now. There are several ways to incorporate the quantum effect of gravity in an indirect manner. A few instances include the usage of non-commutative spacetime, the development of the bumblebee field, and the modification of Heisenberg algebra by converting the Heisenberg uncertainty principle (HUP) to the generalized Heisenberg uncertainty principle (GUP). The existence of a minimal length scale is considered in the central issue since it follows from penetrative string theory. Although the provision of minimal length is absent from the standard Heisenberg algebra, the generalized uncertainty principle (GUP) allows for the introduction of that useful idea into the theory. A further substantial generalization derived from double special relativity (DSR) was ideally suited to describing the minimum measurable length and maximum measurable momentum as they were prominently provided in Eq. (6)

From the studies [42, 69–74], we have seen that the deformation parameter is not always positive. It can be negative, and even it can be regarded as a dynamic variable. With this in view, if we solve Eq. (9) we have a bound for ΔP:

$$\frac{(\eta+\Delta x)}{4\eta^2}\left[1 - \sqrt{1 - \frac{4\eta^2}{(\eta+\Delta x)^2}}\right] \le \Delta p \le \frac{(\eta+\Delta x)}{4\eta^2}\left[1 + \sqrt{1 - \frac{4\eta^2}{(\eta+\Delta x)^2}}\right]. \tag{23}$$

Since $\frac{4\eta^2}{(\eta+\Delta x)^2} \ll 1$, the left-hand side of the inequality can be considered as a small correction over the Heisenberg uncertainty corresponding to p, and the right-hand side can be regarded as an upper bound value to the uncertainty in momentum. So it reveals that Δp cannot be erratic so far increasing momentum uncertainty is considered. considered.

4.1 Computation of temperature and heat capacity

With this information, let us now study the effect of GUP on the thermodynamic properties of the Schwarzschild black hole surrounded by quintessence matter. In the semiclassical notion, the entropy (S) can be considered as a function of the hole area enclosed within the event horizon A. So, the characteristic temperature of the black hole can be expressed as follows [75]

$$T = \frac{\kappa}{8\pi}\frac{\mathrm{d}A}{\mathrm{d}S}, \tag{24}$$

where κ is the surface gravity at the outer horizon. It is defined as

$$\kappa = \lim_{r\to r_H}\sqrt{-\frac{g^{11}}{g^{00}}}\frac{(g^{00})'}{g^{00}} = \frac{1}{r_H}\left(1+\frac{3\xi\tau_q}{r_H^{3\tau_q+1}}\right). \tag{25}$$

Here upper prime ($'$) is indicating the differentiation with respect to r. If a black hole absorbs a particle, then its changes in the area are proportional to the mass and size of the particle mass which is associated with the uncertainties in momentum and position. This minimal change in the area will be reflected in the entropy. However, the change of entropy in this situation cannot be smatter than $ln2$ [75]. Therefore, we can write

$$\frac{\mathrm{d}A}{\mathrm{d}S} \simeq \frac{(\Delta A)_{\min}}{(\Delta S)_{\min}} \simeq \frac{\varepsilon}{ln2}(\Delta x)(\Delta p). \tag{26}$$

Here ε is used as a calibration factor. Now we proceed tp calculate $\frac{\mathrm{d}A}{\mathrm{d}S}$. To calculate this, we can consider position uncertainty as the diameter of the black hole $\Delta x \simeq 2r_H$. From Eq. (23), we can have the uncertainty in Δp. With this we have

$$\frac{\mathrm{d}A}{\mathrm{d}S} \simeq \frac{\varepsilon r_H}{2ln2}\frac{(\eta+2r_H)}{\eta^2}\left[1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right]. \tag{27}$$

Upon substituting of Eqs. (25) and (27) in Eq. (24), the following expression for T results

$$T = \frac{\varepsilon r_H}{16\pi ln2}\frac{(\eta+2r_H)}{\eta^2}\left[1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right]\left(1+\frac{3\xi\tau_q}{r_H^{3\tau_q+1}}\right), \tag{28}$$

In the absence of the quintessence and the correction due to the GUP $\eta = \xi = 0$, and $\xi_q = 0$ then Eq. (28) turns into $T = \frac{\varepsilon}{16\pi r_H ln2}$ that should correspond to the Hawking temperature, $T = \frac{1}{4\pi r_H}$ [1, 2]. Therefore, the calibration factor in this situation turns out to be $\varepsilon = 4ln2$. Hawking temperature with the correction due to GUP can be written as (28)

$$T = \frac{1}{4\pi}\frac{(\eta+2r_H)}{\eta^2}\left[1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right]\left(1+\frac{3\xi\tau_q}{r_H^{3\tau_q+1}}\right). \tag{29}$$

Setting only $\eta \to 0$, in Eq. (29), we get the Hawking temperature of Schwarzschild black hole surrounded by the quintessence when no correction due to the generalization of Heisenberg uncertainty principle is absent.

$$T = \frac{1}{4\pi r_H}\left(1+\frac{3\xi\tau_q}{r_H^{3\tau_q+1}}\right). \tag{30}$$

The characteristics temperature of a black hole must be real valued which supplies the interesting information that the radius of the black hole has to satisfy the condition

$$r_H \leq \sqrt[(3\tau_q+1)]{-3\xi\tau_q}, \tag{31}$$

which implies that it is bounded from the above. A careful look reveals that for $\xi > 0$ and $\tau_q < 0$, the radius of the horizon has a positive root unless the quintessence state parameter takes the value $\tau_q = \frac{-1}{3}$. For $\tau_q = \frac{-1}{3}$, the horizon radius is not bounded from above. In the presence of quintessence matter encompassing the black hole, the greatest possible event horizon may obtain. It can be seen in the way that the presence of quintessence has an enhancing tendency of entropy.

Furthermore, Eq. (29) ensures that the modified Hawking temperature depends not only on the properties of the black hole but also it acquires a correction due to the use of GUP and that gives rise to one more constraint.

$$r_H \geq \frac{\eta}{2}. \tag{32}$$

Equation (32) confirms the lowest value of the radius of the black hole. Therefore, maximal temperature along with the GUP correction is found out as

$$T = \frac{1}{2\pi\eta}\left[1+3\xi\tau_q\left(\frac{\eta}{2}\right)^{(-3\tau_q-1)}.\right] \tag{33}$$

If the surroundings of the black hole are free from quintessence matter ξ tales a vanishing value and it is noticed that the temperature of the black hole with GUP correction will remain restricted within the range

$$0 < T \leq \frac{1}{2\pi\eta}. \tag{34}$$

Depending on the allowed values of the equation of state parameter of the surrounding medium, the temperature of the black hole shows different possibilities when the correction due to GUP is present. First consider the case when $\tau_q = -1$. The GUP-corrected temperature of the Schwarzschild black hole surrounded by the quintessence matter is found to be

$$T_{(\tau_q=-1)} = \frac{1}{4\pi}\frac{(\eta+2r_H)}{\eta^2}\left(1-3\xi r_H^2\right)\left[1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right], \tag{35}$$

and the event horizon radius can only take values lying in following range

$$\frac{1}{\sqrt{3\xi}} \geq r_H \geq \frac{\eta}{2}. \tag{36}$$

As a result, the temperature will lie within the range

$$0 \leq T_{(\tau_q=-1)} = \frac{4-3\xi\eta^2}{8\pi\eta}. \tag{37}$$

Note that, in the absence of the GUP, there is no lower bound of the radius. Therefore, there is no upper limit for temperature.

For $\tau_q = \frac{-2}{3}$, the GUP-corrected temperature of the Schwarzschild black hole in presence of surrounding matter will be

$$T_{(\tau_q=-2/3)} = \frac{1}{4\pi}\frac{(\eta+2r_H)}{\eta^2}(1-2\xi r_H)\left[1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right]. \tag{38}$$

The permitted range of event horizon radius will be

$$\frac{1}{2\xi} \geq r_H \geq \frac{\eta}{2}. \tag{39}$$

As a consequence, the temperature will lie in the range

$$0 \leq T_{(\tau_q=-2/3)} = \frac{1-\xi\eta}{2\pi\eta}. \tag{40}$$

For $\tau_q = -1/3$, the GUP-corrected temperature results to be

$$T_{(\tau_q=-1/3)} = \frac{1}{4\pi}\frac{(\eta+2r_H)}{\eta^2}(1-\xi)\left[1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right]. \tag{41}$$

In this situation, an upper bound on the horizon radius ceases to emerge. Hence, temperature of the black hole will be restricted within the following range.

$$0 \leq T_{(\tau_q=-1/3)} = \frac{1-\xi}{2\pi\eta}. \tag{42}$$

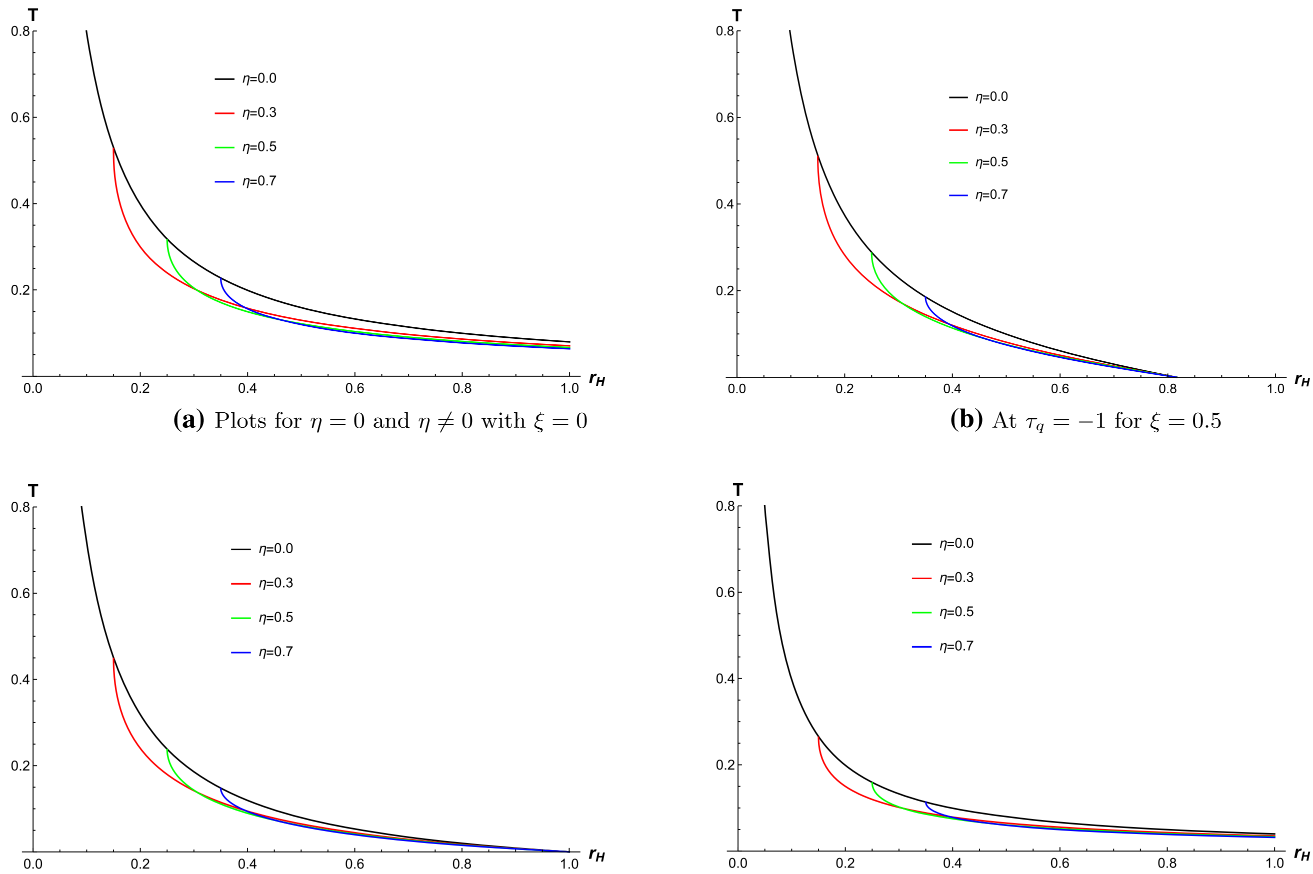
In Fig. 1, we have the temperature of the black hole versus the horizon radius in the usual, i.e., when the Heidelberg uncertainty principle (HUP) is maintained and in the GUP-motivated situation. In the absence of quintessence matter, temperature has no upper bound in the usual HUP-involved case, whereas there exists upper bound of temperature for the GUP-motivated case.

The black holes are considered as thermodynamical system. So it is instructive to calculate the correction due to the GUP on the heat capacity of the Schwarzschild black hole in the presence of encompassing quintessence matter. It can be will be computed by the straightforward use of the definition

$$C = \frac{\mathrm{d}M}{\mathrm{d}T}. \tag{43}$$

Using the definition (43), we obtain

$$C = -\frac{2\pi\eta^2\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\left(1+\frac{3\xi\tau_q}{r_H^{3\tau_q+1}}\right)}{\left[\left(2-\frac{3\xi\eta\tau_q(3\tau_q+1)}{r_H^{3\tau_q+2}}-\frac{8\xi\tau_q^2}{r_H^{3\tau_q+1}}\right)\left(1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right)+\frac{4\eta^2}{(\eta+2r_H)^2}\left(\frac{6\xi\tau_q}{r_H^{3\tau_q+1}}+\frac{3\xi\eta\tau_q(3\tau_q+1)}{r_H^{3\tau_q+2}}+\frac{18\xi\tau_q^2}{r_H^{3\tau_q+1}}\right)\right]}. \tag{44}$$



(a) Plots for $\eta = 0$ and $\eta \neq 0$ with $\xi = 0$

(b) At $\tau_q = -1$ for $\xi = 0.5$

(c) At $\tau_q = -2/3$ for $\xi = 0.5$

(d) At $\tau_q = -1/3$ for $\xi = 0.5$

Fig. 1 Plots of temperature (T) versus horizon radius (r_H)

It is noteworthy that the black hole ceases to exchange any radiation with its surrounding space if the heat acquires a vanishing value. This situation corresponds to the *black hole remnant*. It is striking to observed that at $r_{\rm rem} = \frac{\eta}{2}$, Eq. (44) approaches to a vanishing value which transpires that black hole remnant exists. Like other GUP, the extension associated with LQGUP suggests a remnant. An unlimited amount of information may be stored in it. This information lies in the absolute future with respect to an external observer and remains inaccessible forever for the external observer. Therefore, the information laid in the absolute future should not lead to any paradoxes in calculating physical processes observed by external observers. This suggests one of the possible ways for a resolution of the information paradox [76]. Moreover, the stable remnants can serve as dark matter candidates [77, 78]. It agrees with the conjecture of saving a black hole from the danger of the information loss paradox. Actually, it is now accepted that from the perspective of quantum gravity black holes preserves information. The existence of black hole remnants has also been predicted in the context of noncommutative geometry [79, 80]. Therefore, the plausible reason for observing remnants may be thought of as it is due to the quantum correction that has entered into the picture due to the implementation of deformed Heisenberg algebra or non-commutative spacetime setting [79, 80]

Tt is possible to see the presence of remnant temperature of the black hole by substituting $r_{\rm rem} = \frac{\eta}{2}$ in Eq. (29):

$$T_{\rm rem} = \frac{1}{2\pi\eta}\left[1 + 3\xi\tau_q\left(\frac{\eta}{2}\right)^{-3\tau_q-1}\right], \tag{45}$$

and indeed, there exists a remnant mass

$$M_{\rm rem} = \frac{\eta}{4}\left[1 - \xi\left(\frac{\eta}{2}\right)^{-3\tau_q-1}\right]. \tag{46}$$

In the absence of quintessence matter, $\xi = 0$; therefore, GUP-corrected specific heat and the remnant temperature become

$$C = -\frac{\pi\eta^2\sqrt{1 - \frac{4\eta^2}{(\eta+2r_H)^2}}}{\left(1 - \sqrt{1 - \frac{4\eta^2}{(\eta+2r_H)^2}}\right)}, \tag{47}$$

$$T_{\text{rem}} = \frac{1}{2\pi\eta}. \tag{48}$$

It is remarkable that for $\eta \to 0$, Eq. (47) renders standard heat capacity function $C = -2\pi r_H^2$. Remnant temperature is not found to be zero, and correspondingly, remnant mass also ceases to zero.

Let us now focus on the expressions of GUP-corrected specific heat and remnant temperature to study correction entered into these thermodynamical variables for allowed values of the quintessence state parameter if we set $\tau_q = -1$ the specific heat C and the remnant temperature T_{rem} reduce to

$$C = -\frac{2\pi\eta^2\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\left(1-3\xi r_H^2\right)}{\left[\left(2-6\xi\eta r_H-8\xi r_H^2\right)\left(1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right)+\frac{4\eta^2}{(\eta+2r_H)^2}\left(12\xi r_H^2+6\xi\eta r_H\right)\right]}, \tag{49}$$

$$T_{\text{rem}} = \frac{1}{2\pi\eta}\left[1-\frac{3\xi\eta^2}{4}\right]. \tag{50}$$

Setting another allowed value for the state parameter for $\tau_q = -2/3$, the above thermodynamical quantities acquire the following expression.

$$C = -\frac{2\pi\eta^2\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\left(1-2\xi r_H\right)}{\left[\left(2-2\xi\eta r_H-\frac{32}{9}\xi r_H\right)\left(1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right)+\frac{4\eta^2}{(\eta+2r_H)^2}\left(4\xi r_H+2\xi\eta r_H\right)\right]}, \tag{51}$$

$$T_{\text{rem}} = \frac{1}{2\pi\eta}[1-\xi\eta]. \tag{52}$$

One more allowed value of τ_q is there which is $\tau_q = -1/3$; the above thermodynamical quantities with this value of τ_q turn into

$$C = -\frac{2\pi\eta^2\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\left(1-\xi\right)}{\left[\left(2-\frac{8}{9}\xi\right)\left(1-\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right)\right]}, \tag{53}$$

$$T_{\text{rem}} = \frac{1}{2\pi\eta}[1-\xi]. \tag{54}$$

In Fig. 2, we depicted a specific heat function versus the horizon radius. In the absence of quintessence matter, specific heat and lower value of horizon radius increase with the rise of the GUP parameter.

4.2 Computation of entropy, energy density, and isothermal behavior

The entropy of a black hole is an intriguing issue. We now turn to calculate the entropy of the Schwarchlid black hole when it is surrounded by the quintessence matter in the GUP framework. Using the standard definition

$$S = \int \frac{\mathrm{d}M}{T}, \tag{55}$$

we find that (55)

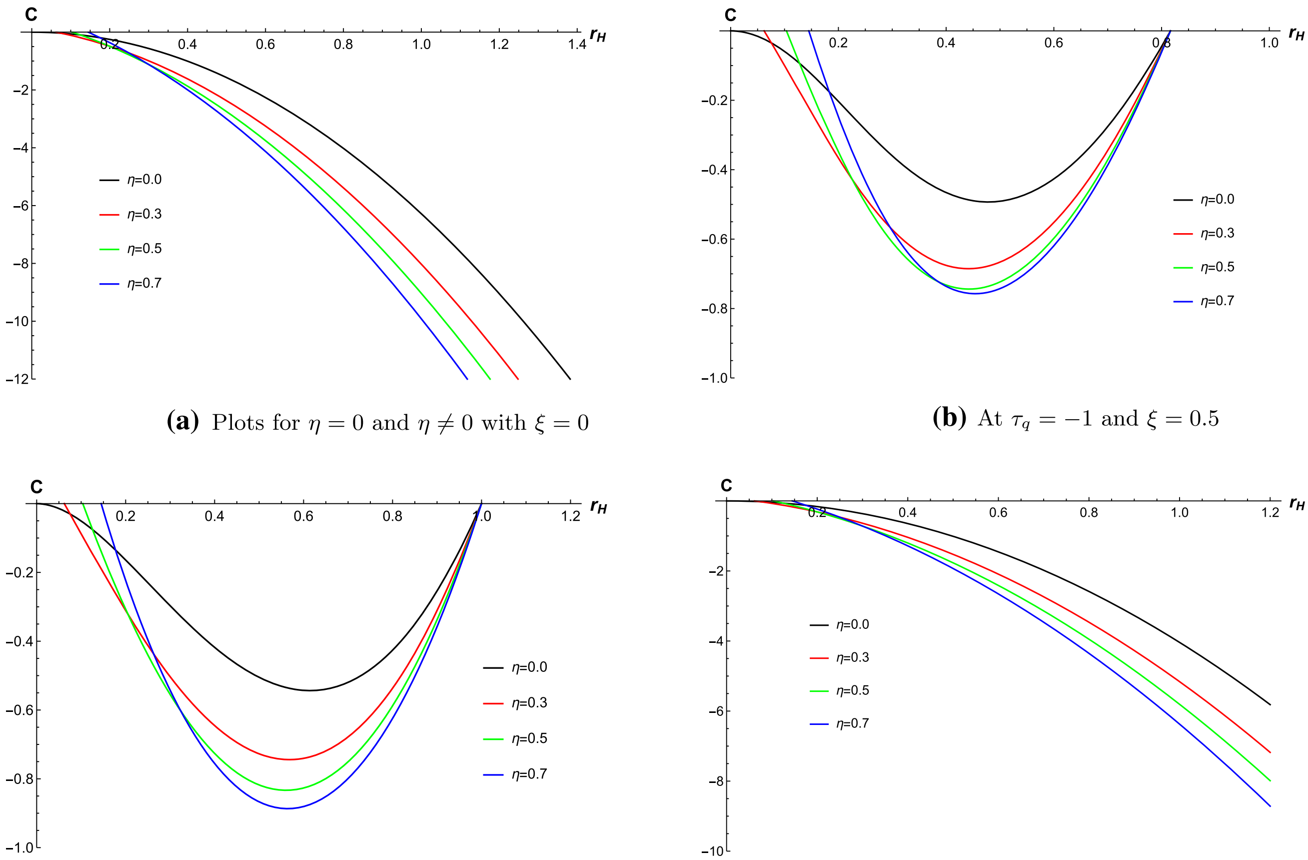
$$S = \frac{\pi}{2}(\eta+2r_H)^2\left[1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right] - 2\pi\eta^2 ln(\eta+2r_H) - 2\pi\eta^2\left[1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right]. \tag{56}$$

by using Eqs. (32) and (29). From Eq. (56), it is observed that quintessence matter surrounding the black hole does not have any influence on the black entropy of the black hole. However, it has crucial dependence on the parameter η. So generalized uncertainty takes significant correction on the entropy of the black hole. In the absence of η, the entropy reduces to the Bekenstein limit

$$S = 4\pi r_H^2 = \frac{A}{4}. \tag{57}$$

Equation (17) enables us to have an energy-matter density of the quintessence which also has crucial dependence on the parameter η associated with the generalization Heisenberg algebra.

$$\rho_q = -\frac{3\xi\tau_q}{2}\left[\frac{1}{8}(\eta+2r_H)^2\left(1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right) - \frac{1}{2}\eta^2 ln(\eta+2r_H) - \frac{1}{2}\eta^2 ln\left(1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right)\right]^{-\frac{(3\tau_q+3)}{2}}. \tag{58}$$



(a) Plots for $\eta = 0$ and $\eta \neq 0$ with $\xi = 0$

(b) At $\tau_q = -1$ and $\xi = 0.5$

(c) At $\tau_q = -2/3$ and $\xi = 0.5$

(d) At $\tau_q = -1/3$ and $\xi = 0.5$

Fig. 2 Plots of specific heat (C) versus horizon radius (r_H)

When $\eta \to 0$, it reduces to

$$\rho_q = -\frac{3\xi\tau_q}{2r_H^{3\tau_q+3}}. \tag{59}$$

Based on different allowed values of quintessence state parameters, the energy-matter density function ρ_q acquires different expressions. For $\tau_q = -1$, Eq. (58) turns into

$$\rho_q = -\frac{3\xi}{2}. \tag{60}$$

Note that it is independent of η. So for this value of the state parameter, the energy-matter density does not acquire any correction due to the generalization of uncertainty. However, for the other allowed values of the state parameter, the situation is different. In those cases, the correction due to the use of generalized uncertainty is given as follows. For $\tau_q = -2/3$, Eq. (58) becomes

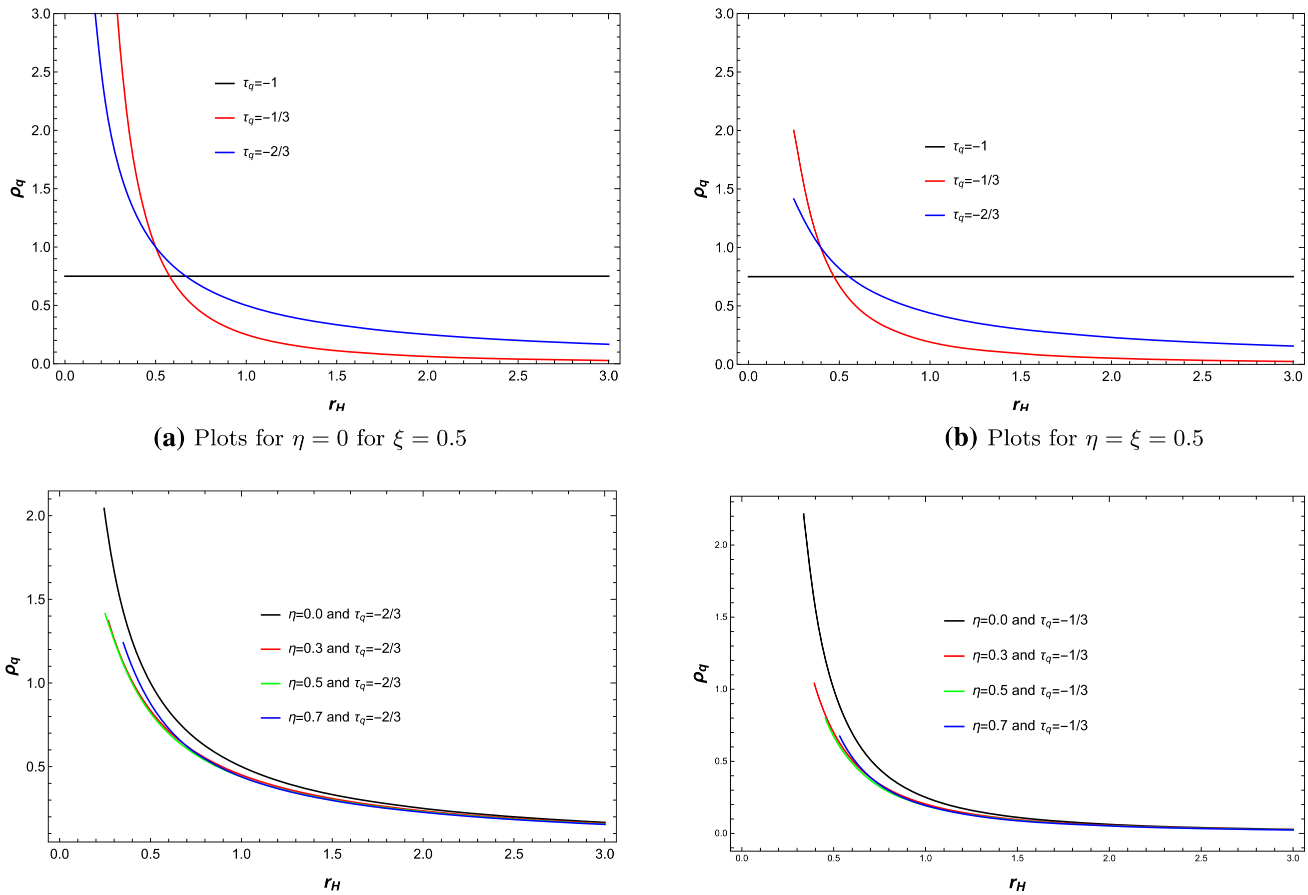
$$\rho_q = \xi\left[\frac{1}{8}(\eta+2r_H)^2\left(1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right) - \frac{1}{2}\eta^2 ln(\eta+2r_H) - \frac{1}{2}\eta^2 ln\left(1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right)\right]^{-\frac{1}{2}}, \tag{61}$$

and in the limit $\eta \to 0$, it reduces to

$$\rho_q = \frac{\xi}{r_H}. \tag{62}$$

For $\tau_q = -1/3$, Eq. (58) acquires the following simplification:

$$\rho_q = \frac{\xi}{2}\left[\frac{1}{8}(\eta+2r_H)^2\left(1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right) - \frac{1}{2}\eta^2 ln(\eta+2r_H) - \frac{1}{2}\eta^2 ln\left(1+\sqrt{1-\frac{4\eta^2}{(\eta+2r_H)^2}}\right)\right]^{-1}, \tag{63}$$



(a) Plots for $\eta = 0$ for $\xi = 0.5$

(b) Plots for $\eta = \xi = 0.5$

(c) At $\tau_q = -2/3$ for $\xi = 0.5$

(d) At $\tau_q = -1/3$ for $\xi = 0.5$

Fig. 3 Plots of energy-matter density of quintessence (ρ_q) versus horizon radius (r_H)

and in the limit $\eta \to 0$, it becomes

$$\rho_q = \frac{\xi}{2r_H^2}. \tag{64}$$

The behavior of the energy-matter density of quintessence with respect to horizon radius is shown in Fig. 3. In Fig. 4a and b, the behavior of energy-matter density for $\eta = 0$ and $\eta \neq 0$, respectively, is shown graphically. We observe that for $\tau_q = -1$ the energy-matter density is independent of horizon radius.

The study of the equation of states of the black hole is also instructive when the generalization of the Heisenberg uncertainty relation is made in use. So attempts are made to derive equations of states with the help of the relations between the pressure and the quintessence matter–energy density which have already been derived in Eqs. (18) and (17). It enables us to express the Mass of the black hole in terms of pressure and horizon radius as follows.

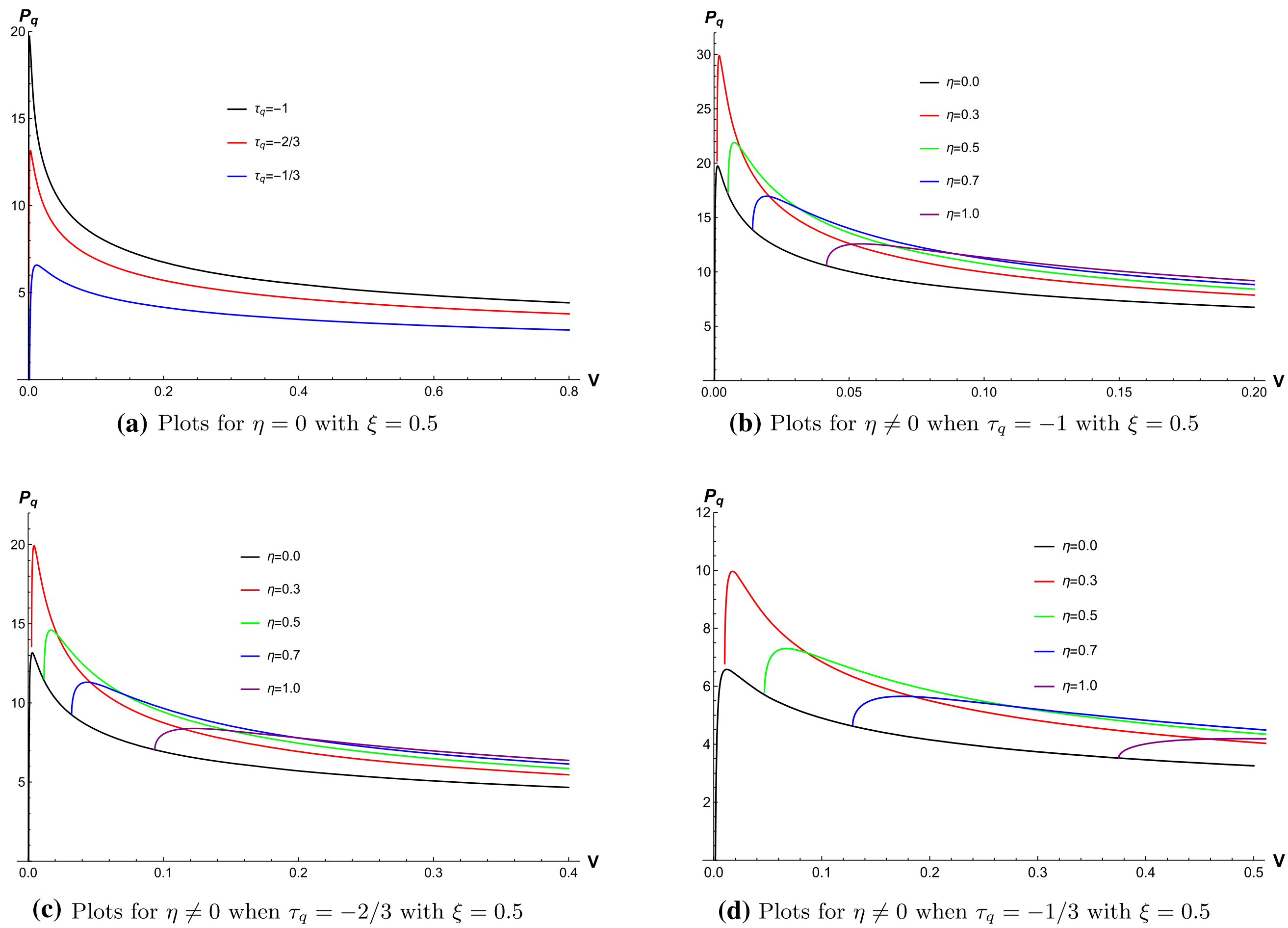
$$M = \frac{r_H}{2} + \frac{P_q r_H^3}{3\tau_q^2}. \tag{65}$$

Using thermodynamic relation $V = \frac{\partial M}{\partial P_q}$, and Eq. (65), we express volume in terms of the radius of the horizon of the black hole:

$$V = \frac{r_H^3}{3\tau_q^2}. \tag{66}$$

Of course, the expression (66) has crucial dependence on the state parameter of the quintessence matter [81]. Then, by the use of Eq. (29) we land on the expression of temperature:

$$T = \frac{1}{4\pi\eta^2}\left(\eta + 2\sqrt[3]{3\tau_q^2 V}\right)\left[1 - \sqrt{1 - \frac{4\eta^2}{(\eta + 2\sqrt[3]{3\tau_q^2 V})^2}}\right]\left(1 - \frac{2P_q(3\tau_q^2 V)^{2/3}}{\tau_q}\right). \tag{67}$$



(a) Plots for $\eta = 0$ with $\xi = 0.5$

(b) Plots for $\eta \neq 0$ when $\tau_q = -1$ with $\xi = 0.5$

(c) Plots for $\eta \neq 0$ when $\tau_q = -2/3$ with $\xi = 0.5$

(d) Plots for $\eta \neq 0$ when $\tau_q = -1/3$ with $\xi = 0.5$

Fig. 4 Isotherm for $T = 1$: pressure (P_q) versus volume (V) curve

Now to find out the equation of state we have to set the desired isotherm. For simplicity, we chose $T = 1$ isotherm. In this specific situation, the equation of state that is obtained from Eq. (67) for non-vanishing η is

$$P_q = \frac{\tau_q}{2(3\tau_q^2 V)^{2/3}}\left[1 - \pi\left(\eta + 2\sqrt[3]{3\tau_q^2 V}\right)\left[1 + \sqrt{1 - \frac{4\eta^2}{(\eta + 2\sqrt[3]{3\tau_q^2 V})^2}}\right]\right]. \tag{68}$$

In the limit $\eta \to 0$, it (68) turns into

$$P_q = \frac{\tau_q}{2(3\tau_q^2 V)^{2/3}}\left[1 - 4\pi(3\tau_q^2 V)^{1/3}\right]. \tag{69}$$

In general, pressure is a real quantity. Therefore, in order to obtain a real-valued pressure for $\eta \neq 0$ the following restriction is to be maintained:

$$V \geq \frac{1}{3\tau_q^2}\left(\frac{\eta}{2}\right)^3, \tag{70}$$

due to the fundamental constraint on $r_H \geq \frac{\eta}{2}$ associated with the GUP framework.

Like the other thermodynamical quantities let us now examine the equation of states for three different values of the quintessence state parameter. First consider the case for $\tau_q = -1$; Eq. (68) with this specified value of τ_q turns into

$$P_q = -\frac{1}{2(3V)^{2/3}}\left[1 - \frac{4\pi\eta^2}{\left(\eta + 2\sqrt[3]{3V}\right)\left[1 - \sqrt{1 - \frac{4\eta^2}{(\eta + 2\sqrt[3]{3V})^2}}\right]}\right]. \tag{71}$$

If now set $\eta = 0$, we get

$$P_q = -\frac{1}{2(3V)^{2/3}}\left[1 - 4\pi\left(\sqrt[3]{3V}\right)\right]. \tag{72}$$

For $\tau_q = -2/3$, Eq. (68) reduces to

$$P_q = -\frac{1}{3(\frac{4V}{3})^{2/3}}\left[1 - \frac{4\pi\eta^2}{\left(\eta + 2\sqrt[3]{\frac{4V}{3}}\right)\left[1 - \sqrt{1 - \frac{4\eta^2}{(\eta+2\sqrt[3]{\frac{4V}{3}})^2}}\right]}\right], \tag{73}$$

, and in the absence of the correction due to the generalization of Heisenberg uncertainty relation, the equation of states will be obtained if we set $\eta = 0$, and in this situation, we get the following simplified version of the equation of states:

$$P_q = -\frac{1}{3(\frac{4V}{3})^{2/3}}\left[1 - 4\pi\left(\sqrt[3]{\frac{4V}{3}}\right)\right]. \tag{74}$$

For the quintessence state parameter $\tau_q = -1/3$, Eq. (68) takes the form

$$P_q = -\frac{1}{6(\frac{V}{3})^{2/3}}\left[1 - \frac{4\pi\eta^2}{\left(\eta + 2\sqrt[3]{\frac{V}{3}}\right)\left[1 - \sqrt{1 - \frac{4\eta^2}{(\eta+2\sqrt[3]{\frac{V}{3}})^2}}\right]}\right], \tag{75}$$

and in the limit $\eta \to 0$ it reduces to the following simplified form.

$$P_q = -\frac{1}{6(\frac{V}{3})^{2/3}}\left[1 - 4\pi\left(\sqrt[3]{\frac{V}{3}}\right)\right]. \tag{76}$$

This completes the description of the equation of state for different allowed values of the quintessence state parameter. In Fig. 4, we have given the plot of the P-V isotherms $T = 1$. Figure 4a portrays the results for $\eta = 0$, and it shows that with the increases in volume, the pressure decreases slowly for the higher quintessence state parameter. In Fig. 4b–d, the effects of the η on the equation of states for the quintessence state parameter $\tau_q = -1, -\frac{2}{3}, -\frac{1}{3}$, respectively, are shown. It is noteworthy that pressure increases to maximum value for any chosen value of the parameter η, but in the asymptotic region it follows the isotheres for $\eta = 0$.

5 Discussion and conclusion

This work discusses the effects of LQGUP on the thermodynamic properties of Schwarzschild black holes in the presence of quintessential matter. LQGUP is used to describe the impact of quantum gravity without going through the formal quantization of gravitational theory. Needless to say, no formal quantization of the theory of gravity is available yet. Because of this, getting quantum correction via GUP is exciting and important in its own right. There are several GUP proposals, but quadratic and linear–quadratic GUP is of particular interest. The first one is related to perturbative string theory, and the second proposal was simulated by making the uncertainty principle compatible with the double special relativity (DSR). In the first proposal, thermodynamic aspects of Kiserev black holes were investigated in [57]. Therefore, an extension with LQGUP was another possibility to explore what we have carried out here in detail.

Using the laws of thermodynamics, we have calculated the heat capacity and entropy function of the LQGUP-corrected black hole. In the presence of this new deformation algebra, a nonzero residual temperature is observed. From this perspective, we also evaluate the black hole equation of state. It turns out that the quantum effect slightly increases the energetic matter density function which is in sharp contrast with the result obtained in [42] by using quadratic GUP. Finally, we examine the pressure-volume isotherms of black holes. We observe the effect of the deformation algebra on the pressure when the volume is small. We graphically compare the results in the context of usual and deformed Heisenberg algebras.

In addition, we show and compare the effect of LQGUP corrections on the thermal properties of black holes for three different values of the quintessential state parameter. Like other GUP proposals, the LQGUP extension too suggests a remnant. It can keep stored an unlimited amount of information. This information may be considered as it is located in the absolute future with respect to an external observer and remains inaccessible forever to the external observer. Therefore, computing physical events viewed by outside observers should not encounter any paradoxes due to information laid out in the absolute future. This suggests one approach to solving the information paradox as noted in [76]. Additionally, the stable remnants can be regarded as potential candidates for dark matter [77, 78]. The existence of a remnant may potentially be associated with the significant phenomenological implications for the finding of black holes at the Large Hadron Collider (LHC) [82, 83]. Therefore, the viable presence of remnants may additionally be belied that it would be due to quantum correction attributable to the implementation of deformed Heisenberg algebra or the calling for a non-commutative spacetime framework [79, 80]

Besides neutrino pair annihilation $\nu\bar{\nu} \rightarrow e^{-}e^{+}$ was studied in the framework of BH surrounding by a quintessence matter in [84, 85]. It was shown in [84] that a shift of the photosphere radius was found to occur in the allowed range of parameters of the quintessence model, as compared to the one computed in the general theory of relativity, and consequently, an enhancement of the rate of admission energy was found to take place. Such an enhancement could be relevant for the generation of gamma-ray bursts in close neutron star binary mergers, for which neutrino pairs annihilation had been proposed as a possible source, and it was argued that the parameter ξ present in the quintessence sector could be constrained. The impact of the generalized uncertainty on neutrino pair annihilation process could be a follow-up investigation.

Data Availability Statement The article is based on analytical calculation. The data used here are generated by numerical computation.

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