



# Dispersive solitary wave solutions for the dynamical soliton model by three versatile analytical mathematical methods

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**Abstract** The solitary wave solutions gained well-reputed significance because of their peculiar characteristics. Solitary waves are spatially localized waves and are found in a variety of natural systems from mathematical physics and engineering phenomena. This manuscript deals the different solitary wave solutions that have a great significance in mathematical physics. Various solutions are recovered in single and combo shapes like bright, dark, singular, bright-dark, and dark-singular solitons by the virtue of the generalized exponential rational function method (GERFM),  $(\frac{G'}{G^2})$ -expansion function method and the generalized Kudryashov method. Besides, the singular periodic wave and rational function solutions are also derived. The physical behavior of the reported results is sketched through several 3 dimensional, and 2 dimensional profiles with the assistance of suitable parameters. The acquired results are valuable in grasping the elementary scenarios of nonlinear sciences as well as in the related nonlinear higher dimensional wave fields. The achieved outcomes have been verified by putting them into the governing equation with the aid of Mathematica. Thus our strategies through the fortress of representative calculations give a functioning and intense mathematical execute for tackling complicated nonlinear wave problems. We anticipate, it will contribute us to observe the waves that occur in nonlinear complicated phenomenas. We believe that this work is timely and will be of interest to a broad range of experts involved in modeling.

## 1 Introduction

This world has many complicated natural phenomena with a broad scope of mathematical applications. The mathematical models of the nonlinear physical phenomena are illustrated explicitly by the nonlinear evolution equations (NLEEs) that have a significant influence on the investigation of nonlinear sciences. So, recently, obtaining exact soliton solutions to NLEEs with the help of computer programs that make repetitious and monotonous mathematical computations easier, has been a marvelous field for analysts and researchers. NLEEs play out an extraordinary part in depicting the actual behavior of real phenomena and dynamical processes in fluid mechanics, optical fibers, material science, geochemistry, ocean engineering, geophysics, mathematical physics, plasma physical science, and numerous other logical regions. Nonlinear science is one of the best astonishing fields for investigators in this bleeding-edge season of science. To track down the analytically or exact results has been the focal point of researchers because of its fundamental commitment to examine the genuine element of the frameworks. As we all know, scholars have devised several methodologies and mathematical tools to aid in the discovery of exact solutions to NLEEs, and each method is tailored to a certain sort of solution [1–8].

Moreover, the concept of soliton or solitary waves is a phenomenon that has attracted the attention of people of all ages. When there is a disturbance in the phenomenon, waves are formed. Soliton interactions occur when two or more solitons come near enough to interact. Soliton waves have risen to prominence among all the waves found in nature due to their fundamental properties that are rarely found in other waves. Due to dispersive effects, the velocity of soliton waves varies with wavelength and is significantly different from the velocity of energy propagation. Moreover, linear effects of soliton waves can be shown dominantly in breaking roll waves on the seashore. Wave spreading effects, also known as dispersive effects, and wave focusing effects, also known as nonlinear effects, have shown a moderate balance in generating waves with a permanent shape. Furthermore, the optical solitons are one of the most significant domains of research in the branch of nonlinear optics. Particularly, the investigation of dispersive optical solitons is getting a lot of consideration in the present days. This tendency is actually continuing, there are numerous new outcomes that are constantly being published in the context of given model. Besides, the exact solutions are necessary for observing the physical properties of mathematical modeled problems. Mathematical techniques are being established in a variety of ways. In previous findings, several analytical schemes have been used to observe the analytic solutions such as the  $(G'/G)$ -expansion method [9], the anstaz approach [10, 11], the trial equation technique [12], the adomian decomposition method [13], the variation iteration method (VIM) [14], the modified  $\exp(-\Phi(\zeta))$ -expansion method [15], the direct algebraic method [16], the extended Fan sub-equation

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method [17], the  $F$ -expansion technique [18], the generalized exponential rational function method [19], new  $\Phi^6$ -model expansion method [20] and several others [21–26].

The key idea of this study is to conceive a variety of soliton solutions in the (2+1)-dimensional soliton equation by employing three analytical methods. In this article we will consider the (2+1)-dimensional soliton equation given by [27]

$$\begin{aligned} i\Psi_t + \Psi_{xx} + \Psi\Phi &= 0, \\ \Phi_t + \Phi_y + (\Psi\Psi^*)_x &= 0, \end{aligned} \tag{1}$$

where  $*$  represents the complex conjugate. The  $\Phi = \Phi(x, y, t)$  is real and  $\Psi = \Psi(x, y, t)$  is imaginary function. Here  $x, y$  and  $t$  represent the spatial domains and time respectively. The governing equation is similar to the integrable Zakharov equation in plasma physics which shows a significant role in several physical applications and governs the behavior of weakly nonlinear ion-acoustic waves in a plasma. The physically most important example involves the interaction between the Langmuir and ion-acoustic waves in plasmas. So far many studies for the (2 + 1)-dimensional soliton have been done in literature, for detail see references [28–32]. The authors attained a few solutions to the above equation. The more effective, novel solitary wave solutions of the given model will be achieved via three proposed methods.

The layout of this manuscript is arranged as follows: The description of GERFM, ( $\frac{G'}{G^2}$ )-expansion function method and the generalized Kudryashov method, is discussed in Sec. 2. Extraction of solitary wave solutions is given in Sec. 2. The Sec. 3 consist of results and discussion along with the graphical representation. The conclusion is revealed in Sec. 4.

## 2 Description of the methods

We present brief description of the proposed methods in this section.

Suppose a NLPDE,

$$\Delta(\Theta, \Theta_t, \Theta_x, \Theta_y, \Theta_{xx}, \Theta_{yy}, \Theta_{xt}, \dots) = 0, \tag{2}$$

where  $\Delta$  is a polynomial in its arguments. We start with hypothesis as:

$$\Theta(x, y, t) = \phi(\zeta) \quad \text{and} \quad \zeta = x + \eta y - \mu t,$$

putting the traveling wave transformation into Eq. (2), yields NODE as,

$$\chi(\phi, \phi', \phi'', \phi''', \dots) = 0, \tag{3}$$

where  $\chi$  is a polynomial function of its arguments and  $'$  expresses the derivative w.r.t  $\zeta$ .

### 2.1 GERFM

First we give a detail description of GERFM [33]. This technique incorporates the following steps.

**Step 1.** The solution of Eq. (3) is written as

$$\phi(\zeta) = \delta_0 + \sum_{k=1}^n \delta_k \sigma(\zeta)^k + \sum_{k=1}^n \rho_k \sigma(\zeta)^{-k}, \tag{4}$$

where

$$\sigma(\zeta) = \frac{p_1 e^{q_1 \zeta} + p_2 e^{q_2 \zeta}}{p_3 e^{q_3 \zeta} + p_4 e^{q_4 \zeta}}. \tag{5}$$

The unknown coefficients  $\delta_0, \delta_k, \rho_k$  ( $1 \leq k \leq n$ ) and constants  $p_i, q_i$  ( $1 \leq i \leq 4$ ) are determined. We use homogeneous balance principle [34] in order to determine the positive integer  $n$  by balancing the nonlinear term and linear term with the highest order in Eq. (3). More precisely, if degree of  $\phi(\zeta)$  is  $deg[\phi(\zeta)] = n$  then other terms will have degree may be obtained as follows:

$$deg[\phi^k(\zeta)] = n + k,$$

$$deg[(\phi(\zeta))^p (\phi^k(\zeta))^s] = np + s(n + k). \tag{6}$$

From above equation we get

$$n + k = np + s(n + k). \tag{7}$$

**Step 2.** We get a cluster of algebraic equations on putting Eq. (4) together with (5) into Eq. (3).

**Step 3.** On solving the cluster of equations, we get the unknown terms and consequently, the required solutions are achieved.

### 2.2 $(\frac{G'}{G^2})$ -expansion function method

Here, we discuss in detail the general property of the proposed method [35].

**Step 1.** Suppose that Eq. (3) has the following form of solution

$$\phi(\zeta) = a_0 + \sum_{k=1}^n \left( \alpha_k \left( \frac{G'}{G^2} \right)^k + \beta_k \left( \frac{G'}{G^2} \right)^{-k} \right), \tag{8}$$

where  $G = G(\zeta)$  holds

$$\left( \frac{G'}{G^2} \right)' = \Upsilon + \Omega \left( \frac{G'}{G^2} \right)^2, \tag{9}$$

with  $\Omega \neq 0, \Upsilon \neq 1$  are integers. The unknown constants  $a_0, \alpha_k, \beta_k$  ( $k = 1, 2, 3, \dots, n$ ) to be calculated latter.

**Step 2.** Thus  $(\frac{G'}{G^2})$ -expansion method provides three types of solutions:

• **Trigonometric solution:**

If  $\Upsilon \Omega > 0$ ,

$$\left( \frac{G'}{G^2} \right) = \sqrt{\frac{\Upsilon}{\Omega}} \left( \frac{\zeta_1 \cos(\sqrt{\Upsilon \Omega} \zeta) + \zeta_2 \sin(\sqrt{\Upsilon \Omega} \zeta)}{\zeta_2 \cos(\sqrt{\Upsilon \Omega} \zeta) - \zeta_1 \sin(\sqrt{\Upsilon \Omega} \zeta)} \right). \tag{10}$$

• **Hyperbolic solution:**

If  $\Upsilon \Omega < 0$ ,

$$\left( \frac{G'}{G^2} \right) = -\frac{\sqrt{|\Upsilon \Omega|}}{\Omega} \left( \frac{\zeta_1 \sinh(2\sqrt{|\Upsilon \Omega|} \zeta) + \zeta_2 \cosh(2\sqrt{|\Upsilon \Omega|} \zeta) + \zeta_1}{\zeta_1 \sinh(2\sqrt{|\Upsilon \Omega|} \zeta) + \zeta_2 \cosh(2\sqrt{|\Upsilon \Omega|} \zeta) - \zeta_1} \right), \tag{11}$$

• **Rational solution:**

If  $\Upsilon = 0, \Omega \neq 0$ ,

$$\left( \frac{G'}{G^2} \right) = \left( -\frac{\zeta_1}{\Omega(\zeta_1 \zeta + \zeta_2)} \right), \tag{12}$$

where  $\zeta_1$  and  $\zeta_2$  are constants.

**Step 3.** On the utilization of Eq. (7) for the homogeneous balance principle the value of  $n$  in Eq. (3) is calculated.

**Step 4.** After putting Eq. (8) together with (9) into Eq. (3) and a set of algebraic system is extracted on the comparison of specific terms.

**Step 5.** The required set of parameters will be obtained after solving the system of algebraic equations through soft computations.

### 2.3 The general properties of generalized Kudryashov method

Here, we discuss in detail the general property of the proposed method [36].

**Step 1:** Suppose that Eq. (3) has the following form of solution

$$\phi(\zeta) = \frac{\sum_{i=0}^n a_i G^i(\zeta)}{\sum_{j=0}^m b_j G^j(\zeta)}, \tag{13}$$

where,  $a_i$  ( $i = 0, 1, 2, 3, \dots, n$ ) and  $b_j$  ( $j = 0, 1, 2, 3, \dots, m$ ) are constants to be determined after ward such that  $a_n \neq 0$  and  $b_m \neq 0$ ,

where  $G = G(\zeta)$  holds

$$G'(\zeta) = G^2(\zeta) - G(\zeta). \tag{14}$$

Moreover, the solution of Eq. (14) has the structure like

$$G(\zeta) = \frac{1}{1 + S e^\zeta}. \tag{15}$$

Here,  $S$  is constant of integration.

**Step 2:** The values of  $n$  and  $m$  are evaluated by using homogeneous balance principle in Eq. (3).

**Step 3:** We obtain a polynomial in  $G(\zeta)$ , after putting Eqs. (13) and (14) into Eq. (3). We get a cluster of an algebraic equations on equating the same powers of  $G(\zeta)$  to zero, and we secure the values of  $a_i$  ( $i = 0, 1, 2, 3, \dots, n$ ) and  $b_j$  ( $j = 0, 1, 2, 3, \dots, m$ ). On the utilization of the obtained values in Eq. (13) with the usage of Eq. (15), we finally find the exact solutions of Eq. (1).

### 3 Extraction of solitary wave solutions

In this section, the application of the proposed methods are utilized for establishing the new soliton solutions to Eq.(1). Suppose the following traveling wave transformation :

$$\Psi(x, y, t) = H(\zeta)e^{i\theta}, \quad \Phi(x, y, t) = R(\zeta) \quad \theta = lx + py + ct, \quad \zeta = L(x + Py - 2lt), \tag{16}$$

where  $l, p, c, L$  and  $P$  are constants;  $H(\zeta)$  and  $R(\zeta)$  represent real functions. By inserting Eq. (16) into Eq. (1), we attain,

$$L^2H''(\zeta) - (c + l^2)H(\zeta) + H(\zeta)R(\zeta) = 0. \tag{17}$$

$$(P - 2l)R'(\zeta) + (H^2(\zeta))' = 0. \tag{18}$$

By integrating Eq. (18), we obtain

$$R(\zeta) = -\frac{H^2(\zeta)}{(P - 2l)} + a. \tag{19}$$

where  $a$  is the constant of the integration and  $P \neq 2l$ . Substituting Eq. (19) into Eq. (18), we get

$$\frac{(a - c - l^2)}{L^2}H(\zeta) - \frac{1}{L^2(P - 2l)}H^3(\zeta) + H''(\zeta) = 0. \tag{20}$$

#### 3.1 Solutions via GERFM

By using the homogeneous balance rule, the highest order derivative  $H''(\zeta)$  and the nonlinear term of the highest order  $H^3(\zeta)$  are balanced via the Eq. (7) in Eq. (20), and from Eq. (20), we get

$$deg[H''(\zeta)] = n + 2 = deg[H^3(\zeta)(H''(\zeta))^s] = 3n + 0(n + 2) \tag{21}$$

which leads to  $n = 1$ . The solution of (20), becomes

$$H(\zeta) = \delta_0 + \delta_1\sigma(\zeta) + \rho_1\sigma(\zeta)^{-1}. \tag{22}$$

**Family-1:** For  $p = [-1, -1, 1, -1]$  and  $q = [1, -1, 1, -1]$ , the Eq. (5) gives

$$\sigma(\zeta) = -\frac{\cosh(\zeta)}{\sinh(\zeta)}. \tag{23}$$

Plugging Eq. (22) and Eq. (23) in Eq. (20), we secure solution sets as follows:

**Set-1:**

$$\delta_0 = 0, \quad \delta_1 = \sqrt{2}\sqrt{L^2(P - 2l)}, \quad \rho_1 = \sqrt{2}\sqrt{L^2(P - 2l)}, \quad c = a - l^2 - 8L^2.$$

- Combo solitons are obtained

$$\Psi_1(x, y, t) = \left( -\sqrt{2L^2(P - 2l)} \tanh(L(-2lt + Py + x)) - \sqrt{2L^2(P - 2l)} \coth(L(-2lt + Py + x)) \right) \times e^{i(lx+py+ct)}. \tag{24}$$

**Set-2:**

$$\delta_0 = 0, \quad \delta_1 = \sqrt{2}\sqrt{L^2(P - 2l)}, \quad \rho_1 = 0, \quad c = a - l^2 - 2L^2.$$

- Singular solitons are obtained

$$\Psi_2(x, y, t) = -\sqrt{2}\sqrt{L^2(P - 2l)} \coth(L(-2lt + Py + x)) \times e^{i(lx+py+ct)}. \tag{25}$$

**Set-3:**

$$\delta_0 = 0, \quad \delta_1 = 0, \quad \rho_1 = \sqrt{2}\sqrt{L^2(P - 2l)}, \quad c = a - l^2 - 2L^2.$$

- Dark solitons are obtained

$$\Psi_3(x, y, t) = -\sqrt{2}\sqrt{L^2(P - 2l)} \tanh(L(-2lt + Py + x)) \times e^{i(lx+py+ct)}. \tag{26}$$

**Family-2:** For  $p = [-i, i, 1, 1]$  and  $q = [i, -i, i, -i]$ , the Eq. (5) provides

$$\sigma(\zeta) = -\frac{\sin(\zeta)}{\cos(\zeta)}. \tag{27}$$

Putting Eq. (22) and Eq. (27) in Eq. (20), we retrieves the solution sets as follows:

**Set-1:**

$$\delta_0 = 0, \delta_1 = iL\sqrt{4l - 2P}, \rho_1 = iL\sqrt{4l - 2P}, c = a - l^2 - 4L^2.$$

- Trigonometric solutions are obtained

$$\Psi_4(x, y, t) = -2iL\sqrt{4l - 2P} \csc(2L(-2lt + Py + x)) \times e^{i(lx+py+ct)}. \tag{28}$$

**Set-2:**

$$\delta_0 = 0, \delta_1 = \sqrt{2L^2(P - 2\sqrt{a - c - 4L^2})}, \rho_1 = \sqrt{2L^2(P - 2\sqrt{a - c - 4L^2})}, \\ l = \sqrt{a - c - 4L^2}.$$

- Periodic solutions are obtained

$$\Psi_5(x, y, t) = -2\sqrt{2}\sqrt{L^2(P - 2\sqrt{a - c - 4L^2})} \csc\left(2L(-2t\sqrt{a - c - 4L^2} + Py + x)\right) \times e^{i(lx+py+ct)}. \tag{29}$$

**Family-3:** For  $p = [1, 0, 1, 1]$  and  $q = [1, 0, 1, 0]$ , the Eq. (5) gives

$$\sigma(\zeta) = \frac{e^\zeta}{e^\zeta + 1}. \tag{30}$$

Substituting Eq. (22) and Eq. (30) in Eq. (20), we achieves the solution set as follows:

**Set-1:**

$$\delta_0 = i\sqrt{2l - P}\sqrt{a - c - l^2}, \delta_1 = -2i\sqrt{2l - P}\sqrt{a - c - l^2}, \rho_1 = 0, L = \sqrt{2}\sqrt{a - c - l^2}.$$

- Exponential function solutions are obtained

$$\Psi_6(x, y, t) = -\frac{i\sqrt{2l - P}\sqrt{a - c - l^2} \left( e^{\sqrt{2}\sqrt{a - c - l^2}(-2lt + Py + x)} - 1 \right)}{e^{\sqrt{2}\sqrt{a - c - l^2}(-2lt + Py + x)} + 1} \times e^{i(lx+py+ct)}. \tag{31}$$

**Family-4:** For  $p = [1 - i, 1 + i, 1, 1]$  and  $q = [i, -i, i, -i]$ , the Eq. (5) provides

$$\sigma(\zeta) = \frac{\cos(\zeta) + \sin(\zeta)}{\cos(\zeta)}. \tag{32}$$

Imposing Eq. (22) and Eq. (32) in Eq. (20), we get solution set as follows:

**Set-1:**

$$\delta_0 = \sqrt{2L^2(P - 2l)}, \delta_1 = 0, \rho_1 = -2\sqrt{2L^2(P - 2l)}, c = a - l^2 + 2L^2.$$

- Trigonometric solutions are obtained

$$\Psi_7(x, y, t) = \frac{\sqrt{2}\sqrt{L^2(P - 2l)}(\sin(L(-2lt + Py + x)) - \cos(L(-2lt + Py + x)))}{\sin(L(-2lt + Py + x)) + \cos(L(-2lt + Py + x))} \times e^{i(lx+py+ct)}. \tag{33}$$

**Family-5:** For  $p = [-3, -1, 1, 1]$  and  $q = [1, -1, 1, -1]$ , the Eq. (5) provides

$$\sigma(\zeta) = \frac{-2 \cosh(\zeta) - \sinh(\zeta)}{\cosh(\zeta)}. \tag{34}$$

Inserting Eq. (22) and Eq. (34) in Eq. (20), we obtain the solution sets as follows:

**Set-1:**

$$\delta_0 = 2\sqrt{2L^2(P - 2l)}, \delta_1 = \sqrt{L^2(P - 2l)}, \rho_1 = 0, c = a - l^2 - 2L^2.$$

- Dark solitons are obtained

$$\Psi_8(x, y, t) = -\sqrt{2}\sqrt{L^2(P - 2l)} \tanh(L(-2lt + Py + x)) \times e^{i(lx+py+ct)}. \tag{35}$$

**Set-2:**

$$\delta_0 = 2\sqrt{2L^2(P - 2l)}, \delta_1 = 0, \rho_1 = 3\sqrt{2L^2(P - 2l)}, c = a - l^2 - 2L^2.$$

- Combo solitons are obtained

$$\Psi_9(x, y, t) = \frac{\sqrt{2}\sqrt{L^2(P - 2l)}(2 \sinh(L(-2lt + Py + x)) + \cosh(L(-2lt + Py + x)))}{\sinh(L(-2lt + Py + x)) + 2 \cosh(L(-2lt + Py + x))} \times e^{i(lx+py+ct)}. \tag{36}$$

**Family-6:** For  $p = [-1, 0, 1, 1]$  and  $q = [0, 1, 0, 1]$ , the Eq. (5) gives

$$\sigma(\zeta) = -\frac{1}{e^\zeta + 1}. \tag{37}$$

Switching Eq. (22) and Eq. (37) in Eq. (20), we recover solution sets as follows:

**Set-1:**

$$\delta_0 = \frac{\sqrt{L^2 \left( P - \sqrt{4a - 2(2c + L^2)} \right)}}{\sqrt{2}}, \delta_1 = \sqrt{2L^2 \left( P - \sqrt{4a - 2(2c + L^2)} \right)}, \rho_1 = 0, l = \sqrt{a - c - \frac{L^2}{2}}.$$

- Exponential function solutions are obtained as

$$\Psi_{10}(x, y, t) = \frac{\sqrt{-L^2 \left( \sqrt{4a - 2(2c + L^2)} - P \right)} \left( \exp \left( L \left( t \left( -\sqrt{4a - 4c - 2L^2} \right) + Py + x \right) \right) - 1 \right)}{\sqrt{2} \left( \exp \left( L \left( t \left( -\sqrt{4a - 4c - 2L^2} \right) + Py + x \right) \right) + 1 \right)} \times e^{i(lx+py+ct)}. \tag{38}$$

**Set-2:**

$$\delta_0 = i\sqrt{2l - P}\sqrt{a - c - l^2}, \delta_1 = 2i\sqrt{2l - P}\sqrt{a - c - l^2}, \rho_1 = 0, L = \sqrt{2}\sqrt{a - c - l^2}.$$

- Exponential function solutions are obtained

$$\Psi_{11}(x, y, t) = i\sqrt{2l - P}\sqrt{a - c - l^2} \left( 1 - \frac{2}{e^{\sqrt{2}\sqrt{a - c - l^2}(-2lt + Py + x)} + 1} \right) \times e^{i(lx+py+ct)}. \tag{39}$$

**Family-7:** For  $p = [-1, 1, 1, 1]$  and  $q = [1, -1, 1, -1]$ , the Eq. (5) provides

$$\sigma(\zeta) = -\frac{\sinh(\zeta)}{\cosh(\zeta)}. \tag{40}$$

Putting Eq. (22) and Eq. (40) in Eq. (20), we get the solution sets as follows:

**Set-1:**

$$\delta_0 = 0, \delta_1 = \frac{\sqrt{2l - P}\sqrt{a - c - l^2}}{\sqrt{2}}, \rho_1 = -\frac{\sqrt{2l - P}\sqrt{a - c - l^2}}{\sqrt{2}}, L = \frac{1}{2}\sqrt{-a + c + l^2}.$$

- Mixed solitons are obtained

$$\Psi_{12}(x, y, t) = \left( \frac{\sqrt{2l - P}\sqrt{a - c - l^2} \coth \left( \frac{1}{2}\sqrt{-a + c + l^2}(-2lt + Py + x) \right)}{\sqrt{2}} - \frac{\sqrt{2l - P}\sqrt{a - c - l^2} \tanh \left( \frac{1}{2}\sqrt{-a + c + l^2}(-2lt + Py + x) \right)}{\sqrt{2}} \right) \times e^{i(lx+py+ct)}. \tag{41}$$

**Set-2:**

$$\delta_0 = 0, \delta_1 = -iL\sqrt{4l - 2P}, \rho_1 = iL\sqrt{4l - 2P}, c = a - l^2 + 4L^2.$$

- The soliton solutions are obtained

$$\Psi_{13}(x, y, t) = 1L\sqrt{4l - 2P} \tanh(L(-2lt + Py + x)) - iL\sqrt{4l - 2P} \coth(L(-2lt + Py + x)) \times e^{i(lx+py+ct)}. \tag{42}$$

**Family-8:** For  $p = [2 - i, 2 + i, 1, 1]$  and  $q = [i, -i, i, -i]$ , the Eq. (5) provides

$$\sigma(\zeta) = \frac{\sin(\zeta) + 2 \cos(\zeta)}{\cos(\zeta)}. \tag{43}$$

Placing Eq. (22) and Eq. (43) in Eq. (20), we retrieve the solution sets as follows:

**Set-1:**

$$\delta_0 = 2\sqrt{2L^2(P - 2l)}, \delta_1 = -\sqrt{2L^2(P - 2l)}, \rho_1 = 0, c = a - l^2 + 2L^2.$$

- Singular periodic solutions are obtained

$$\Psi_{14}(x, y, t) = -\sqrt{2}\sqrt{L^2(P - 2l)} \tan(L(-2lt + Py + x)) \times e^{i(lx+py+ct)}. \tag{44}$$

**Set-2:**

$$\delta_0 = -2\sqrt{2L^2(P - 2l)}, \delta_1 = 0, \rho_1 = 5\sqrt{2L^2(P - 2l)}, c = a - l^2 + 2L^2.$$

- Singular periodic solutions are obtained

$$\Psi_{15}(x, y, t) = \frac{\sqrt{2}\sqrt{L^2(P - 2l)}(1 - 2 \tan(L(-2lt + Py + x)))}{\tan(L(-2lt + Py + x)) + 2} \times e^{i(lx+py+ct)}. \tag{45}$$

**Family-9:** For  $p = [1, 2, 1, 1]$  and  $q = [1, 0, 1, 0]$ , the Eq. (5) recovers

$$\sigma(\zeta) = \frac{e^\zeta + 2}{e^\zeta + 1}. \tag{46}$$

Imposing Eq. (22) and Eq. (46) in Eq. (20), we get the solution sets as follows:

**Set-1:**

$$\delta_0 = -\frac{3\sqrt{L^2(P - 2l)}}{\sqrt{2}}, \delta_1 = 0, \rho_1 = 2\sqrt{2L^2(P - 2l)}, c = a - l^2 - \frac{L^2}{2}.$$

- Exponential function solutions are obtained

$$\Psi_{16}(x, y, t) = \frac{\sqrt{L^2(P - 2l)}(e^{L(-2lt+Py+x)} - 2)}{\sqrt{2}(e^{L(-2lt+Py+x)} + 2)} \times e^{i(lx+py+ct)}. \tag{47}$$

**Set-2:**

$$\delta_0 = -\frac{3\sqrt{L^2(P - 2l)}}{\sqrt{2}}, \delta_1 = \sqrt{2L^2(P - 2l)}, \rho_1 = 0, c = a - l^2 - \frac{L^2}{2}.$$

- Exponential function solutions are obtained

$$\Psi_{17}(x, y, t) = -\frac{\sqrt{L^2(P - 2l)}(e^{L(-2lt+Py+x)} - 1)}{\sqrt{2}(e^{L(-2lt+Py+x)} + 1)} \times e^{i(lx+py+ct)}. \tag{48}$$

**Family-10:** For  $p = [2, 1, 1, 1]$  and  $q = [1, 0, 1, 0]$ , the Eq. (5) retrieves

$$\sigma(\zeta) = \frac{2e^\zeta + 1}{e^\zeta + 1}. \tag{49}$$

Switching Eq. (22) and Eq. (49) in Eq. (20), we obtain the solution sets as follows:

**Set-1:**

$$\delta_0 = -\frac{3\sqrt{L^2(P - 2l)}}{\sqrt{2}}, \delta_1 = 0, \rho_1 = 2\sqrt{2L^2(P - 2l)}, c = a - l^2 - \frac{L^2}{2}.$$

- Exponential function solutions are obtained

$$\Psi_{18}(x, y, t) = \frac{\sqrt{L^2(P - 2l)} (1 - 2e^{L(-2lt+Py+x)})}{\sqrt{2} (2e^{L(-2lt+Py+x)} + 1)} \times e^{i(lx+py+ct)}. \tag{50}$$

**Set-2:**

$$\delta_0 = -\frac{3\sqrt{L^2(P - 2l)}}{\sqrt{2}}, \delta_1 = \sqrt{2L^2(P - 2l)}, \rho_1 = 0, c = a - l^2 - \frac{L^2}{2}.$$

- Exponential function solutions are obtained

$$\Psi_{19}(x, y, t) = \frac{\sqrt{L^2(P - 2l)} (e^{L(-2lt+Py+x)} - 1)}{\sqrt{2} (e^{L(-2lt+Py+x)} + 1)} \times e^{i(lx+py+ct)}. \tag{51}$$

**Family-11:** For  $p = [1, 1, 1, 1]$  and  $q = [0, 0, 1, -1]$ , the Eq. (5) provides

$$\sigma(\zeta) = \frac{2}{e^\zeta + e^{-\zeta}}. \tag{52}$$

Putting Eq. (22) and Eq. (49) in Eq. (20), we get the solution set as follows:

**Set-1:**

$$\delta_0 = 0, \delta_1 = \sqrt{2L^2(2l - P)}, \rho_1 = 0, c = a - l^2 + L^2.$$

- Bright solitons are obtained

$$\Psi_{20}(x, y, t) = \sqrt{2L^2(2l - P)} \operatorname{sech}(L(2lt - Py - x)) \times e^{i(lx+py+ct)}. \tag{53}$$

### 3.2 Solutions via $(\frac{G'}{G^2})$ -expansion method

By applying balance rule via Eq. (21), in Eq. (20) yields,  $n = 1$ . The solution of Eq. (20) as follows:

$$H(\zeta) = \beta_0 + \sum_{j=1}^n \left( \beta_j \left( \frac{G'}{G^2} \right)^j + \delta_j \left( \frac{G'}{G^2} \right)^{-j} \right),$$

$$H(\zeta) = \beta_0 + \beta_1 \left( \frac{G'}{G^2} \right) + \delta_1 \left( \frac{G'}{G^2} \right)^{-1}. \tag{54}$$

Inserting Eq. (54) into Eq. (20) together with Eq. (9) and by equating the coefficients of each powers of  $\left(\frac{G'}{G^2}\right)$  to zero. On solving the system of algebraic equations through Mathematica, we obtain the following solution sets:

**Set - 1 :**  $\beta_0 = 0, \beta_1 = \sqrt{2L^2\Omega^2(P - 2l)}, \delta_1 = 0, c = a - l^2 + 2L^2\Upsilon\Omega.$   
**Set - 2 :**  $\beta_0 = 0, \beta_1 = 0, \delta_1 = \sqrt{2L^2\Upsilon^2(P - 2l)}, c = a - l^2 + 2L^2\Upsilon\Omega.$   
**Set - 3 :**  $\beta_0 = 0, \beta_1 = \sqrt{2L^2\Omega^2(P - 2l)}, \delta_1 = \sqrt{2L^2\Upsilon^2(P - 2l)},$   
 $c = a - l^2 - \frac{6L^2\Upsilon^2\sqrt{L^2\Omega^2(P - 2l)}}{\sqrt{L^2\Upsilon^2(P - 2l)}} + 2L^2\Upsilon\Omega.$

For **Set-1**,

- **Trigonometric solutions:**

If  $\Upsilon\Omega > 0$ ,

$$\Psi_{1,1}(x, y, t) = \frac{\sqrt{2}\sqrt{\Upsilon}\sqrt{L^2\Omega^2(P - 2l)} (\zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}) + \zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}))}{\sqrt{\Omega} (\zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}) - \zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}))} \times e^{i(lx+py+ct)}. \tag{55}$$



• **Hyperbolic solution:**

If  $\Upsilon\Omega < 0$ ,

$$\psi_{1,2}(x, y, t) = -\frac{\sqrt{|\Upsilon\Omega|}\sqrt{-2L^2\Omega^2(2l - P)}(\zeta_1(\sinh(2\zeta\sqrt{|\Upsilon\Omega|}) + \cosh(2\zeta\sqrt{|\Upsilon\Omega|})) + \zeta_2)}{\Omega(\zeta_1(\sinh(2\zeta\sqrt{|\Upsilon\Omega|}) + \cosh(2\zeta\sqrt{|\Upsilon\Omega|})) - \zeta_2)} \times e^{i(lx+py+ct)}. \tag{56}$$

• For soliton solution, take  $\zeta_1 = \zeta_2$ , we get singular wave solution as:

$$\psi_{1,2}(x, y, t) = -\frac{\sqrt{2}\sqrt{|\Upsilon\Omega|}\sqrt{-L^2\Omega^2(2l - P)} \coth(\zeta\sqrt{|\Upsilon\Omega|})}{\Omega} \times e^{i(lx+py+ct)}. \tag{57}$$

• **Rational solutions:**

If  $\Upsilon = 0, \Omega \neq 0$ ,

$$\psi_{1,3}(x, y, t) = -\frac{\sqrt{2}\zeta_1\sqrt{L^2\Omega^2(P - 2l)}}{\Omega(\zeta_1 + \zeta_2)} \times e^{i(lx+py+ct)}. \tag{58}$$

If we take  $\zeta_1 = \zeta_2$ , a rational function solution is obtained

$$\psi_{1,3}(x, y, t) = -\frac{\sqrt{2}\sqrt{-L^2\Omega^2(2l - P)}}{\zeta_2\Omega + \Omega} \times e^{i(lx+py+ct)}. \tag{59}$$

For **Set-2**,

• **Trigonometric solutions:**

If  $\Upsilon\Omega > 0$ ,

$$\psi_{2,1}(x, y, t) = \frac{\sqrt{2}\sqrt{\Omega}\sqrt{L^2\Upsilon^2(P - 2l)}(\zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}) - \zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}))}{\sqrt{\Upsilon}(\zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}) + \zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}))} \times e^{i(lx+py+ct)}. \tag{60}$$

• **Hyperbolic solution:**

If  $\Upsilon\Omega < 0$ ,

$$\psi_{2,2}(x, y, t) = -\frac{\sqrt{2}\Omega\sqrt{-L^2\Upsilon^2(2l - P)}(\zeta_1(\sinh(2\zeta\sqrt{|\Upsilon\Omega|}) + \cosh(2\zeta\sqrt{|\Upsilon\Omega|})) - \zeta_2)}{\sqrt{|\Upsilon\Omega|}(\zeta_1(\sinh(2\zeta\sqrt{|\Upsilon\Omega|}) + \cosh(2\zeta\sqrt{|\Upsilon\Omega|})) + \zeta_2)} \times e^{i(lx+py+ct)}. \tag{61}$$

• For soliton solution, take  $\zeta_1 = \zeta_2$ , we get dark soliton solution as:

$$\psi_{2,2}(x, y, t) = -\frac{\sqrt{2}\Omega\sqrt{-L^2\Upsilon^2(2l - P)} \tanh(\zeta\sqrt{|\Upsilon\Omega|})}{\sqrt{|\Upsilon\Omega|}} \times e^{i(lx+py+ct)}. \tag{62}$$

For **Set-3**,

• **Trigonometric solutions:**

If  $\Upsilon\Omega > 0$ ,

$$\begin{aligned} \psi_{3,1}(x, y, t) = & \frac{\sqrt{2}\sqrt{\Omega}\sqrt{L^2\Upsilon^2(P - 2l)}(\zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}) - \zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}))}{\sqrt{\Upsilon}(\zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}) + \zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}))} \\ & + \frac{\sqrt{2}\sqrt{\Upsilon}\sqrt{L^2\Omega^2(P - 2l)}(\zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}) + \zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}))}{\sqrt{\Omega}(\zeta_2 \cos(\zeta\sqrt{\Upsilon\Omega}) - \zeta_1 \sin(\zeta\sqrt{\Upsilon\Omega}))} \times e^{i(lx+py+ct)}. \end{aligned} \tag{63}$$

• For soliton solution, take  $\zeta_1 = \zeta_2$ , we get periodic wave solution as:

$$\begin{aligned} \psi_{3,1}(x, y, t) = & \frac{1}{\sqrt{\Upsilon}\sqrt{\Omega}} \times \sqrt{2} \sec(2\zeta\sqrt{\Upsilon\Omega}) (\Omega\sqrt{-L^2\Upsilon^2(2l - P)} + \Upsilon\sqrt{-L^2\Omega^2(2l - P)}) \\ & + (\Upsilon\sqrt{-L^2\Omega^2(2l - P)} - \Omega\sqrt{-L^2\Upsilon^2(2l - P)}) \sin(2\zeta\sqrt{\Upsilon\Omega}) \times e^{i(lx+py+ct)}. \end{aligned} \tag{64}$$

• **Hyperbolic solution:**

If  $\Upsilon\Omega < 0$ ,

$$\Psi_{3,2}(x, y, t) = \frac{\Omega^2 \left(-\sqrt{2L^2\Upsilon^2(P-2l)}\right) \left(\zeta_2 - \zeta_1 e^{2\zeta\sqrt{|\Upsilon\Omega|}}\right)^2}{\Omega\sqrt{|\Upsilon\Omega|} \left(\zeta_1^2 e^{4\zeta\sqrt{|\Upsilon\Omega|}} - \zeta_2^2\right)} - \frac{|\Upsilon\Omega|\sqrt{2L^2\Omega^2(P-2l)} \left(\zeta_1 e^{2\zeta\sqrt{|\Upsilon\Omega|}} + \zeta_2\right)^2}{\Omega\sqrt{|\Upsilon\Omega|} \left(\zeta_1^2 e^{4\zeta\sqrt{|\Upsilon\Omega|}} - \zeta_2^2\right)} \times e^{i(lx+py+ct)}. \tag{65}$$

• For soliton solution, take  $\zeta_1 = \zeta_2$ , we get combo dark-singular wave solution as:

$$\Psi_{3,2}(x, y, t) = -\frac{\tanh\left(\zeta\sqrt{|\Upsilon\Omega|}\right) \left(|\Upsilon\Omega|\sqrt{2L^2\Omega^2(P-2l)} \coth^2\left(\zeta\sqrt{|\Upsilon\Omega|}\right) + \Omega^2\sqrt{L^2\Upsilon^2(P-2l)}\right)}{\Omega\sqrt{|\Upsilon\Omega|}} \times e^{i(lx+py+ct)}. \tag{66}$$

For all the above set (1, 2, 3)  $\zeta = L(x + Py - 2lt)$ .

3.3 Solutions via generalized Kudryashov method

By using the homogeneous balance rule, the highest order derivative  $H''(\zeta)$  and the nonlinear term of the highest order  $H^3(\zeta)$  are balanced via the Eq. (7) in Eq. (20), and from Eq. (20), for this method we have

$$\text{deg}[H''(\zeta)] = (n - m) + 2 \tag{67}$$

$$\text{deg}[H^3(\zeta)(H''(\zeta))^s] = 3(n - m) + s(n - m + k). \tag{68}$$

From above two equations we have

$$(n - m) + 2 = 3(n - m) + 0(n - m + k). \tag{69}$$

which leads to

$$n = m + 1. \tag{70}$$

Particularly, if we assigning  $m = 1$ , we get  $n = 2$ . The proposed method has the solution to Eq. (20) as follows

$$H(\zeta) = \frac{a_0 + a_1G(\zeta) + a_2G^2(\zeta)}{b_0 + b_1G(\zeta)}, \tag{71}$$

where  $a_0, a_1, a_2, b_0$  and  $b_1$  are to be determined. Now, solving Eqs. (20) and (71), and following the step 3 of the method, we get the system of equations is obtained. With the assistance of computational software like Mathematica, we solve the system of algebraic expression, and a variety of solution sets is obtained as

**Set-1:**

$$a_1 = -i(2b_0 - b_1)L\sqrt{l - \frac{P}{2}}, a_0 = ib_0L\sqrt{l - \frac{P}{2}}, c = a - l^2 - \frac{L^2}{2}, a_2 = -ib_1L\sqrt{4l - 2P}.$$

On substituting the above values of parameters in Eq. (71) and with the assistance of Eq. (15), and on setting  $S = 1$ , the hyperbolic functions are obtained

$$\Psi_1(x, y, t) = -iL\sqrt{l - \frac{P}{2}} \tanh\left(\frac{1}{2}L(2lt - Py - x)\right) \times e^{i(lx+py+ct)}. \tag{72}$$

**Set-2:**

$$a_1 = -i(2b_0 + b_1)L\sqrt{l - \frac{P}{2}}, a_0 = -\frac{\sqrt{b_0^2(b_0 + b_1)^2(-L^2)(2l - P) + ib_0^2L\sqrt{2l - P}}}{\sqrt{2}b_1},$$

$$c = \frac{1}{2} \left( 2a + \frac{2L \left( b_0(b_0 + b_1)(-L) + \frac{i\sqrt{b_0^2(b_0+b_1)^2(-L^2)(2l-P)}}{\sqrt{2l-P}} \right)}{b_1^2} - 2l^2 - L^2 \right), a_2 = 0.$$

By taking  $S = 1$  and solving Eqs. (71) and (15) together, we get exponential function solution as follows

$$\Psi_2(x, y, t) = - \frac{\frac{\sqrt{b_0^2(b_0+b_1)^2(-L^2)(2l-P)} + ib_0^2L\sqrt{2l-P}}{\sqrt{2}b_1} + \frac{i(2b_0+b_1)L\sqrt{l-\frac{P}{2}}}{e^{L(-2lt+Py+x)}+1}}{\frac{b_1}{e^{L(-2lt+Py+x)}+1} + b_0} \times e^{i(lx+py+ct)}. \tag{73}$$

**Set-3:**

$$a_1 = -2ib_0L\sqrt{4l-2P}, b_1 = -2b_0; a_0 = \frac{-\sqrt{b_0^2L^2(P-2l)} + 3ib_0L\sqrt{2l-P}}{2\sqrt{2}},$$

$$c = a - \frac{3iL\sqrt{b_0^2L^2(P-2l)}}{4b_0\sqrt{2l-P}} - l^2 - \frac{5L^2}{4}, a_2 = 2ib_0L\sqrt{4l-2P}.$$

On selecting  $S = 1$  and solving Eqs. (71) and (15) together, we get soliton solution as

$$\Psi_3(x, y, t) = \frac{1}{b_0 - \frac{2b_0}{-\sinh(2lLt-LPy-Lx)+\cosh(2lLt-LPy-Lx)+1}} \times$$

$$\times \left( \frac{2ib_0L\sqrt{4l-2P}}{(-\sinh(2lLt-LPy-Lx)+\cosh(2lLt-LPy-Lx)+1)^2} + \frac{3ib_0L\sqrt{2l-P}}{2\sqrt{2}} \right.$$

$$\left. - \frac{\sqrt{b_0^2L^2(P-2l)}}{2\sqrt{2}} - \frac{2ib_0L\sqrt{4l-2P}}{-\sinh(2lLt-LPy-Lx)+\cosh(2lLt-LPy-Lx)+1} \right) \times e^{i(lx+py+ct)}. \tag{74}$$

**Set-4:**

$$a_1 = ib_0L\sqrt{4l-2P}, b_1 = 0, a_0 = -ib_0L\sqrt{l-\frac{P}{2}}, c = a - l^2 - \frac{L^2}{2}, a_2 = 0.$$

For  $S = 1$  and solving Eqs. (71) and (15) together, we get the singular wave solution as

$$\Psi_4(x, y, t) = iL\sqrt{l-\frac{P}{2}} \coth\left(\frac{1}{2}L(2lt-Py-x)\right) \times e^{i(lx+py+ct)}. \tag{75}$$

**Set-5:**

$$a_1 = 2ib_0L\sqrt{4l-2P}, b_1 = -2b_0, a_0 = \frac{\sqrt{b_0^2L^2(P-2l)} - 3ib_0L\sqrt{2l-P}}{2\sqrt{2}}$$

$$c = a - \frac{3iL\sqrt{b_0^2L^2(P-2l)}}{4b_0\sqrt{2l-P}} - l^2 - \frac{5L^2}{4}, a_2 = -2ib_0L\sqrt{4l-2P}.$$

In particular, on  $S = -1$  and solving Eqs. (71) and (15) together, the hyperbolic function solution can be expressed as

$$\Psi_5(x, y, t) = \frac{1}{\sqrt{2}b_0(e^{2L(-2lt+Py+x)}-1)} \times \left( -ib_0L\sqrt{2l-P}(3\cosh(L(2lt-Py-x))+1) \right.$$

$$\left. \sqrt{b_0^2L^2(P-2l)} + (\cosh(L(2lt-Py-x))-1) \right) \times e^{i(lx+py+ct)}. \tag{76}$$

**Set-6:**

$$a_1 = -ib_1L\sqrt{4l-2P}, a_0 = 0, b_0 = -\frac{b_1}{2}, c = a - l^2 + L^2, a_2 = ib_1L\sqrt{4l-2P}.$$

If we select  $S = 1$  and solving Eqs. (71) and (15) together, then we obtain singular wave solution as

$$\Psi_6(x, y, t) = -iL\sqrt{4l-2P} \operatorname{csch}(L(2lt-Py-x)) \times e^{i(lx+py+ct)}. \tag{77}$$

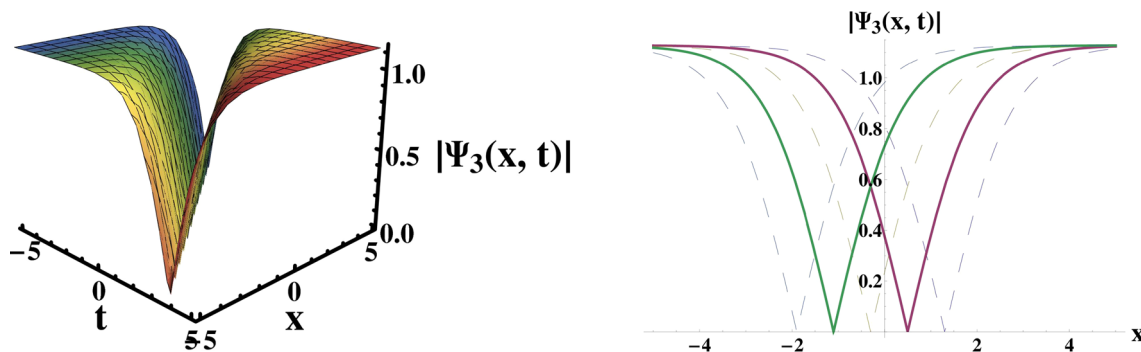


Fig. 1 The 3D, and 2D graphs are presented for Eq. (26)

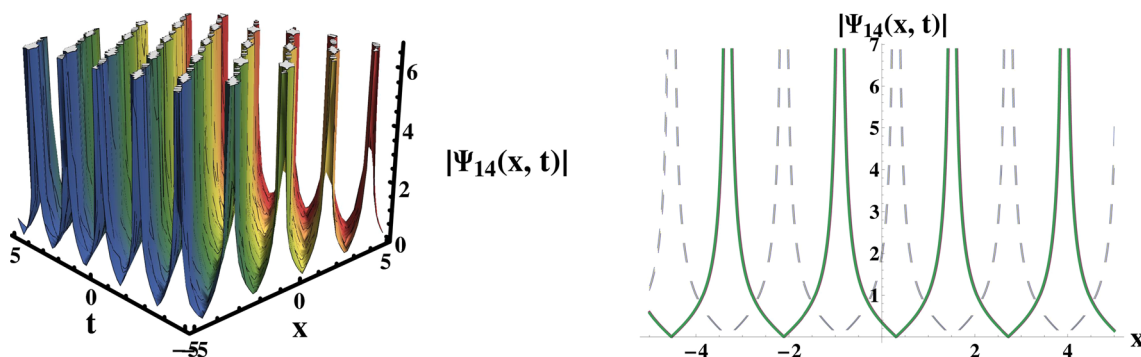


Fig. 2 The 3D, and 2D graphs are presented for Eq. (44)

### 4 Results and discussion

In this section, we will compare our results with other results in the existing research literature after successful utilization of three proposed computational methods to the given dynamical model. Akhtar Chowdhury et al. [27] examine the explicit and periodic solutions by utilizing the double  $(\frac{G'}{G}, \frac{1}{G})$ -expansion method. Besides, in these references [28–32], the authors attained a few solutions to the (2+1)-dimensional soliton equation. However, in this manuscript, our main objective is to extract a variety of solitons in bright, dark, singular, and combo forms along with exponential function, rational and singular periodic solutions. We observe that the retrieved solutions are new and to the best of our knowledge the applications of these techniques to the dynamical soliton equation have not been reported in the literature beforehand. The accomplished outcomes are remarkable and new from the current results in available writing. We encountered that the results presented in this article could help explain the genuine meaning of various nonlinear advancement conditions arising in the many fields of nonlinear sciences especially in mathematical physics. Moreover, these results have some other physical meanings, for instance; the hyperbolic sine function appears in the gravitational potential of a cylinder and the calculation of the Roche limit, the hyperbolic cosine function appears in the shape of a hanging cable (the so-called CATENARY), the hyperbolic tangent appears in the calculation of magnetic moment and special relativity rapidity, and the hyperbolic cotangent appears in the Langevin function for magnetic porosity [37]. By substituting the diverse values to the parameters, variants wave results are discovered from the exact peregrinating wave solution. The dark, periodic and bright soliton solutions, which are provided in Eqs. (26, 62, 72), 44 and (53) as exhibited in figs (1, 4, 5), (2) and (3) respectively. The physically description of some solutions are given below.

### 5 Conclusions

The exploration of this novel effort is to investigate solitary wave solutions in diverse forms as hyperbolic, trigonometric, exponential, and rational function solutions along with well-known soliton solutions in the shapes of bright dark, singular, and multiple solitons by three analytical mathematical methods. The accomplished outcomes are remarkable and new from the current results in available writing. Furthermore, 3D, and 2D graphs have been plotted under the choice of appropriate parameters for getting the physical behavior of some secured solutions. The reported outcomes will be valuable for a comprehensive insight of the dynamics of the mentioned model, and more, the analysis can be enhanced to other nonlinear models. The scrutinized wave’s results are loyal to the researchers and also have imperious applications in mathematical physics. Finally, our solutions have been verified using the

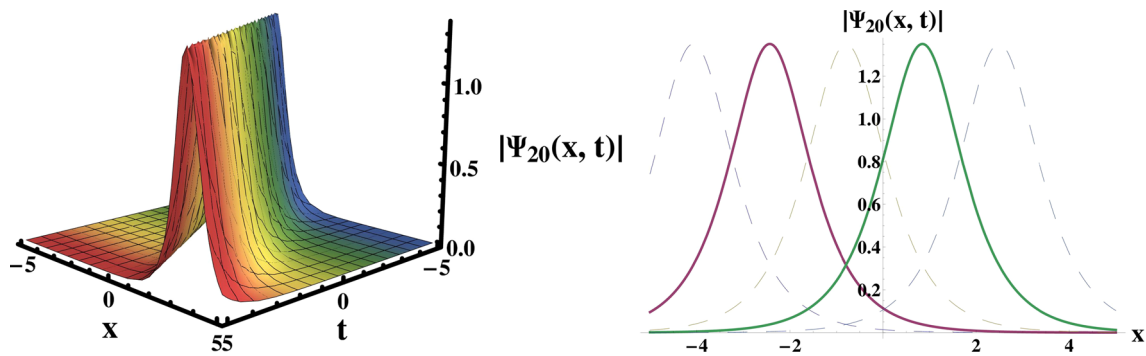


Fig. 3 The 3D, and 2D graphs are presented for Eq. (53)

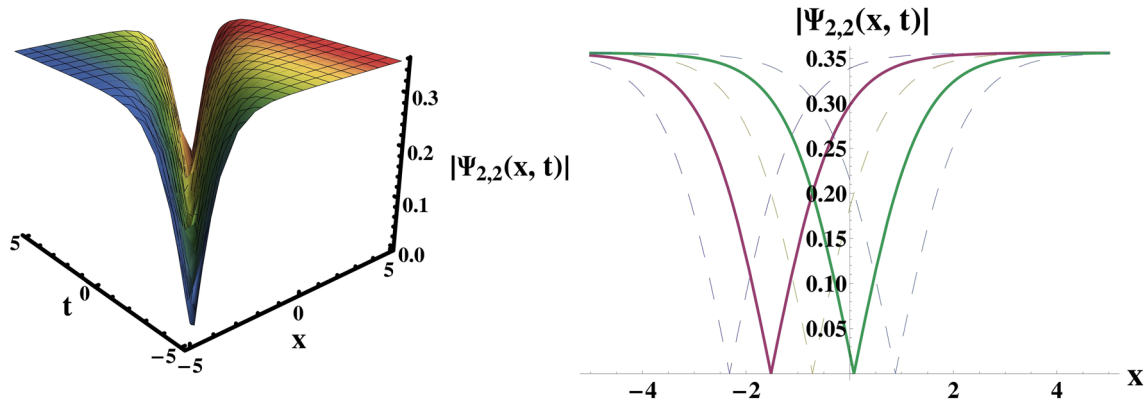


Fig. 4 The 3D, and 2D graphs are presented for Eq. (62)

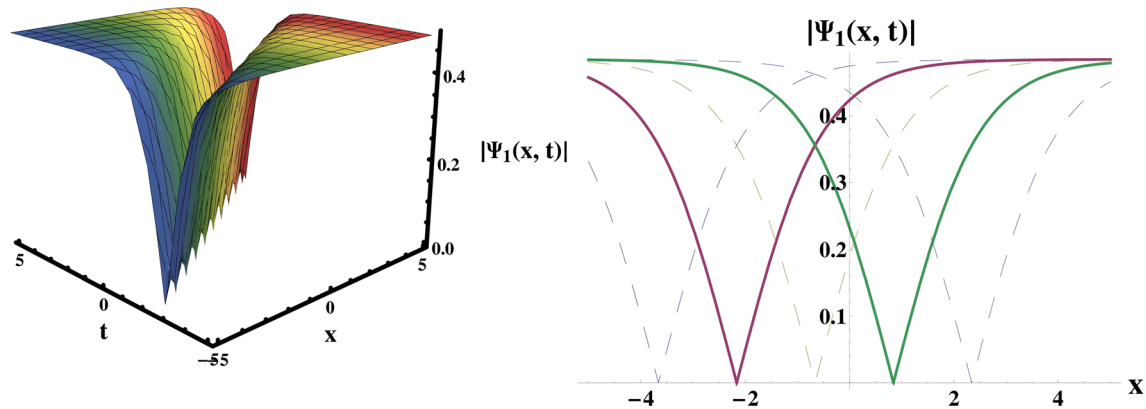


Fig. 5 The 3D, and 2D graphs are presented for Eq. (72)

Mathematica by substituting them back into the original equation. We can assert from the obtained results that the applied techniques are simple, vibrant, and quite well, and will be helpful tool for addressing more highly nonlinear issues in various of fields, especially in physical sciences. Furthermore, our findings are first step toward understanding the structure and physical behavior of complicated structures. We anticipate that our results will be highly valuable in better understanding the waves that occur in the diverse area of physics. We feel that this work is timely and will be of interest to a wide spectrum of experts working on physical and engineering models.

**Data Availability Statement** Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

**Declarations**

**Conflict of interest** The authors declare that they have no competing interests.

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