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About the neutrino oscillation-like effects in general physical systems

On interference between distinguishable particles...

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Abstract We evidence the main theoretical properties determining that neutrino oscillations appear as an interference between fully distinguishable particles. The source of the effect is identified as the many-particle structure of space of states of a quantum field theory. It is underlined that the space of states for neutrinos in the SM is the linear completion of the direct product of the three neutrino Fock spaces. Then, this nature of the state space directly makes clear that the neutrino oscillations become interference effects among non-identical particles; which are exclusively generated by essentially many particle states being always outside the direct product of the three neutrino Fock spaces. This cleanly many-particle effect is then identified as the central reason breaking the usual single particle quantum mechanical rule, not allowing the interference between distinguishable particles. The work also examines the connections with the Vitiello–Blasone analysis about the role of QFT in neutrino oscillations. It is argued that their evaluation corresponds to a more natural representation of the perturbative expansion for the applications in neutrino oscillations. However, the results coincide with the usual and ours simpler evaluations, in the large momentum limit. One conclusion of the work is that similar interference effects should be present in many physical systems. By example, among them are all the ones described by a QFT including at least two kinds of distinguishable particles having similar mass values. For illustrating this point, a band model of a solid is presented which shows oscillations analogous to the neutrino ones, but happening between two electron waves propagating in different bands.

1 Introduction

The interference effect between distinguishable particles had been observed long time ago and started to be investigated in the original works [1–4]. The particular case of such interference effects: the neutrino oscillations, has been and continues to be a relevant theme of research in Particle Physics. Since its prediction by B. Pontecorvo in references [4–6], the

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effect had been theoretically as well as experimentally intensively investigated. The discovery of the real occurrence in nature of these oscillations, in super Kamiokande and Solar neutrino observations, was a breakthrough step [7,8]. After it, an enormous amount of investigations about this effect had been performed [9–17]. The oscillations had been studied either through quantum mechanics (QM) and quantum field theory (QFT) methods [18–21]. In our view the QFT methods had contributed to clarify some of the assumptions which have been done in the QM approaches. In particular, the question about the possibility of defining a Fock space for the flavor eigenfunctions for the electron, muon and tau neutrinos as expressed as linear superpositions of the really stable propagating neutrino models, had been extensively discussed [21]. This aspect constitutes an example of a question related with neutrino oscillation physics that today remain under discussions [22–24]. One point which in our view deserves research attention is related with the fact that the neutrino oscillations look as an interference effect, similar to the one occurring in the QM of a single particle, but occurring between fully distinguishable fermion particles, each one of them described by a wavefunction being in a separate Fock space [4]. This is a peculiar effect if we consider the idea often used in QM presentations about that different (distinguishable) particles do not interfere between them.

Thus, it seems of interest to identify the basic reasons why the interference between neutrinos oscillations violates this rule valid in single particle QM. Another motivating issue is to understand how general these effects can be, in order to estimate the possibility of similar realizations in other physical systems.

In this work we address these two questions. For this purpose a proper use of the space of states in quantum field theory of the SM should be taken into account. A model involving only two distinguishable types (flavors) of relativistic particles is considered for the sake of clearness. In first place, it is argued that to describe interference between two fermion flavors in analogy with the neutrino oscillations measurements, the space of states should not be the direct product of the Fock spaces associated with the two fermion particles. This conclusion is in complete agreement with the usual application of QFT to neutrino oscillations today. This is because such a direct product space of states will be equivalent to impose a superselection rule for defining the allowed physical states. But, such rules are not assumed in the formulation of the SM actually employed to describe the neutrino physics.

Therefore, the space of the states of the SM model is considered here in the usual way: as the linear completion of the direct product of the two neutrino Fock states. That is, after assuming valid the superposition principle: the sum of physical states is also a physical state. This implies that the linear combination with arbitrary coefficients of the external products of the states in each of the two neutrino Fock spaces should be also a physical state. Further, and in order to reproduce the usual results for neutrino oscillations, rotated flavor creation and annihilation operators are defined in the usual way by linear combinations of the creation and annihilation operators for stable propagating neutrino modes solving the Dirac equation. As it is known, these rotated flavor fields at a given time, are linear combinations of the propagating particles modes. Moreover, it is checked that under a measurement in such a propagating states the probability of the initial state oscillates.

Further, we also discuss the connections of our discussion with the one of Vitiello and Blasone in reference [20]. In general, it is shown that the generating functional of the Green functions of the theory based in the propagating neutrinos coincides with the generating functional constructed, by taking the mean value in the vacua defined for electron and muon fields, of the evolution operator expressed in terms of these same fields fields (before the infinite volume limit is taken) [20,25]. This shows that the analysis in [25] gives a most natural representation of the perturbative expansion for applications in neutrino oscillations problems. However, their results coincide with the usual and ours simpler evaluations in the required infinite momentum limit for the actual experimental observations.

Finally, we present a condensed matter model in which oscillation modes appear which are associated with the interference between two electrons pertaining different bands of a solid. The electrons are assumed to have a common quasimomenta and slightly different band energies. The oscillation of the amplitude of measuring the defined superposition state after a time interval is given by the same formula describing the neutrino oscillations. Remarks are also given about a possible way of experimentally generating the considered superposition of many body states. It is argued that creating a superposition of photon waves, being sintonized each of them with the valence to conduction bands energy differences at the chosen quasimomentum of the particles, could be expected to generate the superposed many body states in first-order perturbation theory.

We estimate that the presented discussion clarifies how the usual QFT description of neutrino oscillations allows to predict quantum interference between various distinguishable particles. These conclusions emerge simply, because there is interference between two single particle states being in different Fock spaces (associated with not identical particles). This interference appears between two particles created in two different Fock spaces and added as following the superposition principle for many-body states. Therefore, the neutrino interference is evidenced as a clean and general many-body effect, invalidating the single-particle quantum mechanical rule (Bargmann rule) which excludes the interference between non-identical particles. The presented discussion indicates the validity of what can be evaluated as an important property of any QFT generalization of the Quantum Mechanics: the opening of the possibility for observing interference between many kinds of distinguishable particles.

The plan of the presentation is as follows. In Sect. 2, the two relativistic neutrino free QFT is presented. The Hamiltonian is expressed in terms of the two fields, and the commutation properties among these are written. This allows to define the creation and annihilation operators in momentum space for each of the two flavors and their commutation relations. Section 3 defines the space of states of the QFT associated with the model. Finally, Sect. 4 shows how the many body nature of the space of states determines the oscillation between the two types of neutrinos in the considered model. Next, in Sect. 5 the connection of the evaluation done here and the ones in references [20,25] are discussed. The last Sect. 6 presents the condensed matter model in which it is argued that oscillations modes being similar to the neutrino ones could be measured. The summary reviews the discussion and results.

2 The relativistic two neutrino model

Let us consider the mentioned in the Introduction model of two free relativistic massive fermions with flavor indices $\nu = 1, 2$. We will start from the quantum theory defined by the Hamiltonian operator

$$H = \sum_{\nu=1,2} \int \mathrm{d}\mathbf{x}\, \overline{\psi}_\nu(\mathbf{x},t)(-i\,\gamma.\nabla + m_\nu)\psi_\nu(\mathbf{x},t), \tag{1}$$

expressed in terms of a four components $r = 1, 2, 3, 4$ fermion field $\psi(\mathbf{x},t)$

$$\psi(\mathbf{x},t) = \psi_r(\mathbf{x},t) \equiv \begin{pmatrix} \psi_1(\mathbf{x},t) \\ \psi_2(\mathbf{x},t) \\ \psi_3(\mathbf{x},t) \\ \psi_4(\mathbf{x},t) \end{pmatrix}. \tag{2}$$

The fields in terms of the annihilation and creation operators $b_{_{\nu,s}}(\mathbf{p})$, $b^{+}_{_{\nu,s}}(\mathbf{p})$ for the two types of particles $\nu = 1, 2$ having helicities $s = \pm 1$, and the corresponding annihilation and creation operators for their antiparticles $d_{_{\nu,s}}(\mathbf{p})$, $d^{+}_{_{\nu,s}}(\mathbf{p})$, $\nu = 1, 2$, have the usual expansions

$$\psi_\nu(\mathbf{x}, t) = \sum_{\mathbf{p},\nu,s} (\omega_{\nu,s}(\mathbf{p}, \mathbf{x}) b_{_{\nu,s}}(\mathbf{p}) + v_{\nu,s}(\mathbf{p}, \mathbf{x}) d^{+}_{_{\nu,s}}(\mathbf{p})), \tag{3}$$

$$\psi^{+}_\nu(\mathbf{x}, t) = \sum_{\mathbf{p},\nu,s} (\omega^{*}_{\nu,s}(\mathbf{p}, \mathbf{x}) b^{+}_{_{\nu,r,s}}(\mathbf{p}) + v^{*}_{\nu,s}(\mathbf{p}, \mathbf{x}) d_{_{\nu,s}}(\mathbf{p})), \tag{4}$$

in which the $\omega_{\nu,s}(\mathbf{p})$ are the positive energy solutions of the Dirac equation with helicity $s = \pm 1$ for each of the two flavors $\nu = 1, 2$, defined as [26]

$$w_{\nu,+}(\mathbf{p{,}x}) = \frac{\exp(i\mathbf{p.x})}{\sqrt{L^3}\sqrt{2}\sqrt{2(n_3+1)}} \frac{\sqrt{\epsilon_\nu(\mathbf{p}) + m_\nu}}{\epsilon_\nu(\mathbf{p})} \begin{pmatrix} n_3 + 1 \\ n_1 + in_2 \\ \frac{|\mathbf{p}|}{\epsilon_\nu(\mathbf{p})+m_\nu} \begin{pmatrix} n_3 + 1 \\ n_1 + in_2 \end{pmatrix} \end{pmatrix}, \tag{5}$$

$$w_{\nu,-}(\mathbf{p{,}x}) = \frac{\exp(i\mathbf{p.x})}{\sqrt{L^3}\sqrt{2}\sqrt{2(n_3+1)}} \frac{\sqrt{\epsilon_\nu(\mathbf{p}) + m_\nu}}{\epsilon_\nu(\mathbf{p})} \begin{pmatrix} -n_1 + in_2 \\ n_3 + 1 \\ \frac{|\mathbf{p}|}{\epsilon_\nu(\mathbf{p})+m_\nu} \begin{pmatrix} -n_1 + in_2 \\ n_3 + 1 \end{pmatrix} \end{pmatrix}. \tag{6}$$

The antiparticle functions $v_{\nu,+}(\mathbf{p})$ with helicity $s = \pm 1$ for each of the two flavors $\nu = 1, 2$, have the expressions

$$v_{\nu,+}(\mathbf{p}) = \frac{\exp(-i\mathbf{p.x})}{\sqrt{L^3}\sqrt{2}\sqrt{2(n_3+1)}} \frac{\sqrt{\epsilon_\nu(\mathbf{p}) + m_\nu}}{\epsilon_\nu(\mathbf{p})} \begin{pmatrix} -\frac{|\mathbf{p}|}{\epsilon_\nu(\mathbf{p})+m_\nu} \begin{pmatrix} n_3 + 1 \\ n_1 + in_2 \end{pmatrix} \\ n_3 + 1 \\ n_1 + in_2 \end{pmatrix}, \tag{7}$$

$$v_{\nu,-}(\mathbf{p}) = \frac{\exp(-i\mathbf{p.x})}{\sqrt{L^3}\sqrt{2}\sqrt{2(n_3+1)}} \frac{\sqrt{\epsilon_\nu(\mathbf{p}) + m_\nu}}{\epsilon_\nu(\mathbf{p})} \begin{pmatrix} -\frac{|\mathbf{p}|}{\epsilon_\nu(\mathbf{p})+m_\nu} \begin{pmatrix} -n_1 + in_2 \\ n_3 + 1 \end{pmatrix} \\ -n_1 + in_2 \\ n_3 + 1 \end{pmatrix}. \tag{8}$$

The energies associated with the two types of particles are

$$\epsilon_\nu(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_\nu^2},\ \nu = 1, 2. \tag{9}$$

The field operators and the creation and annihilation ones for the two kinds of particles and antiparticles satisfy

$$\left[\psi_\nu(\mathbf{x{,}}t), \psi^{+}_{\nu'}(\mathbf{x'{,}}t)\right]_{+} = I\delta_{\nu,\nu'}\delta(\mathbf{x\text{-}x'}), \tag{10}$$

$$\left[b_{\nu,s}(\mathbf{p}), b^{+}_{\nu',s'}(\mathbf{p}')\right]_{+} = \delta_{\nu,\nu'}\delta_{s,s'}\delta^{(K)}_{\mathbf{p,p'}}, \tag{11}$$

$$\left[d_{\nu,s}(\mathbf{p}), d^{+}_{\nu',s'}(\mathbf{p}')\right]_{+} = \delta_{\nu,\nu'}\delta_{s,s'}\delta^{(K)}_{\mathbf{p,p'}}, \tag{12}$$

$$\left[b_{\nu,s}(\mathbf{p}), b_{\nu',s'}(\mathbf{p}')\right]_{+} = 0, \tag{13}$$

$$\left[b^{+}_{\nu,s}(\mathbf{p}), b^{+}_{\nu',s'}(\mathbf{p}')\right]_{+} = 0, \tag{14}$$

$$\left[d_{\nu,s}(\mathbf{p}), d_{\nu',s'}(\mathbf{p}')\right]_+ = 0, \tag{15}$$

$$\left[d^+_{\nu,s}(\mathbf{p}), d^+_{\nu',s'}(\mathbf{p}')\right]_+ = 0. \tag{16}$$

where, $\delta^{(K)}_{\mathbf{p},\mathbf{p}'}$ is the Kronecker Delta

$$\delta^{(K)}_{\mathbf{p},\mathbf{p}'} = \begin{matrix} 1 \text{ if } & \mathbf{p} = \mathbf{p}' \\ 0 \text{ if } & \mathbf{p} \neq \mathbf{p}' \end{matrix}, \tag{17}$$

and $\mathbf{p}$ are the momenta satisfying periodicity conditions in a large cubic box having a length size L and volume L^3. That is, if $L = Na$ and N is even, the components of the momenta $\mathbf{p} = (p_1, p_2, p_3)$ are given as

$$\begin{aligned} p_1 &= \frac{2\pi}{a}\frac{m_1}{N}, -\frac{N}{2} \leq m_1 < \frac{N}{2}, \\ p_2 &= \frac{2\pi}{a}\frac{m_2}{N}, -\frac{N}{2} \leq m_2 < \frac{N}{2}, \\ p_3 &= \frac{2\pi}{a}\frac{m_3}{N}, -\frac{N}{2} \leq m_3 < \frac{N}{2}. \end{aligned} \tag{18}$$

3 The space of states in the quantum field theory

After constructed the second quantization of the above defined simple non- relativistic two massive neutrinos system, we will consider the main issue in this work: to investigate the influence of the space of states of the theory, on the possibility for the description of the interference between different distinguishable particles as it occurs between the measured neutrino oscillations.

It can be started by remarking that in the literature, it has been discussed the possibility that when you have distinguishable particles, the correct space of states of the combined system could be the direct product of the Fock spaces which is associated with each of the distinguishable particles. In connection with this view, it should be stressed that this assumption is equivalent to establish a superselection rule not admitting the addition of states of the different Fock species. The establishment of superselection rules in QFT is allowed for sure in some cases [26]. That is the situation with respect to the electric charge in which you can adopt to not allow the superposition of states showing different amounts of electric charge. However, in such cases, the elimination of these type of superpositions is "dynamically" implemented, since the interaction operators conserve the charge of the states over which they act. Therefore, if we assume that the initial states over which the evolution operator acts have a well-defined amount of electric charge, any state after acting over it with evolution operator will have the same eigenvalue of the charge operator. However, in problems where the interaction operators have no property restricting the resulting states to the same physical subspace after their action, it seems not possible to impose such superselection rules. Specifically, for the case of the SM, and in particular for its neutrino sector, there are no superselection rules restricting the space of states.

Then, as it was mentioned, the space of states of the simple QFT model constructed here will be examined. The aim is to determine the conditions for being able to describe the observed neutrino oscillations. Below, in the context of the model constructed in the past section, the space of states will be considered as the linear completion of the direct product of the two Fock spaces, for each of the two distinguishable particles.

Consider the two Fock spaces $\mathcal{F}_\nu$, $\nu = 1, 2$ generated by the before defined creation operators for each of the two particles. The states of a complete basis in the Fock spaces of each separate particles will be indicated as $\left|\Phi_{f_\nu}\right\rangle_{\mathcal{F}_\nu}$, $\nu = 1, 2$ where f_ν is an index for any of the states in the Fock space of type ν . Then, the states in the direct product of the two Fock spaces can be written as

$$
\begin{aligned}
|\Psi\rangle_{\mathcal{F}_1\otimes\mathcal{F}_2} &= \sum_{f_1}\sum_{f_2} C_{f_1} C_{f_2} \left|\Phi_{f_1}\right\rangle_{\mathcal{F}_1} \otimes \left|\Phi_{f_2}\right\rangle_{\mathcal{F}_2} \\
&= \left(\sum_{f_1} C_{f_1} \left|\Phi_{f_1}\right\rangle\right) \otimes \left(\sum_{f_2} C_{f_2} \left|\Phi_{f_2}\right\rangle_{\mathcal{F}_2}\right).
\end{aligned} \tag{19}
$$

But, as mentioned before, this class of states, for a and b different from zero constants, excludes superpositions of the form

$$
a \left|\Phi_{f_1}\right\rangle_{\mathcal{F}_1} \otimes \left|\Phi_{f_2}\right\rangle_{\mathcal{F}_2} + b \left|\Phi_{f_1'}\right\rangle_{\mathcal{F}_1} \otimes \left|\Phi_{f_2'}\right\rangle_{\mathcal{F}_2},
$$

if (f_1, f_2) also differs form (f_1', f_2'). This exclusion is related with the fact that the direct product of Fock states is what is required to implement a superselection rule. The scalar product of two states pertaining to the direct product can be defined as

$$
\langle\Psi \left|\Psi'\right\rangle_{\mathcal{F}_1\otimes\mathcal{F}_2} = \sum_{f_1,} C^*_{f_1,} C'_{f_1,} \times \sum_{f_2} C^*_{f_2} C'_{f_2}, \tag{20}
$$

and normalized states for each component can be defined as separately satisfying

$$
\sum_{f_1,} C^*_{f_1,} C_{f_1,} = 1, \tag{21}
$$

$$
\sum_{f_2} C^*_{f_2} C_{f_2} = 1. \tag{22}
$$

However, the full space of states of the model is defined by the linear completion of the formerly defined direct product space. This linear completion, that will be called as $C(\mathcal{F}_1 \otimes \mathcal{F}_2)$ can be defined as the set of states generated by the arbitrary coefficients C_{f_1,f_2} in the superposition of the form

$$
|\Psi\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)} = \sum_{f_1,f_2} C_{f_1,f_2} \left|\Phi_{f_1}\right\rangle_{\mathcal{F}_1} \otimes \left|\Phi_{f_2}\right\rangle_{\mathcal{F}_2}. \tag{23}
$$

It is evident that such states cannot be always expressed in the form of a direct product of linear spaces

$$
\left(\sum_{f_1} C_{f_1} \left|\Phi_{f_1}\right\rangle\right)_{\mathcal{F}_1} \otimes \left(\sum_{f_2} C_{f_2} \left|\Phi_{f_2}\right\rangle_{\mathcal{F}_2}\right). \tag{24}
$$

The scalar product, assumed that the basis states in both Fock spaces are normalized, is defined as

$$
\langle\Psi \left|\Psi'\right\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)} = \sum_{f_1,f_2} C^*_{f_1,f_2} C'_{f_1,f_2}. \tag{25}
$$

Normalized states satisfy

$$1 = \sum_{f_1, f_2} C^*_{f_1, f_2} C'_{f_1, f_2}. \tag{26}$$

Therefore, in general, and in the absence of superselection rules, the Fock space of any QFT of a system of a number of n_p distinguishable particles (being either bosons or fermions) is interpreted as the whole set of states generated by the arbitrary coefficients $C_{f_1, f_2, \ldots, f_{n_p}}$ of the form

$$|\Psi\rangle_{C(\mathcal{F}_1 \otimes \cdots \otimes \cdot \mathcal{F}_{n_p})} = \sum_{f_1, f_2, \ldots, f_{n_p}} C_{f_1, f_2, \ldots, f_{n_p}} \left|\Phi_{f_1}\right\rangle_{\mathcal{F}_1} \otimes \left|\Phi_{f_2}\right\rangle_{\mathcal{F}_2} \otimes \cdots \otimes \left|\Phi_{f_{n_p}}\right\rangle_{\mathcal{F}_{n_p}}, \tag{27}$$

$$\left\langle\Psi|\Psi'\right\rangle_{C(\mathcal{F}_1 \otimes \cdots \otimes \cdot \mathcal{F}_{n_p})} = \sum_{f_1, f_2} C^*_{f_1, f_2, \ldots, f_{n_p}} C'_{f_1, f_2, \ldots, f_{n_p}}, \tag{28}$$

$$1 = \sum_{f_1, f_2} C^*_{f_1, f_2, \ldots, f_{n_p}} C'_{f_1, f_2, \ldots, f_{n_p}}. \tag{29}$$

4 Neutrino oscillations and the space of states

Let us now consider the QFT defined in Sect. 2. The space of the physical states of the theory was already defined in past section. The many states only including one of the two types of particles (let say of flavor $\nu = 1$ or flavor $\nu = 2$) are described by the Fock space ($\mathcal{F}_1$ or $\mathcal{F}_2$) generated by the creation operators of the specific kind of flavor. Therefore, let us argue below that in adopted space of states defined by the linear completion of the direct product of the two Fock space $C(\mathcal{F}_1 \otimes \mathcal{F}_2)$ the neutrino-like oscillations can be effectively described. Conversely, the oscillations cannot be directly explained if we would like to assume the direct product of the two Fock space as defining the physical space of states for the system

4.1 Space of states $C(\mathcal{F}_1 \otimes \mathcal{F}_2)$

As they are well defined in this space, we will examine the states of the form

$$\begin{aligned} |\Psi\rangle_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} &= \sum_{s=\pm} C_{1,s,} b^+_{1,+1} \, |0\rangle_{\mathcal{F}_1} \otimes |0\rangle_{\mathcal{F}_2} + \left|\Phi_{f_1}\right\rangle_{\mathcal{F}_1} \otimes C_{f_2,s} b^+_{2,+1} \, |0\rangle_{\mathcal{F}_2} \\ &= \sum_{s=\pm} \left(C_{1,s,} b^+_{1,+1}(\mathbf{p}) + C_{f_2,s} b^+_{2,+1}(\mathbf{p}) \right) |0\rangle_{\mathcal{F}_1} \otimes |0\rangle_{\mathcal{F}_2}, \end{aligned} \tag{30}$$

describing states in which a one particle state with negative helicity $s = -1$ is created in the Fock space $\mathcal{F}_1$ (with zero particles in the Fock space $\mathcal{F}_2$) is superposed with a zero particle created in $\mathcal{F}_1$ with one particle with helicity $s = -1$ created in $\mathcal{F}_2$. Both particles have the same momentum **p.** As it can be noted from the second line of the equation, these states are generated by a linear combination of field operators describing two different flavor modes both with a common value of the helicity and momentum. The form of these states was selected in other to more closely represent the situation for the neutrino oscillation measurement. The assumption of the physical nature of these states, then allow to define physical quantities (Hermitian operators constructed in terms of the employed fields) in terms of these superposition of fields, which create particles in different Fock spaces as the above defined ones.

4.2 Flavor rotated fields

It is possible to define now flavor rotated fields, as linear functions of the original fields in terms of which it is possible to define sets of physical quantities as Hermitian operator constructs. These definitions are here discussed in order to further consider measurements, describing quantum oscillations of amplitudes. Let us define the flavor rotated *electron* and *muon* like fields $b_{\nu_e,s}(\mathbf{p})$, $b_{\nu_\mu,s}$ $(\mathbf{p})$ (which are not the stable neutrino fields $b_{1,s}$, $b_{2,s}$) as

$$b_{\nu_e,s}\ (\mathbf{p}) = \left(\cos(\theta)b_{1,s}(\mathbf{p}) + \sin(\theta)b_{2,s}(\mathbf{p})\right), \tag{31}$$

$$b^+_{\nu_e,s}\ (\mathbf{p}) = \left(\cos(\theta)b^+_{1,s}(\mathbf{p}) + \sin(\theta)b^+_{2,s}(\mathbf{p})\right), \tag{32}$$

$$b_{\nu_\mu,s}\ (\mathbf{p}) = \left(-\sin(\theta)b_{1,s}(\mathbf{p}) + \cos(\theta)b_{2,s}(\mathbf{p})\right), \tag{33}$$

$$b^+_{\nu_\mu,s}\ (\mathbf{p}) = \left(-\sin(\theta)b^+_{1,s}(\mathbf{p}) + \cos(\theta)b^+_{2,s}(\mathbf{p})\right). \tag{34}$$

These operators, as the previous ones, also satisfy the following commutation relations

$$\left[b_{\nu_e,s}(\mathbf{p}), b_{\nu_e,s'}(\mathbf{p}')\right]_+ = \delta_{s,s'}\delta^{(K)}_{\mathbf{p},\mathbf{p}'} \tag{35}$$

$$\left[b^+_{\nu_\mu,s}(\mathbf{p}), b^+_{\nu_\mu,s'}(\mathbf{p}')\right]_+ = 0. \tag{36}$$

We will call $b_{\nu_e,,s}(\mathbf{p})$ as the electron neutrino field of helicity s and the $b_{\nu_\mu,,s}(\mathbf{p})$ as the muon neutrino of helicity s. The inverse transformation takes the form

$$b_{1,s}\ (\mathbf{p}) = \left(\cos(\theta)b_{\nu_e,s}(\mathbf{p}) - \sin(\theta)b_{\nu_\mu,s}(\mathbf{p})\right), \tag{37}$$

$$b^+_{1,s}\ (\mathbf{p}) = \left(\cos(\theta)b^+_{\nu_e,s}(\mathbf{p}) - \sin(\theta)b^+_{\nu_\mu,s}(\mathbf{p})\right),$$

$$b_{2,s}\ (\mathbf{p}) = \left(\sin(\theta)b_{\nu_e,,s}(\mathbf{p}) + \cos(\theta)b_{\nu_\mu,s}(\mathbf{p})\right),$$

$$b^+_{2,s}\ (\mathbf{p}) = \left(\sin(\theta)b^+_{\nu_e,s}(\mathbf{p}) + \cos(\theta)b^+_{\nu_\mu,s}(\mathbf{p})\right). \tag{38}$$

These new operators define creation and annihilation operators of the flavor rotated state over the vacuum. By example, the creation of a single particle state with rotated flavor ν_e or ν_μ , momentum $\mathbf{p}$ and helicity s are defined by

$$b^+_{\nu_e,s}(\mathbf{p})\,|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)} = b^+_{\nu_e,s}(\mathbf{p})\,|0\rangle_{\mathcal{F}_1} \otimes |0\rangle_{\mathcal{F}_2}\,, \tag{39}$$

$$b^+_{\nu_\mu,s}(\mathbf{p})\,|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)} = b^+_{\nu_\mu,s}(\mathbf{p})\,|0\rangle_{\mathcal{F}_1} \otimes |0\rangle_{\mathcal{F}_2}\,. \tag{40}$$

Now, it is possible to define the number of rotated flavor particles as the operator

$$\varrho_\theta = \sum_{\mathbf{p}} \sum_{s=\pm 1} (b^+_{\nu_e,s}(\mathbf{p})b_{\nu_e,s}(\mathbf{p}) - b^+_{\nu_\mu,s}(\mathbf{p})b_{\nu_\mu,s}(\mathbf{p})), \tag{41}$$

which has eigenvectors and eigenvalues

$$\varrho_\theta b^+_{\nu_e,s}(\mathbf{p})\,|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)} = b^+_{\nu_e,s}(\mathbf{p})\,|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\,, \tag{42}$$

$$\varrho_\theta b^+_{\nu_\mu,s}(\mathbf{p})\,|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)} = -b^+_{\nu_\mu,s}(\mathbf{p})\,|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\,. \tag{43}$$

Since the mentioned states are eigenfunctions of a physical observable (the Hermitian operator ϱ_θ), the result of the measurement of the rotated flavor eigenvalue should lead to the contraction of the wave-packet to one of the eigenstates of ϱ_θ . Therefore, the probability of the measurement will be the square of the amplitude defined by the scalar product of those eigenstates and the eigenstate of the physical quantity being measured.

It can be remarked that a similar transformation can be also implemented for the antiparticle annihilation and creation operators $d_{\nu,s}(\mathbf{p})$ and $d^+_{\nu,s}(\mathbf{p})$.

4.3 Neutrino oscillations description: $\nu_e \rightarrow \nu_e$

Let assume that an *electron neutrino* with helicity $s = -1$ had been created over the vacuum at time equal to zero defining the state

$$\begin{aligned}\left|\phi_{\nu_e,-1}(0)\right\rangle &= b^+_{\nu_e,-1}(\mathbf{p})\,|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\\ &= \left(\cos(\theta) b^+_{1,s}(\mathbf{p}) + \sin(\theta) b^+_{2,s}(\mathbf{p})\right)|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\,. \end{aligned} \tag{44}$$

Now, consider the evolution of the same *electron neutrino* state after a time t. Then, acting with the evolution operator over the created state at zero time, gives for the state at time t

$$\begin{aligned}\left|\phi_{\nu_e,-1}(t,\mathbf{p})\right\rangle &= U(t)\left|\phi_{\nu_e,-1}(0,\mathbf{p})\right\rangle\\ &= \exp(-iHt)\left|\phi_{\nu_e,-1}(0,\mathbf{p})\right\rangle\\ &= \left(\exp(-i\epsilon_1(\mathbf{p})t)\cos(\theta)a^+_{1,-1}(\mathbf{p}) + \exp(-i\epsilon_2(\mathbf{p})t)\sin(\theta)a^+_{2,-1}(\mathbf{p})\right)|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\,, \end{aligned} \tag{45}$$

where ϵ_1 and ϵ_2 are the energies of the mass eigenvalue neutrinos.

We can now examine the projection amplitude of the above evolved state over the *electron neutrino* state. Then, it is needed to evaluate the scalar product

$$\begin{aligned}{}_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\langle 0|\, b_{\nu_e,-1}(\mathbf{p}')\left|\phi_{\nu_e,-1}(t,\mathbf{p})\right\rangle &= {}_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\langle 0|\left(\cos(\theta)a_{1,-1}(\mathbf{p}) + \sin(\theta)a_{2,-1}(\mathbf{p})\right)\\ &\quad\times\left(\exp(-i\epsilon_1(\mathbf{p})t)\cos(\theta)a^+_{1,s}(\mathbf{p})\right.\\ &\quad\left.+\exp(-i\epsilon_2(\mathbf{p})t)\sin(\theta)a^+_{2,s}(\mathbf{p})\right)|0\rangle_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\\ &= \cos(\theta)^2\exp(-i\epsilon_1(\mathbf{p})t) + \sin(\theta)^2\exp(-i\epsilon_2(\mathbf{p})t). \end{aligned} \tag{46}$$

Therefore, the probability for the detection of the *electron neutrino* mode at any time instant after its creation at zero time, becomes

$$\begin{aligned}P_{\nu_e\rightarrow\nu_e}(t) &= \left|{}_{C(\mathcal{F}_1\otimes\mathcal{F}_2)}\langle 0|\, b_{\nu_e,-1}(\mathbf{p}')\left|\phi_{\nu_e,-1}(t,\mathbf{p})\right\rangle\right|^2\\ &= \left|\cos(\theta)^2\exp(-i\epsilon_1(\mathbf{p})t) + \sin(\theta)^2\exp(-i\epsilon_2(\mathbf{p})t)\right|^2\\ &= (\cos(\theta)^2)^2 + (\sin(\theta)^2)^2\\ &\quad+\cos(\theta)^2\sin(\theta)^2(\exp(-i\epsilon_1(\mathbf{p})t + i\epsilon_2(\mathbf{p})t)\\ &\quad+\exp(i\epsilon_1(\mathbf{p})t - i\epsilon_2(\mathbf{p})t))\\ &= (\cos(\theta)^2)^2 + (\sin(\theta)^2)^2\\ &\quad+2\cos(\theta)^2\sin(\theta)^2\cos\left((\epsilon_1(\mathbf{p}) - \epsilon_2(\mathbf{p}))t\right)\\ &= 1 - 2\cos(\theta)^2\sin(\theta)^2\left(1 - \cos\left((\epsilon_1(\mathbf{p}) - \epsilon_2(\mathbf{p}))t\right)\right)\\ &= 1 - \frac{\sin(2\theta)^2}{2}\left(1 - \cos\left((\epsilon_1(\mathbf{p}) - \epsilon_2(\mathbf{p}))t\right)\right). \end{aligned} \tag{47}$$

Let us consider now the relativistic approximation

$$|\mathbf{p}| \gg m_1, m_2, \tag{48}$$

which allows to derive the following relation

$$\begin{aligned}\epsilon_2(\mathbf{p}) - \epsilon_1(\mathbf{p}) &= \sqrt{m_2^2 + \mathbf{p}^2} - \sqrt{m_1^2 + \mathbf{p}^2} \\ &= |\mathbf{p}| \left(\sqrt{1 + \frac{m_2^2}{\mathbf{p}^2}} - \sqrt{1 + \frac{m_1^2}{\mathbf{p}^2}} \right) \\ &= \frac{1}{2|\mathbf{p}|}(m_2^2 - m_1^2) + \cdots \end{aligned} \tag{49}$$

Then, when the particles are ultra-relativistic, the propagation time for traveling along a distance R is given as

$$t = \frac{R}{c} = R, \tag{50}$$

thanks to the natural units $c = 1$ being used. Henceforth, the probability formula for the transition between an *electron neutrino* state into another *electron neutrino* state as a function of the measurement distance R gets the form

$$\begin{aligned}P_{\nu_e \to \nu_e}(t) &= \left|{}_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} \langle 0| \, b_{\nu_e,-1}(\mathbf{p}') \left|\phi_{\nu_e,-1}(t, \mathbf{p})\right\rangle\right|^2 \\ &= 1 - \frac{\sin(2\theta)^2}{2} \left(1 - \cos\left(-\frac{1}{2|\mathbf{p}|}(m_2^2 - m_1^2) R\right)\right) \\ &= 1 - \frac{\sin(2\theta)^2}{2} \left(1 - \cos\left(-2\pi \frac{R}{L}\right)\right), \\ L &= \frac{4\pi |\mathbf{p}|}{m_2^2 - m_1^2}. \end{aligned} \tag{51}$$

which reproduces the usual formula for the neutrino oscillations in terms of the oscillation and observation distances L and R , the momentum $|\mathbf{p}|$ and the neutrino masses m_1, m_2.

In a similar way it can be evaluated the probability of measuring a muon neutrino in the same state resulting from creating an electron neutrino at zero time. The result is

$$\begin{aligned}P_{\nu_e \to \nu_\mu}(t) &= \left|{}_{C(\mathcal{F}_1 \otimes \mathcal{F}_2)} \langle 0| \, b_{\nu_\mu,-1}(\mathbf{p}') \left|\phi_{\nu_e,-1}(t, \mathbf{p})\right\rangle\right|^2 \\ &= \frac{\sin(2\theta)^2}{2} \left(1 - \cos\left(-2\pi \frac{R}{L}\right)\right) \\ &= 1 - P_{\nu_e \to \nu_e}(t). \end{aligned} \tag{52}$$

Therefore, the discussion presented indicates that the nature of the state space (being the completion of the direct product of the two Fock spaces QFT) is a main reason allowing to explain the interference between distinguishable particles, which neutrino oscillation experiments show to exist. Therefore, the basic property allowing to break the usual single particle quantum mechanical exclusion of the interference between non-identical particles, is essentially the many-body character of the states that produce the oscillations. This many-body nature follows, because the neutrino interfering states are always in two different Fock spaces, which are able to interfere exclusively, because the assumed validity of general superposition principle for many-body states in the associated QFT. Note that the effect cannot occur if the space of states would be the direct product of the two Fock spaces.

Therefore, the main conclusion of this work follows: all the physical systems showing a space of states given by the linear completion of at least two Fock spaces corresponding to non-identical particles, can exhibit analogous of the neutrino oscillations effects, reflecting the interference between the participating distinguishable particles.

5 The link with the Vitiello–Blasone analysis

It is helpful to discuss the connection of the here investigated problems and the existing discussion about the role of QFT in the neutrino oscillations effect [20,25]. In this section, we will argue that the analysis in that works leads to a more natural representation of the perturbative expansion for the applications to neutrino oscillations. However, although their evaluation in general can be more precise, they coincide with the usual and ours simpler calculations in the infinite momentum limit [20,25].

To start, consider the evolution operator associated with neutrinos with a well defined momentum

$$U[\overline{\eta}_\nu, \eta_\nu] = T\left[\exp\left\{i\int_{-T}^{T} \mathrm{d}x \sum_{\nu=1,2}\left(\overline{\eta}_\nu(x)\Psi_\nu(x) + \overline{\Psi}_\nu(x)\eta_\nu(x)\right)\right\}\right], \tag{53}$$

in which the fermion sources $\overline{\eta}_\nu, \eta_\nu$ associated with the propagating neutrino fields defined in (3) and (4) are introduced, $2T$ is a large time interval difference between the period in with the initial state is constituted in terms in free particle states and the final period in which the scattered state is defined in terms of the *out* free states. But, the electron and muon fields can be written in terms of the propagating fields by using the Pontecorvo transformation

$$\begin{aligned}\Psi_e(x) &= \cos(\theta)\Psi_1(x) + \sin(\theta)\Psi_2(x)\\ &= G^{-1}(\theta, t)\Psi_1(x)G(\theta, t), \end{aligned}\tag{54}$$

$$\begin{aligned}\Psi_\mu(x) &= -\sin(\theta)\Psi_1(x) + \cos(\theta)\Psi_2(x)\\ &= G^{-1}(\theta, t)\Psi_2(x)G(\theta, t), \end{aligned}\tag{55}$$

in which $G^{-1}(\theta, t)$ is the time-dependent operator generating those transformations as defined in references [20,25].

But, using the properties of the time ordering rule, the evolution operator can be expressed as a product of ordered infinitesimal time evolution operators during small periods Δt , in the following form

$$\begin{aligned}U[\overline{\eta}_\nu, \eta_\nu] &= T\left[\exp\left\{i\int_{-T}^{T} \mathrm{d}x \sum_{\nu=1,2}\left(\overline{\eta}_\nu(x)\Psi_\nu(x) + \overline{\Psi}_\nu(x)\eta_\nu(x)\right)\right\}\right]\\ &= \prod_{n=0}^{N}\exp\left\{i\ \Delta t\int \mathrm{d}\overrightarrow{x} \sum_{\nu=1,2}\left(\overline{\eta}_\nu(t_n, \overrightarrow{x})\Psi_\nu(t_n, \overrightarrow{x}) + \overline{\Psi}_\nu(t_n, \overrightarrow{x})\eta_\nu(t_n, \overrightarrow{x})\right)\right\},\end{aligned} \tag{56}$$

where the time interval $2T$ had been divided by $N+1$ instants $t_n = -T + \frac{2T}{N}n$, $n = 0, 1, \ldots, N$, with $\Delta t = \frac{2T}{N}$ for a large integer value N. After using relations (54, 55) in (53), the following expression can be written

$$
\begin{aligned}
U[\overline{\eta}_\nu, \eta_\nu] &= G(\theta, T)G^{-1}(\theta, T) \prod_{n=0}^{N} G(\theta, t_n)G^{-1}(\theta, t_n) \exp\left\{ i\Delta t \int \mathrm{d}\overrightarrow{x} \sum_{\nu=1,2} (\overline{\eta}_\nu(t_n, \overrightarrow{x})\Psi_\nu(t_n, \overrightarrow{x}) \right. \\
&\quad \left. + \overline{\Psi}_\nu(t_n, \overrightarrow{x})\eta_\nu(t_n, \overrightarrow{x})) \right\} G(\theta, -T)G^{-1}(\theta, -T) \\
&= G(\theta, T) \prod_{n=0}^{N} \exp\left\{ i\, \Delta t \int \mathrm{d}\overrightarrow{x} \sum_{\nu=1,2} (\overline{\eta}_\nu(t_n, \overrightarrow{x})G^{-1}(\theta, t_n)\Psi_\nu(t_n, \overrightarrow{x})G(\theta, t_{n-1}) \right. \\
&\quad \left. + G^{-1}(\theta, t_n)\overline{\Psi}_\nu(t_n, \overrightarrow{x})G(\theta, t_{n-1})\eta_\nu(t_n, \overrightarrow{x})) \right\} G^{-1}(\theta, -T). \qquad (57)
\end{aligned}
$$

In order to allow approximating some generators associated with contiguous time instant t_n, the following relations will be considered

$$
\Delta t\ G^{-1}(\theta, t_n \pm \Delta t) = \Delta t\ G^{-1}(\theta, t_n) + O(\Delta t^2), \qquad (58)
$$

$$
\Delta t\ G(\theta, t_n \pm \Delta t) = \Delta t\ G(\theta, t_n) + O(\Delta t^2). \qquad (59)
$$

The terms being of second order in Δt can be omitted after limit $\Delta t \to 0$ will be taken for reproducing the time integral in formula (57).

Employing relations (58, 59) expression (57) can be rewritten as follows

$$
\begin{aligned}
U[\overline{\eta}_\nu, \eta_\nu] &= G(\theta, T) \prod_{n=0}^{N} \exp\left\{ i\, \Delta t \int \mathrm{d}\overrightarrow{x} \sum_{\nu=1,2} (\overline{\eta}_\nu(t_n, \overrightarrow{x})G^{-1}(\theta, t_n)\Psi_\nu(t_n, \overrightarrow{x})G(\theta, t_n) \right. \\
&\quad \left. + G^{-1}(\theta, t_n)\overline{\Psi}_\nu(t_n, \overrightarrow{x})G(\theta, t_n)\eta_\nu(t_n, \overrightarrow{x})) \right\} G^{-1}(\theta, -T) \\
&= G(\theta, T) \prod_{n=0}^{N} \exp\left\{ i\, \Delta t \int \mathrm{d}\overrightarrow{x}\, (\overline{\eta}_1(t_n, \overrightarrow{x})\Psi_e(x) + \overline{\Psi}_e(t_n, \overrightarrow{x})\eta_1(t_n, \overrightarrow{x}) \right. \\
&\quad \left. + \overline{\eta}_2(t_n, \overrightarrow{x})\Psi_\mu(x) + \overline{\Psi}_\mu(t_n, \overrightarrow{x})\eta_2(t_n, \overrightarrow{x})) \right\} G^{-1}(\theta, -T). \qquad (60)
\end{aligned}
$$

The above expression leads to the following relation between the evolution operator as expressed in terms of the propagating fields or in terms of the electron and muon fields:

$$
\begin{aligned}
U[\overline{\eta}_\nu, \eta_\nu] = G(\theta, T)\ T \left[\exp\left\{ i \int \mathrm{d}x\, (\overline{\eta}_1(x)\Psi_e(x) + \overline{\Psi}_e(x)\eta_1(t_n, \overrightarrow{x}) \right. \right. . \\
\left. \left. + \overline{\eta}_2(x)\Psi_\mu(x) + \overline{\Psi}_\mu(x)\eta_2(x)) \right\} \right] G^{-1}(\theta, -T). \qquad (61)
\end{aligned}
$$

Note that in expression (61), the auxiliary sources corresponding to the propagating fields are now associated with the electron and muon fields as follows

$$
\begin{aligned}
\overline{\eta}_1 &\to \Psi_e \\
\eta_1 &\to \overline{\Psi}_e, \\
\overline{\eta}_2 &\to \Psi_\mu, \\
\eta_2 &\to \overline{\Psi}_\mu.
\end{aligned} \qquad (62)
$$

We can now also write the corresponding two expressions for Green's functions generating functional as follows. The functional is defined as the mean value in the vacuum of the propagating neutrinos of the evolution operator as expressed in terms of the propagating fields. The vacuum associated with the propagating neutrinos can be written as

$$
|0\rangle_{1,2} = \prod_{\overrightarrow{p}} \left(|0\rangle_{1,\overrightarrow{p}} |0\rangle_{2,\overrightarrow{p}} \right), \qquad (63)
$$

and taking the mean value of the evolution operator and employing formula (61), the result can be written in the form

$$
\begin{aligned}
Z[\overline{\eta}_1,\eta_1,\overline{\eta}_2,\eta_2] &= {}_{1,2}\left\langle 0|U[\overline{\eta}_\nu,\eta_\nu]|0\right\rangle_{1,2} \\
&= {}_{1,2}\,\langle 0|\,G(\theta,T)\,T\left[\exp\left\{i\int \mathrm{d}x(\overline{\eta}_1(t_n,\overrightarrow{x})\Psi_e(x)+\overline{\Psi}_e(t_n,\overrightarrow{x})\eta_1(t_n,\overrightarrow{x})\right.\right. \\
&\quad +\overline{\eta}_2(t_n,\overrightarrow{x})\Psi_\mu(x)+\overline{\Psi}_\mu(t_n,\overrightarrow{x})\eta_2(t_n,\overrightarrow{x}))]G^{-1}(\theta,-T)\,|0\rangle_{1,2}\,.
\end{aligned}
\tag{64}
$$

This expression shows that the evaluation of the oscillations done in reference [25], through calculating the mean value of the time ordered product of field operators in the electron in electron-muon vacuum state

$$
|0,t\rangle_{e,\mu} = G^{-1}(\theta,t)\,|0\rangle_{1,2}\,, \tag{65}
$$

follows from a rearrangement of the original perturbative expansion which is most appropriate to consider neutrino oscillation effects. In our evaluation we have used a simpler, but compatible method, of creating electron neutrino states in the propagating neutrino vacuum and after evolving them in time. In ending, it should be remarked that the evaluation of the oscillations in Sect. 4 and references [20,25], both give oscillation formulae for general values of the momenta, which might differ. However, as it is already noted in reference [25], when the the large momentum approximation $|\overrightarrow{p}| >> m_1, m_2$ is taken, their calculation coincides with the usual evaluation, which is also equal to the one done in Sect. 4.

6 A condensed matter primer of oscillations

In this final section we intend to exemplify how the underlined Many Body nature of the neutrino oscillations implies that similar effects should happen in Condensed Matter Physics and more generally, in a variety of problems that can be described by using Many Body theory. For this purpose, let us consider a semiconductor material showing a single filled valence band and other two empty conduction bands. These energy bands are illustrated in Fig. 1. The Hamiltonian of the model is

$$
\begin{aligned}
H = \int \mathrm{d}\overrightarrow{x}\ &(\Psi_1^+(\overrightarrow{x})\ \epsilon_1(-i\overrightarrow{\nabla})\ \Psi_1(\overrightarrow{x}) \\
&+\Psi_2^+(\overrightarrow{x})\ \epsilon_2(-i\overrightarrow{\nabla})\ \Psi_2(\overrightarrow{x}) \\
&+\Psi_v^+(\overrightarrow{x})\ \epsilon_v(-i\overrightarrow{\nabla})\ \Psi_v(\overrightarrow{x})),
\end{aligned}
\tag{66}
$$

in which the field operators are defined as

$$
\Psi_1(\overrightarrow{x}) = \sum_{\overrightarrow{p}} \frac{1}{\sqrt{V}}\exp(i\ \overrightarrow{p}.\overrightarrow{x})a_1(\overrightarrow{p}), \qquad \Psi_2(\overrightarrow{x}) = \sum_{\overrightarrow{p}} \frac{1}{\sqrt{V}}\exp(i\ \overrightarrow{p}.\overrightarrow{x})a_2(\overrightarrow{p}),
\tag{67}
$$

$$
\Psi_1^+(\overrightarrow{x}) = \sum_{\overrightarrow{p}} \frac{1}{\sqrt{V}}\exp(-i\ \overrightarrow{p}.\overrightarrow{x})a_1^+(\overrightarrow{p}), \quad \Psi_2^+(\overrightarrow{x}) = \sum_{\overrightarrow{p}} \frac{1}{\sqrt{V}}\exp(-i\ \overrightarrow{p}.\overrightarrow{x})a_2^+(\overrightarrow{p}),
\tag{68}
$$

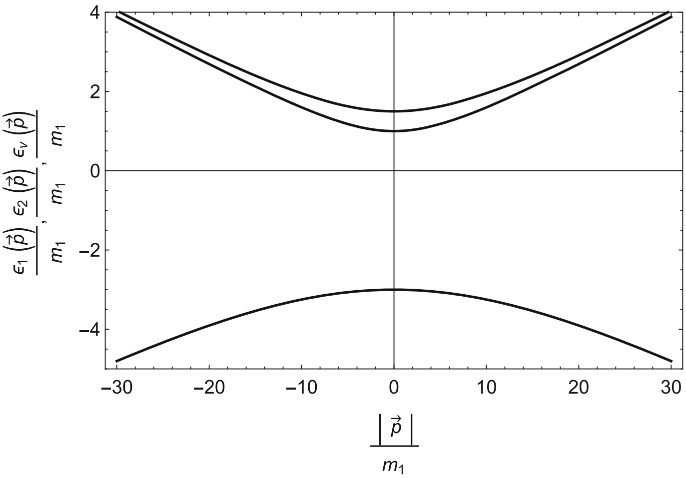

Fig. 1 The figure illustrates the three bands of the model: the two empty conduction bands and the filled valence one. Two electrons with the same momentum but moving in different conduction bands are considered

$$\Psi_v(\overrightarrow{x}) = \sum_{\overrightarrow{p}} \frac{1}{\sqrt{V}} \exp(i\ \overrightarrow{p}.\overrightarrow{x}) a_v^+(\overrightarrow{p}), \quad \Psi_v^+(\overrightarrow{x}) = \sum_{\overrightarrow{p}} \frac{1}{\sqrt{V}} \exp(-i\ \overrightarrow{p}.\overrightarrow{x}) a_v^+(\overrightarrow{p}), \tag{69}$$

and for the dispersion relations of the bands, for definiteness, we will choose with the momentum dependence defined as follows

$$\epsilon_1(\overrightarrow{p}) = \sqrt{m_1^2 + \overrightarrow{p}^2}, \tag{70}$$

$$\epsilon_2(\overrightarrow{p}) = \sqrt{m_2^2 + \overrightarrow{p}^2}, \ \ m_1 > m_2, \tag{71}$$

$$\epsilon_v(\overrightarrow{p}) = -\sqrt{m_v^2 + \overrightarrow{p}^2}. \tag{72}$$

The creation and annihilation operators for electrons in the empty and filled bands satisfy

$$[a_1^+(\overrightarrow{p}), a_1(\overrightarrow{p}')]_+ = \delta^K(\overrightarrow{p}, \overrightarrow{p}'), \tag{73}$$

$$[a_1(\overrightarrow{p}), a_1(\overrightarrow{p}')]_+ = [a_1^+(\overrightarrow{p}), a_1^+(\overrightarrow{p}')]_+ = 0 \tag{74}$$

$$[a_2^+(\overrightarrow{p}), a_2(\overrightarrow{p}')]_+ = \delta^K(\overrightarrow{p}, \overrightarrow{p}'), \tag{75}$$

$$[a_2(\overrightarrow{p}), a_2(\overrightarrow{p}')]_+ = [a_2^+(\overrightarrow{p}), a_2^+(\overrightarrow{p}')]_+ = 0, \tag{76}$$

$$[a_v^+(\overrightarrow{p}), a_v(\overrightarrow{p}')]_+ = \delta^K(\overrightarrow{p}, \overrightarrow{p}'), \tag{77}$$

$$[a_v(\overrightarrow{p}), a_v(\overrightarrow{p}')]_+ = [a_v^+(\overrightarrow{p}), a_v^+(\overrightarrow{p}')]_+ = 0, \tag{78}$$

and all the particular operators in a filled or empty band, anti-commute with all the other types of operators in different bands.

In terms of the defined operators, the Hamiltonian becomes

$$H = \frac{1}{V} \sum_{\overrightarrow{p}} \left(a_1^+(\overrightarrow{p})\, \epsilon_1(\overrightarrow{p})\, a_1(\overrightarrow{p}) + a_2^+(\overrightarrow{p})\, \epsilon_2(\overrightarrow{p})\, a_2(\overrightarrow{p}) + a_v^+(\overrightarrow{p})\, \epsilon_v(\overrightarrow{p})\, a_v(\overrightarrow{p}) \right). \tag{79}$$

The ground state of the system corresponds to the whole valence band filled of electron states and all the conduction bands states empty

$$|\Phi_G\rangle = \prod_{\overrightarrow{p}} \left(|0\rangle_{1,\overrightarrow{p}}|0\rangle_{2,\overrightarrow{p}}\right) \prod_{\overrightarrow{p}} a_v^+(\overrightarrow{p})|0\rangle_{v,\overrightarrow{p}}. \tag{80}$$

The construction above is chosen in the Dirac's sea approach to quantization, in which all the valence band electron states are filled.

Let us now consider a state defined at the initial time as

$$|\Phi(0)\rangle_{\text{osc}} = (\cos(\theta)a_1^+(\overrightarrow{p}) + \sin(\theta)a_2^+(\overrightarrow{p}))\,|\Phi_G\rangle\,. \tag{81}$$

That is, as a superposition of two states created each one by acting over the ground state with a different electron creation operator: The creation of an electron of quasimomentum $\overrightarrow{p}$ in one of the two empty conduction bands and the creation of another electron with the same momentum, but in other, closely laying in energy, conduction band.

This is a state that should be admitted in the Hilbert space of the physical states of the problem. It corresponds with a well-defined charge two units larger than the ground state one. Note that, if we had to define the state by using an electron a creation operator and an also electron but annihilation operator, the application of the superselection rule for the electric charge will not allow the chosen combination, because it superposes states with different charges.

Then, consider the time evolution of the considered state as

$$|\Phi(t)\rangle_{\text{osc}} = \exp(-iH\,t)\,(\cos(\theta)a_1^+(\overrightarrow{p}) + \sin(\theta)a_2^+(\overrightarrow{p}))\,|\Phi_G\rangle\,. \tag{82}$$

The matrix element determining the probability of measuring the initial state $|\Pi(0)\rangle_{\text{osc}}$ after the time interval t had passed is

$$\begin{aligned}
{}_{\text{osc}}\langle\Phi(0)\,|\Phi(t)\rangle_{\text{osc}} &= {}_{\text{osc}}\langle\Phi(0)\,|\exp(-iH\,t)\,(\cos(\theta)a_1^+(\overrightarrow{p}) + \sin(\theta)a_1^+(\overrightarrow{p}))\,|\Phi_G\rangle \\
&= {}_{\text{osc}}\langle\Phi(0)\,|\exp(-i\,\epsilon_1(\overrightarrow{p})t)\;\cos(\theta)a_1^+(\overrightarrow{p}) \\
&\quad + \exp(-i\,\epsilon_2(\overrightarrow{p})t)\;\sin(\theta)a_2^+(\overrightarrow{p})\,|\Phi_G\rangle \\
&= {}_{\text{osc}}\langle\Phi_G\,|(\cos(\theta)a_1(\overrightarrow{p}) + \sin(\theta)a_2(\overrightarrow{p})) \\
&\quad \times (\exp(-i\,\epsilon_1(\overrightarrow{p})t)\;\cos(\theta)a_1^+(\overrightarrow{p}) + \exp(-i\,\epsilon_2(\overrightarrow{p})t)\;\sin(\theta)a_2^+(\overrightarrow{p}))\,|\Phi_G\rangle \\
&= (\exp(-i\,\epsilon_1(\overrightarrow{p})t)\;\cos^2(\theta) + \exp(-i\,\epsilon_2(\overrightarrow{p})t)\;\sin^2(\theta)).
\end{aligned} \tag{83}$$

The evaluation of the squared modulus gives the probability

$$\begin{aligned}
|{}_{\text{osc}}\langle\Phi(0)\,|\Phi(t)\rangle_{\text{osc}}|^2 &= \exp(-i\,\epsilon_1(\overrightarrow{p})t)\;\cos^2(\theta) + \exp(-i\,\epsilon_2(\overrightarrow{p})t)\;\sin^2(\theta)|^2 \\
&= \cos^4(\theta) + \sin^4(\theta) + \cos^2(\theta)\sin^2(\theta) \\
&\quad \times (\exp(i(\epsilon_1(\overrightarrow{p}) - \epsilon_2(\overrightarrow{p}))t) + \exp(-i(\epsilon_1(\overrightarrow{p}) - \epsilon_2(\overrightarrow{p}))t)) \\
&= (\cos^2(\theta) + \sin^2(\theta))^2 \\
&\quad + 2\cos^2(\theta)\sin^2(\theta)(\cos((\epsilon_1(\overrightarrow{p}) - \epsilon_2(\overrightarrow{p}))t) - 1) \\
&= 1 + 2\cos^2(\theta)\sin^2(\theta)(\cos((\epsilon_1(\overrightarrow{p}) - \epsilon_2(\overrightarrow{p}))t) - 1).
\end{aligned} \tag{84}$$

Therefore, we arrive to the mentioned conclusion: the considered semiconductor systems could show oscillation effects, being similar to the neutrino oscillations ones, between particles being in different conduction bands, assumed that some physical processes become able to generate the states $|\Phi(t)\rangle_{\text{osc}}$.

6.1 Possibilities for experimental generation of the states $|\Phi(t)\rangle_{\mathrm{osc}}$

The examined problem involves two electron states. However, they are not the usual two electron states which are generated by the action on the ground state of products of two electron creation operators. On another hand they are analogous to the electron neutrino states which are superpositions of states obtained by acting on the vacuum with two different propagating neutrino creation operators. Thus, the question arises about how feasible could be the creation of such states in the Condensed Matter Physics laboratories. Still we have not a completely clear answer to this question. However, below we comment about one possibility for generating such states.

Since those states are fermion ones, the interaction terms able to generate them in scattering should be even in fermion operators. Let us assume that there is a superposition of two short pulses of photon energy propagating through the considered solid in a given axis, and with their two central photon energies of their energy spectra tuned at values

$$e_1 = \epsilon_1(\overrightarrow{p}) - \epsilon_v(\overrightarrow{p}), \tag{85}$$

$$e_2 = \epsilon_2(\overrightarrow{p}) - \epsilon_v(\overrightarrow{p}). \tag{86}$$

But, in the first approximation, each of the two photon pulses can be expected to separately create states of the form

$$a_1^+(\overrightarrow{p})a_v(\overrightarrow{p})\,|\Phi_G\rangle\,, \tag{87}$$

$$a_1^+(\overrightarrow{p})a_v(\overrightarrow{p})\,|\Phi_G\rangle\,. \tag{88}$$

In one of the states an electron hole pair is created, with the conduction electron being in one of the bands. In the other process, the electron of the created electron hole pair is propagating in the other band.

Henceforth, using the superposition principle, the following state becomes an admissible physical one

$$|\Theta(0)\rangle = a_1^+(\overrightarrow{p})a_v(\overrightarrow{p})\,|\Phi_G\rangle + a_2^+(\overrightarrow{p})a_v(\overrightarrow{p})\,|\Phi_G\rangle\,. \tag{89}$$

Note that this state corresponds to a particular selection $\theta = \frac{\pi}{4}$ in definition (81).

But, the state after evolving during an interval of time t, for afterwards being projected on the same initial state, and also taken the square of the result, leads to the following probability (for the measurement of the state created by the photon pulses after the time t)

$$\begin{aligned}
|\langle\Theta(0)\,|\Theta(t)\rangle\,|^2 &= |\,\langle\Theta(0)\,|\exp(i\,\epsilon_1(\overrightarrow{p})t)a_1^+(\overrightarrow{p})a_v(\overrightarrow{p})\,|\Phi_G\rangle \\
&\quad + \exp(i\epsilon_2(\overrightarrow{p})t)a_2^+(\overrightarrow{p})a_v(\overrightarrow{p})\,|\Phi_G\rangle\,|^2 \\
&= |\,\langle\Phi_G\,|(a_v^+(\overrightarrow{p})(a_1(\overrightarrow{p}) + a_2(\overrightarrow{p}))) \\
&\quad \times (\exp(i\,\epsilon_1(\overrightarrow{p})t)a_1^+(\overrightarrow{p})a_v(\overrightarrow{p})\,|\Phi_G\rangle + \exp(i\epsilon_2(\overrightarrow{p})t)a_2^+(\overrightarrow{p})a_v(\overrightarrow{p}))\,|\Phi_G\rangle\,|^2 \\
&= |\,\langle\Phi_G\,|(a_1(\overrightarrow{p}) + a_2(\overrightarrow{p}))(\exp(i\,\epsilon_1(\overrightarrow{p})t)a_1^+(\overrightarrow{p}) \\
&\quad + \exp(i\epsilon_2(\overrightarrow{p})t)a_2^+(\overrightarrow{p}))\,|\Phi_G\rangle\,|^2 \\
&= 1 + \frac{1}{2}(\cos((\epsilon_1(\overrightarrow{p}) - \epsilon_2(\overrightarrow{p}))t) - 1).
\end{aligned} \tag{90}$$

Therefore, the considered electromagnetic excitation of the system during a short period have the chance of creating the states having a similar effects as the neutrino oscillation. The existence of this interference is determined by the assumption of the validity of the superposition principle in a Many Body or QFT theory.

Before, ending it is helpful to note that the same oscillation result can be obtained if the two different electron bands considered are associated with completely different distinguishable particles. The formal evaluations are almost the same. A slight difference is that in such a case it should also be included two separate valence bands, one for each of the two kinds of particles. Also, the photon excitation should be considered to create two types of particle antiparticle states. We had preferred to present the discussion based on electrons, in order to be closer with the possibility of experimental realization.

Summary

The discussion in this work reproduces the main properties of the neutrino oscillations. The general aim of the exposition was to exhibit the properties of the theory that determine the existence of the interference effects between distinguishable particles. The main conclusion is that the oscillations can be attributed to the superposition principle in many body theories, which allows interference effects between distinguishable particles if no superselection rules act. The nature of these properties indicates that such interference effects should be present in a large variety of physical systems described by a QFT and Many Body theory. For this to occur, the theory should include at least two kinds of distinguishable particles, showing similar mass values and non-exhibiting superselection rules. Therefore, oscillations associated with the interference between non-identical particles should be expected to appear in a large number of physical theories, in Particle as well as in Condensed Matter Physics.

The issue of the connection of the discussion done here with the analysis in reference [20] about the role of QFT in determining the properties of the neutrino oscillations is discussed. We presented a theoretical argue showing that the generating functional of the Green function defined by the mean value (in the vacuum of propagating neutrinos) of the usual evolution operator, coincides with the generating functional determined as the mean value of the evolution operator in terms of the electron and muon operators, but taken in the electron-muon vacua, defined in [25]. This conclusion expresses that the procedure discussed in [25] is furnishing a most natural representation of the perturbative expansion associated with neutrino oscillations. However, it is underlined that although their evaluation of the effect might be more accurate for general values of the momenta, the consideration of large momentum limit reproduces the usual formula for the oscillations, also calculated by us.

Finally, in order to illustrate the argued relevance of the discussion for Condensed Matter Physics, we present a derivation of an oscillation formula between electrons in two different conduction bands. Therefore, it is argued that superposition principle in Many Body theory allows to define interference effects analogous to the neutrino's ones in Condensed Matter systems. Of special interest in this sense looks to be graphene-like materials where even relativistic-like equations become valid.

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