THE EUROPEAN
PHYSICAL JOURNAL PLUS

Reconciling Tsallis holographic dark energy models in modified $f(T, B)$ gravitational framework

M. Zubair[1,a] ![ORCID], **Lala Rukh Durrani**[1,b], **Saira Waheed**[2,c]

[1] Department of Mathematics, COMSATS University Islamabad, Lahore Campus, Lahore, Pakistan
[2] Prince Mohammad Bin Fahd University, Khobar, Kingdom of Saudi Arabia

Abstract In the present paper, we execute a reconstruction of $f(T, B)$ gravity model (where T is the torsion scalar and B is the boundary term) with reference to Tsallis holographic dark energy in flat Friedmann–Lemaitre–Robertson–Walker ($FLRW$) cosmology. Here, we consider two special classes of $f(T, B)$ model and reconstruct their exact forms by taking power and hybrid expansion law's de-Sitter case into account. The obtained solutions are then examined through the graphical analysis of EoS parameter and null energy bound for each model. It is concluded that all reconstructed models favor the current accelerated expansion regime by representing phantom cosmic epoch or de-Sitter model for different scenarios. Also, the graphical analysis of null energy condition indicates its validity only for one case, while for other cases it remains invalid. Furthermore, we explore the stability of these reconstructed models by applying a perturbation technique. It is found that the power law and de-Sitter solutions both show stable behavior against the introduced linear, isotropic and homogeneous perturbations.

Keywords Tsallis holographic dark energy · $f(T, B)$ gravity · Stability and perturbation

1 introduction

The universe is in a constant state of expansion ever since its beginning with the big bang, but this expansion never slowed down as it was expected; instead, it has been inexplicably accelerated over the past 5 billion years [1–3]. This mind boggling scenario introduced two arguments for its justification: The cosmic acceleration could be caused by the mysterious energy component in the universe named as dark energy (DE) and consequently a requisition for modification of gravity described as general relativity (GR) by Einstein. A more pragmatic approach rather than rectifying well-established fundamentals is that the vacuum of space is saturated with dark energy or a repulsive force counteracting the mutual gravitational pull of the galaxies and celestial bodies in the universe. One of the most startling discoveries made from the first hand evidence provided by the cosmological observations from Supernovae

[a] e-mails: mzubairkk@gmail.com; drmzubair@cuilahore.edu.pk (corresponding author)

[b] e-mail: lalarukh_durrani@hotmail.com

[c] e-mail: swaheed@pmu.edu.sa

 ⚛ Springer

Ia (SNe Ia), cosmic microwave background radiation (CMB), large scale structure (LSS), BAO and weak lensing was the fact that our universe consists of almost 5% ordinary matter (atoms, observable matter), 25% dark matter (DM) (a gravitationally interacting form of pressure less and cold non-baryonic matter) and 70% dark energy [4–8]. Cosmologists believe that the cosmic network of DM is responsible for providing the scaffolding upon which the observable universe is formulated. In the absence of DM, the distribution of galaxies in clumps and strands as observed by the astronomers would not be possible. Hence, the quest to unravel the mysteries pertaining DE and DM is considered as one of the most crucial fields of research in cosmology nowadays.

Although much of the observational data endorses the standard model of cosmology, the ΛCDM model, it is challenged owing to its shortcomings (i.e., the fine-tuning problem and the coincidence problem) and numerous dubious assumptions [9–11]. The coincidence problem of late cosmic acceleration illustrates the predicament that precisely in this era, the density of DM and DE denoted by ρ_{DM} and ρ_{DE}, respectively, is of the same order of magnitude even though they have contradistinctive time evolution. All these discrepancies in data have set off a search to understand the cosmic forces at play. Several alternative routes have been proposed so far to overcome the theoretical glitches in the ΛCDM model. One convenient approach to alleviate the aforementioned problem is to replace the constant Λ with the one that is time dependent. Another interesting proposition is to deviate from GR toward modified theories. Over the past two decades, numerous such theories have been devised and studied in literature. The most well-known models of modified gravity include the teleparallel equivalent of GR (TEGR) and its generalized version of $f(T)$ gravity (here T is the torsion scalar), the $f(R)$ theory with R as Ricci scalar, $f(R, \mathcal{T})$ gravity with $\mathcal{T}$ as the trace of the energy momentum tensor, braneworld model, Dvali–Gabadadze–Porrati model, Gauss–Bonnet gravity model, Dirac–Born–Infeld gravity, Brans–Dicke gravity and scalar tensor theories [12–24].

Teleparallel gravity is an interesting version of modified gravity which involves a curvature less spacetime structure with Lagrangian density in terms of the torsion scalar T. Considering the field equations, this theory is equivalent to GR, but in terms of its geometric perspectives, there is a drastic difference. The $f(T)$ gravity which is a natural extension of TEGR has been widely studied in the literature to explain the current cosmic scenario and reconstruction of cosmological models [25–28]. In this respect, Harko et al. introduced an interesting modification by inducing a non-minimal coupling of torsion scalar with the matter field in the standard $f(T)$ action [29–31]. Further, the $f(T, B)$ gravity is another significant contribution made by Bahamonde et al. [32] which is a unique locally Lorentz invariant theory with B ($R = -T + B$) as the torsion boundary term. One interesting aspect of this framework is that it establishes a relation with second-order $f(T)$ gravity and the fourth-order $f(R)$ gravity in certain limits. For example, if the boundary term is neglected, then it transforms into $f(T)$ model, while one can recover $f(R)$ theory if the function uses the profile $f(-T + B)$. The $f(T, B)$ theory has been explored within various cosmic scenarios and proved to be quite successful and interesting [33–43]. In the context of teleparallel theory, the topic of gravitational waves has been explored by Abedi and Capozziello [37] by taking a general framework involving the torsion scalar, the boundary term as well as a scalar field. They concluded that the minimal interaction of boundary term with torsion scalar and the scalar field results in the gravitational waves having the same polarization modes as those of GR. The topic of gravitational waves and its polarization modes has been further explored by Capozziello et al. [38] in $f(T, B)$ gravity, and significant results have been achieved. In another recent work, authors [39] claimed that late time oscillatory behavior of EoS can be explained appropriately in $f(T, B)$ framework, and hence, it can be an interesting can-

didate to modify ΛCDM model using observational data set. Pourbagher and Amani [40] investigated $f(T, B)$ framework by using new agegraphic DE model and viscous fluid and discussed different cosmic measures and thermodynamics using observational constraints. Another significant work in teleparallel theory is by Bahamonde and Camci [41]. They used Noether symmetry technique to find exact solutions for spherically symmetric and static spacetime where different forms of $f(T, B)$ have been considered. Some other interesting and viable contributions in this gravitational framework can be found in the literature [42–46].

Amid the DE proposals, holographic dark energy (HDE) model [47] is the one that has been explored extensively in a variety of DE schemes since the last few decades. Basically, its energy density $\rho_{DE} = 3C^2 M_{pl}^2 L^{-2}$ (where C is a an arbitrary constant, $M_{pl} = \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck mass with G as the gravitational constant, and L is the IR cutoff) is derived using the holographic principle given by Gerard't Hooft [48]. Altering the entropy–area relation or infrared (IR) cutoff, various HDE models have been constructed and analyzed till date. Tavayef et al. [49,50] introduced another dynamical HDE model, namely Tsallis holographic dark energy (THDE) using holographic principal along with the entropy expression proposed by Tsallis and Cirto which is given by $\mathcal{S}_\delta = \gamma A^\delta$, where S denotes entropy, A represents area, γ is arbitrary, and δ is a non-additivity parameter. THDE is found to be stable under specified conditions and also very successful in explaining the late time cosmic expansion. Thereafter in the last few years, THDE has been investigated vigorously applying a variety of IR cutoffs, involving interactions and under the influence of several modified gravities [51–55].

Cosmological reconstruction is one of the most vital schemes utilized in modified gravity to deliver accurate cosmological dispositions. Usually, the reconstruction scheme involves the unification of modified theory and the chosen DE model by comparing their respective energy densities. In [56–63], authors executed the reconstruction of $f(R)$ and $f(R, T)$ gravity theories under different scenarios that can produce realistic cosmology and effectively demonstrate the current DE dominating phases. Sharif and Nazir [64] explored the cosmic evolutionary scenario of generalized ghost pilgrim DE in $f(T, T_G)$ gravity by applying various scale factors. Sharif and Zubair [65] reconstructed the $f(R)$ gravity model by considering the pilgrim DE model as the DE candidate. They studied its behavior under the effect of different IR cutoffs and obtained interesting results. Houndjo and Piattella [66] numerically generated a cosmic regime consistent with the accelerated cosmic expansion by considering the HDE along with two special cases of $f(R, \mathcal{T})$ gravity models. Sharif and Saba [67] analyzed the THDE with IR cutoff as Hubble horizon in $f(G, \mathcal{T})$ gravity by exploring some cosmic diagnostic parameters and phase planes. Also, Waheed [68] recently studied the reconstruction process in $f(T)$ gravity and its different extensions, namely $f(T, G)$, non-minimal interaction of torsion and matter along with $f(T, \mathcal{T})$ theory by taking THDE model into account.

Motivated by the above literature, in the present paper, we are interested to investigate the reconstruction paradigm for THDE using two interesting models of $f(T, B)$ gravity. The paper is organized in this pattern. In Sect. 2, we shall provide a brief review of $f(T, B)$ theory along with the modified Einstein field equations and the THDE model. Section 3 is devoted to study the reconstruction paradigm for $f(T, B)$ models analytically based on THDE model. In each case, we shall apply the power law cosmology and hybrid expansion law (de-Sitter universe) to determine the evolution of cosmological parameters and validity of the null energy condition (NEC). Section 4 provides the stability analysis of all the reconstructed models using the perturbation technique. The last section discusses the conclusion and main findings of this study.

2 Reconstruction of THDE $f(T, B)$ models

In this section, we shall briefly describe the basic ingredients required for this study and also formulate the basic field equations of $f(T, B)$ theory for a flat, homogeneous and isotropic FLRW universe. The locally Lorentz invariant $f(T, B)$ gravity provides an interesting modified framework dependent on the torsion scalar T and divergence of the torsion vector (the vector defined by contracting the first two indices of torsion tensor) which is known as boundary term [39]. The vierbeins $e^a{}_\mu$ in this theory are the orthonormal vectors at each point of the spacetime manifold. They obey the orthogonality relation and are hence related to the metric by $g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab}$, where η_{ab} is the Minkowski metric. It should be noted here that the Latin and Greek letters refer to the tangent space and spacetime indices, respectively. Also, the term $e^a{}_\mu$ is different from the term $e_\mu{}^a$ and they primarily represent inverses of each other. For a spacetime with vanishing curvature and nonzero torsion, we require the Weitzenböck connection in place of the standard Levi–Civita connection which is defined as $W^a_{\mu\nu} = \partial_\nu e^a{}_\mu + \omega^a{}_{b\nu} e^b{}_\mu$. The torsion tensor can be formulated as

$$T^a{}_{\mu\nu} = W^a_{\nu\mu} - W^a_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu} e^b{}_\nu - \omega^a{}_{b\nu} e^b{}_\mu.$$

Also, the contortion tensor and superpotential are defined in terms of torsion tensor as

$$K^{\mu\nu}{}_a = \frac{1}{2}(T_a{}^{\mu\nu} + T^{\nu\mu}{}_a - T^{\mu\nu}{}_a),$$
$$S_a{}^{\mu\nu} = K^{\mu\nu}{}_a - e_a{}^\nu T^{\alpha\mu}{}_\alpha + e_a{}^\mu T^{\alpha\nu}{}_\alpha.$$

Contracting the torsion tensor with the superpotential provides us with the torsion scalar given by $T = T^a{}_{\mu\nu} S_a{}^{\mu\nu}$, which is calculated on $W^a_{\mu\nu}$ in a similar fashion as the Ricci scalar R's dependency on the Levi–Civita connection. Using the contortion tensor, we can obtain a connection between the R and torsion scalar as

$$R = -T + B, \quad B = \frac{2}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} T^\sigma{}_\sigma{}^\mu \right). \tag{1}$$

Here, g is the determinant of the metric tensor. Therefore, with the purpose of combining both $f(R)$ and $f(T)$ gravity, the action for $f(T, B)$ theory of gravity as proposed by Bahamonde et al. [32] is given by

$$\mathcal{S}_{f(T,B)} = \frac{1}{2\kappa^2} \int \sqrt{-g} f(T, B) \mathrm{d}^4 x + \mathcal{S}_m, \tag{2}$$

where $\kappa = \sqrt{8\pi G}$ and $\mathcal{S}_m$ represents the part of action corresponding to the ordinary matter source. If we remove the boundary term, the above action represents the well-known $f(T)$ gravitational framework, while if we choose the specific case $f(-T + B)$, the natural extension of GR, the action of $f(R)$ gravity can be recovered.

The flat FRW spacetime with expansion factor $a(t)$ is described by the following line element

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \tag{3}$$

Considering this metric the tetrad field can be written as

$$e^\nu_\mu = \mathrm{diag}(1, a(t), a(t), a(t)). \tag{4}$$

It is imperative to mention here that for the above choice of diagonal tetrad, the spin connection $\omega^a{}_{b\nu}$ will be zero [69]. Let us consider the source of ordinary matter as perfect fluid which

is defined by the energy–momentum tensor

$$\Theta_{\mu\nu} = (\rho_m + p_m)u_\mu u_\nu - p_m g_{\mu\nu}, \tag{5}$$

where the terms ρ_m, p_m and u_ν stand for the ordinary matter density, pressure and fluid four-velocity, respectively, and satisfy the relation: $u_\mu u^\nu = -1$. In co-moving coordinates, one can pick $u^\mu = \delta_0^\mu$. In the presence of perfect fluid contents, the $f(T, B)$ field equations for FRW geometry can be written as

$$-3H^2(3f_B + 2f_T) + 3H\dot{f}_B - 3\dot{H}f_B + \frac{1}{2}f(T, B) = \kappa^2 \rho_m, \tag{6}$$

$$-3H^2(3f_B + 2f_T) - \dot{H}(3f_B + 2f_T) - 2H\dot{f}_T + \ddot{f}_B + \frac{1}{2}f(T, B) = -\kappa^2 p_m. \tag{7}$$

Here, dot represents the time derivative of the respective function. One can recover the TEGR field equations by replacing the term $f(T, B)$ with $-T$. For FRW geometry, the torsion scalar and boundary term are defined in terms of Hubble parameter using $R = -T + B = 6(2H^2 + \dot{H})$, since $T = 6H^2$ and $B = 6(\dot{H} + 3H^2)$, respectively. These field equations can also be rewritten as

$$3H^2 = -\kappa^2 \left(\frac{\rho_m}{2f_T} + \rho_{TB} \right), \tag{8}$$

$$2\dot{H} = \kappa^2 \left(\frac{\rho_m}{2f_T} + \frac{p_m}{2f_T} + \rho_{TB} + p_{TB} \right), \tag{9}$$

where the terms ρ_{TB} and p_{TB} are defined as:

$$\rho_{TB} = \frac{1}{2f_T \kappa^2} \left[-3H\dot{f}_B + (3\dot{H} + 9H^2)f_B - \frac{1}{2}f(T, B) \right], \tag{10}$$

$$p_{TB} = \frac{1}{2f_T \kappa^2} \left[\frac{1}{2}f(T, B) + \dot{H}(2f_T - 3f_B) - 2H\dot{f}_T - 9H^2 f_B + \ddot{f}_B \right]. \tag{11}$$

Also, the corresponding energy conservation equations are given by

$$\dot{\rho}_m + 3H(1 + \omega_m)\rho_m = 0, \tag{12}$$

$$\dot{\rho}_{TB} + 3H(1 + \omega_{TB})\rho_{TB} = 0. \tag{13}$$

Here, ω_m denotes the ordinary matter equation of state parameter (EoS) and it is simply defined as $p_m = \omega_m \rho_m$, where $0 \leq \omega_m \leq 1$ and ω_{TB} is the EoS parameter of dark energy. The integration of Eq. (12) yields the following expression:

$$\rho_m(t) = \rho_0 a(t)^{-3(1+\omega_m)}. \tag{14}$$

Further, we consider the Tsallis HDE energy density given by [49]:

$$\rho_{DE} = \xi L^{2\delta-4}, \tag{15}$$

where both ξ and δ are arbitrary parameters, while L represents the IR cutoff. It is interesting to mention here that in the limit $\delta = 1$, the Tsallis HDE will be reduced to simple HDE model of DE. In the present case, we undertake the most simple choice for IR cutoff as the Hubble radius (H^{-1}), and consequently, the energy density for THDE takes the form

$$\rho_{DE} = \xi H^{4-2\delta}. \tag{16}$$

There is a bulk of the literature that has discussed cosmic acceleration undertaking different HDE models with Hubble IR cutoff into account. Zadeh et al. [70] explored the Tsallis

HDE model by using three IR cutoffs: Ricci horizon, particle horizon and Granda–Oliveros (GO). It has been concluded that contrary to the simple HDE case, when particle horizon is used as an IR cutoff for Tsallis HDE, it can produce cosmic expansion. Moreover, this horizon also exhibits stability of the model in the presence of an interaction term for some values of the redshift parameter. Tavayef et al. [71] have established that the use of Hubble radius as IR cutoff for Tsallis HDE model can also produce the late time cosmic acceleration even without including interaction of dark cosmic sectors. This result opposes the simple HDE model which is unable to endorse an accelerating cosmos in the absence of interaction term where the Hubble cutoff is taken into account. Also, in another investigation by Jawad et al. [72] of the cosmic expansion using Renyi, Tsallis and Sharma–Mittal HDE models with Hubble IR cutoff in loop quantum cosmology, it is found that models based on Renyi and Sharma–Mittal HDE remained stable in the later epochs, while Tsallis HDE showed unstable behavior. In a research article by Zadeh et al. [73], the authors have considered two choices for the system's IR cutoffs: the age of the universe and the conformal time. From the analysis of different cosmic measures, it is concluded that in non-interacting case, the discussed models are classically unstable (using speed of sound). In this context, Sadri [74] has studied Tsallis HDE model by opting the Hubble as well as the future event horizon as the IR cutoff and concluded that in comparison with the Hubble horizon, the future event horizon leads to a stable model generating cosmic acceleration in the final epochs. Likewise, Maity and Debnath [75] introduced a nonlinear coupling of same HDE models with the cold DM in the framework of flat fractal universe of D-dimensions where IR cutoffs as Hubble radius and GO have been considered. It was concluded that due to the considered interaction term, these models can successfully explain the cosmic expansion.

In a recent study [76], authors examined the features of Tsallis HDE model in the framework of Brans–Dicke gravity by taking Hubble IR cutoff. They have concluded that the obtained results of different cosmic measures like deceleration and EoS are in good agreement with the recent observational data. Similarly, in [77], the researchers showed that by taking Tsallis HDE model with Hubble IR cutoff along with interaction term, the model can exhibit the usual thermal history and DM and DE epochs before resulting in the DE-dominated future eras. It is also observed that different cosmic measures like deceleration parameter, speed of sound and EoS behave more appropriately when the condition of $\delta > 2$ is imposed. Shekh et al. [78] have explored the $f(T, B)$ framework by using Hubble radius as IR cutoff for Renyi, Tsallis and Sharma–Mittal HDE models. It can be concluded that a majority of the above-mentioned literature confirms that this choice of HDE model as well as IR cutoff can successfully lead to cosmic acceleration but remain unstable (using speed of sound). Therefore, our prime target is to explore the behavior of the reconstructed solutions using Hubble radius as IR cutoff and to examine whether the obtained solutions are stable and favor the cosmic acceleration. Also, the second reason for choosing Hubble as the IR cutoff is its simplicity since we are interested in finding the exact solutions and not the numerical interpretations, which will be difficult to obtain in case of a complicated IR cutoff.

3 $f(T, B)$ models

In the present section, we will reconstruct the form of generic function $f(T, B)$ by using power and hybrid expansion laws of scale factor. Here, we shall consider two particular forms of $f(T, B)$ function given by

- $f(T, B) = f_1(T) + f_2(B),$

- $f(T, B) = -T + g(B)$.

For the reconstruction of generic function, correspondence of two densities ρ_{TB} and ρ_{DE} yields a complicated differential equation, so it is necessary to assume some simple form of $f(T, B)$ function. The most simplest choice will be the separable form which can be as sum or product of two functions. The former will provide the minimal interaction of T and B, while the later leads to the direct interaction and of course again can yield a complex equation. Just for the sake of simplicity in calculations, we have assumed the addition form which has been considered already in numerous researches; for instance, one can see the references [33–35,67]. The second case: $f(T, B) = -T + g(B)$ is similar to the models $f(R) = R + F(R)$ and $f(T) = -T + f(T)$ considered in the respective $f(R)$ and $f(T)$ theories [79,80]. It is interesting to mention here that although the first separable case is more general than the second choice, one might get a different model as mentioned in [33].

3.1 $f(T, B) = f_1(T) + f_2(B)$

Firstly, we consider the function $f(T, B)$ as the sum of two independent functions of T and B, i.e., separable in the sum form.

3.1.1 Power law Cosmology

Here, we reconstruct the form of generic function by taking power law form of scale factor into account. It is considered as one of the most interesting and widely used choices of scale factor for the description of accelerated expansion of our cosmos in modified gravity theories. The power law form of scale factor is defined by [81,82]:

$$a(t) = a_0(t_p - t)^{-\mu}; \qquad \mu > 0, \ \ t_p > t, \tag{17}$$

where a_0 is the current value of the scale factor, t_p represents the possible time when a finite singularity in future might occur, and μ is a constant. By using Eq. (15), the Hubble parameter, its time rate, the torsion scalar and boundary term take the form

$$H = \frac{\mu}{t_p - t}, \qquad \dot{H} = \frac{\mu}{(t_p - t)^2}, \qquad T = \frac{6\mu^2}{(t_p - t)^2}, \qquad B = \frac{6\mu(3\mu + 1)}{(t_p - t)^2}.$$

Also, the deceleration parameter given by $q = -1 - \frac{\dot{H}}{H^2}$ leads to the expression $-1 + \frac{1}{\mu}$ which further enables us to set a constraint on the value of parameter μ. It is easy to check that $\mu > 1$ favors the accelerating phases of our cosmos, while a decelerating epoch can be discussed for $\mu \leq 1$.

In order to find the form of generic function $f(T, B)$ that can generate the same cosmology as produced by THDE model, we use the correspondence scheme of energy densities by assuming $\rho_{DE} = \rho_{TB}$. Consequently, we obtain

$$\frac{1}{2 f_T \kappa^2} \left[-3H \dot{f}_B + (3\dot{H} + 9H^2)f_B - \frac{1}{2} f(T, B) \right] = \xi H^{4-2\delta}. \tag{18}$$

For the specified separable form given by $f(T, B) = f_1(T) + f_2(B)$, Eq. (18) can be rewritten as

$$-2B^2 f_{2BB}(B) + (3\mu + 1)Bf_{2B}(B) - (3\mu + 1)f_2(B) = 2(3\mu + 1)K, \tag{19}$$

$$4\xi\kappa^2 \left(\frac{T}{6}\right)^{(2-\delta)} f_{1T}(T) + f_1(T) = 2K, \tag{20}$$

where we have applied the methodology of separation of variables with K as a constant of separation. Here, we have introduced the notations: $f_{1T} = \frac{\partial f_1(T)}{\partial T}$, $f_{1TT} = \frac{\partial^2 f_1(T)}{\partial T^2}$, $f_{2B} = \frac{\partial f_2(B)}{\partial B}$ and $f_{2BB} = \frac{\partial^2 f_2(B)}{\partial B^2}$. The analytical solution for these equations can be found as:

$$f_2(B) = c_1 B^{\frac{3\mu+1}{2}} + c_2 B - 2K, \tag{21}$$

$$f_1(T) = c_3 e^{-\frac{2^{-\delta} 3^{2-\delta} T^{\delta-1}}{\xi(\delta-1)\kappa^2}} + 2K. \tag{22}$$

Thus, in power law cosmology, the function takes the form

$$f(T, B) = c_1 B^{\frac{3\mu+1}{2}} + c_2 B + c_3 e^{-\frac{2^{-\delta} 3^{2-\delta} T^{\delta-1}}{\xi(\delta-1)\kappa^2}}; \quad \delta \neq 1. \tag{23}$$

Here, the constants of integration c_1, c_2 and c_3 can be found by taking the following boundary conditions [83–85] into account

$$f_B|_{t=t_0} = 1, \qquad f_T|_{t=t_0} = 1, \qquad f(T_0, B_0) = B_0 + \epsilon, \tag{24}$$

where $\epsilon = 6H_0^2(3 - \Omega_{DE0})$, while Ω_{DE0} is taken as $1 - \Omega_{M0}$ and H_0 represents the present day value of the Hubble parameter. Therefore, after some algebraic manipulations, we get

$$c_1 = \frac{2B_0^{-\frac{1+3\mu}{2}} T_0^{-\delta} \left(54H_0^2 T_0^\delta \Omega_{DE0} - 162H_0^2 T_0^\delta - 6^\delta \xi \kappa^2 T_0^2\right)}{9(3\mu - 1)},$$

$$c_2 = \frac{T_0^{-\delta} \left(27\mu B_0 T_0^\delta - 9B_0 T_0^\delta - 162\mu H_0^2 T_0^\delta \Omega_{DE0} - 54H_0^2 T_0^\delta \Omega_{DE0} + 486\mu H_0^2 T_0^\delta + 162H_0^2 T_0^\delta + 2^\delta 3^{\delta+1} \mu \kappa^2 \xi T_0^2 + 6^\delta \kappa^2 \xi T_0^2\right)}{9B_0(3\mu-1)},$$

$$c_3 = \left(-2^\delta\right) 3^{\delta-2} \kappa^2 \xi T_0^{2-\delta} e^{\frac{2^{-\delta} 3^{2-\delta} T_0^{\delta-1}}{\xi(\delta-1)\kappa^2}}.$$

Here, if we substitute our reconstructed solution (23) in the Friedmann Eq. (8), we get:

$$3H^2 = \kappa^2 \left(\frac{\kappa^2 \xi \rho_m}{c_3 2^{1-\delta} 3^{2-\delta} T^{\delta-2}} e^{\frac{2^{-\delta} 3^{2-\delta} T^{\delta-1}}{\xi(\delta-1)\kappa^2}} - \xi H^{-2\delta+4}\right) \tag{25}$$

Considering Eq. (12), we arrive at:

$$3\mu^2 t^{-2} \sim 2e^{\frac{3\left(\frac{\mu^2}{t^2}\right)^{\delta-1}}{2(\delta-1)}} \left(\frac{\mu^2}{t^2}\right)^{2-\delta} \left((-t)^{-\mu}\right)^{-3(\omega_m+1)} + \left(-\frac{\mu}{t}\right)^{4-2\delta} \tag{26}$$

For a dominating THDE, the power law solution will exist only when the constraint $-2 = 2\delta - 4$ is imposed which further gives $\delta = 1$ and hence corresponds to simple HDE model. Further, if we consider matter-dominated epoch, then constraint $-2 = 3\mu(1+\omega_m)$ should be imposed which further leads to $\mu = -\frac{2}{3(1+\omega_m)}$. It is worthy to note here that for the matter-dominated case, one must consider $\delta \approx 1$, not the constraint $\delta = 1$, as the exponential term on the right-hand side of Eq. (26) satisfies the condition $e^{\frac{3\left(\frac{\mu^2}{t^2}\right)^{\delta-1}}{2(\delta-1)}} \to 0$ as $\delta \to 1$. Thus, in this case, the effect of matter term in the field equations thoroughly vanishes if $\delta = 1$ is imposed. Therefore, in order to maintain the presence of HDE fluid within a power law cosmology, the evolution must occur during a matter-dominated phase imposing the constraint $\delta \approx 1$ and $\mu = -\frac{2}{3(1+\omega_m)}$ which, in our case, will reduce to $-\frac{2}{3}$ as we are considering a pressureless matter source ($\omega_m = 0$). This condition ensuring the matter-dominated case $\mu = -\frac{1}{3} < 1$ is also suggested by the deceleration parameter. Consequently, to maintain consistency with
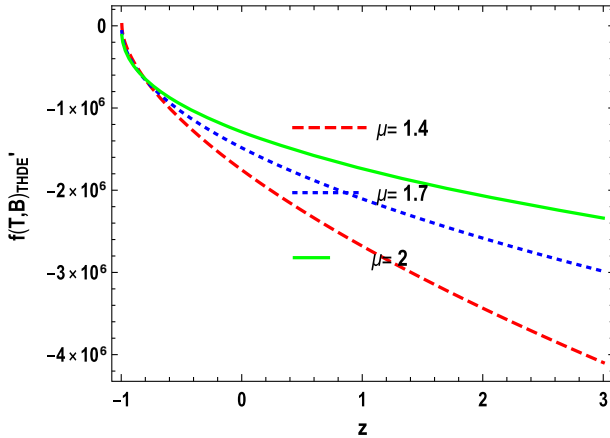
Fig. 1 The graphs show the behavior of $f(T, B)$ function versus redshift parameter for various choices of μ, $\delta = 1$ and $\xi = 3.5$ under the influence of power law cosmology

the field equations and assuming a DE dominating epoch, we assume $\mu > 1$ and choose $\delta = 1$ in Eq. (20) and obtain a new $f(T, B)$ function which can be expressed as follows

$$f(T, B) = c_1 B^{\frac{3\mu+1}{2}} + c_2 B + c_3 T^{-\frac{3}{2\xi\kappa^2}}. \tag{27}$$

Consequently, the constants take the form

$$c_1 = \frac{B_0^{-\frac{3\mu}{2}-\frac{1}{2}} \left(9H_0^2\Omega_{\text{DE0}} - 27H_0^2 + \kappa^2\xi T_0\right)}{3(3\mu - 1)},$$

$$c_2 = -\frac{-9B_0\mu + 3B_0 + 54H_0^2\mu\Omega_{\text{DE0}} + 18H_0^2\Omega_{\text{DE0}} - 162H_0^2\mu - 54H_0^2 + 6\kappa^2\mu\xi T_0 + 2\kappa^2\xi T_0}{3B_0(3\mu - 1)},$$

$$c_3 = \frac{1}{3}(-2)\kappa^2\xi T_0^{\frac{3}{2\kappa^2\xi}+1}.$$

The EoS parameter ω_{TB} is formulated using Eqs. (13) and (16) as

$$\omega_{TB} = -1 + \frac{2(\delta - 2)}{3\mu}. \tag{28}$$

Now, we shall examine the graphical behavior of the reconstructed model using some cosmological quantities. Here, the graphs are plotted against the redshift parameter z by using the relation $\frac{a(t)}{a_0} = (z+1)^{-1}$. The graph for the function $f(T, B)_{THDE}$ is plotted using three different values of the parameter μ: 1.4, 1.7 and 2 along with $\Omega_{m0} = 0.27$ and $H_0 = 67.3$ (taken from the recent available Planck data [86]). Figure 1 shows that $f(T, B)_{THDE}$ takes negative gradually increasing values versus redshift parameter.

The plots of ω_{TB} for four varied choices of μ are provided in Fig. 2 (left panel). It can be seen that for $\mu > 1$, the EoS parameter takes negative values satisfying $-1.5 < \omega_{TB} < -1$ for the entire cosmic evolution and hence corresponds to the phantom cosmic epoch. In this plot, we have also displayed the evolution of ω_{TB} by taking $\mu = -\frac{2}{3}$ with $\delta = 0.99$. The graph displays a positive EoS parameter and hence depicting a matter-dominated era, which is in conformity with the results obtained in Eq. (26). Hence, graphical description of reconstructed model validates the discussion provided earlier in the text.
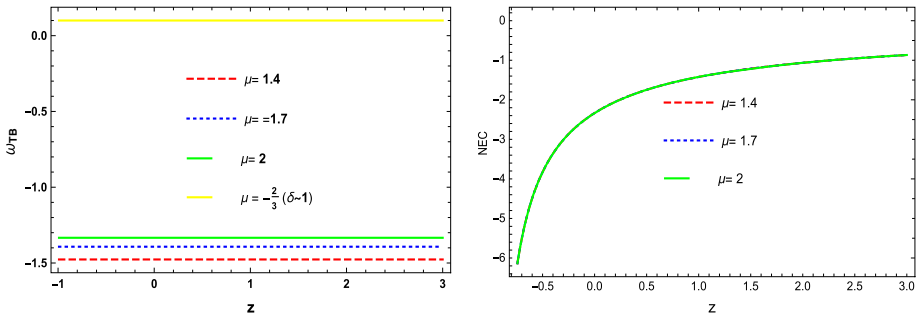
Fig. 2 The plots indicate the evolution of ω_{TB} and NEC for $f(T, B)$ model in power law cosmology for different choices of $\mu = 1.4$, 1.7 and 2 with $\delta = 1$ and the matter-dominated case with $\mu = -\frac{2}{3}$ and $\delta = 0.99$. Here, we have used $\Omega_{m0} = 0.27$ and $H_0 = 67.3$

Next, we have explored the behavior of null energy condition (NEC) for the reconstructed $f(T, B)$ model. The validation of NEC ensures that the energy density of the universe remains within its bounds and its violation implies that it escalates without limits causing expansion to accelerate relentlessly leading to the Big Rip of the universe. In FRW cosmology with perfect fluid contents, the NEC implies $\rho + p \geq 0$, and consequently, we can write

$$\text{NEC}: \ \rho_m + \rho_{TB} + p_m + p_{TB} \geq 0, \tag{29}$$

which, in the present case, can be calculated as

$$\text{NEC}: \ \xi H^{2-\delta} \frac{2(\delta - 2)}{3\mu} \geq 0 \Rightarrow \frac{2\xi(\delta - 2)}{3\mu} \left(\mu(1 + z)^{-1/\mu} \right)^{2-\delta} \geq 0. \tag{30}$$

The curves of NEC for three different choices of μ are provided in the right panel of Fig. 2. It is seen that this condition is not satisfied for all values of redshift parameter which again endorses the phantom regime.

3.1.2 Hybrid Expansion law

In this section, we shall investigate the reconstructed form of generic function based on THDE model in separable form $f(T, B) = f_1(T) + f_2(B)$ by taking the hybrid scale factor [87]. The hybrid law of expansion factor is defined as

$$a(t) = a_0 \left(\frac{t}{t_p} \right)^{\mu} e^{\nu(\frac{t}{t_p} - 1)}, \tag{31}$$

where μ and ν are arbitrary nonnegative constant values. Furthermore, a_0 is the present value of scale factor and t_0 gives the current age of cosmos. Here, we can proceed by taking two possibilities of these parameters into account. First, by assuming $\nu = 0$, it will give us the simple power law solution, and secondly, the choice $\mu = 0$ will result in the exponential solution. Also, for $\mu, \nu \neq 0$, we encounter $q = -1 - \frac{\mu t_p^2}{(\mu t_p + \nu t)^2}$ which shows that $q \to -1$ as $t \to \infty$. Now, we reconstruct the $f(T, B)$ function by considering the above specified cases. The case with $\nu = 0$ holds a striking resemblance, in terms of equations, with the power law cosmology; hence, it has been omitted from the manuscript.
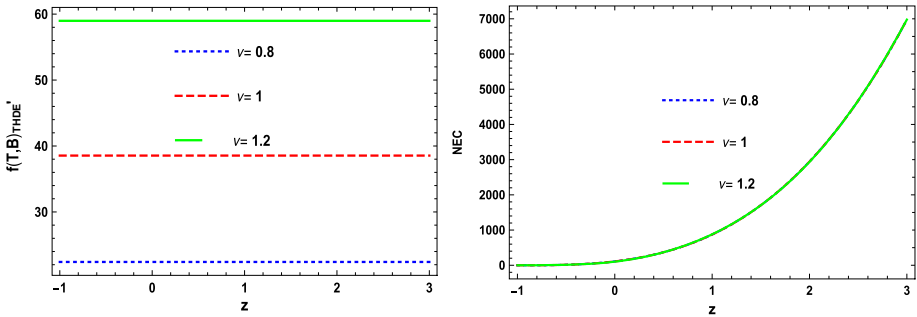
- **De-Sitter Case** ($\mu = 0$)

Fig. 3 Left curve shows the behavior of $f(T, B)$ model for de-Sitter cosmology and the evolution of NEC for different values of $\nu = 0.8$, 1 and 1.2 along with $\xi = 3.5$ and $\delta = 1.4$

Here, we assume the exponential form of scale factor which refers to the de-Sitter model of our cosmos. The de-Sitter solution holds a substantial place in cosmology and has been widely examined in the literature. In the present study, this choice of scale factor leads to $a(t) = a_0 e^{\nu\left(\frac{t}{t_p} - 1\right)}$, which further yields a constant form of $f(T, B)$ function and a steady observationally correct value for the EoS parameter $\omega_{TB} = -1$. For this choice, we obtain

$$H = \frac{\nu}{t_p}, \qquad \dot{H} = 0, \qquad T = \frac{6\nu^2}{t_p{}^2}, \qquad B = \frac{18\nu^2}{t_p{}^2}. \tag{32}$$

It is worthwhile to mention here that since $\dot{H} = 0$, therefore the deceleration parameter leads to $q = -1$, which abides by the perfect cosmological principle. Using the correspondence scheme of energy densities, the resulting differential equations are given by

$$B f_{2B}(B) - f_2(B) = 2K, \quad 4\xi\kappa^2 \left(\frac{T}{6}\right)^{(2-\delta)} f_{1T}(T) + f_1(T) = 2K.$$

The solutions of the above equations lead to the following form of generic function

$$f(T, B) = c_1 B + c_2 e^{-\frac{2^{-\delta}3^{2-\delta}T^{\delta-1}}{\xi(\delta-1)\kappa^2}}. \tag{33}$$

Further, by using the boundary conditions, the value of c_1 and c_2 can be found as

$$c_1 = \frac{T_0^{-\delta}\left(9B_0 T_0^\delta - 54H^2 T_0^\delta \Omega_{\mathrm{DE0}} + 162H_0^2 T_0^\delta + \kappa^2\xi 6^\delta\kappa^2 T_0^2\right)}{9B_0},$$

$$c_2 = -2^\delta 3^{\delta-2}\xi\kappa^2 T_0^{2-\delta} e^{\frac{2^{-\delta}3^{2-\delta}T_0^{\delta-1}}{\xi(\delta-1)\kappa^2}}.$$

The graph in Fig. 3 (left panel) indicates the evolution of a positive constant $f(T, B)$ function against redshift parameter. The EoS parameter ω_{TB}, in this case, is also a constant equal to -1 and hence coinciding with the ΛCDM model for the entire evolution history. The NEC is unconditionally verified and approaches to 0 which is in agreement with EoS parameter recommending the ΛCDM model as shown in Fig. 3 (right panel).

3.2 $f(T, B) = -T + g(B)$

Here, we shall discuss the reconstructed form of generic function based on THDE model by taking both power and hybrid expansion laws when function $f(T, B)$ is assumed as the difference of torsion scalar and an unknown function of the boundary term B.

3.2.1 Power law Cosmology

Here, we shall reconstruct the form of $f(T, B) = -T + g(B)$ based on THDE model by using power law form of scale factor. Using the correspondence scheme of energy densities, we get the following differential equation:

$$-2B^2 g_{BB}(B) - (3\mu+1)g(B) + (3\mu+1)Bg_B(B) + \mu B = -4\kappa^2 \xi (3\mu+1) \left(\frac{\mu B}{6(3\mu + 1)} \right)^{2-\delta},$$
(34)

which is a second-order differential equation with B as the sole dependent variable. The corresponding solution can be obtained by integration as follows

$$g(B) = C_1 B^{\frac{3\mu+1}{2}} + C_2 B + \frac{2^{\delta+1}\kappa^2 \xi B^{2-\delta}(9\mu + 3)^\delta}{9(\delta - 1)(3\mu + 1)(2\delta + 3\mu - 3)} + \frac{\mu B \ln B}{1 - 3\mu},$$

and consequently, our model takes the form:

$$f(T, B) = -T + C_1 B^{\frac{3\mu+1}{2}} + C_2 B + \frac{2^{\delta+1}\kappa^2 \xi B^{2-\delta}(9\mu + 3)^\delta}{9(\delta - 1)(3\mu + 1)(2\delta + 3\mu - 3)} + \frac{\mu B \ln B}{1 - 3\mu}.$$
(35)

The constants of integration C_1 and C_2 have been calculated similarly using the initial condition (24), which are as follows

$$C_1 = \frac{2B_0^{-\frac{3\mu+1}{2}} \left(\frac{2^{\delta+1} 3^\delta \kappa^2 (3\mu-1)\xi B_0^{2-\delta}(3\mu+1)^{\delta-1}}{2\delta+3\mu-3} + 9B_0\mu - 9(3\mu - 1)\left(T_0 - 6H_0^2 \Omega_{DE0}\right) \right)}{9(1 - 3\mu)^2},$$

$$C_2 = \frac{9\mu(3\mu - 1)\log(B_0) + 9\mu(9\mu - 8) + 9 + \frac{9(9\mu^2 - 1)(T_0 - 6H_0^2\Omega_{DE0})}{B_0} - \frac{2^{\delta+1} 3^\delta \kappa^2 (3\mu-1)\xi B_0^{1-\delta}(3\mu+1)^{\delta-1}}{\delta-1}}{9(1 - 3\mu)^2}.$$

It is important to mention here that the above solution will be valid only for $\delta \neq 1$, $\mu \neq \pm\frac{1}{3}$ as well as $2\delta + 3\mu - 3 \neq 0$.

Applying the same technique as before, we use Eqs. (35) and (15) in (8) and obtain the following result:

$$3H^2 = \frac{1}{2}\kappa^2 \left(a^{-3(\omega_m+1)} + 2\xi H^{4-2\delta} \right)$$
(36)

which after further simplification can be written in terms of t as follows

$$3\mu^2 t^{-2} \sim \frac{1}{2}\left((-t)^{-\mu} \right)^{-3(\omega_m+1)} + \left(-\frac{\mu}{t} \right)^{4-2\delta}$$
(37)

Similar to the previous case, this model also reduces to the HDE model with $\delta = 1$. Also, for a matter-dominated scenario the choice of $\mu = -\frac{2}{3}$ must be imposed. In view of these constraints, new $f(T, B)$ function obtained from Eq. (34) can be written as

$$f(T, B) = -T + C_1 B^{\frac{3\mu+1}{2}} + C_2 B - \frac{\mu B \left(2\kappa^2\xi + 3\right)\left((3\mu - 1)\ln(3B(1 - 3\mu)) + 2\right)}{3(1 - 3\mu)^2},$$
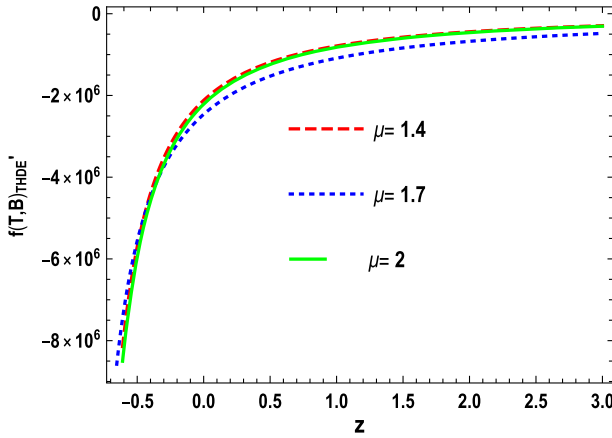(38)

Fig. 4 The graph indicates the behavior of $f(T, B)$ function against z for three values of μ, $\delta = 1$ and $\xi = 3.5$ for power law cosmology
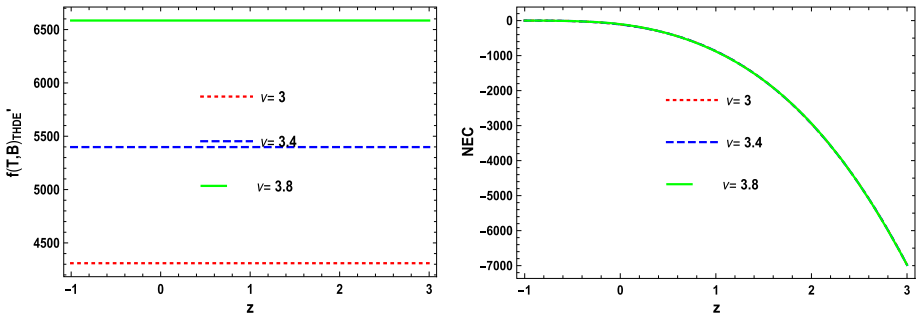

Fig. 5 The graphs indicate the behavior of $f(T, B)$ and NEC against the redshift parameter for three different values of ν along with $\delta = 0.8$ and $\xi = 3.5$ for de-Sitter case

which is valid as long as $\mu \neq -\frac{1}{3}$. To discuss the graphical behavior of cosmic parameters, we have plotted the graphs using $\delta = 1$ and three different choices of $\mu = 1.4, 1.7$ and 2 (similar to the previous case of power law cosmology). This case has resulted in an increasing negative-valued function as provided in graph of Fig. 4.

In this case, NEC is violated in this case, i.e., $\rho + p < 0$, which coincides with our result for EoS parameter $\omega_{TB} < -1$ in Fig. 2, depicting the phantom-like regime of our cosmos.

3.2.2 Hybrid Expansion law

Here, we study the reconstruction of specific model $f(T, B) = -T + g(B)$ based on THDE model by taking the hybrid expansion law in consideration.

- **De-Sitter Case** ($\mu = 0$)
  Considering the de-Sitter solution for the second choice of $f(T, B)$ model, the correspondence scheme generates the following differential equation

$$\frac{B}{2} g_B(B) + \frac{B}{6} - \frac{1}{2} g(B) + 2\xi \kappa^2 \left(\frac{B}{18}\right)^{(2-\delta)} = 0. \tag{39}$$

This differential equation has one dependent variable B, and hence, after evaluating the solution, the mathematical expression for the reconstructed function of $f(T, B)$ can be expressed as

$$f(T, B) = -T + \frac{2^\delta 9^{\delta-2} \kappa \xi B^{2-\delta}}{\delta - 1} + BC_1 - \frac{1}{3} B \ln(B),$$ (40)

where

$$C_1 = \frac{-\frac{2^\delta 9^{\delta-2} \kappa \xi B_0^{2-\delta}}{\delta-1} + \frac{1}{3} B_0 (\ln(B_0) + 3) - 6H_0^2 \Omega_{\mathrm{DE0}} + T_0}{B_0}.$$ (41)

The EoS parameter ω_{TB} is also turned as a constant value equals to -1. Next, the graph of the reconstructed function $f(T, B)$ assumes all positive values, while the behavior of NEC can be observed from Fig. 5. It is evident that NEC also depicts a ΛCDM approach since it approaches to 0 as $z \to -1$. It is interesting to mention here that for this de-Sitter solution, the NEC is violated completely as shown in Fig. 5, thus suggesting the rapid interminable cosmic expansion in this case.

4 Perturbations and stability

In this section, we intend to investigate the stability of the THDE-based reconstructed $f(T, B)$ models against the introduced linear, isotropic, homogeneous perturbations. Let us consider a natural solution and its related functions defined by

$$H(t) = H_i(t), \quad T_i = 6H_i^2(t), \quad B_i = 6\dot{H}_i(t) + 18H_i^2(t).$$ (42)

Using Eqs. (6) and (42), it can be written as

$$-3H_i^2(3f_B^i + 2f_T^i) + 3H_i \dot{f}_B^i - 3\dot{H}_i f_B^i + \frac{1}{2} f^i = \kappa^2 \rho_{mi},$$ (43)

$$\dot{\rho}_{mi} + 3H_i(1 + \omega)\rho_{mi} = 0.$$ (44)

The perturbed Hubble parameter and the energy density are defined by

$$H(t) = H_i(t)(1 + \lambda(t)), \qquad \rho_m(t) = \rho_{mi}(1 + \lambda_m(t)),$$ (45)

where $\lambda(t)$ and $\lambda_m(t)$ correspond to the dark energy and matter energy density perturbations, respectively. With the help of above equations, the function $f(T, B)$ can be expressed as a power series in terms of T and B as follows

$$f(T, B) = f^i + f_T^i(T - T_i) + f_B^i(B - B_i),$$ (46)

where we have neglected the higher-order terms. Here, the superscript indicates the values of $f(T, B)$ function and its derivatives in reference to Eq. (42). Following the technique adopted by Bahamonde et al. [33], we use Eqs. (45) and (46) in the field Eq. (43) and the continuity Eq. (44), and consequently, we obtain the following set of equations:

$$a_0 \ddot{\lambda}(t) + a_1 \dot{\lambda}(t) + a_2 \lambda(t) = a_m \lambda_m(t),$$ (47)

$$\dot{\lambda}_m(t) + 3H_i \lambda(t) = 0.$$ (48)

Here, the coefficients a_0, a_1, a_2 and a_3 are defined as a linear combination of $f(T, B)$ function and its derivatives. The expression for these coefficients is given as follows
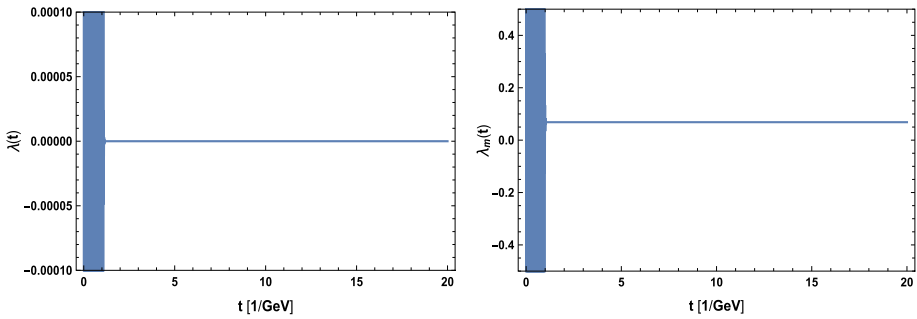
Fig. 6 The left and right graphs provide the evolution of $\lambda(t)$ (dark energy density perturbation) and $\lambda_m(t)$ (matter density perturbation) against time for $\mu = 1.4$, $\delta = 1$ and $\xi = 3.5$ in case of power law form of scale factor
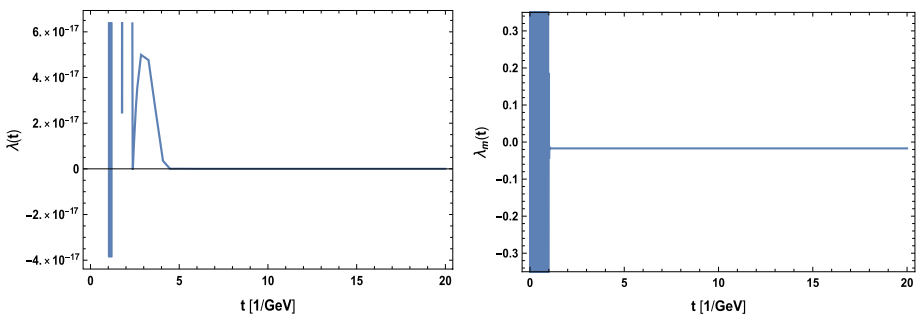

Fig. 7 Left and right plots illustrate the evolution of perturbations $\lambda(t)$ and $\lambda_m(t)$ in de-Sitter universe versus time for $\nu = 0.8$, $\delta = 1.4$ and $\xi = 3.5$

$$
\begin{aligned}
a_0 &= -18H_i^2 f_{TB}^i T_i + 324H_i^4 f_{BB}^i + 54H_i^2 \dot{H}_i f_{BB}^i - 36H_i^2 f_B^i - 12H_i^2 f_{TT}^i T_i \\
&\quad +216H_i^4 f_{TB}^i + 36H_i^2 \dot{H}_i f_{TB}^i - 24H_i^2 f_T^i \\
&\quad +6H_i f_{TB}^i T_i + 6H_i f_{TB}^i T_i - 108H_i^3 f_{BB}^i - 18H_i \dot{H}_i f_{TB}^i - 216H_j^2 \dot{H}_i f_{BB}^i \\
&\quad -18H_i \ddot{H}_i f_{BB}^i + 6H_i \dot{f}_B^i - 6\dot{H}_i T_i f_{TB}^i \\
&\quad +108H_i^2 \dot{H}_i f_{BB}^i + 18H_i^2 f_{BB}^i - 9\dot{T} f_B^i + f_B^i T_i - 18H_i^2 f_B^i - 3\dot{H}_i f_B^i, \\
a_1 &= 54H_i^3 f_{BB}^i + 36H_i^3 f_{TB}^i T_i + 6H_i f_{TB}^i T_i - 18H_i^2 f_{BB}^i - 108H_i^3 f_{BB}^i - 6H_i f_{BB}^i, \\
a_2 &= -18H_i^2 f_{BB}^i, \qquad a_m = \kappa^2 \rho_m.
\end{aligned}
$$

Now, we will use the reconstructed models to find the solution of Eqs. (47) and (48) (as discussed in the previous sections) for investigating their stability in each case.

- *Case 1 $f(T, B) = f_1(T) + f_2(B)$*
  In case of power law cosmology, we find the growth of perturbation parameters for dark energy-dominated era by solving Eqs. (47) and (48). We have used Eq. (27) along with the boundary conditions $\lambda'(1) = 0.2$, $\lambda(1) = 0.1$ and $\lambda_m(1) = 0.1$ to find plot our solution. The growth of perturbations $\lambda(t)$ and $\lambda_m(t)$ with reference to the imposed initial conditions by undertaking $\delta = 1$ and $\mu = 1.4$ is provided in Fig. 6 (right and left panels). It can be easily observed that the oscillating curves of $\lambda(t)$ and $\lambda_m(t)$ decay in the future ($\lambda(t)$, $\lambda_m(t) = 0$, for $t \geq 1$) and hence yield stability of the obtained solutions in this case. It is worthwhile to mention here that for $\mu > 1.4$, the model does not show
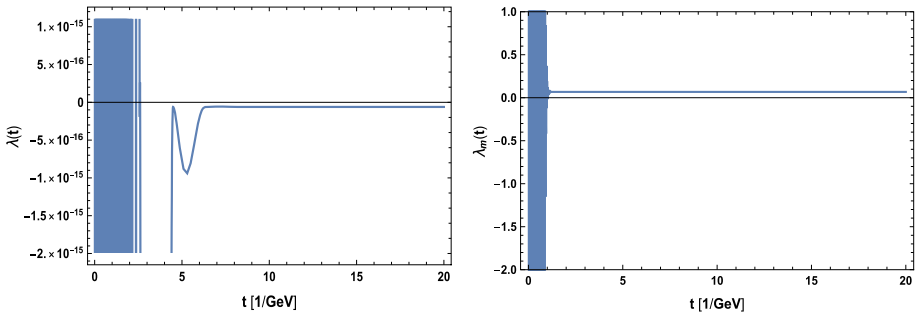
Fig. 8 The graphs indicate the evolution of $\lambda(t)$ and $\lambda_m(t)$ against time for $\mu = 1.4$, $\delta = 1$ and $\xi = 3.5$ in case of power law solution of scale factor
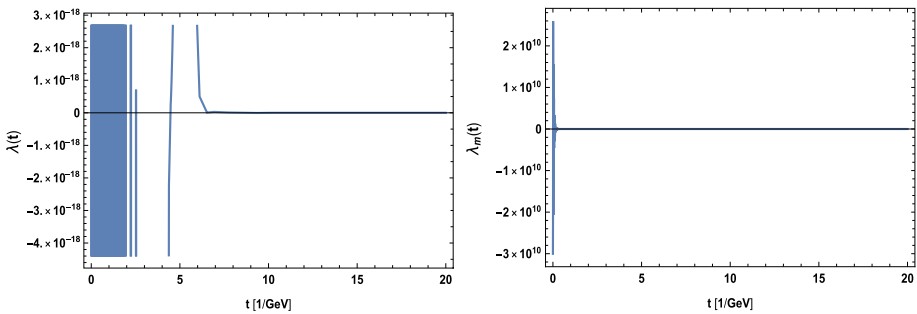

Fig. 9 Graphs show the evolution of $\lambda(t)$ and $\lambda_m(t)$ in de-Sitter cosmology against time for $\nu = 3.8$, $\delta = 0.8$ and $\xi = 3.5$

stability, while the cosmological parameters show acceptable behavior.

Further, we investigate the stability of reconstructed $f(T, B)$ model for hybrid scale factor's de-Sitter like $f(T, B)$ model given by Eq. (33). The graphical illustration of introduced perturbations $\lambda(t)$ and $\lambda_m(t)$ is provided in the left and right panels of Fig. 7. Figure shows that $\lambda(t)$ and $\lambda_m(t)$ oscillate in initial cosmic epochs but show a stable behavior in future eras ($\lambda(t) = 0$, for $t \geq 5$ and $\lambda_m(t) = 0$, for $t \geq 1$), hence providing a stable model in this case.

- **Case 2** $f(T, B) = -T + g(B)$

Here, we explore the stability of power law solution for the reconstructed form of $f(T, B) = -T + g(B)$ model which is given in Eq. (36). For this model, we find the solution of Eqs. (47) and (48) by using the model presented in Eq. (38) along with the boundary conditions $\lambda'(1) = 0.2$, $\lambda(1) = 0.1$ and $\lambda_m(1) = 0.1$. Figure 8 shows the evolution of perturbations $\lambda(t)$ and $\lambda_m(t)$ versus time for $\delta = 1$ and $\mu = 1.4$. It can be noticed that the trajectories of both these functions indicate oscillating behavior in the earlier cosmic stages which further reach to a constant value ≈ 0 in future time. It is seen that $\lambda(t) \to 0$ in far future around $t > 7$ and $\lambda_m(t) \to 0$ for $t > 1$ and thus corresponds to a stable model which is in contrast to the findings of Ghaffari [53] where THDE braneworld model was found classically unstable.

Lastly, for the de-Sitter $f(T, B)$ model, we analyze the evolution of these perturbation parameters by following a similar approach and the resulting graphs are provided in Fig. 9. It can be easily observed that for $\lambda(t)$, these perturbations exhibit oscillating behavior in the beginning but remain stable and approach zero in future times ($t > 6$), whereas the graph

Table 1 Model behavior comparison

$f(T, B)$ Models	ω_{TB}	NEC	Density perturbation
$f(T, B) = f(T) + f(B)$			
Power law case	Phantom	Invalid	Stable
De-Sitter case	ΛCDM	Validated	Stable
$f(T, B) = -T + f(B)$			
Power law case	Phantom	Violated	Stable
De-Sitter case	ΛCDM	Violated	Stable

for $\lambda_m(t)$ decays very early in time and hence indicating that the model is stable in this case. It is worthwhile to mention here that for $\delta > 1$, the model is found to be unstable against linear isotropic perturbations.

5 Concluding remarks

The investigation for an interesting and successful candidate of DE that can describe the complete evolution of our cosmos and solve this mysterious puzzle effectively is one of the challenging problems in modern cosmology. Among different DE models, modified gravity theories have the disposition to explain the combined early and late time cosmological eras accurately and these candidates can also imitate DE character. Teleparallel theory of gravity and its different extensions are considered as one of the most effective tools in providing ample solutions that can provide interesting results in this regard. The present paper will provide a significant contribution in this respect. In this manuscript, we have considered the well-known $f(T, B)$ modified gravitational framework and reconstructed the possible forms of generic function present in its Lagrangian that can generate the same cosmology as produced by the THDE model. This model has been recently proposed in the literature and gained much attention of the researchers. For this purpose, we have considered two models of $f(T, B)$ gravity and used the correspondence scheme of reconstruction in each case. For the sake of simplicity, we have considered two specific choices of scale factor , i.e., power law expansion and hybrid law (de-Sitter universe). In order to explore the physical significance of the reconstructed solutions, we have analyzed the behavior of some cosmological measures graphically. The obtained results are summarized in Table 1 and explained as follows.

- The evolution trajectories for the reconstructed $f(T, B)$ model under the influence of THDE for power law solutions depict negative behavior for the discussed cases $f(T, B) = f(T) + f(B)$ and $f(T, B) = -T + f(B)$, whereas the de-Sitter solutions represent positive-valued functions in both these cases.
- Equation of state parameter ω_{TB} gives a constant value and exhibits phantom-like behavior in power law cosmology. We have also presented a matter dominating scenario by assuming $\mu = -\frac{2}{3}$ which is in accordance with the constraint derived from the field equations for making the reconstructed solution consistent. As obvious, the de-Sitter scenario for both cases only hence portrayed a ΛCDM like EoS parameter.
- It is seen that the NEC is invalid for both reconstructed generic functions in power law cosmology case. For de-Sitter cosmology, it is valid for the first case, whereas, for the model $f(T, B) = -T + g(B)$, the NEC is thoroughly invalid.

- We have also studied the stability of these models against linear and homogeneous perturbations. The power law cosmology has been proved to be a good fit in $f(T, B)_{THDE}$ gravity as for both cases, we have found the reconstructed models exhibiting stable behavior. An interesting finding is that if we consider power law cosmology, the Friedmann equations are only satisfied if THDE is reduced to HDE. Furthermore, the de-Sitter cosmology also satisfied the stability criteria for both reconstructed models which is in compliance with the results obtained by Bahamonde et al. [33].

These reconstructed models demonstrate an appropriate behavior which is consistent with the current cosmic observations as shown by the graphical analysis of different cosmological parameters in each case. Thus, it can be concluded that our reconstructed models are cosmologically viable and stable at cosmological level and found to be physically interesting. It would be worthwhile to reconstruct the form of generic function by taking some other forms of $f(T, B)$ model or scale factor into account.

References

1. A.G. Riess et al., Astrophys. J. **116**, 1009 (1998)
2. S. Perlmutter et al., Astrophys. J. **517**, 565 (1999)
3. S. Perlmutter et al., Astrophys. J. **598**, 102 (2003)
4. E. Hawkins et al., Mon. Not. R. Astron. Soc. **346**, 78 (2003)
5. S. Hanany et al., Astrophys. J. Lett. **545**, L5 (2000)
6. E. Komatsu et al., Astrophys. J. Suppl. **192**, 18 (2011)
7. D.J. Eisenstein et al., Astrophys. J. **633**, 560 (2005)
8. B. Jain, A. Taylor, Phys. Rev. Lett. **91**, 141302 (2003)
9. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989)
10. P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559 (2003)
11. M. Roos, *Introduction to Cosmology*, 4th edn. (Wiley, 2015)
12. T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010)
13. S. Capozziello, S.I. Nojiri, S.D. Odintsov, A. Troisi, Phys. Lett. B **639**, 135 (2006)
14. R. Ferraro, F. Fiorini, Phys. Lett. B **702**, 75 (2011)
15. T. Harko et al., Phys. Rev. D **84**, 024020 (2011)
16. K. Hayashi, T. Shirafuji, Phys. Rev. D **19**, 3524 (1979)
17. J.W. Maluf, Ann. Phys. **525**, 339 (2013)
18. G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B **485**, 208 (2000)
19. S. Nojiri, S.D. Odintsov, Phys. Lett. B **631**, 1 (2005)
20. M. Alishahiha, E. Silverstein, D. Ton, Phys. Rev. D **70**, 123505 (2004)
21. C. Brans, R.H. Dicke, Phys. Rev. C **124**, 925 (1961)
22. R. Ferraro, F. Fiorini, Phys. Rev. D **75**, 084031 (2007)
23. V. Faraoni, *Cosmology in Scalar-Tensor Gravity*, (Kluwer Academic Publishers, Dordrecht, 2004)
24. M. Zubair, F. Kousar, S. Bahamonde, Phys. Dark Univ. **14**, 116 (2016)
25. E.V. Linder, Phys. Rev. D **81**, 127301 (2010)
26. P. Wu, H. Yu, Phys. Lett. B **693**, 415 (2010)
27. S. Bahamonde, S. Capozziello, Eur. Phys. J. C **77**, 107 (2017)
28. M.H. Daouda, M.E. Rodrigues, M.J.S. Houndjo, Eur. Phys. J. C **72**, 1893 (2012)
29. T. Harko, F.S.N. Lobo, G. Otalora, E.N. Saridakis, Phys. Rev. D **89**, 124036 (2014)
30. M. Zubair, S. Waheed, Astrophys. Space Sci. **360**, 68 (2015)
31. M. Zubair, S. Waheed, Astrophys. Space Sci. **355**, 361 (2015)
32. S. Bahamonde, C.G. Bohmer, M. Wright, Phys. Rev. D **92**, 104042 (2015)
33. S. Bahamonde, M. Zubair, G. Abbas, Phys. Dark Univ. **19**, 78 (2018)
34. A.D.L. Cruz-Dombriz, D. Saez-Gomez, Class. Quant. Gravit. **29**, 245014 (2012)
35. M. Zubair, S. Waheed, M.A. Fayyaz, I. Ahmad, Eur. Phys. J. Plus **133**, 452 (2018)
36. M. Zubair, S. Bahamonde, M. Jamil, Eur. Phys. J. C **77**, 472 (2017)

37. H. Abedi, S. Capozziello, Eur. Phys. J. C **78**, 474 (2018)
38. S. Capozziello, M. Capriolo, L. Caso, Eur. Phys. J. C **80**, 156 (2020)
39. C. Escamilla-Rivera, J. Levi Said, Class. Quant. Grav. **37**, 165002 (2020)
40. A. Pourbagher, A. Amani, Astrophys. Space Sci. **364**, 140 (2019)
41. S. Bahamonde, U. Camci, Symmetry **11**, 1462 (2019)
42. M. Zubair, L.R. Durrani, Eur. Phys. J. Plus **135**, 1 (2020)
43. G. Farrugia, J.L. Said, V. Gakis, E.N. Saridakis, Phys. Rev. D **97**, 124064 (2018)
44. G. Farrugia, J.L. Said, A. Finch, Universe **6**, 34 (2020)
45. S. Bhattacharjee, Phys. Dark Univ. **30**, 100612 (2020)
46. M. Caruana, G. Farrugia, J.L. Said, Eur. Phys. J. C **80**, 640 (2020)
47. M. Li, Phys. Lett. B **603**, 1 (2004)
48. G. t Hooft, Conf. Proc. C **930308**, 284 (1993)
49. M. Tavayef et al., Phys. Lett. B **781**, 195 (2018)
50. C. Tsallis, L.J.L. Cirto, Eur. Phys. J. C **73**, 2487 (2013)
51. M.A. Zadeh et al., arXiv:1901.05298v1 (2019)
52. M. Abdollahi Zadeh, A. Sheykhi, H. Moradpour, Gen. Relat. Gravit. **51**, 12 (2019)
53. S. Ghaffari et al., Phys. Dark Univ. **23**, 100246 (2019)
54. G. Varshney, U.K. Sharma, A. Pradhan, New Astro. **70**, 36 (2019)
55. S. Rani, A. Jawad, K. Bamba, I.U. Malik, Symmetry **11**, 509 (2019)
56. S.I. Nojiri, S.D. Odintsov, Gen. Relat. Gravit. **38**, 1285 (2006)
57. S. Capozziello, V.F. Cardone, A. Troisi, Phys. Rev. D **71**, 043503 (2005)
58. S. Nojiri, S.D. Odintsov, M. Sami, Phys. Rev. D **74**, 086005 (2006)
59. M. Sharif, M. Zubair, Gen. Relat. Gravit. **46**, 1723 (2014)
60. S. Nojiri, S.D. Odintsov, J. Phys. Conf. Ser. **66**, 012005 (2007)
61. S. Nojiri, Mod. Phys. Lett. A **25**, 859 (2010)
62. D. Saez-Gomez, Gen. Relat. Gravit. **41**, 1527 (2009)
63. S. Nojiri, S.D. Odintsov, A. Toporensky, P. Tretyakov, Gen. Relat. Gravit. **42**, 1997 (2010)
64. M. Sharif, K. Nazir, Mod. Phys. Lett. A **31**, 1650148 (2016)
65. M. Sharif, M. Zubair, Astrophys. Space Sci. **353**, 699 (2014)
66. M.J.S. Houndjo, O.F. Piattella, Int. J. Mod. Phys. D **21**, 1250024 (2012)
67. M. Sharif, S. Saba, Symmetry **11**, 92 (2019)
68. S. Waheed, Eur. Phys. J. Plus **135**, 11 (2020)
69. N. Tamanini, C.G. Boehmer, Phys. Rev. D **86**, 044009 (2012)
70. M.A. Zadeh, A. Sheykhi, H. Moradpour, K. Bamba, Eur. Phys. J. C **78**, 940 (2018)
71. M. Tavayef, A. Sheykhi, K. Bamba, H. Moradpour, Phys. Lett. B **781**, 195 (2018)
72. A. Jawad et al., Symmetry **10**, 635 (2018)
73. M.A. Zadeh, A. Sheykhi, H. Moradpour, Mod. Phys. Lett. A **34**, 1950086 (2019)
74. E. Sadri, Eur. Phys. J. C **79**, 762 (2019)
75. S. Maity, U. Debnath, Eur. Phys. J. Plus **134**, 514 (2019)
76. Y. Aditya, S. Mandal, P.K. Sahoo, D.R.K. Reddy, Eur. Phys. J. C **79**, 1020 (2019)
77. A. Al Mamon, A.H. Ziaie, K. Bamba, Eur. Phys. J. C **80**, 974 (2020)
78. S.H. Shekh, P.H.R.S. Moraes, P.K. Sahoo, Universe **7**, 67 (2021)
79. Y.S. Song, W. Hu, I. Sawicki, Phys. Rev. D **75**, 044004 (2007)
80. S. Nesseris, S. Basilakos, E.N. Saridakis, L. Perivolaropoulos, Phys. Rev. D **88**, 103010 (2013)
81. S. Nojiri, S.D. Odintsov, Int. J. Geom. Methods Mod. Phys. **4**, 115 (2007)
82. H.M. Sadjadi, Phys. Rev. D **73**, 063525 (2006)
83. S. Capozziello, V.F. Cardone, A. Troisi, Phys. Rev. D **71**, 043503 (2005)
84. M.J.S. Houndjo, Int. J. Mod. Phys. D **21**, 1250003 (2012)
85. M. Sharif, M. Zubair, Astrophys. Space Sci. **349**, 529 (2014)
86. P.A.R. Ade et al., Astron. Astrophys. **571**, A1 (2014)
87. O. Akarsu, S. Kumar, R. Myrzakulov, M. Sami, L. Xu, JCAP **01**, 022 (2014)
88. N. Aghanim et al., arXiv:1807.06209 (2018)