Eur. Phys. J. Plus (2021) 136:651
https://doi.org/10.1140/epjp/s13360-021-01633-3

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

Variable thermal conductivity and hyperbolic two-temperature theory during magneto-photothermal theory of semiconductor induced by laser pulses

A. M. S. Mahdy[1,2,a], **Kh. Lotfy**[2,5,b], **A. El-Bary**[3,c], **Ismail M. Tayel**[4,d]

[1] Department of Mathematics, College of Science, Taif University, P. O. Box 11099, Taif 21944, Saudi Arabia
[2] Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt
[3] Arab Academy for Science, Technology and Maritime Transport, P.O. Box 1029, Alexandria, Egypt
[4] Department of Mathematics, College of Science Al-Zulfi, Majmaah University, Al Majmaah 11952, Saudi Arabia
[5] Department of Mathematics, Faculty of Science, Taibah University, Madinah, Saudi Arabia

Received: 3 February 2021 / Accepted: 1 June 2021

Abstract The main goal of this work is to investigate the interaction impact between three propagated waves. In this case, the elastic wave, plasma wave and thermal waves are obtained in the context of the hyperbolic generalized two-temperature theory. The governing equations are studied during the photothermal theory. The impact of external magnetic field and laser pulse are obtained which they fall on the outer surface of a semiconductor medium. The thermal conductivity of semiconductor material is investigated in a variable case. When the coupled between photothermal theory and thermoelasticity theory is occurred, three various models of the photo-thermoelasticity theory are obtained. The integral transforms technique in two-dimensional (2D) deformation is applied to solve the main equations. The double Fourier and Laplace transforms with some initial conditions are used as example of integral transforms technique. The inversion of the double transforms with some thermal-elastic-mechanical-plasma boundary conditions is applied numerically to obtain the complete solutions. Some comparisons during three various models in external magnetic field with variable thermal conductivity Si (silicon) material of photo-thermoelasticity theory are performed.

1 Introduction

Semiconductor materials have many applications in modern mechanical and physical engineering. Before 1950, scientists interested in semiconductors only studied them as elastic media. However, with the development many distinct properties of semiconductors have been discovered. One of the most important characteristics that is taken into account is the internal structures with microelectronics process. Over time a photothermal (PT) technique

[a] e-mail: amattaya@tu.edu.sa
[b] e-mail: khlotfy_1@yahoo.com (corresponding author)
[c] e-mail: aaelbary@aast.edu
[d] e-mail: i.tayel@mu.edu.sa

was used to analysis the waves propagation of semiconductor media. On the other hand, the photo-acoustic (PA) analysis of semiconductor is used in modern technology also. The sensitive analysis processes are using PT and PA when the thermal diffusivity is important phenomena [1, 2]. The impact of magnetic field and laser pulses of semiconductor medium generates exited electrons on the outer surface during the electronic deformation (ED), and the carrier density with plasma waves is obtained.

The importance of the theory of thermal elasticity appeared in the last century when Biot was developed the coupled theory of thermoelasticity (CD) [3]. Biot [3] removed the contradiction in the uncoupled thermoelasticity theory. Biot [3] presented a model that studies the interference between the strain and temperature that dependent on the Fourier's law with infinite speeds of wave propagation. To remove the unacceptable physical meaning which introduced by Biot, Lord and Shulman (LS) [4] developed a new model. In this model, the Fourier's law is replaced by another approximation when they insert a single relaxation time into the heat equation. On the other hand, Green and Lindsay (GL) [5] introduced another generalized thermoelasticity model which the governing equations of this model contain two relaxation times. A lot of scholars used LS and GL models in many application problems of thermoelasticity media [6–10]. Marin et al. [11, 12] studied the chemical reaction, the heat and mass transfer impact and the thermoelasticity models on third-grade MHD fluid flow and blood flow with variable reactive index with some applications during anisotropically tapered arteries.

The photothermal theory appeared in the second half of the last century when a semiconductor material was exposed to an incident pulse laser beam [13, 14]. Due to the thermal effect of laser pulse, the thermal waves and elastic waves are generated in process called the thermoelastic deformation (TE) [15]. In this case, the TE and ED mechanisms appear altogether during the photo-excited transport processes [16]. However, the interaction processes between plasma and thermo-elastic waves in semiconductor material are obtained [17]. Quintanilla and Tien [18] studied a short-pulse laser heating in the context of the heat transfer mechanism to investigate the structural stability of the medium. In the beginning of this century, Youssef et al. [19, 20] introduced a new model of the linear generalized thermoelasticity which depends on the two distinct temperatures. This model named two-temperature theory. Lotfy et al. [21–29] applied the two-temperature theory to develop the photo-thermoelasticity theory during TE and ED deformation with different thermal memories (relaxation times) and several external fields. Many authors [30–35] investigated the photo-thermo-elastic wave propagation in a non-homogenous semiconductor media with memory responses. After that, Youssef and El-Bary [36] introduced a new model in the generalized thermoelasticity theory to modify the paradox which it found in [19]. In this model, the two temperatures depend on the two distinct accelerations and called the hyperbolic two-temperature model.

In this work, a novel mathematical-physical 2D deformation model is investigated under the hyperbolic two-temperature theory. The problem is studied in the photo-thermoelasticity theory during photo-excited processes of a semiconductor thin film. The physical properties of the medium depend on the gradient in temperature which leads to the variable in thermal conductivity. The elastic medium exposed to external magnetic field during a pulsed laser. The numerical integral transforms technique under the Fourier and Laplace transformations is used. On the other hand, by using a numerical inversion method of Fourier and Laplace transforms the main physical fields are obtained. Many comparisons between the physical quantities with three various models of photo-thermoelasticity theory, namely (CD, LS, LG), are performed under the effect of magnetic field. Silicon (Si) material is used to validate of the numerical results when the thermal conductivity is changed.

2 Basic equations

The semiconductor medium is homogenous, linear and isotropic which it exposed to a laser pulses and external magnetic field. When the thermal conductivity is variable, the medium is studied during 2D deformation. The main four physical quantities in this case are $T\ (\vec{r}, t)$, $\phi(\vec{r}, t)$, $\overset{\rightharpoonup}{u}\ (\vec{r}, t)$ and $N\ (\vec{r}, t)$ which refer to the distribution of thermodynamic temperature (thermal waves), the distribution of conductive temperature, the displacement distribution (elastic waves) and the plasma waves (carrier density), respectively ($\vec{r}$ is the space vector and t refers to the time). The induced magnetic field $h(x, y, z)$ generates when the initial external magnetic field in y-direction $H = H_0 + h$, $\overset{\rightharpoonup}{H} = (0, H_0, 0)$ falls on the external surface of medium. In case of a slowly moving, the Maxwell's equations for electromagnetic medium are used with neglected the density of charge. In this case the particle velocity $\dot{\overset{\rightharpoonup}{u}}$ of the semiconductor elastic medium is taken into account and the electromagnetic can be written as [35]:

$$\left.\begin{aligned} &\overset{\rightharpoonup}{J} = \text{curl}\ \overset{\rightharpoonup}{h} - \varepsilon_0 \dot{\overset{\rightharpoonup}{E}},\ \ \text{curl}\,\vec{E} = -\mu_0 \dot{\overset{\rightharpoonup}{H}}, \\ &\overset{\rightharpoonup}{E} = -\mu_0(\dot{\overset{\rightharpoonup}{u}} \times \overset{\rightharpoonup}{H}),\ \text{div}\ \overset{\rightharpoonup}{H} = 0 \end{aligned}\right\}. \tag{1}$$

The μ_0 and ε_0 are the magnetic constant permeability and the electric permeability, respectively, the electric field is $\overset{\rightharpoonup}{E}$, the vector of current density is $\overset{\rightharpoonup}{J} = (J_1, J_2, J_3)$ and the dot notation refers to the time differentiation. The current density components can be obtained from Eq. (1) in terms of displacement when eliminating $\overset{\rightharpoonup}{h}$ and $\overset{\rightharpoonup}{E}$ which leads to the following (see the schematic figure) [30–32]

$$\left.\begin{aligned} &\overset{\rightharpoonup}{J} = \begin{vmatrix} \overset{\rightharpoonup}{i} & \overset{\rightharpoonup}{j} & \overset{\rightharpoonup}{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & h & 0 \end{vmatrix} - \varepsilon_0(\mu_0 H_0 \ddot{w}, 0, -\mu_0 H_0 \ddot{u}) = \\ &\left.\begin{aligned} &J_x = -\left(\frac{\partial h}{\partial z} + \mu_0 H_0 \varepsilon_0 \ddot{w}\right), \quad J_y = 0, \\ &J_z = \frac{\partial h}{\partial x} + \mu_0 H_0 \varepsilon_0 \ddot{u}, \end{aligned}\right\} \end{aligned}\right\}. \tag{2}$$

$$\left.\begin{aligned} &\overset{\rightharpoonup}{E} = -\mu_0 \begin{vmatrix} \overset{\rightharpoonup}{i} & \overset{\rightharpoonup}{j} & \overset{\rightharpoonup}{k} \\ \dot{u} & 0 & \dot{w} \\ 0 & H_0 & 0 \end{vmatrix}, \\ &E_x = \mu_0\, H_0\, \dot{w},\ E_y = 0,\ E_z = -\mu_0\, H_0\, \dot{u}. \end{aligned}\right\}. \tag{3}$$

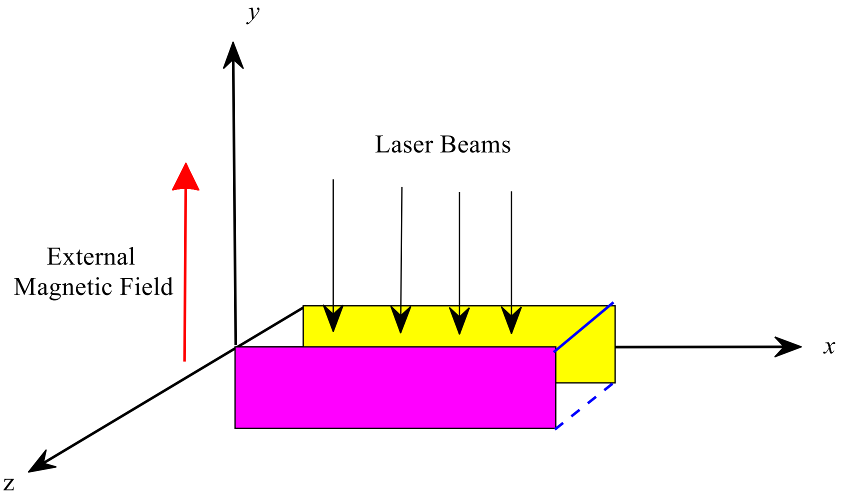
The Lorentz's force $\overset{\rightharpoonup}{F} = \mu_0(\overset{\rightharpoonup}{J} \times \overset{\rightharpoonup}{H})$ can be obtained from the electromagnetic force Eqs. (1) and (2) as [31]:

$$\begin{aligned} \overset{\rightharpoonup}{F} = \mu_0(\overset{\rightharpoonup}{J} \times \overset{\rightharpoonup}{H}) &\equiv \left(-\mu_0 H_0 \frac{\partial h}{\partial x} - \varepsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 u}{\partial t^2}, 0, -\mu_0 H_0 \frac{\partial h}{\partial z} - \varepsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 w}{\partial t^2}\right) \\ &\equiv F_i = (F_x, 0, F_z). \end{aligned} \tag{4}$$

On the other hand, a laser pulse has an input a non-Gaussian function Q which it is taken the following form [40] (see the schematic figure):

$$Q = \frac{I_0 \gamma' t}{2\pi a^2 t_0^2} \exp\left(-\frac{z^2}{a^2} - \frac{t}{t_0} - \gamma' x\right). \tag{5}$$

where I_0, a, t_0, and γ' refer to the absorbed energy for unit area, the radius of beam of laser, the rise time of pulse and the absorption coefficient of the semiconductor medium, respectively.



The coupled equations (governing equations) that describe the interaction between plasma, thermal and elastic impacts under the effect of magnetic field in the context of the hyperbolic two-temperature theory can be constructed in tensor form as [5, 19, 37–39]:

$$\frac{\partial N(r,t)}{\partial t} = D_E N_{,ii}(r,t) - \frac{N(r,t)}{\tau} + \kappa\ T(r,t), \tag{6}$$

$$\rho C_e \left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial T(r,t)}{\partial t} = (K\phi_{,i}(r,t))_{,i} + \frac{E_g}{\tau} N(r,t) + \gamma T_0 \left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t} u_{,i}(r,t) - \rho Q\right), \tag{7}$$

$$\rho \frac{\partial^2 \vec{u}(\vec{r},t)}{\partial t^2} = \mu u_{i,jj}(r,t) + (\mu+\lambda) u_{i,jj}(r,t) - \gamma\left(1 + \nu_0 \frac{\partial}{\partial t}\right)(T_{,i}(r,t) - \delta_n N_{,i}(r,t)) + F_i. \tag{8}$$

$$\frac{\partial^2 T(r,t)}{\partial t^2} - \frac{\partial^2 \varphi(r,t)}{\partial t^2} = -\lambda\, \varphi_{,ii}(r,t). \tag{9}$$

In a classical theory of the two temperatures, Eq. (9) can be rewritten as:

$$T(r,t) - \phi(r,t) = -a\, \phi_{,ii}(r,t). \tag{10}$$

In the above equations, λ is arbitrary positive small parameter which describes the hyperbolic two-temperature case and $a > 0$ is the two-temperature parameter. The quantity κ is

Table 1 .

Name (unit)	Symbol	value
Absolute temperature (K)	$T0$	800
Lamé's constants (N/m^2)	λ, μ	$3.64 \times 10^{10}, 5.46 \times 10^{10}$
Density (kg/m^3)	ρ	2330
The photogenerated Carrier lifetime (s)	τ	5×10^{-5}
The electronic deformation coefficient (m^3)	d_n	-9×10^{-31}
The carrier diffusion coefficient (m^2/s)	D_E	2.5×10^{-3}
The energy gap (eV)	Eg	1.11
The pulse rise time (ps)	t_0	9
The linear thermal expansion coefficient (K^{-1})	α_t	4.14×10^{-6}
The sample thermal conductivity (Wm^{-1} K^{-1})	k	150
The recombination velocities (m/s)	s	2
Specific heat at constant strain (J/(kg K))	C_e	695
The radius of the beam (µm)	r	100
The absorption depth of heating energy (m^{-1})	γ'	10−3
The absorbed energy (J)	I_0	105

a nonzero thermal activation coupling parameter ($\kappa = \frac{\partial N_0}{\partial T} \frac{T}{\tau}$) which describes the equilibrium case of the medium when the carrier concentration is N_0 at high temperature [40–42]. However, the elastic and thermal relaxation times are ν_0 and τ_0, but the quantities n_0, n_1 are a dimensionless chosen parameters which taken according to the photo-thermoelasticity models [43, 44]. On the other hand, the other physical quantities will be defined in Table 1.

The constitutive (stress–strain) relations can be given in the following form [24, 25]:

$$\sigma_{ij} = \mu u_{i,j}(r,t) + (\mu + \lambda) u_{i,j}(r,t) - (3\lambda + 2\mu)\left(\alpha_T \left(1 + \nu_0 \frac{\partial}{\partial t}\right) T(r,t) + d_n N(r,t)\right). \tag{11}$$

When physical properties of the inner structure of the medium change with gradient temperature, then the thermal conductivity K can be chosen as variable. In this case, K can be chosen a linear function of the thermodynamical temperature as follows:

$$K(T) = K_0(1 + K_1 T). \tag{12}$$

where K_0 is thermal conductivity when the medium independent to temperature at $K_1 = 0$ (K_1 very small negative parameter). From Eq. (10) with using Eq. (12) the following relation can be given:

$$K(T) = K_0(1 + K_1(\varphi - \tilde{\lambda}\nabla^2\varphi)). \tag{13}$$

The value $K_1\tilde{\lambda}\nabla^2\varphi$ can be neglected because it is very small value; in this case, the thermal conductivity parameter (Eq. 13) can be rewritten as:

$$K(T) = K_0(1 + K_1\phi) \text{ or } K(\phi) = K_0(1 + K_1\phi). \tag{14}$$

On the other hand, the following maps can be used for more suitable form as [32]:

$$\hat{\phi} = \frac{1}{K_0}\int_0^{\phi} K(\Gamma)\mathrm{d}\Gamma, \tag{15}$$

$$\hat{T} = \frac{1}{K_0}\int_0^{T} K(\Gamma)\mathrm{d}\Gamma. \tag{16}$$

Using the differentiation notation $\frac{\partial}{\partial x_i}$ for all both sides of the maps Eq. (15) and (16), the following relations can be obtained:

$$K_0\frac{\partial\hat{\phi}}{\partial x_i} = K(\phi)\frac{\partial\phi}{\partial x_i} \overset{\text{or in tensor form}}{\Longleftrightarrow} K_0\hat{\phi}_{,i} = K(\phi)\phi_{,i}. \tag{17}$$

Using again the differentiating operator on Eqs. (17) with respect to x_i yields:

$$K_0\frac{\partial^2\hat{\phi}}{\partial x_i^2} = \frac{\partial}{\partial x_i}\left(K(\phi)\frac{\partial\phi}{\partial x_i}\right) \overset{\text{or in tensor form}}{\Longleftrightarrow} K_0\hat{\phi}_{,ii} = (K(\phi)\phi_{,i})_{,i}\,. \tag{18}$$

On the other hand, by using the same way, the temperature (T) can be expressed in the following form:

$$K_0\hat{T}_{,i} = K(T)T_{,i}. \tag{19}$$

The time-differentiation operator can be applied of Eq. (16) as:

$$K_0\frac{\partial\hat{T}}{\partial t} = K(T)\frac{\partial T}{\partial t}. \tag{20}$$

The equation of motion (8) under the mapping transformation Eqs. (18), (19) and (20) can be rewritten as:

$$\rho\frac{\partial^2\vec{u}}{\partial t^2} = \mu\nabla^2\vec{u} + (\mu+\lambda)\nabla(\nabla\cdot\vec{u}) - \gamma\left(1+\nu_0\frac{\partial}{\partial t}\right)(1+K_1T)^{-1}\frac{\partial\hat{T}}{\partial x_i} - \delta_n\nabla N + \vec{F}. \tag{21}$$

Applying the expand technique on the third term in the right-hand side of the above equation when neglected the nonlinear terms, yields:

$$\begin{aligned}\gamma\left(1+\nu_0\frac{\partial}{\partial t}\right)(1+K_1T)^{-1}\frac{\partial\hat{T}}{\partial x_i} &= \gamma\left(1+\nu_0\frac{\partial}{\partial t}\right)(1-K_1T++(K_1T)^2-\ldots\ldots)\frac{\partial\hat{T}}{\partial x_i}\\ &= \gamma\left(1+\nu_0\frac{\partial}{\partial t}\right)\frac{\partial\hat{T}}{\partial x_i}.\end{aligned} \tag{22}$$

Therefore, Eq. (21) can be expressed in tensor form as:

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \mu\frac{\partial^2 u_i}{\partial x_m^2} + (\mu+\lambda)\frac{\partial^2 u_m}{\partial x_i^2} - \gamma\left(1+\nu_0\frac{\partial}{\partial t}\right)\frac{\partial\hat{T}}{\partial x_i} - \delta_n\frac{\partial N}{\partial x_i} + F_i. \tag{23}$$

To analyze the governing equation during 2D (xz-plane) electronic-elastic deformation, consider the displacement vector which can be expressed in two components as $\vec{u} = (u_x, 0, u_z)$, $u_x(x,z,t)$ and $u_z(x,z,t)$.

$$\rho\left(\frac{\partial^2 u}{\partial t^2}\right) = (2\mu+\lambda)\frac{\partial^2 u}{\partial x^2} + (\mu+\lambda)\frac{\partial^2 w}{\partial x \partial z} + \mu\frac{\partial^2 u}{\partial z^2} - \gamma\frac{\partial \hat{T}}{\partial x} - \delta_n\frac{\partial N}{\partial x} - \mu_0 H_0\frac{\partial h}{\partial x} - \varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}\Bigg\}, \quad (24)$$

$$\rho\left(\frac{\partial^2 w}{\partial t^2}\right) = (\mu)\frac{\partial^2 w}{\partial x^2} + (\mu+\lambda)\frac{\partial^2 u}{\partial x \partial z} + (2\mu+\lambda)\frac{\partial^2 w}{\partial z^2} - \gamma\frac{\partial \hat{T}}{\partial y} - \delta_n\frac{\partial N}{\partial z} - \mu_0 H_0\frac{\partial h}{\partial z} - \varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 w}{\partial t^2}\Bigg\}. \quad (25)$$

On the other hand, the heat Eq. (7) under the mapping with neglected the nonlinear term can be rewritten in tensor form as:

$$\frac{1}{k}\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial \hat{T}}{\partial t} = \frac{\partial^2\hat{\phi}}{\partial x_i^2} + \frac{E_g}{K_0\tau}N(\vec{r},t) + \frac{\gamma T_0}{K_0}\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x_i}\frac{\partial u_i}{\partial t} - \rho Q\right). \quad (26)$$

where $\frac{K_0}{\rho C_e} = k$.

The hyperbolic two-temperature Eq. (10) under the mapping transformation when neglected the nonlinear term can be represented as:

$$\frac{\partial^2\hat{T}}{\partial t^2} - \frac{\partial^2\hat{\varphi}}{\partial t^2} = -\tilde{\lambda}\frac{\partial^2\hat{\varphi}}{\partial x_m^2}. \quad (27)$$

Similarly, the coupled plasma-thermal Eq. (6) can be rewritten under the influence of mapping transformation as [43]:

$$\frac{\partial N}{\partial t} = D_E\frac{\partial^2 N}{\partial x_m^2} - \frac{N}{\tau} + \kappa\hat{T}. \quad (28)$$

3 Formulation of the problem

According to the Helmholtz's theorem, the displacement vector (with components u_x and u_z) can be described in terms of the scalar function and vector function ($\Pi(x,\ z,\ t)$ and $\psi(x,\ z,\ t)$, $\vec{u} = \text{grad}\,\Pi + \text{rot}\,\psi$), as:

$$u_x = \frac{\partial\Pi}{\partial x} + \frac{\partial\psi}{\partial z},\ u_z = \frac{\partial\Pi}{\partial z} - \frac{\partial\psi}{\partial x}, \quad (29)$$

For more simplification, the following non-dimensional quantities can be used:

$$(x', z',\ u'_x, u'_z) = \frac{(x, z,\ u_x, u_z)}{C_T t^*},\ (t', \upsilon'_0, \tau'_0) = \frac{(t,\ \upsilon_0, \tau_0)}{t^*}(\hat{T}',\ \hat{\phi}') = \frac{\gamma(\hat{T}, \hat{\phi})}{2\mu+\lambda},\ \sigma'_{ij} = \frac{\sigma_{ij}}{\mu}$$
$$N' = \frac{\delta_n N}{2\mu+\lambda},\ (\Pi', \psi') = \frac{(\Pi, \psi)}{(C_T t^*)^2},\ h' = \frac{h}{\rho C_T^2},\ Q' = \frac{Q}{T_0 C_e t^*}. \quad (30)$$

The above non-dimension quantities which defined in Eq. (30) and using Eq. (29) with ignored the primes for the main equations yield:

$$\left(\nabla^2 - q_1^* - q_2^*\frac{\partial}{\partial t}\right)N + \varepsilon_3\hat{T} = 0, \quad (31)$$

$$\nabla^2\hat{\varphi} - \left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial \hat{T}}{\partial t} + \varepsilon_2 N + \varepsilon_1\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2\Pi = \varepsilon_1\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)Q, \tag{32}$$

$$\left(\alpha\nabla^2 - R_H\frac{\partial^2}{\partial t^2}\right)\Pi - \left(1 + \nu_0\frac{\partial}{\partial t}\right)\hat{T} - N = 0, \tag{33}$$

$$\left(\nabla^2 - R_H\coprod^2\frac{\partial^2}{\partial t^2}\right)\psi = 0, \tag{34}$$

$$\hat{\hat{T}} - \hat{\hat{\phi}} = -a\,\nabla^2\hat{\phi}.$$

where

$$q_1^* = \frac{K_0 t^*}{D_E\rho\tau C_e}, q_2^* = \frac{K_0}{D_E\rho C_e}, \varepsilon_1 = \frac{\gamma^2 T_0 t^{*2}}{K_0\rho}, \varepsilon_2 = \frac{\alpha_T E_g t^*}{d_n\rho\tau C_e},\ \varepsilon_3 = \frac{d_n K_0\kappa t^*}{\alpha_T\rho C_e D_E}, a = \frac{\bar{\lambda}}{C_L^2}$$

$$C_T^2 = \frac{2\mu+\lambda}{\rho}, C_L^2 = \frac{\mu}{\rho}, \coprod^2 = \frac{C_T^2}{C_L^2}, \delta_n = (2\mu+3\lambda)d_n,\ t^* = \frac{K_0}{\rho C_e C_T^2}, \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} = \nabla^2. \tag{36}$$

The parameters ε_1, ε_2 and ε_3 represent the coupled relation between thermal-elastic, thermal, energy with thermal-electric properties.

During the elastic 2D deformation, the non-dimension stress–strain relations under the mapping transformation can be obtained as follows:

$$\sigma_{xx} = \frac{(2\mu+\lambda)}{\mu}\frac{\partial^2\Pi}{\partial x^2} + \frac{\lambda}{\mu}\frac{\partial^2\Pi}{\partial z^2} + 2\frac{\partial^2\psi}{\partial x\partial z} - \frac{(2\mu+\lambda)}{\mu}\left(\left(1+\nu_0\frac{\partial}{\partial t}\right)\hat{T} + N\right), \tag{37}$$

$$\sigma_{zz} = \frac{(2\mu+\lambda)}{\mu}\frac{\partial^2\Pi}{\partial z^2} + \frac{\lambda}{\mu}\frac{\partial^2\Pi}{\partial x^2} - 2\frac{\partial^2\psi}{\partial x\partial z} - \frac{(2\mu+\lambda)}{\mu}\left(\left(1+\nu_0\frac{\partial}{\partial t}\right)\hat{T} + N\right), \tag{38}$$

$$\sigma_{xz} = \frac{\partial^2\psi}{\partial z^2} + 2\frac{\partial^2\Pi}{\partial x\partial z} - \frac{\partial^2\psi}{\partial x^2}. \tag{39}$$

The initial conditions which used in this problem can be constructed as:

$$\begin{aligned} u_x(x,\ t)|_{t=0} &= \left.\frac{\partial u_x(x,\ t)}{\partial t}\right|_{t=0} = 0,\ u_z(x,t)|_{t=0} = \left.\frac{\partial u_z(x,\ t)}{\partial t}\right|_{t=0} \\ &= 0\ \ \sigma_{ij}(x,t)\big|_{t=0} = \left.\frac{\partial\sigma_{ij}(x,\ t)}{\partial t}\right|_{t=0} = 0 \\ N(x,t)|_{t=0} &= \left.\frac{\partial N(x,\ t)}{\partial t}\right|_{t=0} = 0,\ \ \hat{T}(x,t)\big|_{t=0} \\ &= \left.\frac{\partial\hat{T}(x,\ t)}{\partial t}\right|_{t=0} = 0,\ \ \hat{\phi}(x,t)\big|_{t=0} = \left.\frac{\partial\hat{\phi}(x,\ t)}{\partial t}\right|_{t=0} = 0. \end{aligned} \tag{40}$$

4 The solution in Laplace and Fourier transform

The Laplace and Fourier transformations can be used to convert the time–space domain to Laplace and Fourier domain by using the following definition for any function $\aleph(x, z, t)$ as:

$$L(\aleph(x, z, t)) = \overline{\aleph}(x, z, s) = \int_0^\infty \aleph(x, z, t) \exp(-st) \mathrm{d}t, \tag{41}$$

$$F(\overline{\aleph}(x, z, s)) = \tilde{\aleph}(\xi, z, s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty \overline{\aleph}(x, z, s) \exp(-i\xi x) \mathrm{d}t. \tag{42}$$

Using the above definition Eqs. (41) and (42) with applied them of all the main Eqs. (31)–(35), yield:

$$(D^2 - \alpha_1)\tilde{N} + \varepsilon_3 \tilde{T} = 0, \tag{43}$$

$$(D^2 - b^2)\tilde{\phi} - \omega^* \tilde{T} + \varepsilon_2 \tilde{N} + \alpha_2 (D^2 - b^2)\tilde{\Pi} = \Theta(\xi, s) e^{-\gamma' x}, \tag{44}$$

$$(\alpha D^2 - \alpha_3)\tilde{\Pi} - \alpha_7 \tilde{T} - \tilde{N} = 0, \tag{45}$$

$$(D^2 - \alpha_4^2)\tilde{\psi} = 0, \tag{46}$$

$$(D^2 - A_1)\tilde{\phi} + \beta^* \tilde{T} = 0. \tag{47}$$

On the other hand, the stress–strain relations (37)–(39) under the impact of Laplace and Fourier transformation can be expressed as:

$$\tilde{\sigma}_{xx} = (\alpha_5 D^2 - \alpha_6 b^2)\tilde{\Pi} + 2ibD\tilde{\psi} - \alpha_5(\alpha_7 \tilde{T} + \tilde{N}), \tag{48}$$

$$\tilde{\sigma}_{zz} = (-\alpha_5 b^2 + \alpha_6 D^2)\tilde{\Pi} - 2ibD\tilde{\psi} - \alpha_5(\alpha_7 \tilde{T} + \tilde{N}), \tag{49}$$

$$\tilde{\sigma}_{xz} = 2ibD\tilde{\Pi} - (D^2 + b^2)\tilde{\psi}. \tag{50}$$

where

$$\begin{aligned}
&D = \frac{\mathrm{d}}{\mathrm{d}x}, \alpha_1 = \xi b^2 + q_1^* + s\, q_2^*, \alpha_2 = \varepsilon_1(n_1 + n_0\tau_0 s), \alpha_3 = \xi^2 + R_H s^2, \alpha_4^2 = \xi^2 + s^2 R_H \coprod^2, \\
&\omega^* = s(n_1 + n_0\tau_0 s), \alpha_5 = \frac{(2\mu + \lambda)}{\mu}, \alpha_6 = \frac{\lambda}{\mu}, A_1 = \xi^2 + \beta^*, \beta^* = \frac{s^2}{a} \\
&\Theta(z, t) = \left(\frac{I_0 \gamma' \varepsilon_1 (n_1 + n_0\tau_0 s)}{2\pi\, a^2 t_0^2 (s + \frac{1}{t_0})^2} \right) e^{-\frac{\xi^2}{4a^2}}.
\end{aligned} \tag{51}$$

Eliminating the above system Eqs. (43)–(45) and (47) in $\tilde{\Pi}(x)$, $\tilde{\phi}(x)$, $\tilde{T}(x)$, and $\tilde{N}(x)$, however the six-order ordinary non-homogeneous differential equation can be obtained in the quantity $\tilde{T}(x)$ as:

$$[D^6 - ED^4 + FD^2 - G]\, \tilde{T}(x) = L_1 \Theta(\xi, s) \exp(-\gamma' x). \tag{52}$$

where

$$\left.\begin{aligned}
&E = [(\alpha_1 + \alpha_3 + b^2)\beta^* + (\alpha_1 + \alpha_3 + A_1)\omega^* - \varepsilon_2\varepsilon_3 - \alpha_2(\alpha_1^* + \alpha_7(A_1 + b^2))]/\alpha A_2 \\
&F = [(\alpha_1\alpha_3 + (\alpha_1 + \alpha_3)b^2)\beta^* + (\alpha_1 A_3 + \alpha_3 A_1)\omega^* - \varepsilon_2\varepsilon_3 A_3 - \alpha_2(\alpha_1^* A_1 + b^2 A_4)]/\alpha A_2 \\
&G = [\alpha_1\alpha_3 b^2 \beta^* + \alpha_1\alpha_3 A_1 \omega^* - \varepsilon_2\varepsilon_3\alpha_3 A_1 - \alpha_2 \alpha_1^* A_1 b^2]/\alpha A_2 \\
&\alpha_1^* = \alpha_1\alpha_7 + \varepsilon_3, A_2 = \beta^* + \omega^* - \alpha_2\alpha_7, A_3 = \alpha_3 + A_1, A_4 = (\alpha_1^* + \alpha_7 A_1),
\end{aligned}\right\}. \tag{53}$$

The factorization form of homogeneous Eq. (52) can be decomposed in in terms of temperature as:

$$\left(D^2 - k_1^2\right)\left(D^2 - k_2^2\right)\left(D^2 - k_3^2\right)\tilde{T}(x) = 0. \tag{54}$$

The characteristic equation of Eq. (54) when the real roots (k_n^2 (Re(k_n) > 0, $n = 1, 2, 3$) are taken into account can be rewritten as:

$$\Delta^6 - E\Delta^4 + F\Delta^2 - G = 0. \tag{55}$$

On the other hand, the factorization form of the non-homogenous Eq. (52) can be expressed as:

$$\left(D^2 - k_1^2\right)\left(D^2 - k_2^2\right)\left(D^2 - k_3^2\right)\tilde{T}(x) = L_1\Theta(\xi, s)\exp(-\gamma' x). \tag{56}$$

However, Eq. (46) can be factorized and it has the following characteristic equation:

$$\left(D^2 - k_4^2\right)\tilde{\psi}(x) = 0. \tag{57}$$

But k_4^2 is the real root of Eq. (57).

According to the linearity, the solution of Eq. (56) can be expressed as:

$$\tilde{T}(x) = \sum_{n=1}^{3} \Re_n(s, \xi)\, e^{-k_n x} + L_2 e^{-\gamma' x}. \tag{58}$$

where $L_1 = \gamma'^6 + (\alpha_1 A_1 + \alpha_3 A_1 + \alpha_1\alpha_3)\gamma'^2 - (\alpha_1 + \alpha_3 + A_1)\gamma'^4 - \alpha_1\alpha_3 A_1$, $L_2 = \left(\frac{\Theta L_1}{\gamma'^6 - E\gamma'^4 + F\gamma'^2 - G}\right)$.

From the relations between $\tilde{T}(x)$ and the other quantities, according to the linearity the main quantities take the following form:

$$\tilde{\Pi}(x) = \sum_{n=1}^{3} \Re'_n(s, \xi) e^{-k_n x} + L_3 e^{-\gamma' x}, \tag{59}$$

$$\tilde{N}(x) = \sum_{n=1}^{3} \Re''_n(s, \xi)\, e^{-k_n x} + L_4 e^{-\gamma' x}, \tag{60}$$

$$\tilde{\phi}(x) = \sum_{n=1}^{3} \Re'''_n(s, \xi) e^{-k_n x} + L_5 e^{-\gamma' x}, \tag{61}$$

$$\tilde{\psi}(x) = \Re_4(s, \xi)\, e^{-k_4 x}. \tag{62}$$

where $L_3 = L_2 \frac{(\alpha_7\gamma'^2 - \alpha_1^*)}{(\gamma'^2 - \alpha_1)(\gamma'^2 - \alpha_3)}$, $L_4 = \frac{-\varepsilon_3 L_2}{(\gamma'^2 - \alpha_1)}$, $L_5 = -\frac{\beta^* L_2}{\gamma'^2 - A_1}$ and $\Re_4$, $\Re_n$, $\Re'_n$, $\Re''_n$, $\Re'''_n$ ($n = 1, 2, 3$) are the main unknown parameters that depend on (s, ξ).

The displacement components can be obtained as:

$$\tilde{u}_x(x) = D\tilde{\Pi} + i\, b\, \tilde{\psi}, \tag{63}$$

$$\tilde{u}_x(x) = -\sum_{n=1}^{3} \Re'_n(s, \xi)\, k_n e^{-k_n x} - \gamma' L_3 e^{-\gamma' x} + ib\Re_4(s, \xi)\, e^{-k_4 x}, \tag{64}$$

$$\tilde{u}_z(x) = i\, b\, \tilde{\Pi} - D\, \tilde{\psi}, \tag{65}$$

$$\tilde{u}_z(x) = ib\left\{\sum_{n=1}^{3} \Re'_n(s, \xi)\, e^{-k_n x} + L_3 e^{-\gamma' x}\right\} + \Re_4(s, \xi) k_4 e^{-k_4 x}. \tag{66}$$

The relations between the quantities $\Re_n'$, $\Re_n''$, $\Re_n'''$ and $\Re_n$ can be constructed from Eqs. (59)–(61) and Eqs. (43)–(45) as:

$$\Re_n'(s,\xi)=\frac{(\alpha_7 k_n^2-\alpha_1^*)}{(k_n^2-\alpha_1)(k_n^2-\alpha_3)}\Re_n(s,\xi),\quad n=1,\ 2,3. \tag{67}$$

$$\Re_n''(s,\xi)=\frac{-\varepsilon_3}{(k_n^2-\alpha_1)}\Re_n(s,\xi),\quad n=1,\ 2,3, \tag{68}$$

$$\Re_n'''(s,\xi)=-\frac{\beta^*}{k_n^2-A_1}\Re_n(s,\xi),\quad n=1,\ 2,\ 3. \tag{69}$$

In this case, the main physical quantities can be represented in terms of $\Re_n$ as:

$$\tilde{\Pi}(x)=\sum_{n=1}^{3}\frac{(\alpha_7 k_n^2-\alpha_1^*)}{(k_n^2-\alpha_1)(k_n^2-\alpha_3)}\Re_n\, e^{-k_n x}+L_3 e^{-\gamma' x}, \tag{70}$$

$$\tilde{N}(x)=\sum_{n=1}^{3}\frac{-\varepsilon_3}{(k_n^2-\alpha_1)}\Re_n\, e^{-k_n x}+L_4 e^{-\gamma' x}, \tag{71}$$

$$\tilde{\phi}(x)=\sum_{n=1}^{3}\frac{-\beta^*}{k_n^2-A_1}\Re_n\, e^{-k_n x}+L_5 e^{-\gamma' x}, \tag{72}$$

$$\tilde{\sigma}_{xx}=\sum_{n=1}^{3}h_n\,\Re_n\, e^{-k_n x}+\chi_1 e^{-\gamma' x}-2ibk_4\Re_4 e^{-k_4 x}, \tag{73}$$

$$\tilde{\sigma}_{zz}=\sum_{n=1}^{3}h_n'\,\Re_n\, e^{-k_n x}+\chi_2 e^{-\gamma' x}+2ibk_4\Re_4 e^{-k_4 x}, \tag{74}$$

$$\tilde{\sigma}_{xz}=\sum_{n=1}^{3}h_n''\,\Re_n\, e^{-k_n x}+\chi_3 e^{-\gamma' x}-(k_4^2+b^2)\Re_4 e^{-k_4 x}, \tag{75}$$

$$\tilde{u}_x(x)=-\sum_{n=1}^{3}\frac{k_n(\alpha_7 k_n^2-\alpha_1^*)}{(k_n^2-\alpha_1)(k_n^2-\alpha_3)}\Re_n\, e^{-k_n x}-\gamma' L_3 e^{-\gamma' x}+ib\Re_4 e^{-k_4 x}, \tag{76}$$

$$\tilde{u}_z(x)=\sum_{n=1}^{3}\frac{ib(\alpha_7 k_n^2-\alpha_1^*)}{(k_n^2-\alpha_1)(k_n^2-\alpha_3)}\Re_n\, e^{-k_n x}+ibL_3 e^{-\gamma' x}+k_4\Re_4 e^{-k_4 x}. \tag{77}$$

where

$$h_n=\frac{(\alpha_5 k_n^2-\alpha_6 b^2)(\alpha_7 k_n^2-\alpha_1^*)}{(k_n^2-\alpha_3)(k_n^2-\alpha_1)}+\left(\frac{\varepsilon_3}{(k_n^2-\alpha_1)}-\alpha_7\right)\alpha_5,\ \chi_3=-2ib\gamma' L_3,\ h_n''=-2ibk_n$$

$$h_n'=-\frac{(\alpha_5 b^2-\alpha_6 k_n^2)(\alpha_7 k_n^2-\alpha_1^*)}{(k_n^2-\alpha_3)(k_n^2-\alpha_1)}+\left(\frac{\varepsilon_3}{(k_n^2-\alpha_1)}-\alpha_7\right)\alpha_5,\ \chi_1$$

$$=L_3(\alpha_5\gamma'^2-\alpha_6 b^2)-(L_4+\alpha_7 L_2)\alpha_5,$$

$$\chi_2=L_3(\alpha_6\gamma'^2-\alpha_5 b^2)-(L_4+\alpha_7 L_2)\alpha_5.$$

5 Boundary conditions

To calculate the unknown parameters $\Re_n (n=1,2,3,4)$ analytically, some boundary conditions are applied at the free surface of the elastic semiconductor medium. During these conditions, the effect of external magnetic field and laser pulses in the context of the photo-hyperbolic two-temperature-thermoelasticity theory when the thermal conductivity is of variable is considered.

(I) The isothermal condition:
The thermal condition can be chosen at the boundary surface $x = 0$ as an isothermal case:

$$\frac{\partial \tilde{\bar{T}}(0, \xi, s)}{\partial x} = 0 \text{ or } \sum_{n=1}^{3} k_n \Re_n(s, \xi) = -\gamma' L_2. \tag{78}$$

(II) The mechanical ramp type can be applied for the normal stress component at the surface $x = 0$, when used the Fourier and Laplace transform, which yields:

$$\left.\begin{array}{l} \sigma_{xx}(x, z, t) = \begin{cases} 0 & t \leq 0 \\ \frac{t}{t_0} & 0 < t \leq t_0 \\ 1 & t > t_0 \end{cases} \Rightarrow \tilde{\sigma}_{xx}(0, \xi, s) = \tilde{F}(\xi) \dfrac{\left(1 - e^{-st_0}\right)}{t_0 s^2}, \\ \sum_{n=1}^{3} h_n \Re_n - 2ibk_4 \Re_4 = -\chi_1 + \tilde{F}(\xi) \dfrac{\left(1 - e^{-st_0}\right)}{t_0 s^2}. \end{array}\right\}. \tag{79}$$

(III) The other mechanical conditions can be chosen when the traction component of the stress is free at $x = 0$, when using the Fourier and Laplace transform as:

$$\left.\sigma_{xz} = 0 \Rightarrow \tilde{\sigma}_{xz} = 0, \Rightarrow \sum_{n=1}^{3} h''_n \Re_n - (k_4^2 + b^2)\Re_4 = -\chi_3 \right\}. \tag{80}$$

(IV) The plasma condition at the surface $x = 0$ can be taken during the transport processes in the context of the recombination processes. On the other hand, the photo-generated (plasma) condition can be represented by carrier density as:

$$\left.\tilde{N}(a, s) = \frac{\Sigma}{D_e} \tilde{R}(s) \Rightarrow \sum_{n=1}^{3} \frac{-\varepsilon_3 k_n}{(k_n^2 - \alpha_1)} \Re_n = -\left(\frac{s}{D_e} N + \gamma' L_4\right)\right\}. \tag{81}$$

where Σ and $R(s)$ are a chosen parameter and the Heaviside unit step function, respectively. By using the system of above boundary condition equations the values of $\Re_n (n = 1, 2, 3, 4)$ can be obtained with helping the MATLAB 2018a computer programing.

6 Fourier–Laplace transforms inversion

The Honig and Hirdes method is used to obtain the inversion form of Fourier and Laplace transformation.

In this case, the inverse of Fourier transform can be written as:

$$F^{-1}(\tilde{\zeta}(x, \xi, s)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{\zeta}(x, \xi, s) \exp(i\xi x)\, \mathrm{d}t = \overline{\zeta}(x, z, s). \tag{82}$$

On the other hand, the Riemann-sum approximation method is used to obtain the inverse of Laplace transform in numerical form. In this case, the inverse of Laplace transform in time domain for a function $\overline{\zeta}(x, z, s)$ can be written as:

$$\zeta(x, z, t') = L^{-1}\{\overline{\zeta}(x, z, s)\} = \frac{1}{2\pi i} \int_{n-i\infty}^{n+i\infty} \exp(st') \overline{\zeta}(x, z, s) \mathrm{d}s. \tag{83}$$

where $s = n + i\mathrm{M}(n,\ \mathrm{M} \in R)$, in this case Eq. (83) can be rewritten as follows:

$$\zeta(x, z, t') = \frac{\exp(nt')}{2\pi} \int_{\infty}^{\infty} \exp(i\beta t)\overline{\zeta}(x, z, n + i\beta)\mathrm{d}\beta. \tag{84}$$

After that, the expansion in the closed interval $[0,\ 2t']$ of the Fourier series can be used for large integer N when $nt' \approx 4.7$, therefore:

$$\zeta(x, z, t') = \frac{e^{nt'}}{t'}\left[\frac{1}{2}\overline{\zeta}(x, z, n) + \mathrm{Re}\sum_{k=1}^{N}\overline{\zeta}\left(x, z, n + \frac{ik\pi}{t'}\right)(-1)^n\right]. \tag{85}$$

where $i = \sqrt{-1}$ and Re refers to the real part.

To obtain the basic temperature T and the conductive temperature ϕ under the impact of Laplace and Fourier transformation the map Eqs. (15) and (16) are used. In this case, the following relation can be obtained:

$$\hat{T} = \frac{1}{K_0}\int_0^T K_0(1 + K_1 T)\mathrm{d}T = T + \frac{K_1}{2}T^2 \Rightarrow T = \frac{1}{K_1}\left[\sqrt{1 + 2K_1\hat{T}} - 1\right]. \tag{86}$$

7 Validation

7.1 The two-temperature theory

The two-temperature theory in classical case can be investigated in the framework of the thermoelasticity theory during photothermal excitation processes when the second time differentiation is ignored and Eq. (9) can be represented as [47]:

$$T - \varphi = -\tilde{\lambda}\nabla^2\varphi. \tag{87}$$

In this case $\tilde{\lambda} = a > 0$ that refers to the two-temperature parameter. In this case the hyperbolic two-temperature theory is not taken into account.

7.2 The one temperature theory

The one-temperature theory is a special case from the two-temperature theory which it can be obtained when it is ignored the two-temperature parameter $a = 0$ in Eq. (87). Therefore, the conductive temperature must be equal the thermodynamic temperature and the heat conduction Eq. (7) can be represented as [21]:

$$\rho C_e\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T(r, t)}{\partial t} = (KT_{,i}(r, t))_{,i} + \frac{E_g}{\tau}N(r, t) + \gamma T_0\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}u_{,i}(r, t) - \rho Q\right). \tag{88}$$

7.3 Laser pulses effect

The hyperbolic two-temperature theory in the context of the photo-thermoelasticity theory under the impact of magnetic field is investigated only when the influence of the laser pulse is neglected. In this case the heat input function can be put as $Q = 0$. However, the heat conduction Eq. (7) can be rewritten as [42]:

$$\rho C_e\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T(r,t)}{\partial t} = (K\phi_{,i}(r,t))_{,i} + \frac{E_g}{\tau}N(r,t) + \gamma T_0\left(n_1 + n_0\tau_0\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}u_{,i}(r,t). \tag{89}$$

7.4 The variable thermal conductivity

In case of independent the thermal conductivity of the temperature when $K_1 = 0$, in this case the variable thermal conductivity is ignored and equal to the classical thermal conductivity $K = K_0$. However, the maps transformation are vanished also.

7.5 The hyperbolic two temperatures in the generalized thermoelasticity theory

When the plasma wave which considering by the carrier density $N(\vec{r}, t)$ is vanished. In this case the free electrons on the free surface of the semiconductor medium effects are neglected, i.e.$N = 0$ and the main equation in this investigation is studied in the generalized hyperbolic two-temperature thermoelasticity theory under the influence of magnetic field with laser pulses [38].

7.6 The generalized case of the photo-thermoelasticity theories

The photo-thermoelasticity theory in generalized case is studied when the thermal relaxation times (thermal memories) are variable according to the photo-thermoelasticity models. In this case, the three models of photo-thermoelasticity can be represented namely the following:

(i) The CD theory is obtained when the following parameters take the values $n_1 = 1, n_0 = 0, \nu_0 = \tau_0 = 0$, [3].
(ii) The LS theory is obtained when the following parameters take the values $n_1 = n_0 = 1, \nu_0 = 0, \tau_0 > 0$, [4].
(iii) The GN theory is obtained when the following parameters take the values $n_1 = 1, n_0 = 0, \nu_0 \geq \tau_0 > 0$, [5].

7.7 The impact of magnetic field

The generalized hyperbolic two-temperature theory during the photo-thermoelasticity theory under the effect of the laser pulse is studied only when the impact of the magnetic field is neglected. In this case, the Lorentz's force $\vec{F} = \mu_0(\vec{J} \times \vec{H}) = 0$ is neglected.

8 Numerical results and discussions

After evaluated the numerical inversion of Laplace and Fourier transform, the complete solutions of the main quantities are obtained. In this case, the computer programing numerically can be used to carry out the numerical simulations and the description of the behavior of the main fields is made. To make this simulation, the physical constants of silicon (Si) material are used as an example of a plate of semiconductor medium. On the other hand, the laser beam physical properties and generalized hyperbolic two-temperature theory are used in the numerical simulations under the effect of the magnetic field when the variable thermal conductivity is taken into account. The physical and optical constants for the Si medium are considered in SI unit which they displayed in Table 1 as [43, 44].

The first figure (Fig. 1) has been divided into a number of subfigures which it displays the impact of the thermal conductivity when it is variable. In this case, the thermal conductivity depends on a linear function of temperature and clarification the effect of it on the main physical fields. The wave distributions can be taken in the direction of the horizontal distance x in the generalized hyperbolic two temperatures. The physical fields are investigated under the impact of magnetic field and a non-Gaussian laser pulses according to the photo-thermoelasticity (GL) model. However, three different values of the thermal conductivity (variable) parameters are taken into account. On the other hand, when the thermal conductivity independent to the temperature (with solid black lines) can be obtained at $K_1 = 0.0$ and $K_0 = K$. But in the variable cases of the thermal conductivity, two cases are studied, the first case is taken at $K_1 = -0.03$ (dotted red lines) and the second case is taken at $K_1 = -0.06$ (dashed blue lines). The amplitude values of T satisfy the thermal condition for three cases of K_1 which they start at $x = 0$ (isothermal). The thermo-dynamical T temperature distributions increase with exponential behavior for all different cases in the first range which they take a smooth distributions until to arrive to the maximum value. It is due to the thermal impact of external magnetic field and the laser pulses according to the photothermal excitation. In the second range, the amplitudes of the temperature T decrease with a smooth shape also when the curves coincide with the increasing in the horizontal distance. The conductive temperature distribution is shown in the second subfigure which it takes the same behavior of temperature distribution, it is due to the proportional between the temperature and conductive temperature. The third subfigure shows the horizontal displacement distribution u_x which it represents the elastic wave. When $x = 0$ the component u_x distributions start at three different values because the roughly surface, in the first range all curves according to K_1 increase near the surface reach a peak maximum values with a great affinity at the surface. However, the second range of the curves of u_x distributions decreases according to the values of K_1 until they arrive to the minimum values with the increasing in the horizontal distance. On the other hand, the distributions of the mechanical waves σ_{xx} are illustrated in the fourth subfigure. The σ_{xx} distributions satisfy the ramp mechanical type condition, all curves under three values of K_1 start from a maximum positive values in the beginning is due to ramp mechanical type and magnetic field with thermal laser pulses impact. Far away from the surface, σ_{xx} distributions decrease with smooth shape until arrive to the minimum values and all curves distributions coincide. The σ_{xz} distributions (tangent stress) are shown in the fifth subfigure, the distributions begin at zero minimum values for all different cases of K_1 are due to zero ramp mechanical load. The plasma waves distributions are shown in the last subfigure which describe the magnitude of distributions of carrier density N according to three different cases of thermal conductivity K_1. The plasma wave distributions satisfy the condition in the context of the plasma recombination processes. At the beginning the plasma N distributions increase smoothly to reach the maximum values which is due to the effect of magnetic field and photo-excited thermal laser. On the other hand, in the second range, the plasma wave distributions decrease to arrive to the minimum values, again all distributions increase and decrease periodically until the waves are damped.

Figure 2 illustrates in dimensionless the wave propagation of main physical fields distributions ($T, u_x, \phi, u_x, \sigma_{xx},\ \sigma_{xz}$ and N) at the plane z $= -1$ against the horizontal distance x. The three models in photo-thermoelasticity theory are shown according to the variation of thermal memories (thermal relaxation times) when a variable thermal conductivity $K_1 = -0.03$ under the effect of external magnetic field with laser pulses when the hyperbolic two-temperature parameter is present. According to a thermal memories, CD model which represented by

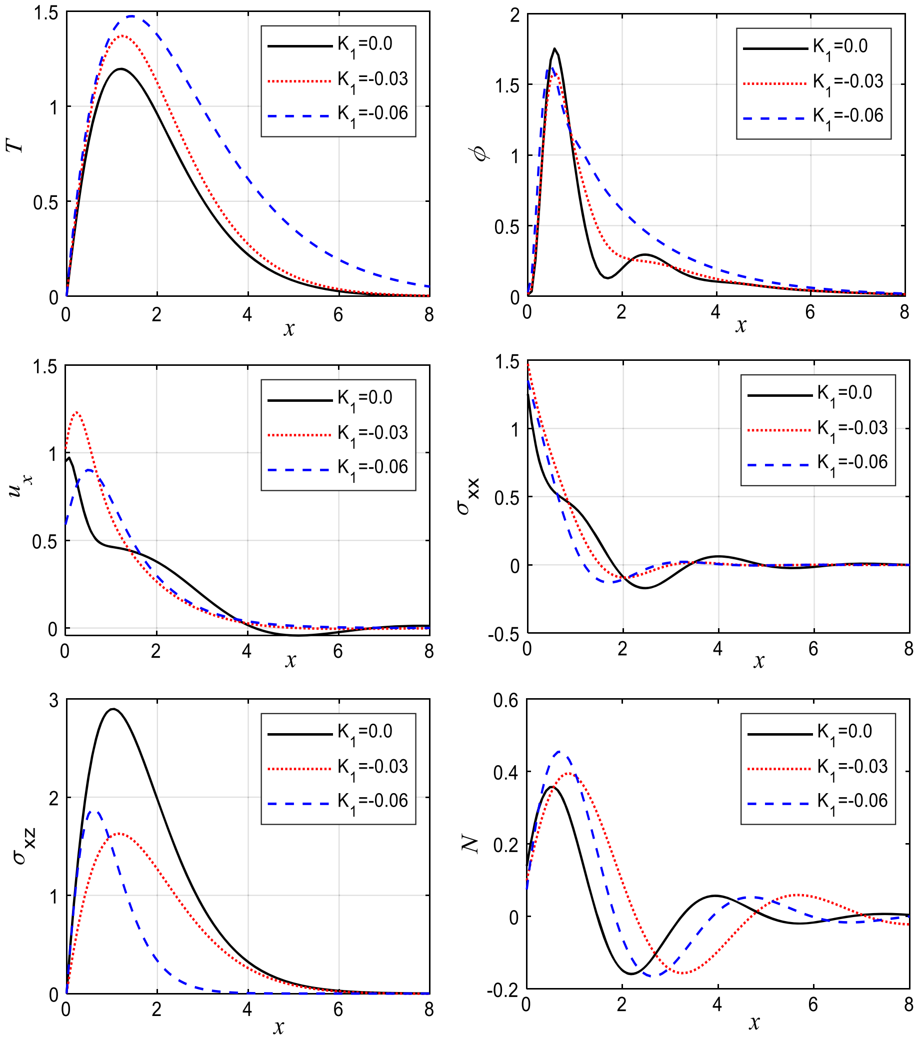


Fig. 1 The main physical fields against the horizontal distance in various cases of thermal conductivity under GL model and magnetic field with laser pulses in hyperbolic two-temperature field

solid lines can be obtained when the thermal memories are ignored, the LS model which represented by dotted lines is studied when introduced a one relaxation time at $\tau_0 = 0.00002\,s$. On the other hand, the general case is taken in GL model which represented by dashed lines in this case the two relaxation times are presented when $\tau_0 = 0.00002\,s,\ \nu_0 = 0.00003\,s$. In this category, the change in the amplitude of all distributions of the main physical fields increases according to the increasing in the thermal relaxation times.

Figure 3 shows the wave propagation of the main physical quantities against the distance in the horizontal direction under the effect of external magnetic field. All calculations are made under the effect of laser pulses in the generalized GL model when the hyperbolic two-temperature theory is taken into account. Figure 3 studies two different cases, the first case when the magnetic field is absent which refers by WOMG (without magnetic field) in the

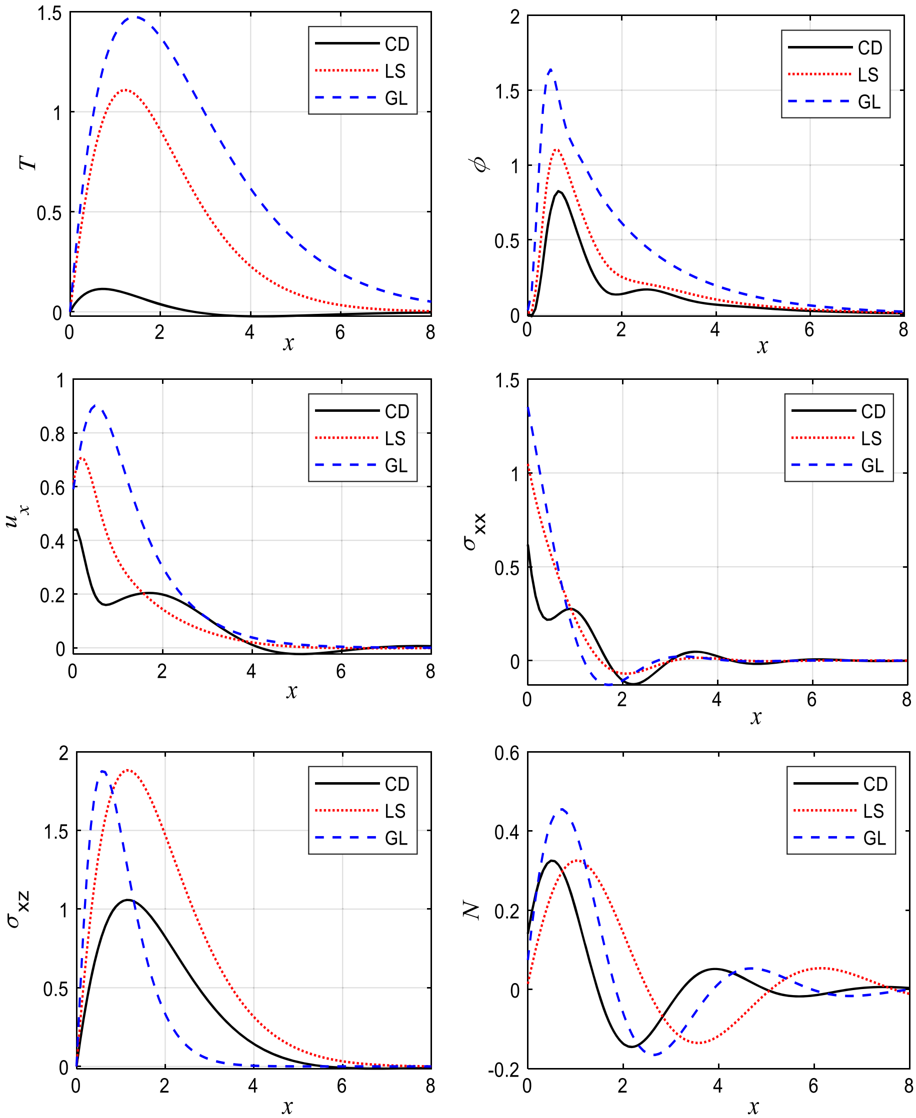


Fig. 2 The main physical fields against the horizontal distance under generalized three models in photo-thermoelasticity theory under the effect of magnetic field with laser pulses when $K_1 = -0.06$ in hyperbolic two-temperature field

same range. On the other hand, the second case is studied when the magnetic field is present which is referred by WMF (with magnetic field). The wave propagation curves for all main distributions coincide at infinity when the horizontal distance x is in increasing case. That is due to the hyperbolic two-temperature effect with a finite speed of distribution waves. The magnetic field has a great influence in all physical fields.

Figure 4 shows the impact of three different cases according to the two-temperature parameter for the main physical fields against the horizontal distance. All numerical results

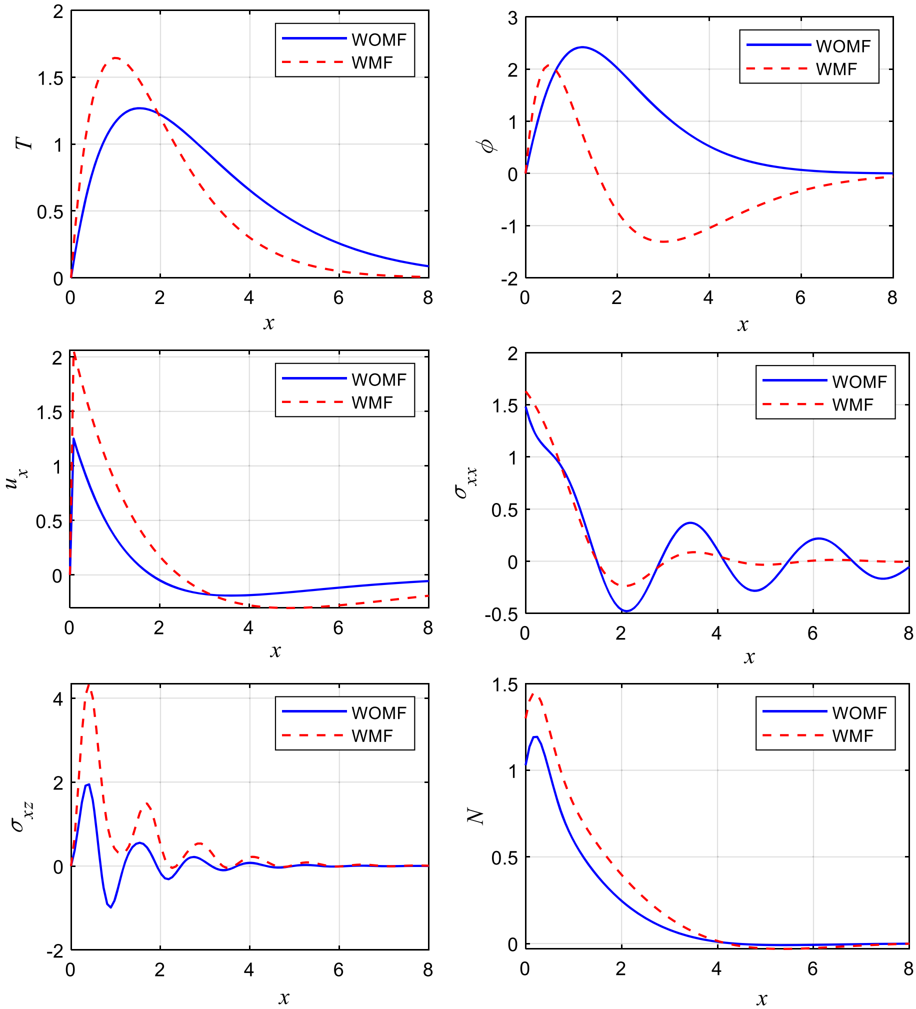


Fig. 3 The main physical fields against the horizontal distance with generalized GL model under the impact of laser pulses in hyperbolic two-temperature field

in this category are made under the effect of magnetic field and the laser pulses in generalized GL model when $K_1 = -0.06$. The solid lines curves in this category express the first case when T and ϕ are equal when the heat supply is absent which can be named one-temperature (OT) model. The dotted lines curves refer to the classical two-temperature model (CTT) which is taken when the heat supply is absent also. The dashed lines curves show the general model which named the hyperbolic two-temperature (HTT) model; this model illustrates the finite speed of wave propagation. A clear significant effects in this figure are observed according to the three different cases of the hyperbolic two-temperature theory.

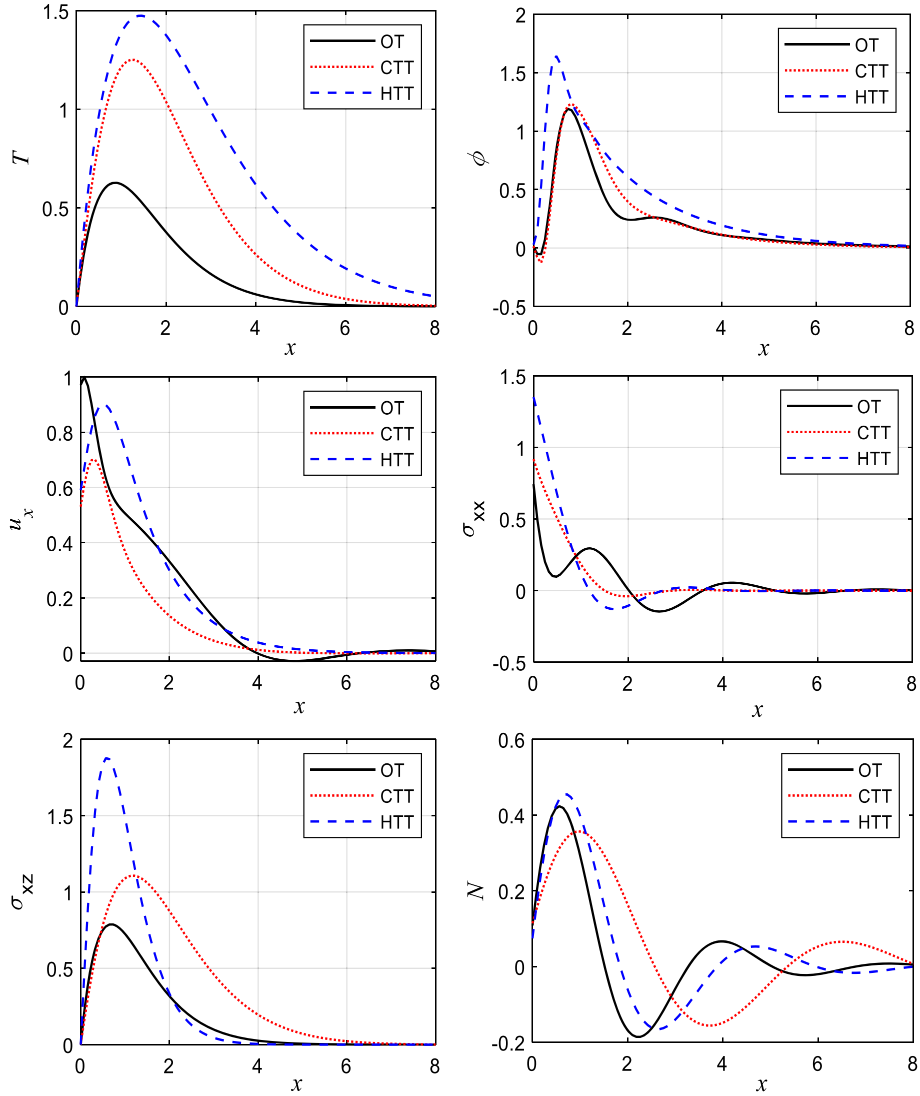


Fig. 4 The main physical fields against the horizontal distance in generalized GL model with laser pulses and magnetic field in one temperature, two temperatures and hyperbolic two temperatures when $K_1 = -0.06$

9 Conclusions

The problem concludes the effect of the different relaxation times according to the generalized photo-thermoelasticity models. This investigation is studied under the effect of magnetic field when the thermal conductivity is in change case (depend on the temperature). The problem is investigated under the impact of a novel model that called the hyperbolic two-temperature model. The impact of laser pulses is taken into account according to a non-Gaussian laser model. The thermal memories with a negative parameter constant of thermal conductivity, effect of magnetic field and laser pulses in hyperbolic two temperatures have more significant influence with the improving processes in the waves propagation of the main fields. These observations are very important when studying the semiconductor media during the photothermal excitation in the context of the generalized thermoelasticity theory. The obtained results with numerical calculations are graphed which they are useful for scientists and engineering to carry out many applications in modern physic, mechanical engineering and plasma design.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. A. Mandelis, Photoacoustic and thermal wave phenomena in semiconductors. 1987.
2. D. Almond, P. Patel, *Photothermal Science and Techniques*, vol. 10 (Springer, Berlin, 1996)
3. M. Biot, Thermoelasticity and irreversible thermodynamics. J. Appl. Phys. **27**, 240–253 (1956)
4. H. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity. J. Mech. Phys. Solids. **15**, 299–309 (1967)
5. A. Green, K. Lindsay, Thermoelasticity. J. Elast. **2**, 1–7 (1972)
6. D. Chandrasekharaiah, Thermoelasticity with second sound: a review. Appl. Mech. Rev. **39**, 355–376 (1986)
7. D. Chandrasekharaiah, Hyperbolic thermoelasticity: a review of recent literature. Appl. Mech. Rev. **51**, 705–729 (1998)
8. K. Lotfy, S. Abo-Dahab, Two-dimensional problem of two temperature generalized thermoelasticity with normal mode analysis under thermal shock problem. J. Comput. Theor. Nanosci. **12**(8), 1709–1719 (2015)
9. M. Othman, K. Lotfy, The influence of gravity on 2-D problem of two temperature generalized thermoelastic medium with thermal relaxation. J. Comput. Theor. Nanosci. **12**(9), 2587–2600 (2015)
10. M. Othman, K. Lotfy, Effect of rotation on plane waves in generalized thermo-microstretch elastic solid with one relaxation time. Multidisc. Model. Mater. Struct. **7**(1), 43–62 (2011)
11. A. Khan, S. Bukhari, M. Marin, R. Ellahi, Effects of chemical reaction on third-grade MHD fluid flow under the influence of heat and mass transfer with variable reactive index. Heat Transfer Res. **50**(11), 1061–1080 (2019)
12. M. Bhatti, M. Marin, A. Zeeshan, R. Ellahi, S. Abdelsalam, Swimming of motile gyrotactic microorganisms and nanoparticles in blood flow through anisotropically tapered arteries. Front. Phys. **8**, 1–12 (2020)
13. J.P. Gordon, R.C.C. Leite, R.S. Moore et al., Long-transient effects in lasers with inserted liquid samples. J. Appl. Phys. **36**, 3–8 (1965)
14. L.B. Kreuzer, Ultralow gas concentration infrared absorption spectroscopy. J. Appl. Phys. **42**, 2934–2943 (1971)
15. A.C. Tam, *Ultrasensitive Laser Spectroscopy* (Academic Press, New York, 1983), pp. 1–108
16. A.C. Tam, Applications of photoacoustic sensing techniques. Rev. Mod. Phys. **58**, 381–431 (1986)
17. A.C. Tam, *Photothermal Investigations in Solids and Fluids* (Academic Press, Boston, 1989), pp. 1–33
18. D.M. Todorović, P.M. Nikolić, A.I. Bojičić, Photoacoustic frequency transmission technique: electronic deformation mechanism in semiconductors. J. Appl. Phys. **85**, 7716–7726 (1999)
19. Y.Q. Song, D.M. Todorovic, B. Cretin et al., Study on the generalized thermoelastic vibration of the optically excited semiconducting microcantilevers. Int. J. Solids Struct. **47**, 1871–1875 (2010)
20. T.Q. Quintanilla, C.L. Tien, Heat transfer mechanism during short-pulse laser heating of metals. ASME J. Heat Transfer. **115**, 835–841 (1993)
21. H.M. Youssef, Theory of two-temperature-generalized thermoelasticity. IMA J. Appl. Math. **71**, 383–390 (2006)
22. H.M. Youssef, E.A. Al-Lehaibi, State-space approach of two-temperature generalized thermoelasticity of one-dimensional problem. Int. J. Solids Struct. **44**, 1550–1562 (2007)

23. K. Lotfy and A. A. El-Bary, photothermal excitation for a semiconductor medium due to pulse heat flux and volumetric source of heat with thermal memory, Waves in Random and Complex Media. (2019) https://doi.org/10.1080/17455030.2019.1662511
24. K. Lotfy, M. Gabr, A. El-Bary, Surface wave propagation in two- dimensional models of generalized two temperature thermoelasticity with gravity effect. Arch. Thermodyn. **41**(1), 219–243 (2020)
25. K. Lotfy, W. Hassan, A.A. El-Bary, M.A. Kadry, Response of electromagnetic and Thomson effect of semiconductor mediu due to laser pulses and thermal memories during photothermal excitation. Results Phys. **16**, 102877 (2020)
26. A.K. Khamis, A.A. El-Bary, K. Lotfy, A. Bakali, Photothermal excitation processes with refined multi dual phase-lags theory for semiconductor elastic medium. Alex. Eng. J. **59**, 1–9 (2020)
27. M. Yasein, K. Lotfy, N. Mabrouk, A.A. El-Bary, W. Hassan, Response of thermo- electro-magneto semiconductor elastic medium to photothermal excitation process with Thomson influence. SILICON **12**, 2789–2798 (2020)
28. K. Lotfy, A. El-Bary, W. Hassan, A. Alharbi, M. Almatrafi, Electromagnetic and Thomson effects during photothermal transport process of a rotator semiconductor medium under hydrostatic initial stress. Results Phys. **16**, 102983 (2020)
29. M. Othman, K. Lotfy, On the plane waves of generalized thermo- microstretch elastic half-space under three theories. Int. Commun. Heat Mass Transfer **37**(2), 192–200 (2010)
30. M.I.A. Othman, S. Abo-dahab, K. Lotfy, Gravitational effect and initial stress on generalized magneto-thermo-microstretch elastic solid for different theories. Appl. Math. Comput. **230**, 597–608 (2014)
31. K. Lotfy, Two temperature generalized magneto-thermoelastic interactions in an elastic medium under three theories. Appl. Math. Comput. **227**, 871–888 (2014)
32. S. Mondal, A. Sur, Photo-thermo-elastic wave propagation in an orthotropic semiconductor with a spherical cavity and memory responses. Waves Random Complex Medium. (2020). https://doi.org/10.1080/17455030.2019.1705426
33. M. Ezzat, Hyperbolic thermal-plasma wave propagation in semiconductor of organic material, Waves Random Complex Medium (2020). https://doi.org/10.1080/17455030.2020.1772524
34. A. Hobiny, I. Abbas, Analytical solutions of photo-thermo-elastic waves in a non-homogenous semiconducting material. Results Phys. **10**, 385–390 (2018)
35. C. Itu, A. Öchsner, S. Vlase, M. Marin, Improved rigidity of composite circular plates through radial ribs. Proc. Inst. Mech. Eng. Part L—J. Mater.-Des. Appl. **233**(8), 1585–1593 (2019)
36. H. Youssef, A. El-Bary, Theory of hyperbolic two-temperature generalized thermoelasticity. Mater. Phys. Mech. **40**, 158–171 (2018)
37. A. Mandelis, M. Nestoros, C. Christofides, Thermoelectronic-wave coupling in laser photothermal theory of semiconductors at elevated temperatures. Opt. Eng. **36**, 459–468 (1997)
38. D.M. Todorović, Plasma, thermal, and elastic waves in semiconductors. Rev. Sci. Instrum. **74**, 582–585 (2003)
39. A.N. Vasil'ev, V.B. Sandomirskii, Photoacoustic effects in finite semiconductors. Sov. Phys. Semicond. **18**, 1095–1099 (1984)
40. K. Lotfy, A.A. El-Bary, R.S. Tantawi, Effects of variable thermal conductivity of a small semiconductor cavity through the fractional order heat-magneto-photothermal theory. Eur. Phys. J. Plus **134**, 280 (2019)
41. C. Christofides, A. Othonos, E. Loizidou, Influence of temperature and modulation frequency on the thermal activation coupling term in laser photothermal theory. J. Appl. Phys. **92**, 1280–1285 (2002)
42. Y.Q. Song, J.T. Bai, Z.Y. Ren, Study on the reflection of photothermal waves in a semiconducting medium under generalized thermoelastic theory. Acta. Mech. **223**, 1545–1557 (2012)
43. A. Hobiny, I.A. Abbas, A study on photothermal waves in an unboundedsemiconductor medium with cylindrical cavity. Mech. Time-Depend Mater. **6**, 1–12 (2016)
44. K. Lotfy, The elastic wave motions for a photothermal medium of a dual-phase-lag model with an internal heat source and gravitational field. Can. J. Phys. **94**, 400–409 (2016)