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Numerical simulation of mixed convection in a lid-driven trapezoidal cavity with flexible bottom wall and filled with a hybrid nanofluid

Bader Alshuraiaan[1,a], **Ioan Pop**[2]

[1] Mechanical Engineering Department, College of Engineering and Petroleum, Kuwait University, 13060 Safat, Kuwait
[2] Department of Mathematics, Babeş-Bolyai University, 400084 Cluj-Napoca, Romania

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Abstract Mixed convection in a lid-driven trapezoidal cavity with flexible bottom wall and filled with a hybrid nanofluid is analyzed numerically in this study. Hybrid nanofluid that is a combination of Al_2O_3-Cu/Water is employed in this investigation. Finite element analysis is utilized in this study using the fluid–structure-interface Multiphysics of COMSOL. The results presented in this investigation showed that both Reynolds number and volume fraction of nanoparticles significantly affect flow and heat transfer characteristics inside the cavity. The results revealed that FSI model has more profound effect on the heat transfer compared with rigid wall model. Further, the present results showed that FSI was more feasible to enhance heat transfer compared with the addition of nanoparticles. This study confirms the promising applications of FSI model in enhancing heat transfer characteristics.

List of symbols

c_p	Specific heat
$\ddot{\boldsymbol{d}}_s$	Acceleration of the solid domain
E*	Elasticity
$\mathbf{f}_s^B$	Solid body force
g	Gravity
Gr	Grashof number
k	Thermal conductivity
p	Pressure
Pr	Prandtl number
Re	Reynolds number
$\mathbf{u}$	Velocity vector
u	X-component velocity
U	Non-dimensional velocity in X-direction
T	Temperature

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[a] e-mail: alshuraiaan@yahoo.com (corresponding author)

v	y-velocity component
V	Non-dimensional velocity in Y-direction
$\mathbf{w}$	Mesh velocity
x,y	Coordinates

Greek Letters

ρ	Density
β	Thermal expansion coefficient
$\boldsymbol{\phi}$	Volume fraction
μ	Viscosity

Subscripts

C	Cold
H	Hot
hnf	Hybrid nanofluid
s	Solid
1	Al_2O_3
2	Cu

1 Introduction

Enhancing the performance of thermal systems was the main objective of many researchers over many decades since it has a direct impact on carbon footprints. In recent years, many researchers have started to consider using nanofluids in thermal systems in order to enhance the heat transfer processes because of their higher thermal properties. More recently, the use of hybrid nanofluids emerged as a better alternative since they provide much more augmentation in the process of heat transfer in many applications such as electronic cooling, heat exchangers, solar systems, refrigeration systems, nuclear reactors etc. Many studies presented extensive reviews on nanofluids with a focus on characteristics, thermophysical properties, stability of the suspension, applications and many other aspects. A detailed review on nanofluid applications, thermophysical properties, common geometries and boundary conditions reported in the literature is presented by Khanafer and Vafai [1] where they highlighted the great potential of using nanofluids in porous media. Ghadimi et al. [2] presented general stabilization methods and suggested different measurement techniques during preparation of nanofluids. A comprehensive review on estimating the thermal conductivity of nanofluids using intelligence methods is presented by Ramezanizadeh et al.[3] where they suggested that this important property depends on many factors such as the temperature, synthesis method, dimension and concentration of nanoparticles. Sundar et al. [4] presented a comprehensive review on viscosity of nanofluids including empirical and theoretical correlations and the effects of temperature and structure of nanoparticles on this significant property. Preparation and thermophysical properties including density and heat capacity are presented in review paper by Asadi et al. [5], a wide range of useful correlations of such properties are summarized in this study. Salman et al. [6] presented a thorough review on heat transfer characteristics in microtubes and microchannels using nanofluids where they summarized the influence of many parameters on both experimental and numerical studies in the literature. Many other reviews [7–10] considered different aspects of nanofluids characteristics for different flow regimes and configurations.

More recently, a new type of nanofluids with enhanced thermophysical properties is called hybrid nanofluids was the focus of many studies. Hybrid nanofluids are made by decomposing two or more nanoparticles into conventional fluids. A state-of-the-art review presented by Huminic and Huminic [11] states clearly, based on experimental studies, that using hybrid nanofluids can achieve heat transfer enhancement in thermal processes for various types of heat exchangers. Their study recommended the need of more studies in order to find more precise correlations for thermophysical properties of hybrid nanofluids. Babu et al. [12] presented a general review on hybrid nanofluids where they summarized many relevant characteristics on the implantation of nanofluids in thermal applications such as synthesis methods, preparation, stability and thermophysical estimation of hybrid nanofluids. Another review on heat transfer and friction factor is presented by Sundar et al. [13], where a set of Nusselt number and friction factor correlations are proposed for different nanofluids combinations. Sajid and Ali [14] presented a critical review on thermal conductivity of hybrid nanofluids and summarized correlations of thermal conductivity developed by previous studies. Hybrid nanofluids preparation, stability, thermal characteristics, and behavior are discussed in analogous studies [15–17].

Studies related to mixed convection in nanofluids considered many different configurations and boundary conditions as reported in Esfe et al. [18]. Mixed convection for a nanofluid of Cu-water in a cavity with a hot thick-wavy sidewall and opposite moving wall while the top and bottom walls are kept adiabatic is investigated numerically by Pal et al. [19]. The same authors [20] considered a similar problem with different orientation where the wavy wall is placed in the bottom side of the cavity, where it is evident by comparing results of the two studies that higher rates of heat transfer are for the case of heating from the side. Buoyancy-driven convection in a square enclosure filled with a nanofluid with periodic oscillating temperature on a vertical wall with insulated top–bottom walls and cold vertical wall is numerically investigated by Han et al. [21]. Mixed convection of nonhomogeneous nanofluids in a double lid-driven enclosure with a top insulated wavy surface and heated-cooled side walls with bottom adiabatic wall is numerically investigated by Alsabery et al. [22]. Their analysis shows that the presence of a wavy wall degrades the heat transfer rate in the cavity. Cho et al. [23] investigated the mixed convection of nanofluids in a cavity with heated-cooled wavy side walls and adiabatic top–bottom walls. Their results showed that a wavy cavity to the inside gives more heat transfer values. Mixed convection of nanofluids in a cavity with a wavy heated bottom with the presence of an inner solid block is investigated by Azizul et al. [24] where they found that the rate of heat transfer is higher as the number of waves increases. The problems of natural and mixed convection using nanofluids in square cavities with different boundary conditions are considered by many other studies [25–28]. Many studies [29–34] considered a trapezoidal shape cavity to investigate the problem of natural or mixed convection using nanofluids where it is evident that introducing an inclined wall can enhance the heat transfer process.

Using hybrid nanofluids has promising results in natural and mixed convection heat transfer as shown by many studies [35–37]. For example, Takabi and Salehi [38] investigated the problem of natural convection using hybrid nanofluids in a cavity with wavy side walls where it is found that the presence of hybrid nanofluids along with wavy sides gives relatively a better performance when compared to the heat transfer in a square cavity with pure fluids. Introducing wavy, inclined, or trapezoidal shapes cavities are shown to achieve better heat transfer when using hybrid nanofluids [39–41]. Another emerging option to enhance the process of heat transfer in cavities is the application of flexible walls to allow a smother path for the flow as shown by Al-Amiri and Khanafer [42]. Khanafer and Vafai [43] considered the fluid–structure-interaction "FSI" for application in porous media, their results showed that the presence of a flexible wall in a domain enhances the process of heat transfer. Alsabery et al. [44] considered the FSI problem of natural convection in a square cavity with flexible right-side wall, their results showed that the presence of flexible wall result in higher rates of heat transfer. Mixed convection in a square cavity heated from below with a flexible adiabatic vertical wall and cold moving top wall is investigated numerically by Selimefendigil et al. [45] where they show that the presence of a flexible wall significantly enhances the heat transfer. Selimefendigil and Oztop [46] studied the presence of a flexible segment in a triangular cavity filled with nanofluids, their results reveal that the presence of the flexible segment enhances the process of heat transfer in the triangular cavity. Raisi and Arvin [47] investigated numerically transient natural convection in an air-filled square cavity based on the effects of fluid–structure interaction (FSI). The Prandtl number of air was assumed to be 0.71. A thin deformable baffle was horizontally placed in the center of the cavity, and the top wall of the cavity was also flexible. They showed that increasing the baffle length had mixed effects on the thermal performance of the system. Khanafer [48] conducted a numerical investigation of steady laminar mixed convection flow and heat transfer in a lid-driven cavity with a flexible heated bottom surface. Moreover, the heated bottom wall was characterized by rectangular and sinusoidal wavy profiles for a rigid wall analysis. Flexible bottom wall case was found to exhibit substantial heat transfer improvement (61.4%) compared with a flat bottom wall case at Grashof number of 104 and $\mathrm{Re} < 400$.

A careful review to the above-mentioned literature reveals that the combination of mixed convection, hybrid nanofluids, inclined cavities and flexible walls gives much better performance than the combination of natural convection, nanofluids, square cavities with rigid walls. The applications of elastic wall combined with nanofluids never studied before in the literature inside a trapezoidal enclosure. Actual enclosures occurring in practice often have the shapes differing from rectangular ones. The present geometry with flexible walls can be utilized in solar energy collector to enhance its performance. Therefore, the main goal of this study is to numerically study the problem of mixed convection in a trapezoidal cavity filled with hybrid nanofluids with the presence of a flexible wall using various pertinent parameters such as Reynolds number and volume fraction of hybrid nanofluid. Many studies considered the effect of inclination of the side walls and this part will not be considered in the present study.

2 Mathematical formulation

Consider laminar, two-dimensional, mixed convection flow and heat transfer in a lid-driven trapezoidal cavity as shown in Fig. 1. The inclined walls are kept insulated while the upper moving wall is kept at a low temperature of T_C. The bottom wall is assumed to be flexible with a high temperature T_H. The upper wall is moving to the right to boost the fluid motion inside the cavity as the fluid is expected to have a clockwise rotation. The working fluid is hybrid nanofluid that consists of Al_2O_3-Cu/Water. Alumina (Al_2O_3) is the most common nanoparticle used by many researchers in their experimental works because it has many beneficial properties such as chemical inertness and a great deal of stability. While Al_2O_3 exhibits lower thermal conductivity with respect to the metallic nanoparticles, it can be expected that the addition of metal nanoparticles (such as Cu) into a nanofluid com-

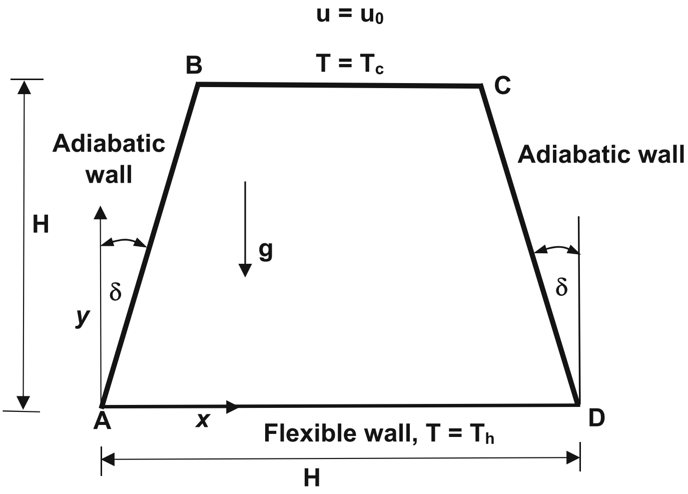

Fig. 1 Schematic diagram of the trapezoidal cavity, coordinates and boundary conditions under consideration

Table 1 Water and hybrid nanofluid (Al_2O_3–Cu) thermophysical properties [37]

Fluid	Water	Al_2O_3	Cu
Property			
Density	997.1	3970	8933
Viscosity	8.9×10^{-4}	–	–
Heat capacity	4179	765	385
Thermal expansion	21×10^{-5}	0.85×10^{-5}	1.67×10^{-5}
Thermal conductivity	0.613	40	401

Table 2 Comparison of the average Nusselt number at various Reynolds and Grashof numbers

References	$Gr = 10^4$		
	Re = 100	Re = 400	Re = 1000
Iwatsu et al. [50]	1.34	3.62	6.29
Sharif [51]	–	3.82	6.50
Present	1.38	3.77	6.56
References	$Gr = 10^6$		
	Re = 100	Re = 400	Re = 1000
Iwatsu et al. [50]	1.02	1.22	1.77
Sharif [51]	–	1.17	1.81
Present	1.02	1.17	1.77

posed based on Al_2O_3 nanoparticles can enhance the thermophysical properties of this mixture.

The governing equations utilized in this investigation after taking into consideration the above assumptions can be written as [49]:

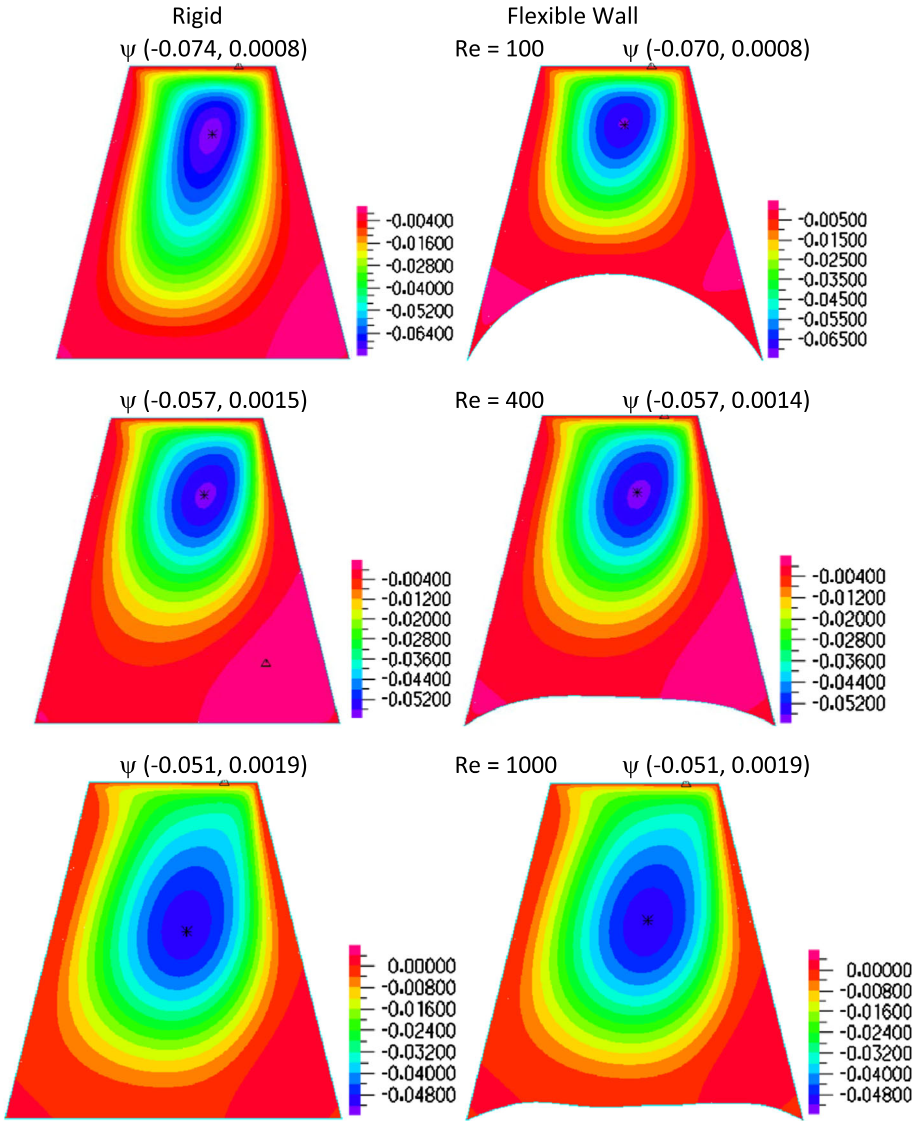

Fig. 2 Streamlines comparison between rigid wall model and flexible bottom wall model in the absence of nanoparticles for various values of Reynolds number (Gr = 10^4, Pr = 0.71, E* = 5000)

2.1 Continuity

$$\nabla.\mathbf{u} = 0 \tag{1}$$

2.2 Momentum

$$\rho_{\mathrm{hnf}}(\mathbf{u} - \boldsymbol{w}).\nabla\mathbf{u} = -\nabla p + \mu_{\mathrm{hnf}}\nabla^2\mathbf{u} + \rho_{\mathrm{hnf}} g_y \beta_{\mathrm{hnf}}(T - T_c) \tag{2}$$

2.3 Energy

$$\left(\rho c_p\right)_{\mathrm{hnf}}(\mathbf{u} - \boldsymbol{w}).\nabla T = k_{\mathrm{hnf}}\nabla^2 T \tag{3}$$

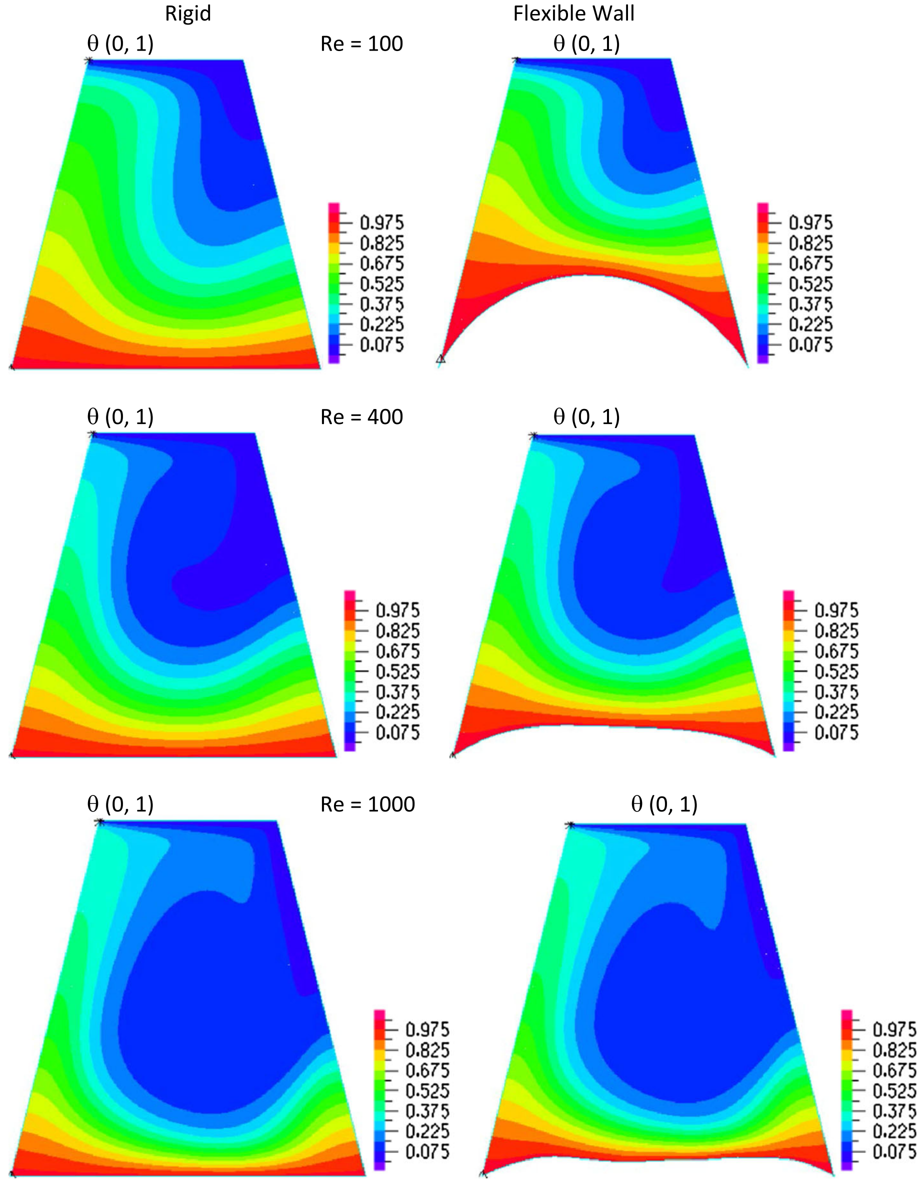

Fig. 3 Isotherms comparison of between rigid wall model and flexible bottom wall model in the absence of nanoparticles for various values of Reynolds number (Gr $= 10^4$, Pr $= 0.71$, E* $= 5000$)

The flexible solid wall is governed by the following equation [42, 43]:

$$\rho_s \ddot{\boldsymbol{d}}_s - \nabla \cdot \boldsymbol{\sigma}_s^{\text{total}} = \mathbf{f}_s^B \tag{4}$$

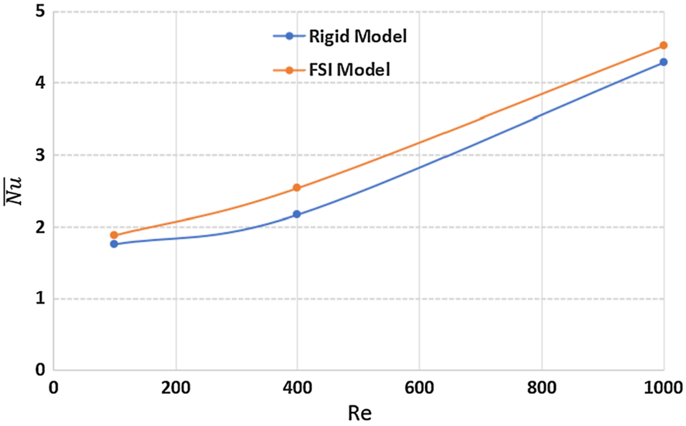

Fig. 4 Comparison of the average Nusselt number between the rigid walls model and flexible bottom wall model in the absence of nanoparticles for various values of Reynolds number ($\mathrm{Gr} = 10^4$, $\mathrm{Pr} = 0.71$)

where $\ddot{\boldsymbol{d}}_s$ indicates the rate of change of the solid domain velocity w, $\mathbf{f}_s^B$ is the solid body force, ρ_s is the solid density, $\mathbf{u}$ is the velocity vector, and $\boldsymbol{\sigma}_s$ is the stress. The boundary conditions can be written as:

$$\text{Inclined wall “AB”}: \ u = 0, \quad \frac{\partial T}{\partial y} = 0 \tag{5}$$

$$\text{Inclined wall “CD”}: \ u = 0, \quad \frac{\partial T}{\partial y} = 0 \tag{6}$$

$$\text{Bottom wall “DA”}: \ u = 0, T = T_h \tag{7}$$

$$\text{Top wall “BC”}: \ u = u_0, \ T = T_c \tag{8}$$

Thermophysical properties of the hybrid nanofluid as reported by Aly and Pop [50] are:

$$\varphi = \varphi_1 + \varphi_2 \tag{9}$$

$$\frac{\mu_{\text{hnf}}}{\mu_f} = (1 - \varphi_1 - \varphi_2)^{-2.5} \tag{10}$$

$$\frac{\rho_{\text{hnf}}}{\rho_f} = 1 - \varphi_1 - \varphi_2 + \frac{\varphi_1\rho_1 + \varphi_2\rho_2}{\rho_f} \tag{11}$$

$$\frac{(\rho\beta)_{\text{hnf}}}{(\rho\beta)_f} = 1 - \varphi_1 - \varphi_2 + \frac{\varphi_1(\rho\beta)_1 + \varphi_2(\rho\beta)_2}{(\rho\beta)_f} \tag{12}$$

$$\frac{\left(\rho C_p\right)_{\text{hnf}}}{\left(\rho C_p\right)_f} = 1 - \varphi_1 - \varphi_2 + \frac{\varphi_1\left(\rho C_p\right)_1 + \varphi_2\left(\rho C_p\right)_2}{\left(\rho C_p\right)_f} \tag{13}$$

$$\frac{k_{\text{hnf}}}{k_f} = \frac{\frac{\varphi_1 k_1 + \varphi_2 k_2}{\varphi_1 + \varphi_2} + 2k_f + 2(\varphi_1 k_1 + \varphi_2 k_2) - 2(\varphi_1 + \varphi_2)k_f}{\frac{\varphi_1 k_1 + \varphi_2 k_2}{\varphi_1 + \varphi_2} + 2k_f - (\varphi_1 k_1 + \varphi_2 k_2) + (\varphi_1 + \varphi_2)k_f} \tag{14}$$

$$\alpha_{\text{hnf}} = \frac{k_{\text{hnf}}}{\left(\rho C_p\right)_{\text{hnf}}} \tag{15}$$

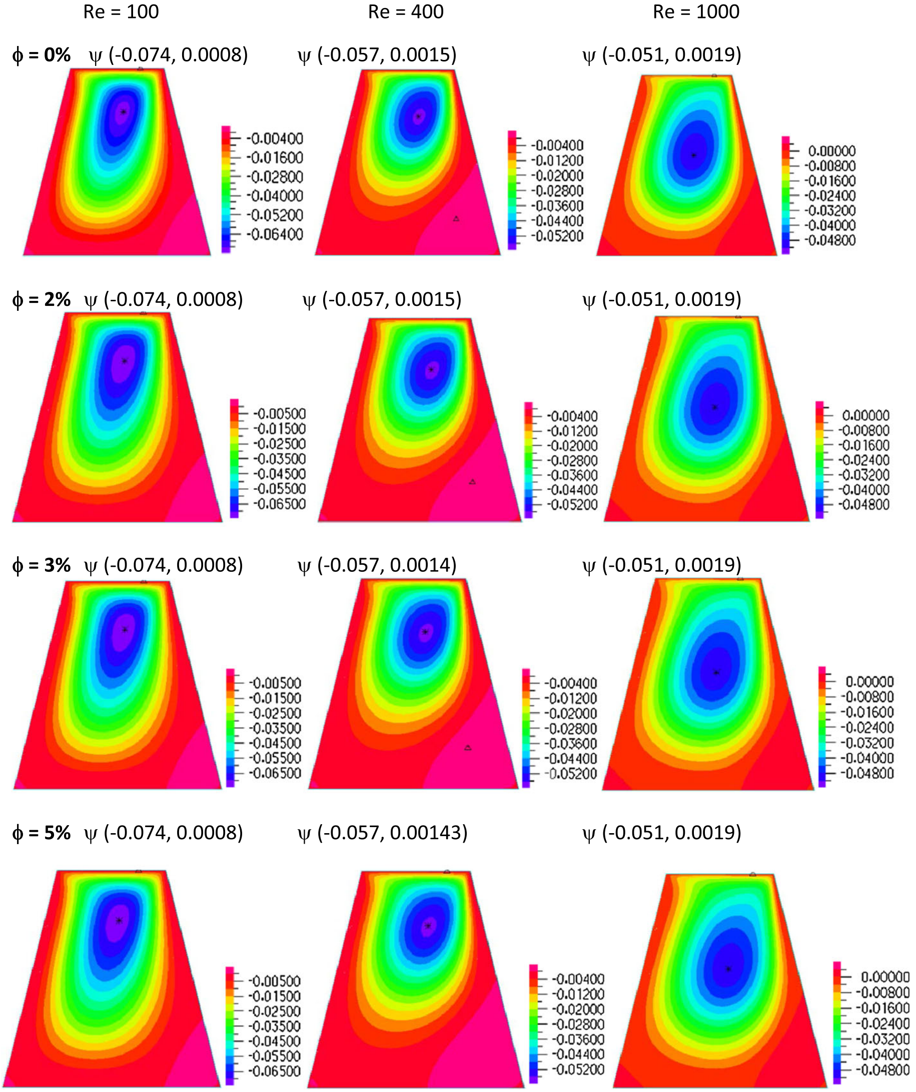

Fig. 5 Effect of varying the volume fraction of the hybrid nanofluid on the streamlines for various values of Reynolds number (Gr $= 10^4$, Pr $= 0.71$)

where $\boldsymbol{\phi}_1$ and $\boldsymbol{\phi}_2$ are the volume fractions of the Al_2O_3 and Cu nanoparticles, respectively, and ϕ is the total volume fraction of both nanoparticles. Equations (1–3) can be converted to nondimensional form by using the following dimensionless parameters:

$$X = \frac{x}{H},\ Y = \frac{y}{H},\ U = \frac{u}{u_0},\ V = \frac{v}{u_0},\ W = \frac{w}{u_0},\ P = \frac{p}{\rho_f u_0^2},$$

$$\theta = \frac{T - T_c}{T_h - T_c},\ \mathrm{Pr} = \frac{\mu_f C_p}{k_f},\ \mathrm{Ri} = \frac{g\beta_f H \Delta T}{u_0^2},\ \mathrm{Re} = \frac{\rho_f u_0 H}{\mu_f}, \tag{16}$$

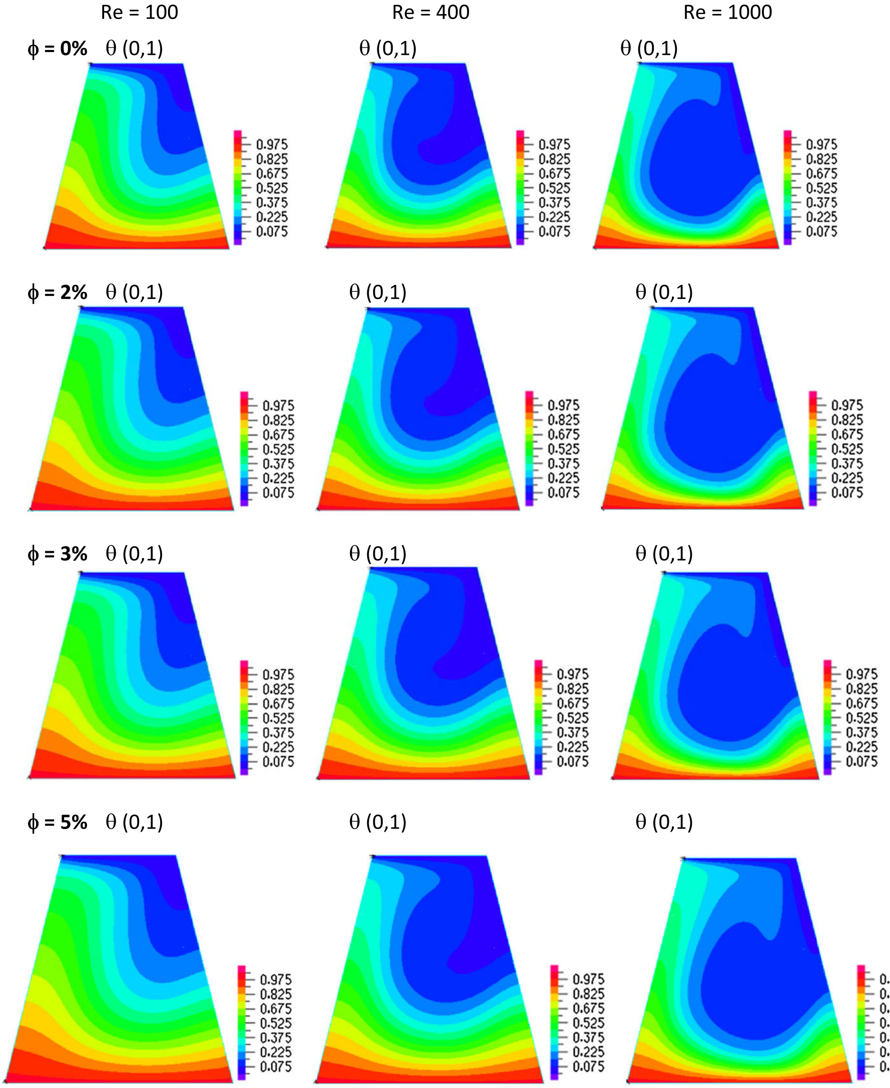

Fig. 6 Effect of varying the volume fraction of the hybrid nanofluid on the isotherms for various values of Reynolds number ($\mathrm{Gr} = 10^4$, $\mathrm{Pr} = 0.71$)

Utilizing the above dimensionless parameters, the non-dimensional form of the governing equations can be expressed as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{17}$$

$$(U - W)\frac{\partial U}{\partial X} + (V - W)\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\rho_f}{\rho_{\mathrm{hnf}}}\frac{\mu_{\mathrm{hnf}}}{\mu_f}\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{18}$$

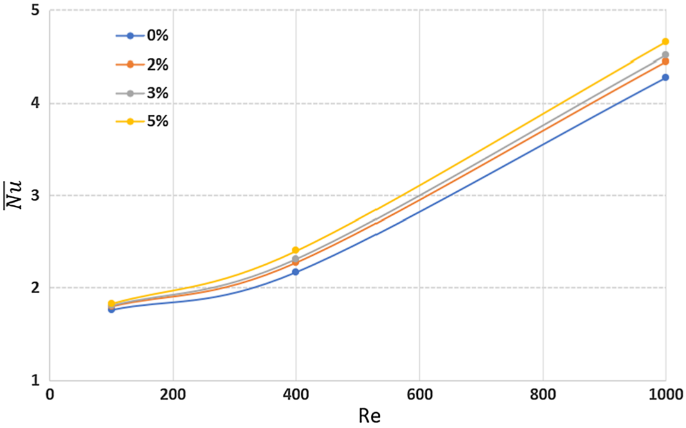

Fig. 7 Effect of varying the volume fraction of hybrid nanofluid on the average Nusselt number for various value Reynolds number (Gr = 10^4, Pr = 0.71)

$$(U-W)\frac{\partial V}{\partial X}+(V-W)\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\rho_f}{\rho_{\mathrm{hnf}}}\frac{\mu_{\mathrm{hnf}}}{\mu_f}\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{\mathrm{hnf}}}{(\rho\beta)_f}\mathrm{Ri}\,\theta \tag{19}$$

$$(U-W)\frac{\partial \theta}{\partial X}+(V-W)\frac{\partial \theta}{\partial Y}=\frac{\alpha_{\mathrm{hnf}}}{\alpha_f}\frac{1}{\mathrm{RePr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{20}$$

The numerical values of the thermophysical properties for water, Al_2O_3, and Cu are given in Table 1 [37]. The interface boundary conditions must satisfy the displacement and traction equations [42, 43]. The average Nusselt number over the top cold wall can be written as:

$$\overline{Nu}=-\frac{k_{\mathrm{hnf}}}{k_f}\int_B^C\frac{\partial\theta}{\partial Y}dX \tag{21}$$

3 Computational methodology

The partial nonlinear governing Eqs. (17–20) used in this investigation are solved using the Galerkin-weighted residual approach of the finite element method. The package 'COMSOL 5.4′ with the feature of fluid–structure interaction Multiphysics that accounts for the moving mesh because of the presence of the flexible bottom wall is utilized in this study. A free unstructured mesh of triangular elements with fine mesh near the center of the cavity and extremely fine mesh next to the boundaries is selected to perform the computations. A mesh size of 251 by 251 was found satisfactory with a convergence criterion of error less than 10^{-6} for velocity components and temperature.

4 Code validation

Present results are validated against two previous studies of Iwatsu et al. [51] and Sherif [52] and as shown in Table 2 in terms of the average Nusselt number for various values

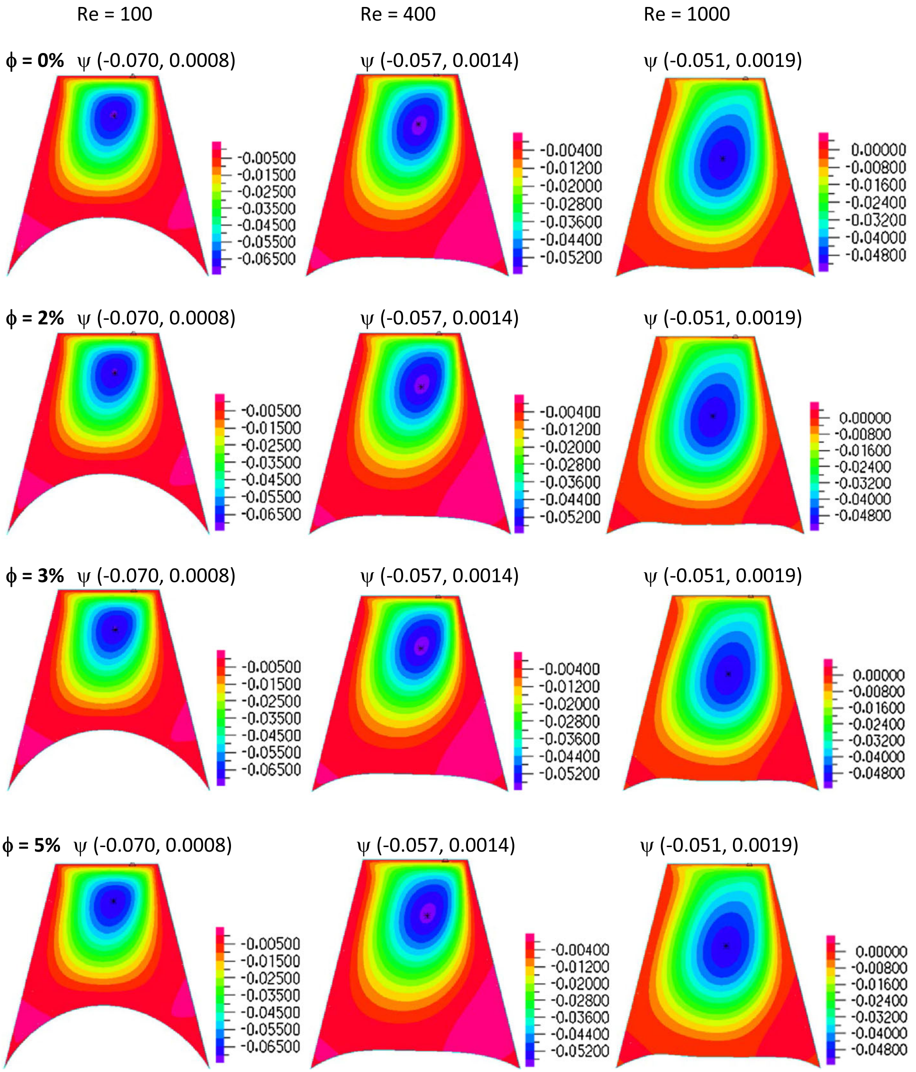

Fig. 8 Effect of varying the volume fraction of the hybrid nanofluid on the streamlines for various values of Reynolds number assuming flexible bottom wall ($Gr = 10^4$, $Pr = 0.71$, $E^* = 5000$)

of Reynolds and Grashof numbers. Excellent agreement found between current results and these two studies.

5 Discussion of results

A numerical study of mixed convective flow and heat transfer was conducted in a trapezoidal cavity filled with a hybrid nanofluid and assumed a flexible bottom wall. To study the feasibility of using a flexible wall model compared with a rigid bottom wall model, the effect of introducing a flexible wall on the fluid flow and heat transfer inside the trapezoidal cavity

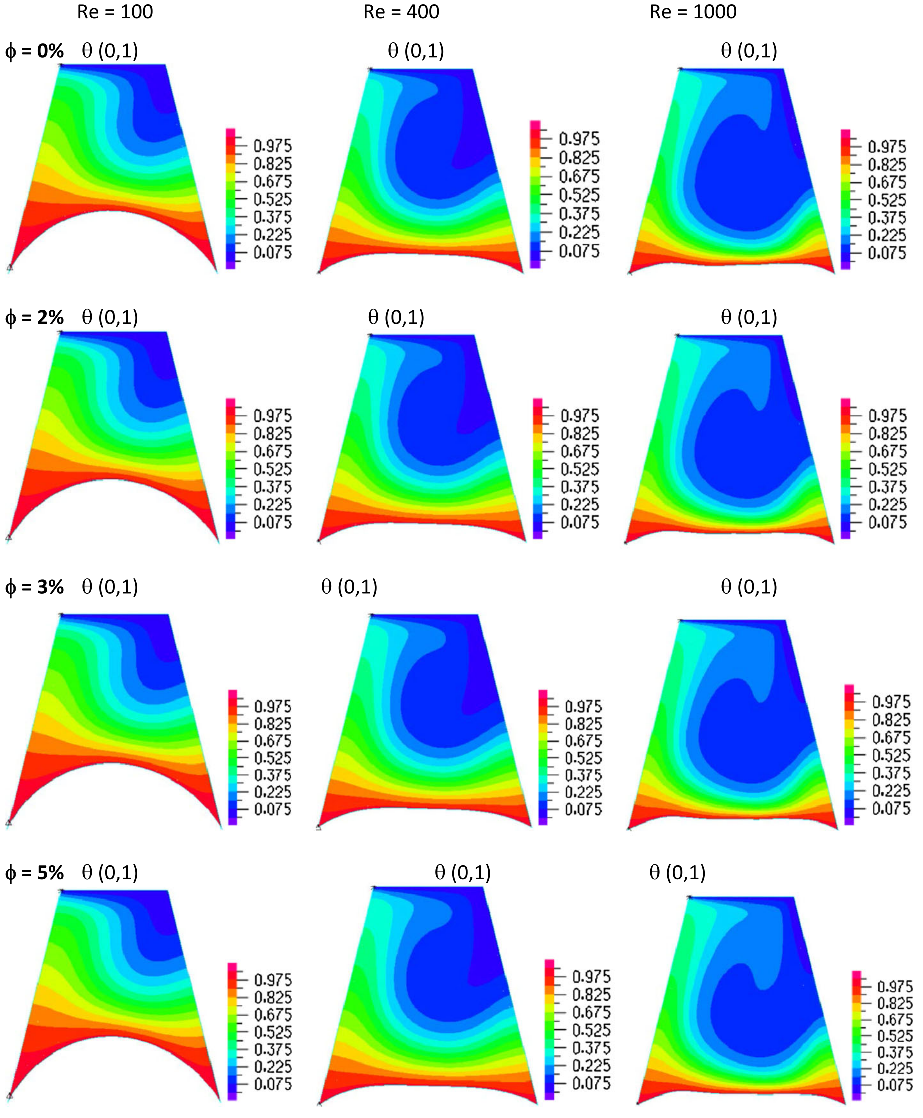

Fig. 9 Effect of varying the volume fraction of the hybrid nanofluid on the isotherms for various values of Reynolds number assuming flexible bottom wall (Gr = 10^4, Pr = 0.71, E* = 5000)

without the presence of nanoparticles in the base fluid against the model with fixed walls is demonstrated in Figs. 2, 3 and 4. Figure 2 shows that the streamlines are highly affected by the presence of the flexible bottom wall at low values of Reynolds and this can be attributed to a smaller value of the slip velocity on the top wall of the cavity. For a Reynolds number of 100, the bottom wall is bulged significantly inside the cavity compared with other values of Reynolds numbers. This can be attributed to a lower pressure applied on the surface. As the velocity of the top wall increases, the center of circulation of the flow moves down toward the bottom wall which pushes the flexible wall outward. This causes the lower bottom surface to

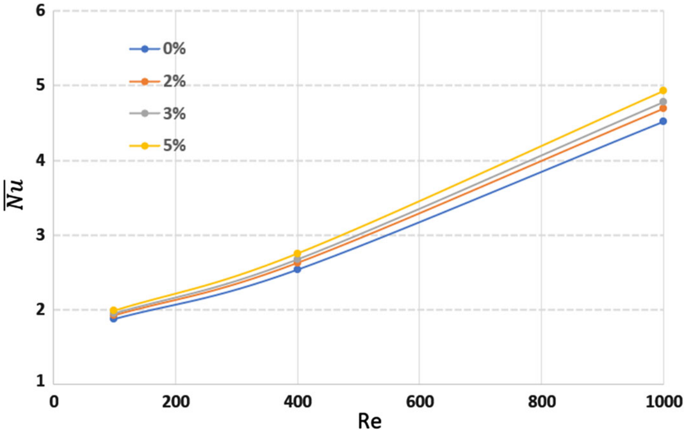

Fig. 10 Effect of varying the total volume fraction of hybrid nanofluid on the average Nusselt number for various values of Reynolds number assuming flexible bottom wall ($Gr = 10^4$, $Pr = 0.71$, $E^* = 5000$)

be almost horizontal. The same applies to isotherms as depicted from Fig. 3 where a thinner thermal boundary layer is present for higher values of Reynolds number for fixed values of Grashof number and modulus of elasticity of the flexible wall. Figure 3 illustrates that as Reynolds number increases, the cold fluid penetrates further toward the center of the cavity until it fills most of the cavity for a Reynolds number of 1000. This indicates that forced convection is dominant compared with natural convection heat transfer model. Figure 4 clearly shows that the presence of a flexible wall results in higher rates of heat transfer inside the cavity. This figure clearly shows that an increase in Reynolds number results in an increase in the average Nusselt number with higher heat transfer rates with the presence of a flexible wall compared with a rigid wall model. Therefore, it is highly recommended for an industrial application that requires better thermal management is to utilize flexible walls.

Figures 5, 6 and 7 show the effect of introducing nanoparticles to the fluid inside a cavity assuming rigid walls. In these figures and following figures, three values of the total volume fraction are used, namely 2%, 3% and 5%. A fixed volume fraction of 1% for Al_2O_3 is used while different volume fractions for Cu of 1%, 2% and 4% are used. It is evident that increasing the total volume fraction does not have a pronounced effect on the streamlines and isotherms for fixed values of Reynolds number as illustrated in Figs. 5 and 6. However, increasing this fraction have a noticeable effect on the average Nusselt number as shown in Fig. 7 because of the resultant higher value of the effective thermal conductivity of the hybrid nanofluid. This figure shows similar behavior to those mentioned earlier in Fig. 4 where increasing the Reynolds number for a fixed value of Grashof number results in higher rates of heat transfer in terms of the average Nusselt number. This clearly shows the importance of using nanoparticles in enhancing the heat transfer characteristics within a thermal system.

The combined effect of introducing both the flexible wall and the hybrid nanofluid on the fluid flow and heat transfer behavior in the cavity is shown in Figs. 8, 9 and 10. Similar behavior to those in Figs. 2, 3, 4, 5, 6 and 7 is found with relatively higher temperature gradients next to the hot and cold walls which results in significantly higher Nusselt number values as a result of the presence of both effects. It is evident that the presence of the flexible wall has more effect than introducing nanoparticles to the fluid on the fluid flow behavior as depicted from Fig. 8. Similar consequences are found on the streamlines as shown in Fig. 9. Similar results for the effect of Reynolds number on the average value of the Nusselt number

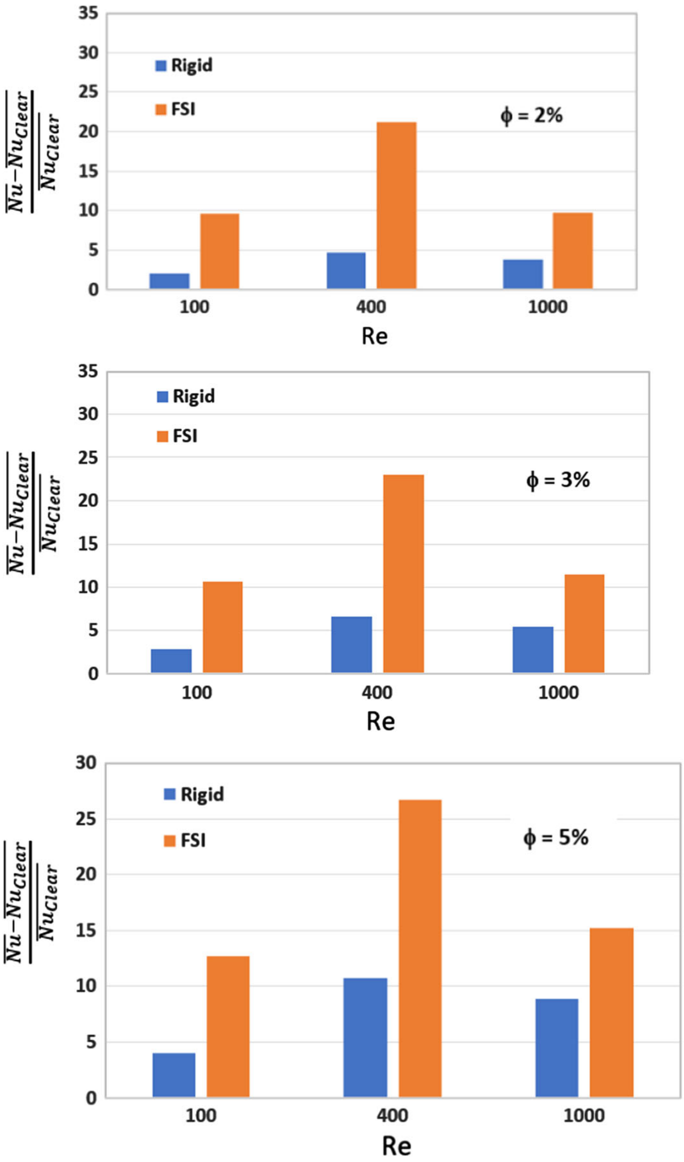

Fig. 11 Comparison of the relative average Nusselt number between rigid walls and FSI models for various values of Reynolds number and total volume fraction of nanoparticles (Gr = 10^4, Pr = 0.71, E* = 5000)

are found in Fig. 10 with those discussed earlier in Fig. 4 and Fig. 7 with shifted values as a result of introducing both effects. Figure 11 shows the relative increase in the averaged Nusselt number of the combined effect compared to those of a clear fluid in a rigid walls cavity. This relative change is more pronounced at moderate values of Reynolds number since at small and large values of Reynolds number the percentage increase in the Nusselt number is smaller because of the domination of one effect over the other. However, it is evident that introducing both the flexible wall along with nanoparticles results in more enhancement to the heat transfer process within the trapezoidal cavity. It is very interesting to note from Fig. 11 that FSI contributes significantly to heat transfer enhancement compared with adding nanoparticles.

6 Conclusions

The effect of introducing a flexible wall along with hybrid nanofluid to a lid-driven trapezoidal cavity is numerically investigated in this study. The feature of fluid–structure-interaction in COMSOL is used in this investigation to analyze the presence of a flexible wall in the trapezoidal enclosure. Varying Reynolds number and the volume fraction of the hybrid nanofluid showed that the presence of a flexible wall besides using a denser hybrid nanofluid results in a significant increase in the heat transfer inside the cavity. However, the effect of introducing the flexible wall alone in a cavity with clear fluid has more pronounced effect on the enhancement of the heat transfer if compared to the presence of a hybrid nanofluid in a cavity with rigid walls. This study paves the road for the researchers in the area of thermal management to utilize elastic wall in the presence of nanoparticles.

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