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Analytical solitons with the Biswas-Milovic equation in the presence of spatio-temporal dispersion in non-Kerr law media

Muhammad Tahir[a] and Aziz Ullah Awan

Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore, Pakistan

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Abstract. In this paper, we concentrate on the optical soliton solutions of spatially temporal Biswas-Milovic equation with power law and dual-power law nonlinearities. This model accounts for several imperfections for the transmission of solitons through optical fibers. The first integral method is employed in order to construct singular and dark 1-soliton solutions from the governing Biswas-Milovic equation with spatio-temporal dispersion. Some parametric restrictions are also enumerated which guarantee the existence of these soliton solutions. Moreover, the obtained results are demonstrated by 3D and 2D plots.

1 Introduction

Nonlinear partial differential equations (PDEs) are often used as models to describe several major physical phenomena and assume a perceptible part in numerous areas of engineering. Recently, various analytical traveling wave solutions of nonlinear PDEs are obtained, which are important to reveal their rheological properties. In the last three decades, a variety of powerful and productive methods have been introduced and utilized to construct variety of exact traveling wave solutions of nonlinear PDEs [1–13].

The theory of optical solitons is potentially the most energetic and striking field of research in theoretical physics, telecommunication engineering and in nonlinear fiber optics [14–23].

In nonlinear sciences, the dynamics of soliton propagation is comprehensively addressed by nonlinear Schrödinger equation [24–34]. This equation works as a basic model to describe optical pulse transmission in nonlinear mediums when the pulse width is above 100 femto-seconds. However, as the intensity of the incident light increases, it produce pulses of shorter width and the effect of non-Kerr nonlinearity become vital. To accomodate the higher order nonlinear and dispersive effects, Biswas and Milovic proposed the generalized form of nonlinear Schrödinger equation that describes the dynamics of optical solitons more comprehensively [35–43]. The Biswas-Milovic equation along with spatio-temporal dispersion is given by [44]

$$i(v^m)_t + a(v^m)_{xt} + b(v^m)_{xx} + c\mathcal{F}(|v|^2)v^m = 0, \tag{1}$$

where t is the temporal variable, x denotes the non-dimensional distance along the fiber and $v(x,t)$ is designated for wave profile of the soliton. First term in eq. (1) denotes the evolution term, while the real constants a, b and c are the coefficients of spatio-temporal dispersion (STD), group velocity dispersion (GVD) and nonlinear term, respectively. The functional $\mathcal{F}$, in general, denotes the non-Kerr law nonlinearity.

In the present work, we analyze eq. (1) for two types of nonlinear mediums through power law and dual-power law nonlinearities. The first integral method is implemented which give various soliton solutions of eq. (1).

The rest of the paper is structured as follows. In sect. 2, key concepts of the first integral method are introduced. In sect. 3, exact soliton solutions of eq. (1) are formulated with two types of nonlinearity. Finally, the paper is concluded in sect. 4.

[a] e-mail: rajtahir39@gmail.com

2 First integral method

The first integral method was introduced by Feng [45] and has been applied successfully to construct various analytic solutions [46–51].

We summarize this method in the following steps:

Assume that, we have a nonlinear PDE of the form

$$G(v, v_t, v_x, v_{tt}, v_{xt}, v_{xx}, \cdots) = 0, \tag{2}$$

where G is a polynomial in $v(x,t)$.

By substituting $v(x,t) = U(\zeta)$, where $\zeta = x - \nu t$, eq. (2) is transmuted into an ordinary differential equation (ODE):

$$H(U, -\nu U', U', \nu^2 U'', -\nu U'', U'', \cdots) = 0, \tag{3}$$

where $U' = \frac{\mathrm{d}U}{\mathrm{d}\zeta}$, $U'' = \frac{\mathrm{d}^2U}{\mathrm{d}\zeta^2}$ and so on.

Next, we assume that

$$S(\zeta) = U(\zeta), \qquad T(\zeta) = U'(\zeta), \tag{4}$$

which leads to a system of nonlinear ODEs

$$\begin{cases} S'(\zeta) = T(\zeta), \\ T'(\zeta) = J\left(S(\zeta), T(\zeta)\right). \end{cases} \tag{5}$$

By applying the division theorem of two variables in the complex domain C, which is followed by Hilbert-Nullstellensatz theorem [52], we can find the first integral to eq. (5) which condenses eq. (3) into a first order integrable ODE. By resolving this equation, an exact solution to (2) can be determined.

3 Soliton solutions

To tackle eq. (1) with first integral method, we apply the following transformation:

$$v(x,t) = U(\zeta)e^{i\psi(x,t)}, \tag{6}$$

where $U(\zeta)$ denotes the wave profile and

$$\zeta = x - \nu t, \tag{7}$$

where ν denotes soliton velocity and $\psi(x,t)$ represents the phase component

$$\psi(x,t) = -\kappa x + \omega t + \epsilon, \tag{8}$$

where κ and ω represent the frequency and wave number of the soliton, respectively, while ϵ is the phase constant.

Inserting eqs. (6)–(8) into eq. (1) and breaking into imaginary and real parts, we get

$$\nu = \frac{m(2b\kappa - a\omega)}{am\kappa - 1}, \quad am\kappa - 1 \neq 0 \tag{9}$$

and

$$(b - a\nu)(U^m)'' - m(\omega + bm\kappa^2 - am\omega\kappa)U^m + cF(|U|^2)U^m = 0, \tag{10}$$

where $' = \frac{\mathrm{d}}{\mathrm{d}\zeta}$.

Equation (9) provides the speed of soliton and eq. (10) can be evaluated to construct the wave profile if the functional is identified. In the succeeding sections, eq. (1) is analyzed for two kinds of nonlinear mediums by first integral approach.

3.1 Power law nonlinearity

The power law nonlinearity arises in theory of turbulence, plasma physics and in nonlinear fiber optics. This law of nonlinearity is also presented in several process such as semiconductors and higher-order photons. For the power law nonlinearity, eq. (1) becomes

$$i(v^m)_t + a(v^m)_{xt} + b(v^m)_{xx} + c(|v|^{2n})v^m = 0, \tag{11}$$

and eq. (10) reduces to

$$(b - a\nu)(U^m)'' - m(\omega + bm\kappa^2 - am\omega\kappa)U^m + cU^{2n+m} = 0. \tag{12}$$

To attain the closed form solutions, we substitute

$$U = V^{1/2n}. \tag{13}$$

This transformation reduces eq. (12) to an ODE

$$(b - a\nu)\{m(m-2n)(V')^2 + 2mnVV''\} - 4mn^2(\omega + bm\kappa^2 - am\omega\kappa)V^2 + 4n^2cV^3 = 0. \tag{14}$$

3.1.1 First integral approach

In this section, we analyze eq. (14) by applying the first integral method. By substituting $S(\zeta) = V(\zeta)$ and $T(\zeta) = V'(\zeta)$ in eq. (14), we have the following two-dimensional autonomous system

$$\begin{cases} S'(\zeta) = T(\zeta), \\ T'(\zeta) = \left(1 - \dfrac{m}{2n}\right)\dfrac{T^2}{S} + \dfrac{2n}{S(b-a\nu)}\left[\left(\omega + bm\kappa^2 - am\omega\kappa\right)S^2 - \left(\dfrac{c}{m}\right)S^3\right]. \end{cases} \tag{15}$$

To avoid the singular line $S = 0$, we use the transformation

$$\mathrm{d}\zeta = S\,\mathrm{d}\eta. \tag{16}$$

Then, the system in (15) becomes

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}\eta} = ST, \\ \dfrac{\mathrm{d}T}{\mathrm{d}\eta} = \left(1 - \dfrac{m}{2n}\right)T^2 + \dfrac{2n}{(b-a\nu)}\left[\left(\omega + bm\kappa^2 - am\omega\kappa\right)S^2 - \left(\dfrac{c}{m}\right)S^3\right]. \end{cases} \tag{17}$$

Assume that $S(\eta)$ and $T(\eta)$ are nontrivial solutions to system (17) and $Q(S,T) = \sum_{i=0}^{M}\alpha_i(S)T^i$ is an irreducible polynomial in C, such that

$$Q\left(S(\eta), T(\eta)\right) = \sum_{i=0}^{M}\alpha_i(S(\eta))T^i(\eta) = 0, \tag{18}$$

where $\alpha_i(S)$ $(i = 0, 1, \ldots, M)$ are polynomials of S and $\alpha_M(S) \neq 0$. Notice that $(\mathrm{d}Q/\mathrm{d}\eta)$ is a polynomial of S and T, and $Q(S(\eta), T(\eta)) = 0$ infers that $(\mathrm{d}Q/\mathrm{d}\eta)|_{(17)} = 0$.

By the division theorem, there exists a polynomial $g(S) + h(S)T$ in C, such that

$$\frac{\mathrm{d}Q}{\mathrm{d}\eta} = \frac{\mathrm{d}Q}{\mathrm{d}S}\frac{\mathrm{d}S}{\mathrm{d}\eta} + \frac{\mathrm{d}Q}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}\eta} = [g(S) + h(S)T]\sum_{i=0}^{M}\alpha_i(S)T^i. \tag{19}$$

Taking $M = 1$ in (18) and using eqs. (17) and (19), we get

$$\left(\sum_{i=0}^{1}\alpha_i'(S)T^i\right)ST + \alpha_1(S)\left[\left\{1 - \frac{m}{2n}\right\}T^2 + \frac{2n(\omega + bm\kappa^2 - am\omega\kappa)}{b - a\nu}S^2 - \frac{2nc}{m(b-a\nu)}S^3\right] = \{g(S) + h(S)T\}\sum_{i=0}^{1}\alpha_i(S)T^i, \tag{20}$$

where primes denote differentiation with respect to S. By comparing the coefficients of T^i $(i = 2, 1, 0)$ in eq. (20), we obtain:

– T^2 coeff,

$$S\alpha_1'(S) = \alpha_1(S)\left\{h(S) - \left(1 - \frac{m}{2n}\right)\right\}; \tag{21}$$

– T^1 coeff,

$$S\alpha_0'(S) = \alpha_1(S)g(S) + \alpha_0(S)h(S); \tag{22}$$

– T^0 coeff,

$$\alpha_1(S)\left[\left\{\frac{2n(\omega + bm\kappa^2 - am\omega\kappa)}{b - a\nu}\right\}S^2 - \left\{\frac{2nc}{m(b-a\nu)}\right\}S^3\right] = \alpha_0(S)g(S). \tag{23}$$

Since $\alpha_i(S)$ $(i = 0, 1)$ are polynomials, from eq. (21) we deduce that $\alpha_1(S)$ is constant and $h(S) = 1 - m/2n$. Let us take $\alpha_1(S) = 1$. Matching the degrees of $g(S)$ and $\alpha_0(S)$, we found that $\deg(g(S)) = 1$ and $\deg(\alpha_0(S)) = 2$. Assume that

$$g(S) = A_0 + A_1 S, \qquad A_1 \neq 0, \tag{24}$$

and

$$\alpha_0(S) = B_0 + B_1 S + B_2 S^2, \qquad B_2 \neq 0, \tag{25}$$

where A_0, A_1, B_0, B_1, B_2 are arbitrary constants to be recognized.

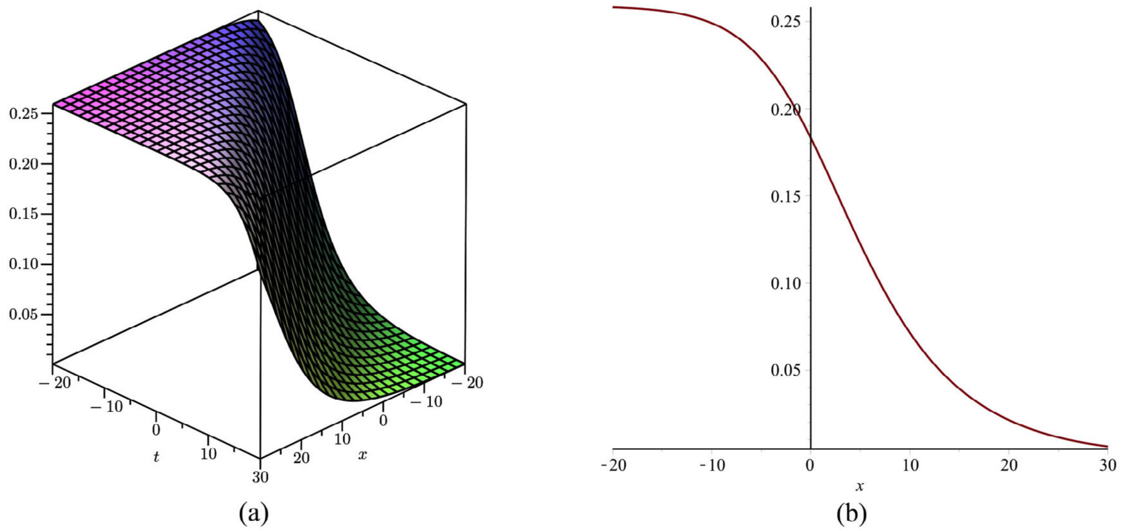

Fig. 1. (a) 3D representation of dark soliton solution (30) with $a = 1$, $b = 1.9$, $c = 1$, $\kappa = 1$, $\omega = 2.9$, $\epsilon = 1$, $m = 3$, $n = 1$ and $-20 \leq x \leq 30$, $-20 \leq t \leq 20$. (b) 2D representation of dark soliton solution (30) with $t = 0$ and $-20 \leq x \leq 30$.

Replacing eqs. (24) and (25) into (22), we attain

$$A_0 = \left(\frac{m}{2n} - 1\right) B_0, \qquad A_1 = \left(\frac{m}{2n}\right) B_1. \tag{26}$$

Inserting these values in (24), we obtain

$$g(S) = \left(\frac{m}{2n} - 1\right) B_0 + \left(\frac{m}{2n}\right) B_1 S. \tag{27}$$

Substituting $\alpha_0(S)$, $\alpha_1(S)$ and $g(S)$ into (23) and equating all the coefficients of S^j to zero, we attain a nonlinear system of equations, which recovers

$$B_0 = 0, \qquad B_1 = \pm 2n\sqrt{\frac{\omega + bm\kappa^2 - am\kappa\omega}{m(b - a\nu)}}, \qquad B_2 = \mp \frac{2nc}{m\sqrt{m(b - a\nu)(\omega + bm\kappa^2 - am\kappa\omega)}}. \tag{28}$$

Using these values in eq. (18) and combining with system (17), we obtain

$$S'(\zeta) = \mp 2n\sqrt{\frac{\omega + bm\kappa^2 - am\kappa\omega}{m(b - a\nu)}} S \pm \frac{2nc}{m\sqrt{m(b - a\nu)(\omega + bm\kappa^2 - am\kappa\omega)}} S^2. \tag{29}$$

By solving eq. (29), we determine exact solutions to eq. (14). Exact solutions to eq. (11) can then be written as follows.

1) Dark 1-soliton solution:

$$v(x,t) = \left[\pm \frac{m(\omega + bm\kappa^2 - am\kappa\omega)}{2c}\left\{1 \pm \tanh\left(\frac{n\sqrt{\omega + bm\kappa^2 - am\kappa\omega}}{\sqrt{m(b - a\nu)}}\left(x - \frac{m(2b\kappa - a\omega)}{am\kappa - 1}t\right)\right)\right\}\right]^{1/2n} \times \exp\left(i\left\{-\kappa x + \omega t + \epsilon\right\}\right). \tag{30}$$

2) Singular 1-soliton solution:

$$v(x,t) = \left[\pm \frac{m(\omega + bm\kappa^2 - am\kappa\omega)}{2c}\left\{1 \pm \coth\left(\frac{n\sqrt{\omega + bm\kappa^2 - am\kappa\omega}}{\sqrt{m(b - a\nu)}}\left(x - \frac{m(2b\kappa - a\omega)}{am\kappa - 1}t\right)\right)\right\}\right]^{1/2n} \times \exp\left(i\left\{-\kappa x + \omega t + \epsilon\right\}\right). \tag{31}$$

The validity of these solutions holds for

$$(\omega + bm\kappa^2 - am\kappa\omega)(b - a\nu) > 0. \tag{32}$$

In figs. 1(a) and (b), the 3D and 2D graphs of the dark soliton solution (30) are presented while, in figs. 2(a) and (b), the 3D and 2D graphs of the singular soliton solution (31) are presented, along with specified values of parameters.

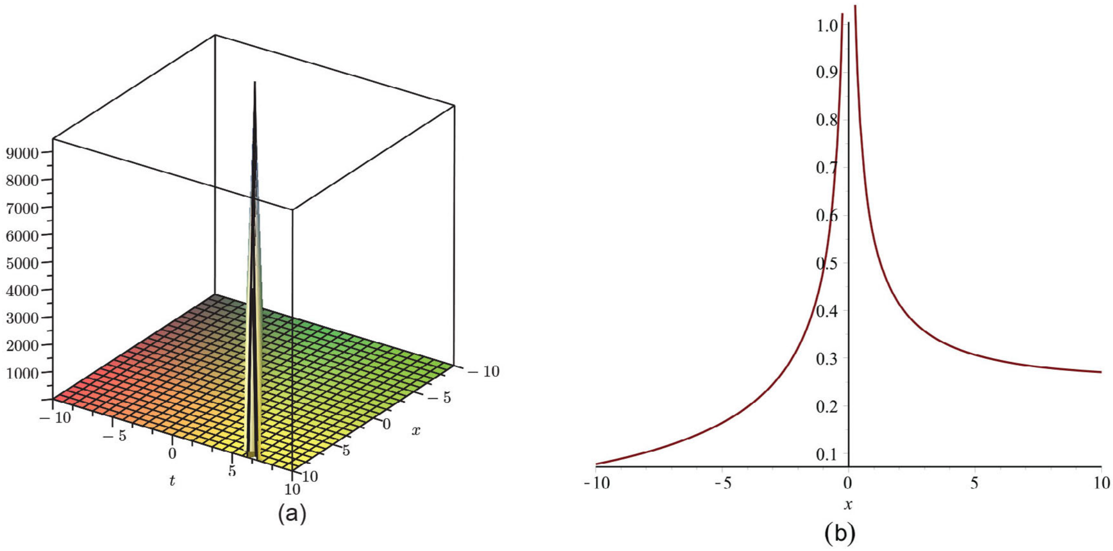

Fig. 2. (a) 3D representation of singular soliton solution (31) with $a = 1$, $b = 1.9$, $c = 1$, $\kappa = 1$, $\omega = 2.9$, $\epsilon = 1$, $m = 3$, $n = 1$ and $-10 \le x$, $t \le 10$. (b) 2D representation of singular soliton solution (31) with $t = 0$ and $-10 \le x \le 10$.

3.2 Dual-power law nonlinearity

Dual-power law nonlinearity is utilized to explicate the saturation phenomena of the nonlinear refractive index. Moreover, this law works as a basic model to define the soliton dynamics in photovoltaic-photorefractive materials such as $LiNbO_3$ [53].

For dual-power law nonlinearity, eq. (1) becomes

$$i(v^m)_t + a(v^m)_{xt} + b(v^m)_{xx} + \left(c_1|v|^{2n} + c_2|v|^{4n}\right) v^m = 0, \tag{33}$$

and eq. (10) reduces to

$$(b - a\nu)(U^m)'' - m(\omega + bm\kappa^2 - am\omega\kappa)U^m + (c_1U^{2n} + c_2U^{4n})U^m = 0. \tag{34}$$

To attain the closed form solutions, we replace

$$U = V^{1/2n}. \tag{35}$$

This transformation reduces eq. (34) to an ODE:

$$m(b - a\nu)\left[(m - 2n)(V')^2 + 2nVV''\right] - 4n^2\left[m(\omega + bm\kappa^2 - am\omega\kappa)V^2 - c_1V^3 - c_2V^4\right] = 0. \tag{36}$$

3.2.1 First integral approach

In this subsection, we apply the first integral method to solve eq. (36). By substituting $S(\zeta) = V(\zeta)$ and $T(\zeta) = V'(\zeta)$ in eq. (36), we have the following two-dimensional autonomous system:

$$\begin{cases} S'(\zeta) = T(\zeta), \\ T'(\zeta) = \left(1 - \dfrac{m}{2n}\right)\dfrac{T^2}{S} + \dfrac{2n}{S(b - a\nu)}\left[(\omega + bm\kappa^2 - am\omega\kappa)S^2 - \dfrac{c_1}{m}S^3 - \dfrac{c_2}{m}S^4\right]. \end{cases} \tag{37}$$

To avoid the singular line $S = 0$, we use the transformation

$$\mathrm{d}\zeta = S\,\mathrm{d}\eta. \tag{38}$$

Then, the system in (37) becomes

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}\eta} = ST, \\ \dfrac{\mathrm{d}T}{\mathrm{d}\eta} = \left(1 - \dfrac{m}{2n}\right)T^2 + \dfrac{2n}{b - a\nu}\left[(\omega + bm\kappa^2 - am\omega\kappa)S^2 - \dfrac{c_1}{m}S^3 - \dfrac{c_2}{m}S^4\right]. \end{cases} \tag{39}$$

Suppose that $M=1$ in (18) and using eqs. (39) and (19), we get

$$\left[\sum_{i=0}^{1}\alpha_i'(S)T^i\right]ST+\alpha_1(S)\left(1-\frac{m}{2n}\right)T^2+\frac{2n\alpha_1(S)}{b-a\nu}\left[(\omega+bm\kappa^2-am\omega\kappa)S^2-\frac{c_1}{m}S^3-\frac{c_2}{m}S^4\right]=[g(S)+h(S)T]\sum_{i=0}^{1}\alpha_i(S)T^i. \tag{40}$$

Balancing the coefficients of T^i ($i=2,1,0$) in eq. (40), we obtain

– T^2 coeff,

$$S\alpha_1'(S)=\alpha_1(S)\{h(S)-(1-m/2n)\}; \tag{41}$$

– T^1 coeff,

$$S\alpha_0'(S)=\alpha_1(S)g(S)+\alpha_0(S)h(S); \tag{42}$$

– T^0 coeff,

$$\frac{2n\alpha_1(S)}{b-a\nu}\left[(\omega+bm\kappa^2-am\omega\kappa)S^2-\frac{c_1}{m}S^3-\frac{c_2}{m}S^4\right]=\alpha_0(S)g(S). \tag{43}$$

Since $\alpha_i(S)$ ($i=0,1$) are polynomials, from eq. (41) we deduce that $\alpha_1(S)$ is constant and $h(S)=1-m/2n$. For the sake of convenience, we take $\alpha_1(S)=1$. Matching the degrees of $g(S)$ and $\alpha_0(S)$, we found that $\deg(g(S))=\deg(\alpha_0(S))=2$. Assume that

$$g(S)=A_0+A_1S+A_2S^2,\qquad A_2\neq 0, \tag{44}$$

and

$$\alpha_0(S)=B_0+B_1S+B_2S^2,\qquad B_2\neq 0, \tag{45}$$

where A_0, A_1, A_2, B_0, B_1, B_2 are arbitrary constants.

Replacing eqs. (44) and (45) into (42), we obtain

$$A_0=\left(\frac{m}{2n}-1\right)B_0,\qquad A_1=\frac{m}{2n}B_1,\qquad A_2=\left(\frac{m}{2n}+1\right)B_2. \tag{46}$$

Inserting these values in (44), we obtain

$$g(S)=\left(\frac{m}{2n}-1\right)B_0+\frac{m}{2n}B_1S+\left(\frac{m}{2n}+1\right)B_2S^2. \tag{47}$$

Substituting $\alpha_0(S)$, $\alpha_1(S)$ and $g(S)$ into (43) and equating all the coefficients S^j to zero, we attain a nonlinear system of equations, which reveals

$$\begin{aligned}B_0=0,\qquad B_1=\mp\frac{nc_1\sqrt{m+2n}}{(m+n)\sqrt{mc_2(a\nu-b)}}\,,\qquad B_2=\pm\frac{2n\sqrt{c_2}}{\sqrt{m(m+2n)(a\nu-b)}}\,,\\ \omega=\frac{c_1^2(m+2n)}{4c_2(m+n)^2(am\kappa-1)}+\frac{bm\kappa^2}{am\kappa-1}\,.\end{aligned} \tag{48}$$

Substituting these values in eq. (18) and combining with system (39), we obtain

$$S'(\zeta)=\pm\frac{nc_1\sqrt{m+2n}}{(m+n)\sqrt{mc_2(a\nu-b)}}S\mp\frac{2n\sqrt{c_2}}{\sqrt{m(m+2n)(a\nu-b)}}S^2. \tag{49}$$

By solving eq. (49), we determine the exact solutions to eq. (36). Exact solutions to eq. (33) can then be written as follows.

1) Dark 1-soliton solution:

$$\begin{aligned}v(x,t)=&\left[\pm\frac{c_1(m+2n)}{4c_2(m+n)}\left\{1\pm\tanh\left(\frac{nc_1\sqrt{m+2n}}{2(m+n)\sqrt{mc_2(a\nu-b)}}(x-\frac{m(2b\kappa-a\omega)}{am\kappa-1}t)\right)\right\}\right]^{1/2n}\\ &\times\exp\left(i\left\{-\kappa x+\left(\frac{c_1^2(m+2n)}{4c_2(m+n)^2(am\kappa-1)}+\frac{bm\kappa^2}{am\kappa-1}\right)t+\epsilon\right\}\right).\end{aligned} \tag{50}$$

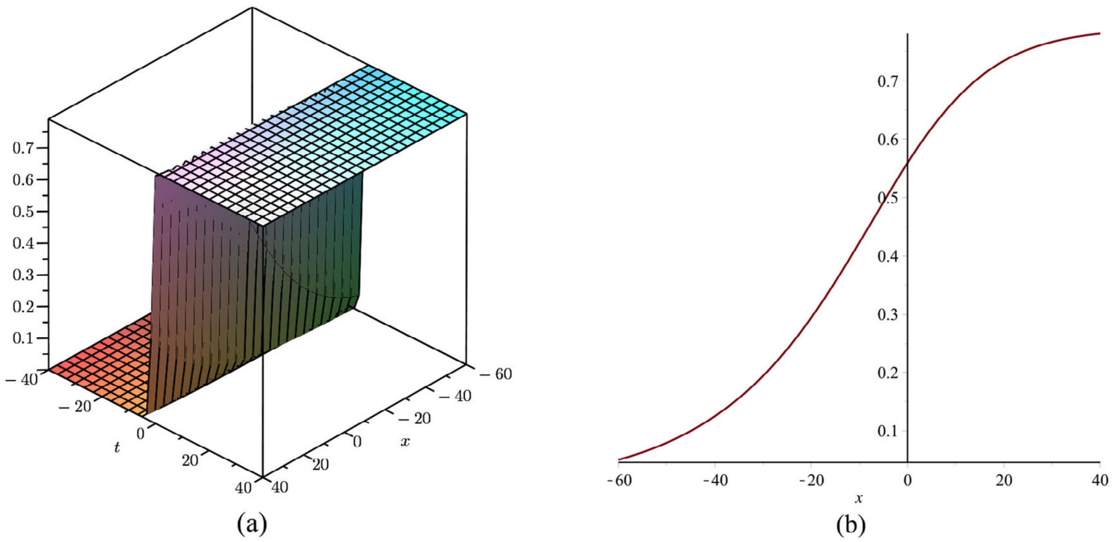

Fig. 3. (a) 3D representation of dark soliton solution (50) with $a = 1$, $b = -50$, $c_1 = 1$, $c_2 = 1$, $\kappa = 1$, $\epsilon = 1$, $m = 3$, $n = 1$ and $-60 \le x \le 40$, $-40 \le t \le 40$. (b) 2D representation of dark soliton solution (50) with $t = 0$ and $-60 \le x \le 40$.

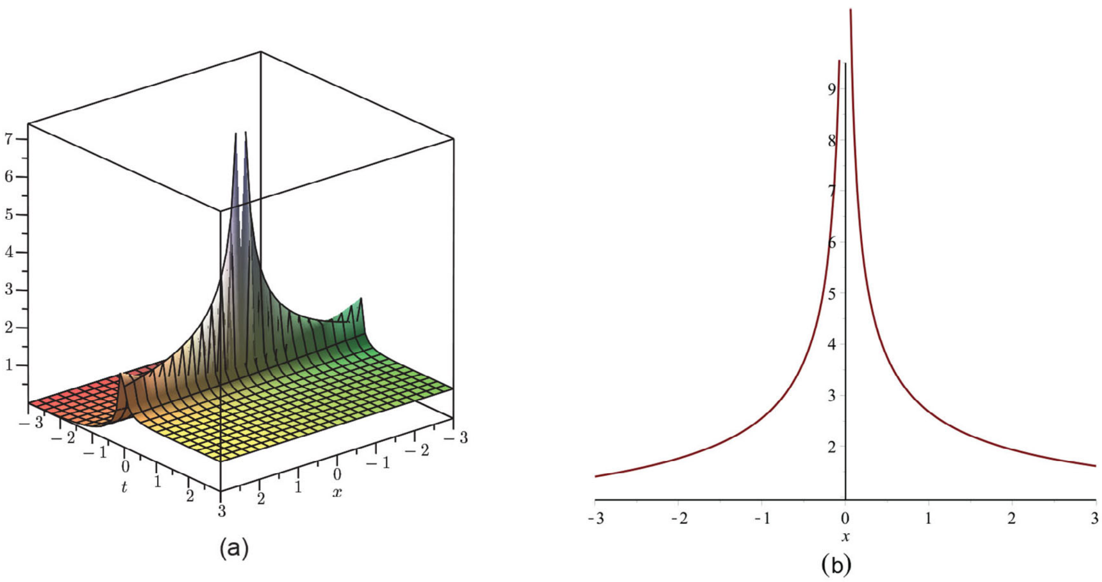

Fig. 4. (a) 3D representation of singular soliton solution (51) with $a = 1$, $b = -50$, $c_1 = 1$, $c_2 = 1$, $\kappa = 1$, $\epsilon = 1$, $m = 3$, $n = 1$ and $-3 \le x, t \le 3$. (b) 2D representation of singular soliton solution (51) with $t = 0$ and $-3 \le x \le 3$.

2) Singular 1-soliton solution:

$$v(x,t) = \left[\pm\frac{c_1(m+2n)}{4c_2(m+n)}\left\{1 \pm \coth\left(\frac{nc_1\sqrt{m+2n}}{2(m+n)\sqrt{mc_2(a\nu-b)}}(x - \frac{m(2b\kappa - a\omega)}{am\kappa - 1}t)\right)\right\}\right]^{1/2n}$$
$$\times \exp\left(i\left\{-\kappa x + \left(\frac{c_1^2(m+2n)}{4c_2(m+n)^2(am\kappa-1)} + \frac{bm\kappa^2}{am\kappa-1}\right)t + \epsilon\right\}\right). \tag{51}$$

These solutions are valid for

$$c_2(a\nu - b) > 0. \tag{52}$$

Note. By assuming $M = 2$ in eq. (18) and using eqs. (39) and (19), we obtain the soliton solutions of eq. (33) identical to (50) and (51).

In figs. 3(a) and (b) the 3D and 2D graphs of the dark soliton solution (50) are presented while, in figs. 4(a) and 4(b), the 3D and 2D graphs of the singular soliton solution (51) are presented, along with specified values of parameters.

Remark 1. When $n = 1$ in eq. (11), we get Kerr-law nonlinearity and the corresponding results follow.

Remark 2. When $n = 1/2$ in eq. (33), we get quadratic-cubic law nonlinearity and the corresponding results follow.

Remark 3. When $a = 0$ in eqs. (50) and (51), the results recovered are same as those created in [43].

4 Conclusion

In this work, we successfully established optical soliton solutions for Biswas-Milovic equation in addition with STD. This model is investigated in two nonlinear mediums through power law and dual-power law nonlinearities, which accounts for several imperfections for the propagation of solitons in nonlinear optical fibers. We effectively applied the first integral method and retrieved dark and singular 1-soliton solutions in association with some parametric restrictions, from the governing Biswas-Milovic equation with spatio-temporal dispersion. Moreover, for the physical interpretation the 3D and 2D graphs of the obtained solutions are presented. These novel results are important for practical problems in nonlinear optics.

Conflict of interest

There is no potential conflict of interest.

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